

Feedback System (H.1)

Block Diagram

20150617

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IVP

$$\left\{ \begin{array}{l} \frac{d^2}{dt^2} y(t) + 3 \frac{d}{dt} y(t) + 2y(t) = x(t) \\ y(0) = k_0, \quad y'(0) = k_1 \end{array} \right.$$

$$\left\{ \begin{array}{l} (y''(t) + 3y'(t) + 2y(t)) = x(t) \\ y(0) = k_0, \quad y'(0) = k_1 \end{array} \right.$$

$$(s^2 Y(s) - s y(0) - y'(0)) + 3(s Y(s) - y(0)) + 2 Y(s) = X(s)$$

$$(s^2 + 3s + 2) Y(s) = s y(0) + y'(0) + 3 y(0) + X(s)$$

$$(s^2 + 3s + 2) Y(s) = k_0 s + k_1 + 3k_0 + X(s)$$

$$Y(s) = \underbrace{\frac{k_0 s + k_1 + 3k_0}{(s^2 + 3s + 2)}}_{\text{Zero input Rsp}} + \underbrace{\frac{X(s)}{(s^2 + 3s + 2)}}_{\text{Zero state Rsp}}$$

Transfer function

Zero state

$$H(s) = \frac{Y(s)}{X(s)} \quad \dots \text{"zero state"}$$

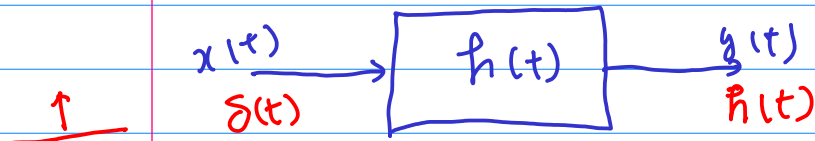
$$= \frac{Y(s)}{X(s)} = \frac{1}{(s^2 + 3s + 2)}$$

$$Y(s) = \frac{k_0 s + k_1 + 3k_2}{(s^2 + 3s + 2)} + \frac{X(s)}{(s^2 + 3s + 2)}$$

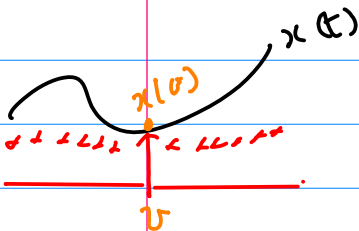
Zero state ($y(0) = y'(0) = 0$)

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s^2 + 3s + 2)}$$

Impulse Response



$$\begin{array}{ll} \delta(t-v) & h(t-v) \\ a \cdot \delta(t-v) & a \cdot h(t-v) \\ x(v) \delta(t-v) & x(v) h(t-v) \\ x(v) \delta(t-v) & x(v) h(t-v) \end{array}$$



$$\int_{-\infty}^{+\infty} x(v) \delta(t-v) dt$$

$$\int_{-\infty}^{+\infty} x(v) h(t-v) dv = y(t)$$

$x(t) * h(t)$

$$x(v) = 0 \quad \underline{v < 0}$$

$$h(t-v) = 0 \quad t-v < 0$$

$$t < v$$

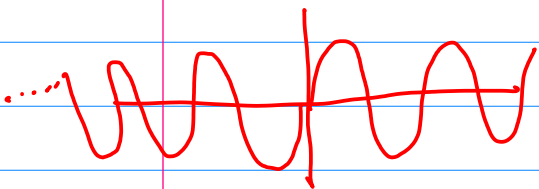
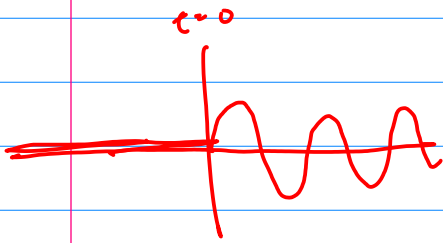
$$\underline{0 < v < t}$$

$x(t)$ → $h(t)$ → $y(t) = \int_0^{\infty} h(\nu) x(t-\nu) d\nu$ $t-\nu \geq 0$
 $y(t) = \int_0^{\infty} h(\nu) e^{s(t-\nu)} d\nu$
 $= e^{st} \int_0^{\infty} h(\nu) e^{-s\nu} d\nu$

$s = \sigma + j\omega$

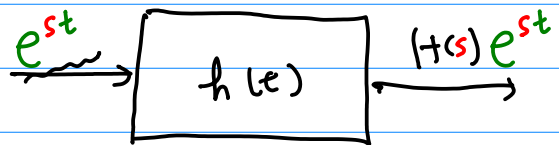
e^{st}

$e^{st} \cdot \int_0^{\infty} h(\nu) e^{-s\nu} d\nu$
||
 $H(s)$



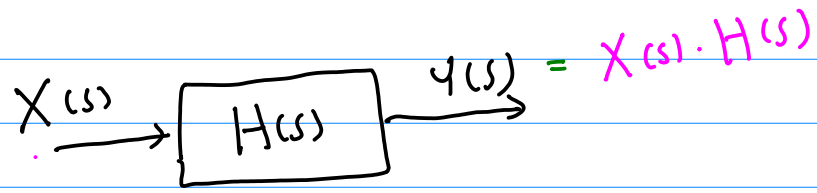
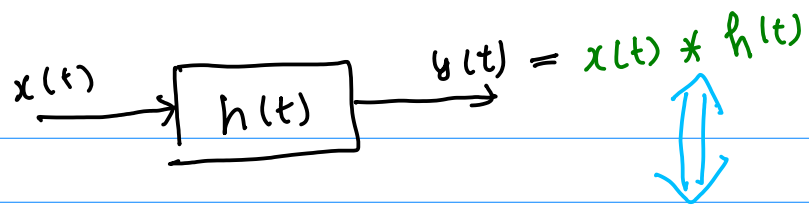
$F(s) = \int_0^{\infty} f(t) e^{-st} dt$

