Angle Recoding 2. Wu
3. MVR

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② MVR (Modified Vector Rotational)

two modifications

① repeatition of elementary angles

each micro-rotation of elementary angle can be performed repeatedly

- more possible combinations
- smaller ξ_m

② confinement of total micro-rotation number

confine the iteration number in the micro-rotation phase to R_m ($R_m \ll W$)

The role of R_m is quite similar to the number of non-zero digit N_D in CSD recoding scheme the angle quantization error

$$\boxed{\xi_{m,MVR} \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i))}$$

the rotational sequence

$s(i) \in \{0, 1, \cdots, W-1\}$

the micro-rotation angle
in the i-th iteration

the directional sequence

$\alpha(i) \in \{-1, 0, +1\}$

the direction of the i-th
micro-rotation of $a(s(i))$

$$\alpha(i)\, a(s(i)) = \tilde{\theta}(j)$$

the rotational sequence

$s(i) \in \{0, 1, \cdots, W-1\}$ [0, 3, 6, 7]

the directional sequence

$\alpha(i) \in \{-1, 0, +1\}$ [1, -1, -1, 1]

atan(2^0) - atan(2^{-3}) - atan(2^{-6}) + atan(2^{-7})

$$\alpha(i)\, a(s(i)) = \tilde{\theta}(j)$$

MVR-CORDIC Algorithm with $R_m = 4$	Greedy Algorithm	3	$\overline{\alpha} = [1\ \ -1\ \ -1\ \ -1]$ $\overline{s} = [0\ \ 3\ \ 6\ \ 7]$	$5.2891 * 10^{-4}$
	Semi-greedy Algorithm ($D = 2$)	4	$\overline{\alpha} = [1\ \ -1\ \ -1\ \ 1]$ $\overline{s} = [0\ \ 3\ \ 5\ \ 7]$	$5.2033 * 10^{-4}$
	TBS Algorithm	5	$\overline{\alpha} = [1\ \ 1\ \ -1\ \ -1]$ $\overline{s} = [1\ \ 2\ \ 4\ \ 7]$	$2.5911 * 10^{-4}$

```
>> s = [0, 3, 6, 7]
>> alpha = [1, -1, -1, -1]
>> sum(atan(2.^(-s) .* alpha))
ans =  0.63761
>>
```

```
>> s = [0, 3, 5, 7]
>> alpha = [1, -1, -1, 1]
>> sum(atan(2.^(-s) .* alpha))
ans =  0.63762
```

```
>> alpha = [1, 1, -1, -1]
>> s = [1, 2, 4, 7]
>> sum(atan(2.^(-s) .* alpha))
ans =  0.63840
```

AQ & MVR CORDIC

$$\xi_{m,MVR} \triangleq \theta - \left[\sum_{j=0}^{R_m - 1} \alpha(j)\, a(s(j)) \right]$$

the rotational sequence $s(j)$

$$j = 0, 1, 2, \cdots, R_m - 1$$

$$\downarrow$$

$$s(j) \in \{0, 1, \cdots, W-1\}$$ rotational sequence

determines the micro-rotation angle $a(s(j))$ in the j-th iteration

the directional sequence $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the j-th micro-rotation of $a(s(j))$

$$\alpha(j)\, a(s(j)) = \tilde{\theta}(j)$$

$i =$	$0, \boxed{1, 2,} 3, \cdots, W-1$	
$s(j) =$	$0, \boxed{1, 2,} 3, \cdots, W-1$	rotational sequence
$\alpha(j) =$	$-1, \boxed{0, 0,} +1, \cdots, -1$	directional sequence
$j =$	$0, -, -, 1, \cdots, R_m-1$	effective iteration number

$R_m \ll W$

sub-angle $(\alpha(j)\, a(s(j)))$ ~ $\tilde{\theta}(j)$

$$\xi_{m,AR} = \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)|$ the effective iteration number

EAS formed by MVR-CORDIC
is the same as AR
also performs AQ

the EAS consists of all possible values of $\tilde{\theta}(j)$

the EAS S_1 in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \quad s^* \in \{0, 1, \cdots, N-1\} \}$$

