Preprint

A card game for Bell's theorem and its loopholes

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1 | Comments on the Manuscript

1.1 On the pedagogical spirit

I understand the intent to avoid calculations and technical demonstrations to do with the physics of the problem at stake. Nevertheless, since this manuscript is intended for teachers, they may not be experts on the subject and may find it much more difficult to search for the bibliography and become fluent on the matter, than to read through the matter if you provide the theoretical and experimental background. The danger behind not providing it, is that teachers at the end would just transmit a card game, with certain rules, with no knowledge, or an incorrect interpretation of the knowledge, of what the motivation behind the card game really is.

So, first of all, I recommend a theoretical background for:

- The causal structure of Special Relativity (SR);
- Bohm's thought experiment, and the EPR paradox;
- Bell's inequality;
- Bell's theorem;

Next, an explanation of the three loopholes you will use. Namely,

- The locality loophole;
- The freedom-of-choice/superdeterminism loophole;
- The detector efficiency loophole;

that does not have to be too technical, just an explanation of how each of these loopholes would allow for a local realist interpretation of Quantum Mechanics (QM).

Next, the card game, how the card game mimics an experiment; which experiment (initial state and measurements); and how each cheat in the game represents one of the loopholes of the experiment. Please explain the strategies (α and β) before making claims about them.

At the end you can explain how the quantum realm might seem to get advantage of the loopholes as if it were "cheating" but that, by now, we know it doesn't, given that some (or all?) of the loopholes have been experimentally closed. Explain which and how, and give your conclusions on the superdeterminism loophole.

1.2 A claim on experimental results

The conundrum of Bell's theorem is that, entangled particles in an actual experiment manage to win with a probability of 3/4. (p.5)

This claim is made when talking about the solitaire version of the game that would be mapped to an experiment.

The rules of the game are:

- Three different measurements can be performed;
- Different measurements must be performed on the two different particles;
- The particles must choose form two different answers for each possible measurement;

The experiment that I propose to follow these rules is:

- Three different measurements to be performed:

$$\hat{\sigma}_x \quad , \quad \hat{\sigma}_y \quad , \quad \hat{\sigma}_z \,;$$

- Different measurements must be performed on the two different particles, say

$$\hat{\sigma}_z^1 \hat{\sigma}_x^2 \,;$$

- Each particle, as we know, can result in two different states:

$$|\uparrow\rangle_1^z \quad , \quad |\downarrow\rangle_1^z \quad , \quad |\uparrow\rangle_2^x \quad , \quad |\downarrow\rangle_2^x \,;$$

Now, let's say we begin with a singlet state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big[|\uparrow\rangle_1^z|\downarrow\rangle_2^z - |\downarrow\rangle_1^z|\uparrow\rangle_2^z\Big]$$

so we can calculate the state that would come out of the measurement $\hat{\sigma}_z^1\hat{\sigma}_x^2$, namely:

$$\begin{aligned} \hat{\sigma}_z^1\hat{\sigma}_x^2|\psi\rangle &= \hat{\sigma}_z^1\hat{\sigma}_x^2\frac{1}{\sqrt{2}}\Big[|\uparrow\rangle_1^z\frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_2^x - |\downarrow\rangle_2^x\Big) - |\downarrow\rangle_1^z\frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_2^x + |\downarrow\rangle_2^x\Big)\Big] \\ &= \frac{1}{2}\Big[|\uparrow\rangle_1^z\Big(|\uparrow\rangle_2^x + |\downarrow\rangle_2^x\Big) + |\downarrow\rangle_1^z\Big(|\uparrow\rangle_2^x - |\downarrow\rangle_2^x\Big)\Big] \end{aligned}$$

and we can see that the four possible outcomes:

$${}^z\langle\uparrow|^x\langle\uparrow| \quad , \quad {}^z\langle\uparrow|^x\langle\downarrow| \quad , \quad {}^z\langle\downarrow|^x\langle\uparrow| \quad , \quad {}^z\langle\downarrow|^x\langle\downarrow|$$

have equal probability of $\frac{1}{4}$.

This is also the result for the other two possible measurements,

$$\hat{\sigma}_x\hat{\sigma}_y \quad , \quad \hat{\sigma}_y\hat{\sigma}_z \,.$$

Winning with a probability of $\frac{3}{4}$ means that the states where the projection of the spin of the particles is different would overall have a probability of $\frac{3}{4}$, and not of $\frac{1}{2}$ as in this experiment. So, where does the $\frac{3}{4}$ come from? Please provide the experiment that yields this result.

Also, please clarify what it means for particles to win, since in the typical case of the maximally entangled Bell state, particles *answer differently to the same question and vice-versa.* So what is the rule for particles to score a point in your proposed experiment?

1.3 α and β strategies

There are different explanations of these two strategies throughout the text. The explanations appear too late, after claims about them have already been made. Please provide one clear explanation of each strategy soon enough, for the reader to follow through the claims.