$= \int_0^{\infty} f(\nu) e^{-s\nu} d\nu$



$h(t) \leftrightarrow H(s)$

Laplace Transform



$$y^{(3)}(t) + a y^{(2)}(t) + b y^{(1)}(t) + c y(t) = m x^{(1)}(t) + n x(t)$$

Zero-state assumed

Transfer Fun. $\hat{y}(s) = \hat{x}(s) H(s)$

$$y(0) = y'(0) = y''(0) = 0$$

$$x(0) = 0$$

$$x(t)$$

$$s^3 Y(s) + a s^2 Y(s) + b s Y(s) + c Y(s) = m s X(s) + n X(s)$$

$$(s^3 + a s^2 + b s + c) Y(s) = (m s + n) X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{m s + n}{s^3 + a s^2 + b s + c}$$

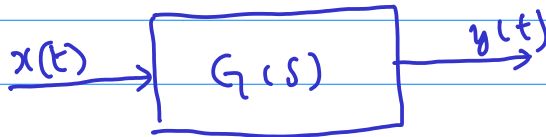
$$y'' + 3y' + 2y = x$$

Zero state

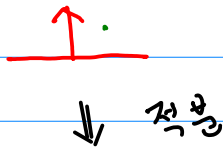
$$s^2 Y(s) + 3sY(s) + 2Y(s) = X(s)$$

$$(s^2 + 3s + 2) Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+2)}$$



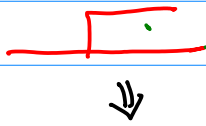
Impulse 함수



impulse 응답

$g(t)$
 \downarrow 적분

Step 함수



step 응답

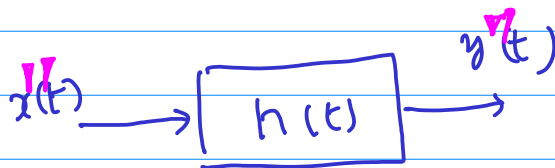
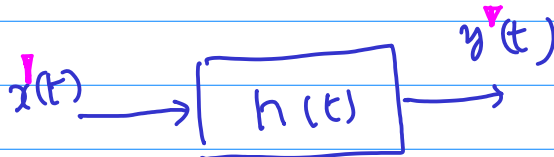
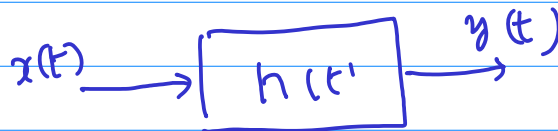
$\int_0^t g(\tau) d\tau$
 \downarrow 적분

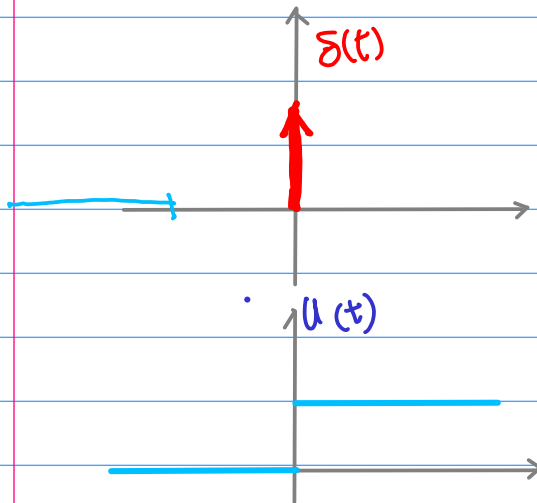
ramp 함수



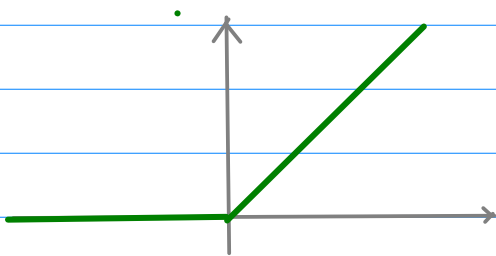
ramp 응답

$\int_0^t \int_0^{\tau} g(z) dz dt$

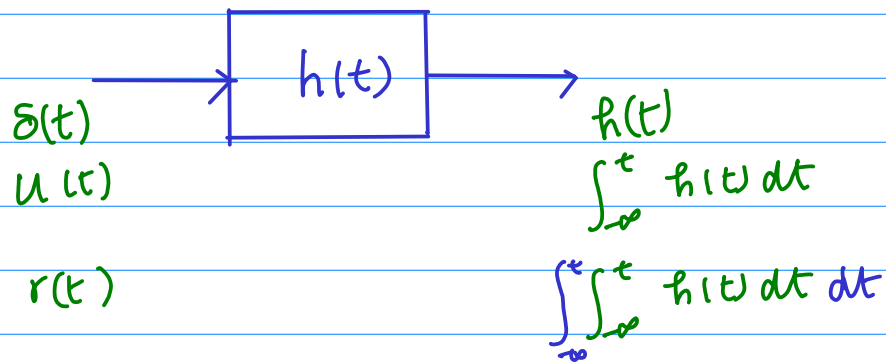




$$u(t) = \int_{-\infty}^t \delta(t) dt$$



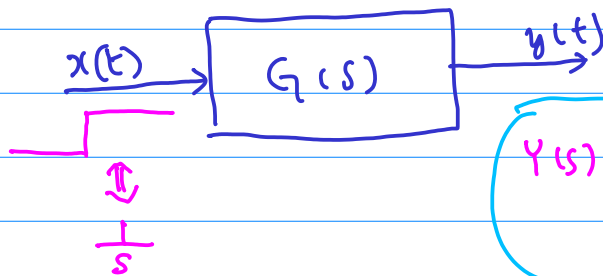
$$r(t) = \int_{-\infty}^t u(t) dt$$



$$s^2 Y(s) + 3s Y(s) + 2Y(s) = X(s)$$

$$(s^2 + 3s + 2) Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s-1)(s+2)}$$