The major difference

1) the total number of sub-angles N_A

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of R_m

$N_A = R_m$

2) the sub-angle θ_i corresponds to $\alpha(j)\, a(s(j))$

$$\theta_j = \alpha(j)\, a(s(j)) = \tilde{\theta}_j$$

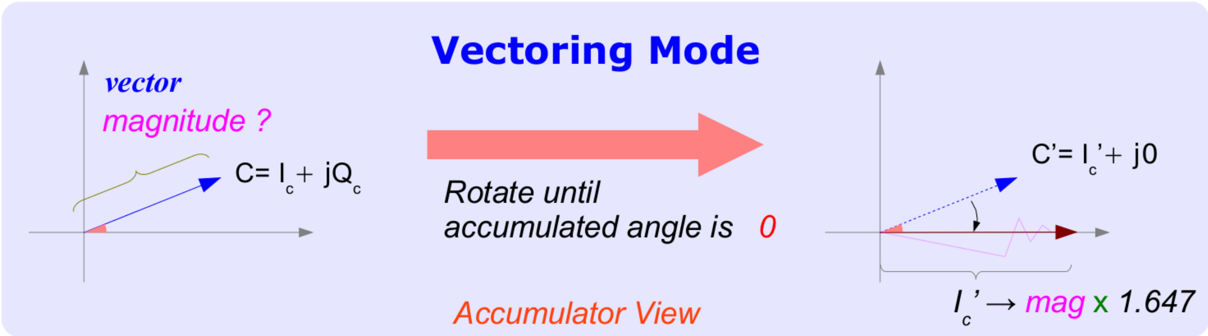
MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles θ_i, θ_i

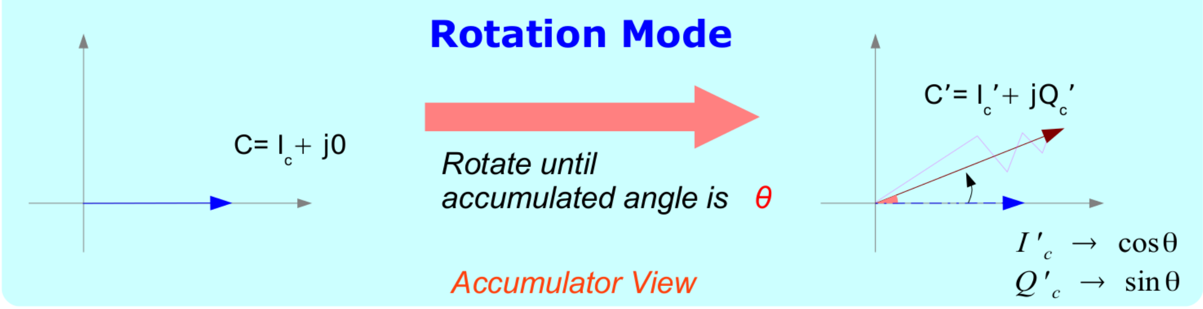
2) fixed total micro-rotation Number R_m

* Vector Rotation Mode

* and the rotation angles are known in advance



vector rotation mode



Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

Three Searching Algorithm

1. the selective prerotation
2. the selective scaling
3. iteration-tradeoff scheme

Optimization Problem

EAS point of view

Given θ, find the combination of R_m elementary angles from EAS S_1, such that the angle quantization error $|\xi_{m,MUR}|$ is minimized.

semi-greedy algorithm
trade offs between computational complexities and performance

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key issue in the MVR-CORDIC
is to find the best sequences of
$s(i)$ and $\alpha(i)$ to minimize $|\xi_m|$
subject to the constraint that
the total iteration number is confined to R_m

1) Greedy Algorithm
2) Exhaustive Algorithm
3) Semigreedy Algorithm

1) Greedy Algorithm

try to approach the target rotation angle, θ, step by step
in each step, decisions are made on $\alpha(i)$ and $s(i)$
by choosing the best combination of $\alpha(i)\, a(s(i))$
so as to minimize $|\xi_m|$

$\alpha(i)$ and $s(i)$ are determined such that
the error function $J(i) = |\theta(i) - \alpha(i)\, a(s(i))|$ is minimited