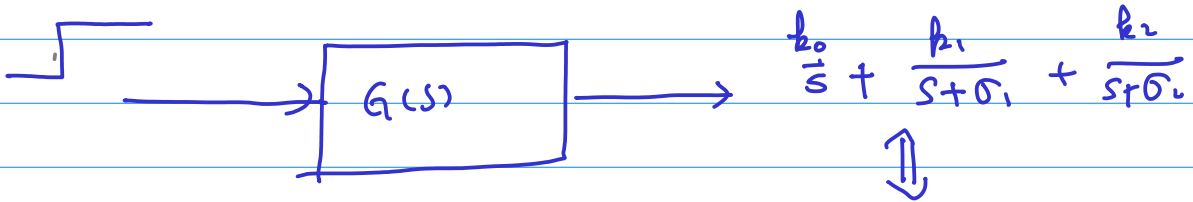


$$Y(s) = G(s) \cdot X(s) = \frac{1}{s(s+1)(s+2)}$$

$$y(t) = \mathcal{L}^{-1} \left(\frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+2)} \right)$$



$$= \underbrace{A \cdot 1}_{y_p} + \underbrace{B e^{-1 \cdot t} + C e^{-2t}}_{y_h}$$



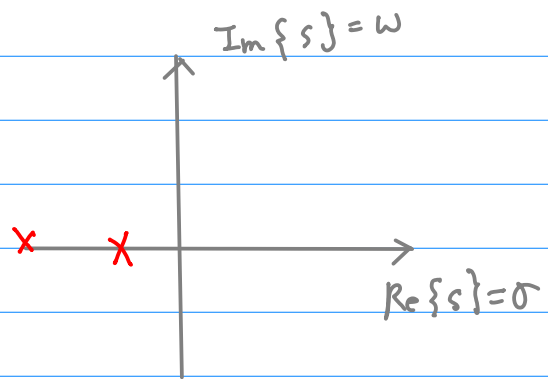
$$G(s) = \frac{1}{s^2 + 3s + 2}$$

$$k_0 + k_1 e^{-\sigma_1 t} + k_2 e^{-\sigma_2 t}$$

$$\sigma_1, \sigma_2 \Leftarrow \text{분모} = 0 \quad (s^2 + 3s + 2) = 0$$

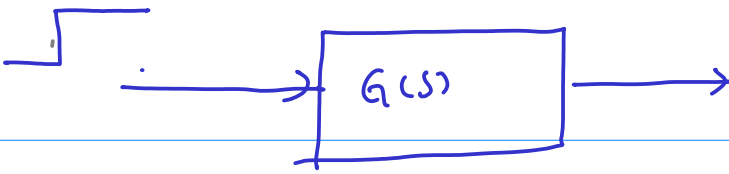
분모가 0이 되면 s 두 개의 pole

$$s = \underbrace{\sigma} + j \underbrace{\omega}$$



σ_1, σ_2 는 서로 다른 두 실수

$$\sigma_1 = \sigma_2 \quad \text{중첩}$$

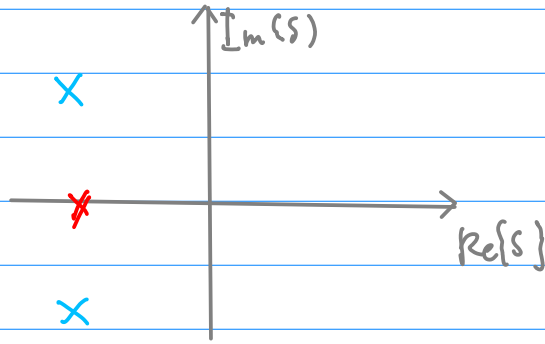


$$\frac{1}{s(s-\sigma_1)^2} = \frac{k_0}{s} + \frac{k_1}{(s-\sigma_1)} + \frac{k_2}{(s-\sigma_1)^2}$$

⇕

$$k_0 + k_1 e^{-\sigma_1 t} + k_2 t e^{-\sigma_1 t}$$

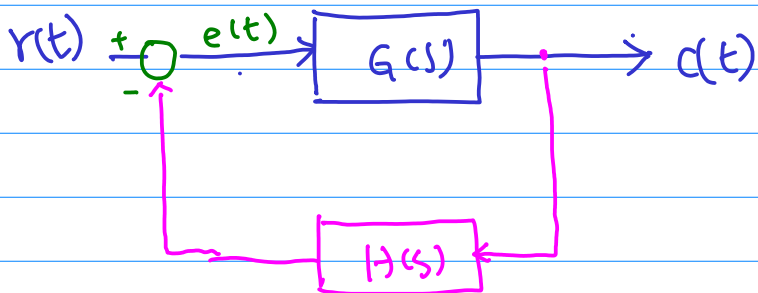
$$G(s) = \frac{1}{s^2 + 3s + 2}$$



Feedback

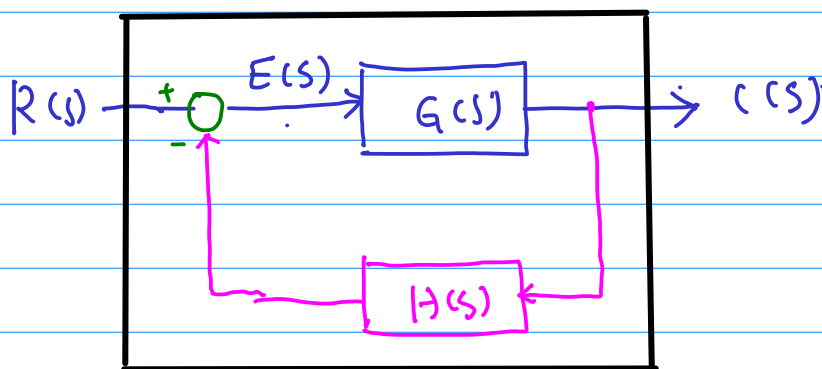


$$c(s) = G(s) \cdot R(s)$$

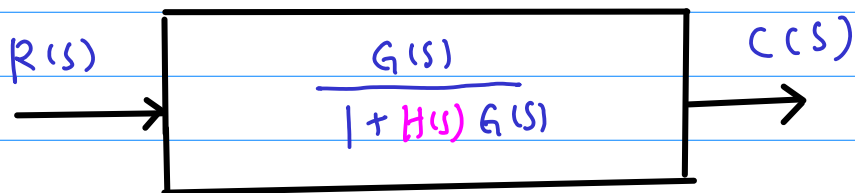


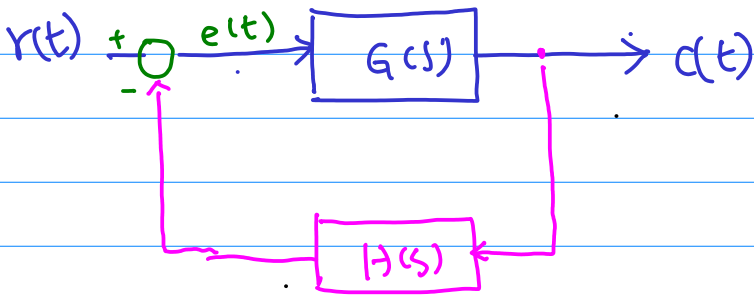
$$C(s) = G(s) \cdot E(s)$$

$$E(s) = R(s) - H(s)C(s)$$



$$\begin{aligned} \frac{C(s)}{R(s)} &= \frac{G(s) \cdot E(s)}{E(s) + H(s)C(s)} \\ &= \frac{G(s) \cdot E(s)}{E(s) + H(s)G(s) \cdot E(s)} \\ &= \frac{G(s)}{1 + H(s)G(s)} \end{aligned}$$





$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - H(s)C(s)$$

$$E(s) + H(s)C(s) = R(s)$$

$$\frac{E(s)}{R(s)} = \frac{E(s)}{E(s) + H(s)C(s)}$$

$$= \frac{E(s)}{E(s) + H(s)G(s)E(s)}$$

$$= \frac{1}{1 + H(s)G(s)}$$

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - H(s)C(s)$$

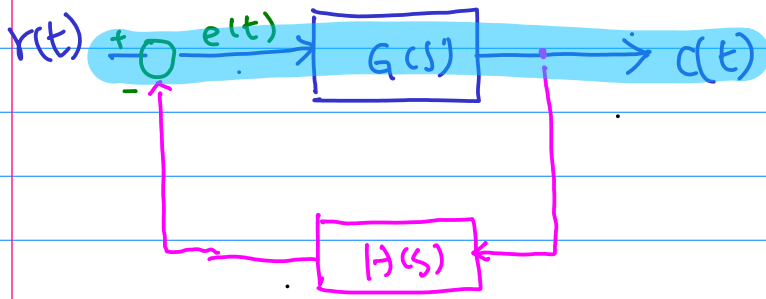
$$\frac{E(s)}{R(s)} = \frac{R(s) - H(s)C(s)}{R(s)}$$

$$= 1 - \frac{H(s)G(s)E(s)}{R(s)}$$

$$\frac{E(s)}{R(s)} + H(s)G(s)\frac{E(s)}{R(s)} = 1$$

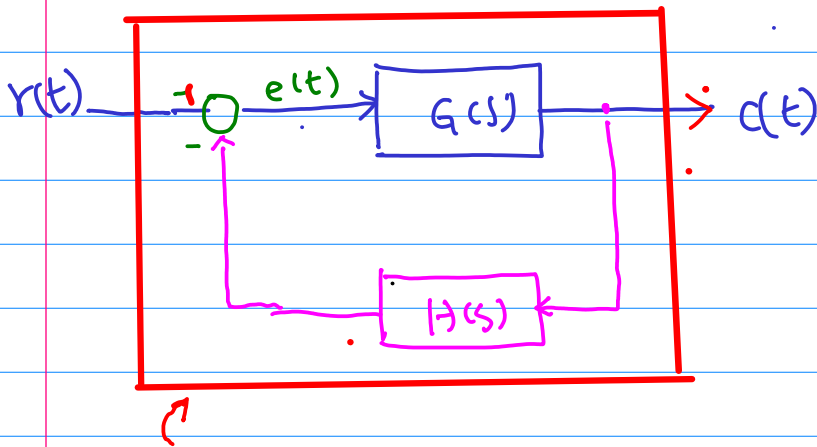
$$(1 + H(s)G(s))\frac{E(s)}{R(s)} = 1$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + H(s)G(s)}$$

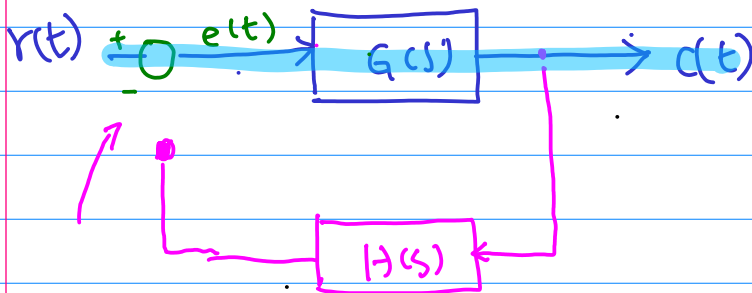


forward transfer fn : $G(s)$

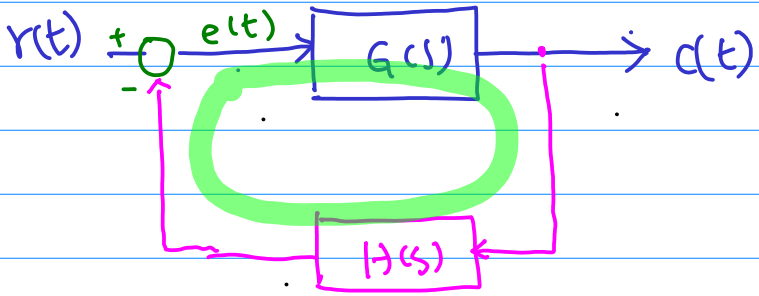
feedback transfer fn : $H(s)$



closed loop transfer fn : $\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$