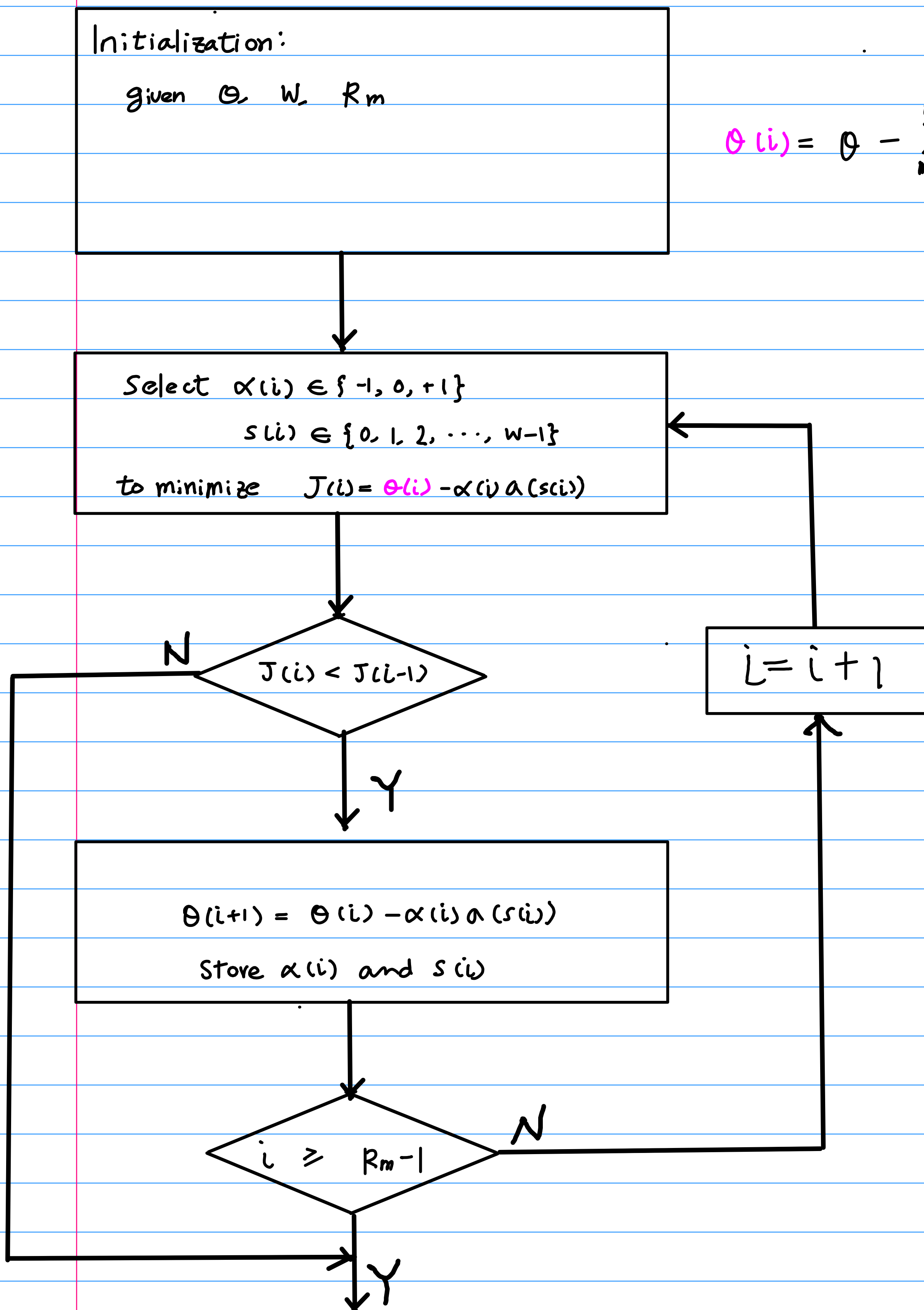
$\theta(i)$: the <u>residue</u> angle in the i-th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m)\, a(s(m))$$

the searching is terminated
if no further improvements can be found
$J(i) \geq J(i-1)$
$\alpha(R_m - 1)$ and $s(R_m - 1)$ are determined
at the end of the searching

the greedy algorithm terminates
only when the residue angle error
cannot be further reduced.