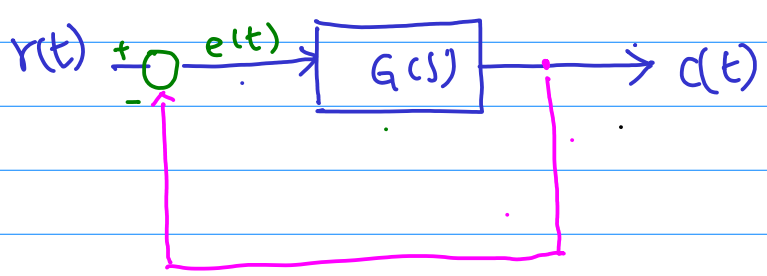


Open loop transfer fn : $G(s)$

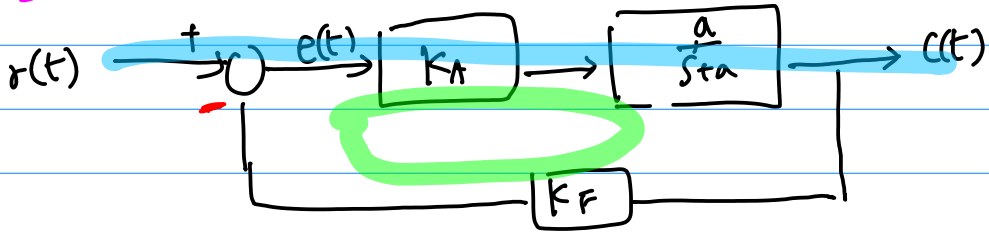


loop transfer fn : $G(s)H(s)$

Unit feed back $H(s) = 1$



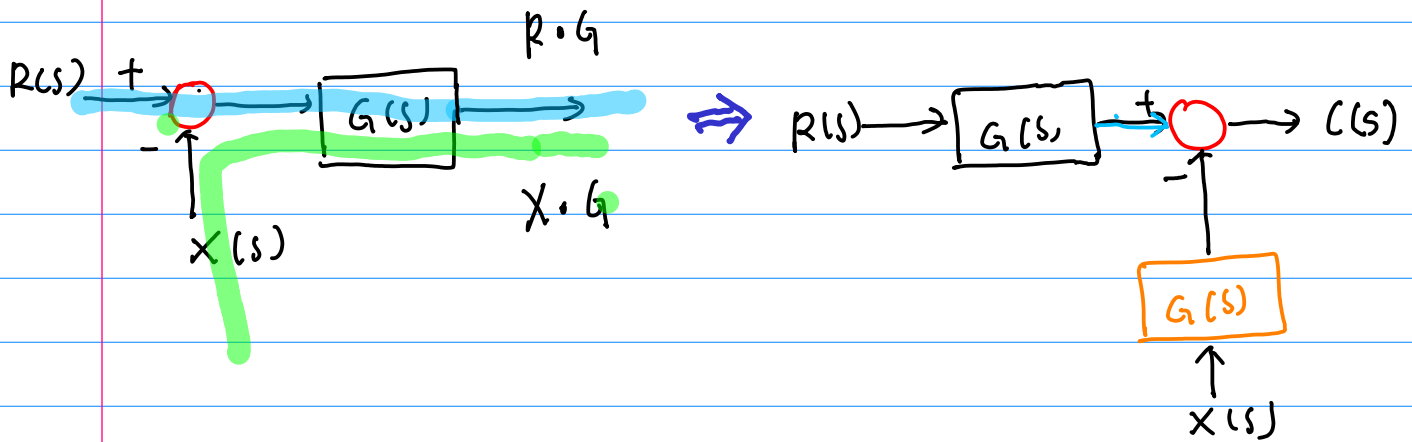
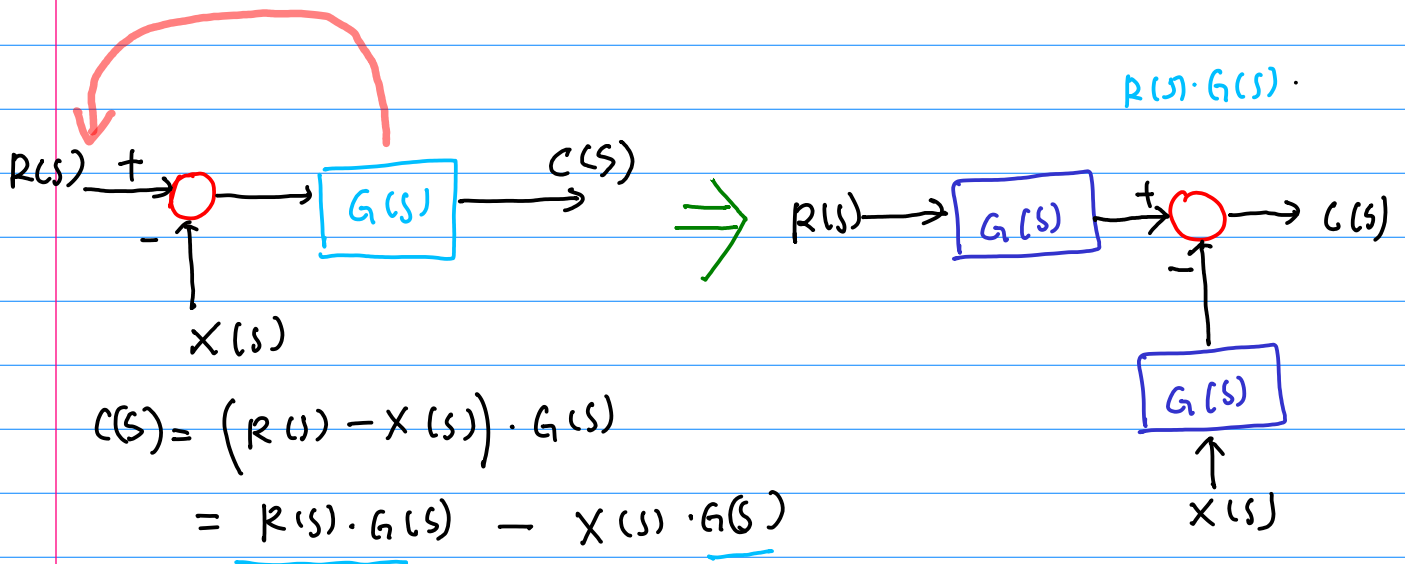
2장 2-10



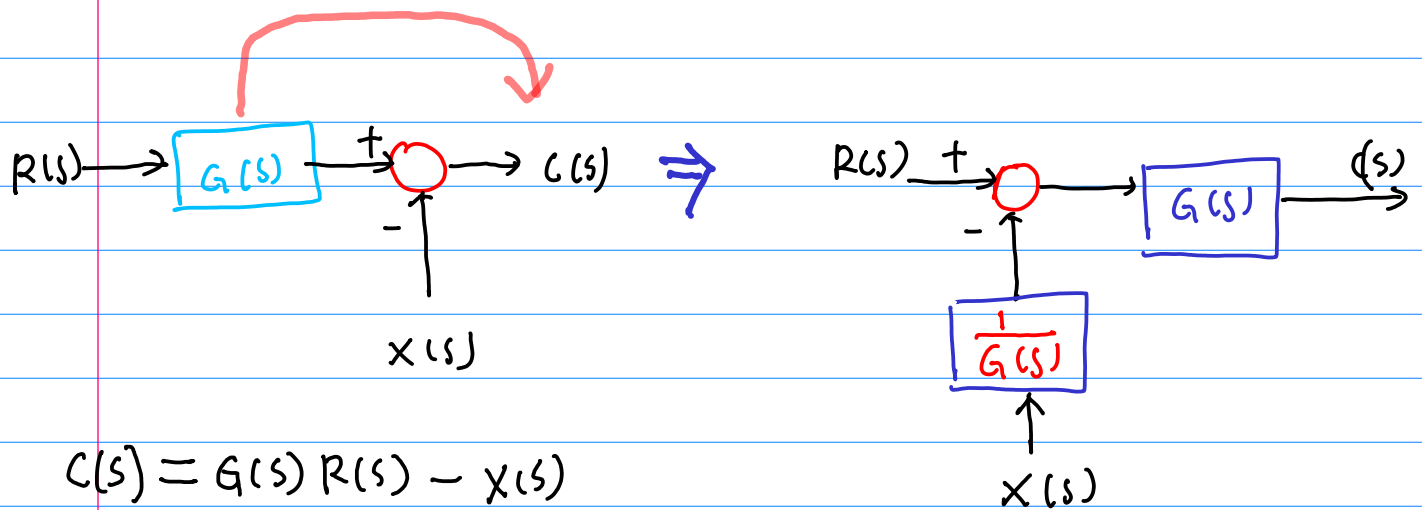
— $\frac{K_A a}{(s+a)}$
○ $-\frac{K_A K_F a}{(s+a)}$

$$G_{cl}(s) = \frac{\text{blue}}{1 - \text{green}} = \frac{\frac{K_A a}{(s+a)}}{1 - \left(-\frac{K_A K_F a}{(s+a)}\right)} = \frac{K_A a}{s+a + K_A K_F a}$$