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m)\, a(s(m))$$



2) Exhaustive Algorithm

search for the entire solution space

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i))$$

in a <u>single</u> step

decisions for $\alpha(i)$ and $s(i)$, $0 \leq i \leq R_m - 1$

by minimizing the error. function

$$J = \left| \theta - \sum_{i=0}^{R_m-1} \alpha(i)\, a(s(i)) \right|$$

global optimal solution

Initialization:

given Θ, W, R_m

let $\Theta(0) = \Theta$,

$i = 0$

$J(-1) = \infty$

Select $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \cdots, W-1\}$

for $0 \le i \le R_m - 1$

to minimize $J(i) = \Theta - \sum_{i=0}^{R_m - 1} \alpha(i)\, a(s(i))$

Store $\alpha(i)$ and $s(i)$

for $0 \le i \le R_m - 1$

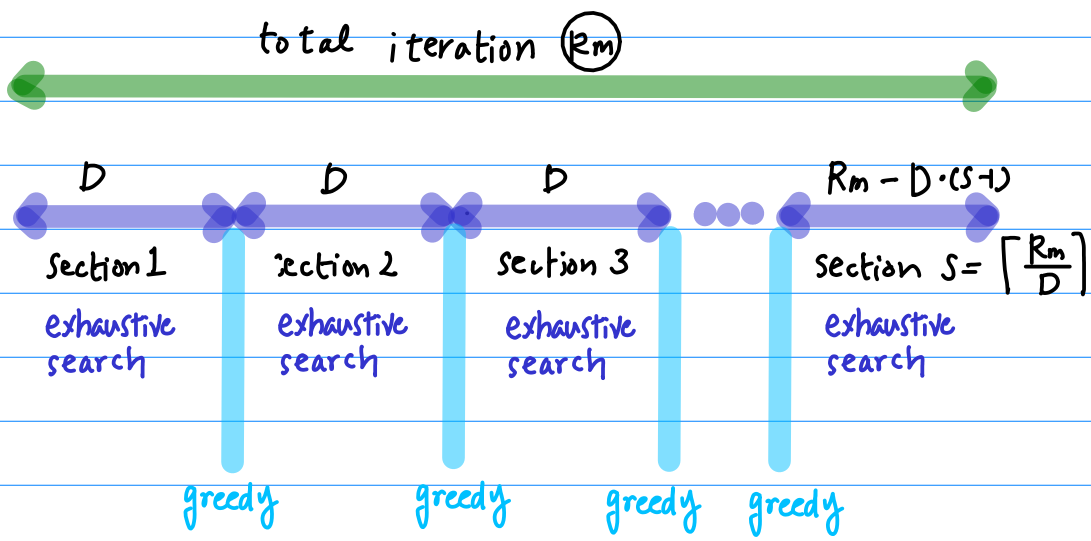
$$(3 \cdot W) \cdot (3 \cdot W) \cdots (3 \cdot W)$$
$$= 3^{R_m} \cdot W^{R_m}$$

3) Semi-greedy Algorithm

a combination of greedy and exhaustive algorithm

the search space of $\alpha(i)$ and $S(i)$ for $0 \leq i \leq R_m - 1$
are divided into several sections
with D iterations as a segment
$\downarrow$ block length $\downarrow$ block