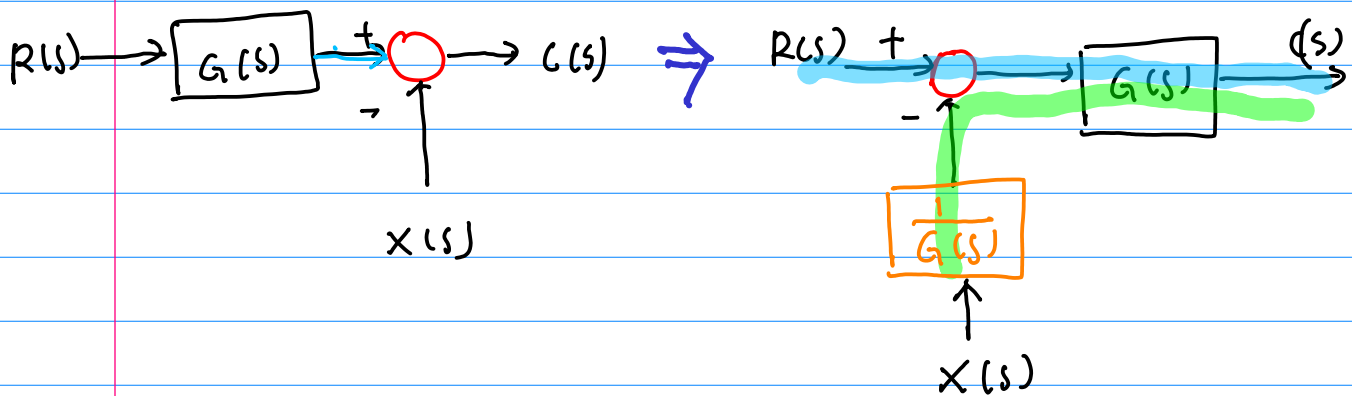
① Moving $G(s)$ backward



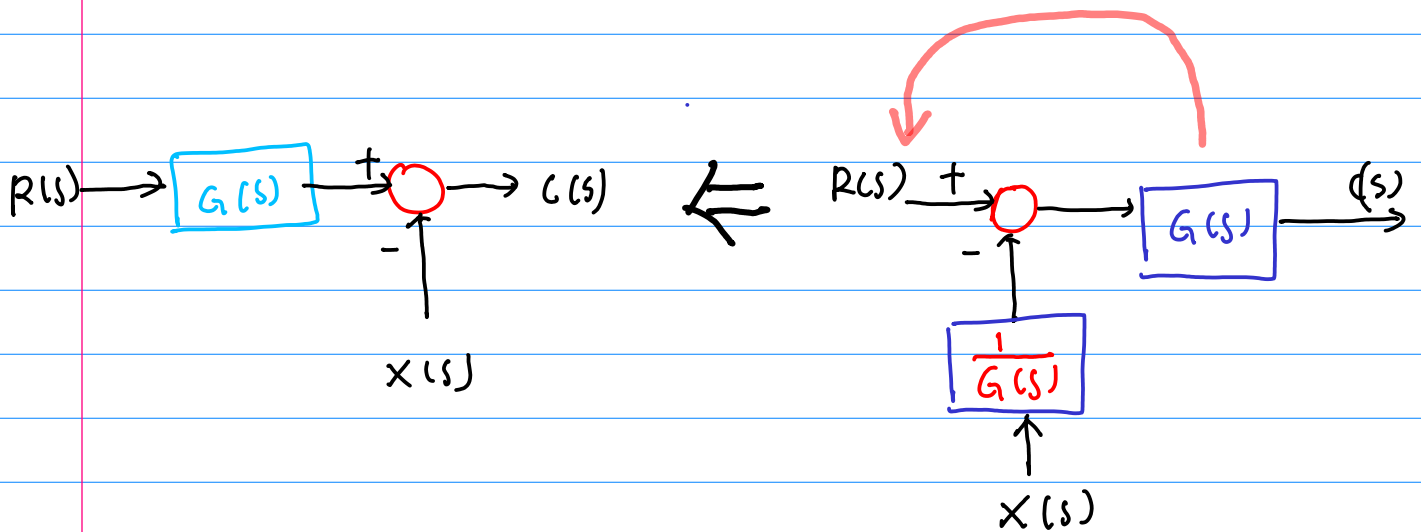
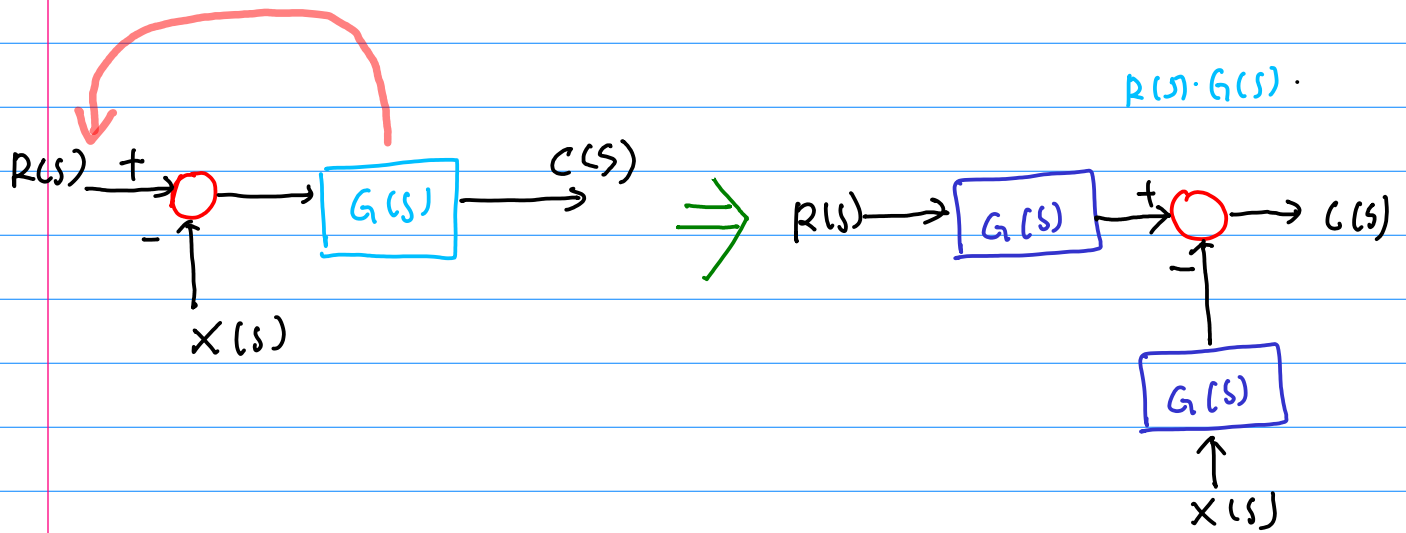
② Moving $G(s)$ forward