the segmentation scheme



in the i-th block

decision of $\alpha(k)$ and $s(k)$ for $iD \le k \le (i+1)D-1$

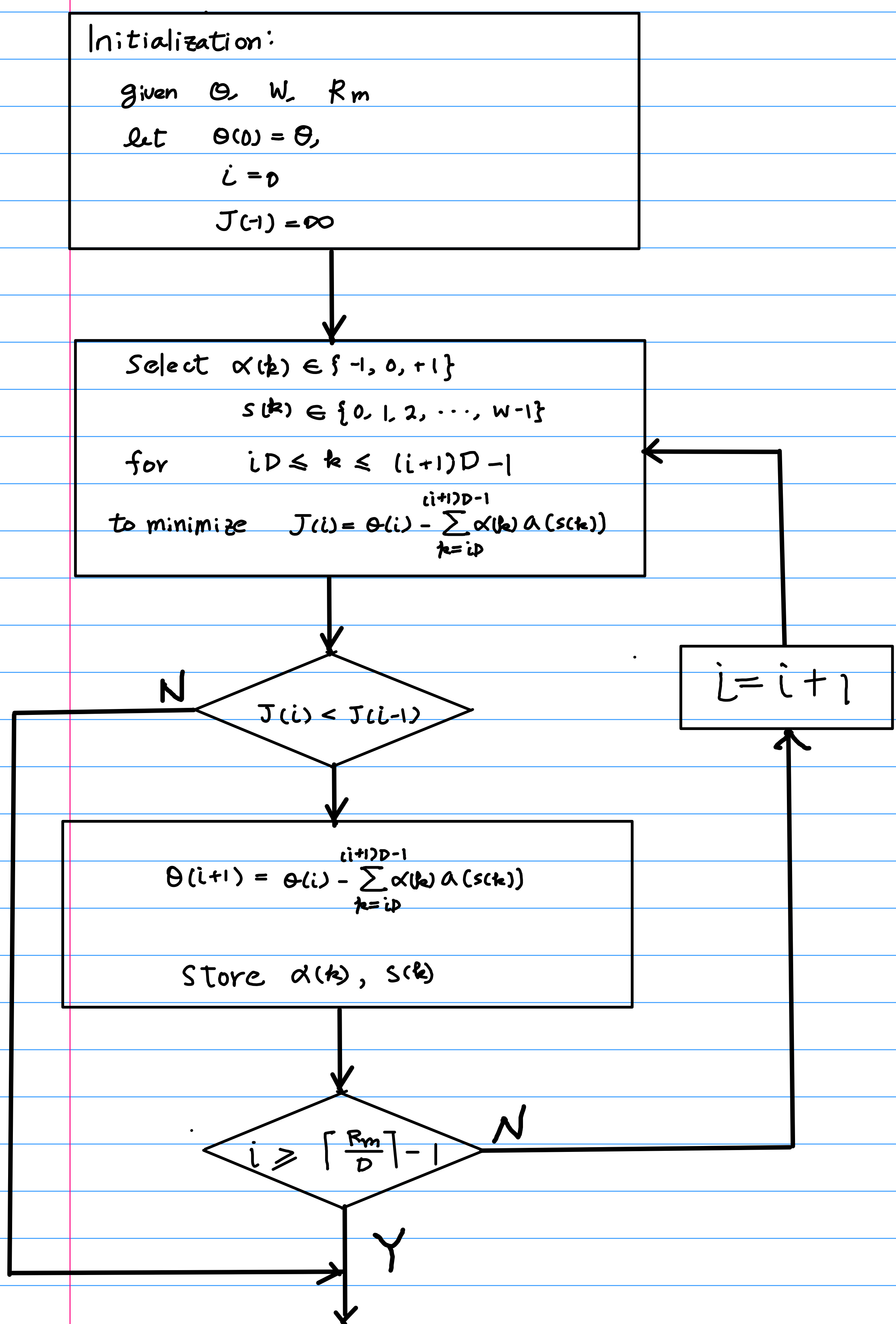
minimizes $$J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k)\, a(s(k)) \right|$$

where $$\theta(i) = \theta - \sum_{m=0}^{i-1} \left[\sum_{k=mD}^{(m+1)D-1} \alpha(k)\, a(s(k)) \right]$$

the residue angle in the i-th step

$$S = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[\sum_{k=0\cdot D}^{1\cdot D-1} \alpha(k)\, a(s(k)) + \sum_{k=1D}^{2\cdot D-1} \alpha(k)\, a(s(k)) + \cdots + \sum_{k=2D}^{S\cdot D-1} \alpha(k)\, a(s(k)) \right]$$


Initialization:
given Θ, W, R_m
let $\Theta(0) = \Theta$,
$i = 0$
$J(-1) = \infty$
Select $\alpha(k) \in \{-1, 0, +1\}$
$s(k) \in \{0, 1, 2, \cdots, W-1\}$
for $iD \leq k \leq (i+1)D-1$
to minimize $J(i) = \Theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k)\, a(s(k))$
$J(i) < J(i-1)$
N
$\Theta(i+1) = \Theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k)\, a(s(k))$
Store $\alpha(k)$, $s(k)$
$i \geq \lceil \frac{R_m}{D} \rceil - 1$
N
Y
$i = i+1$