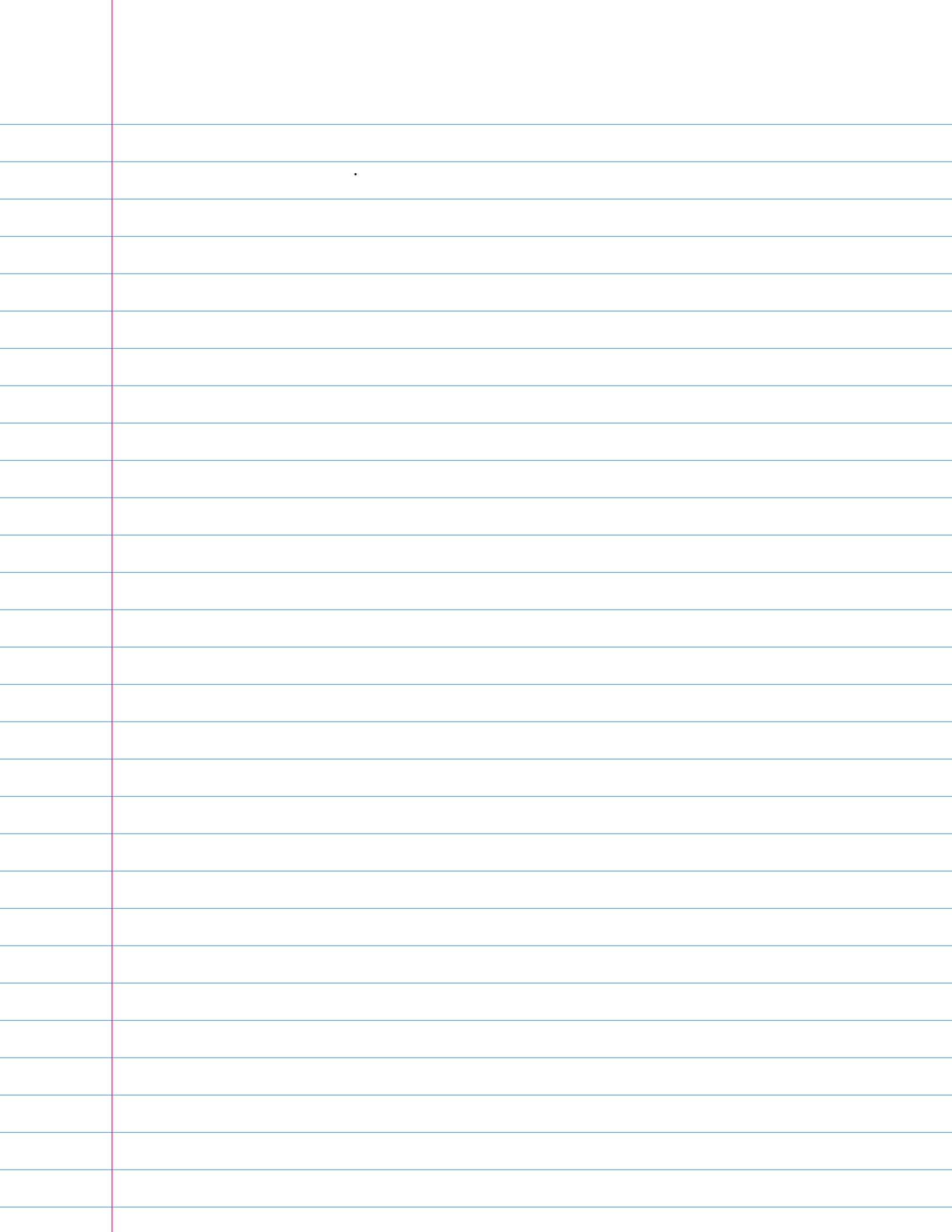


$$\begin{aligned} C(s) &= G(s)R(s) - X(s) \\ &= G(s) \left(R(s) - \frac{X(s)}{G(s)} \right) \end{aligned}$$

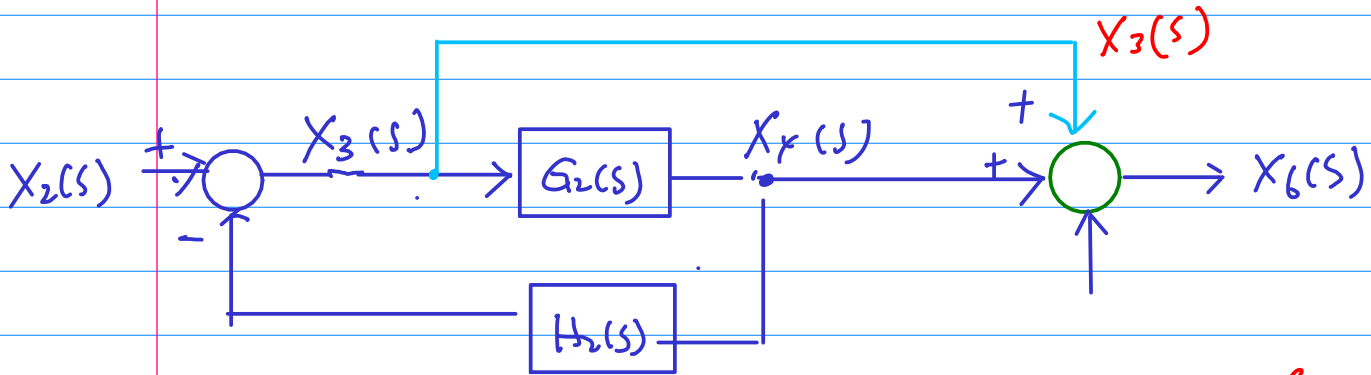


* moving backward

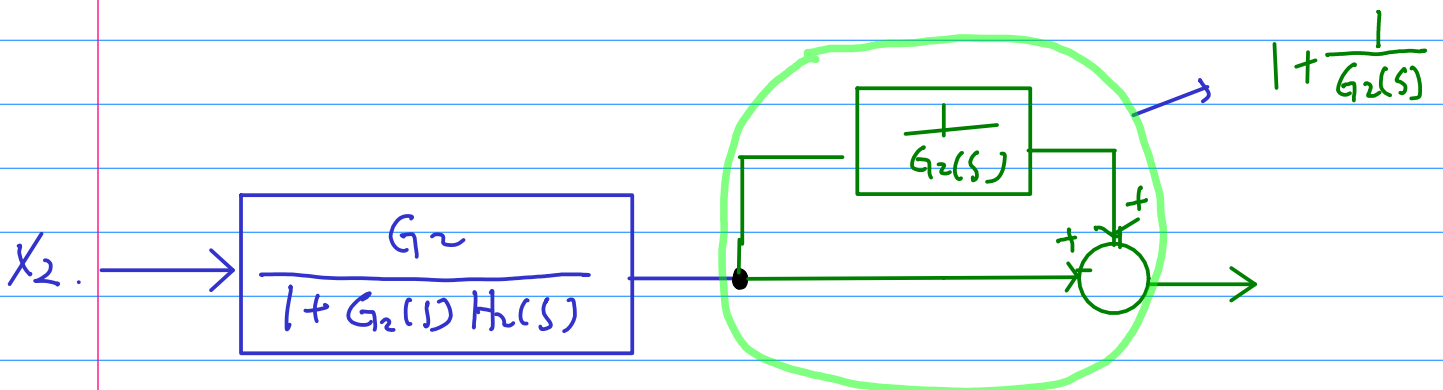
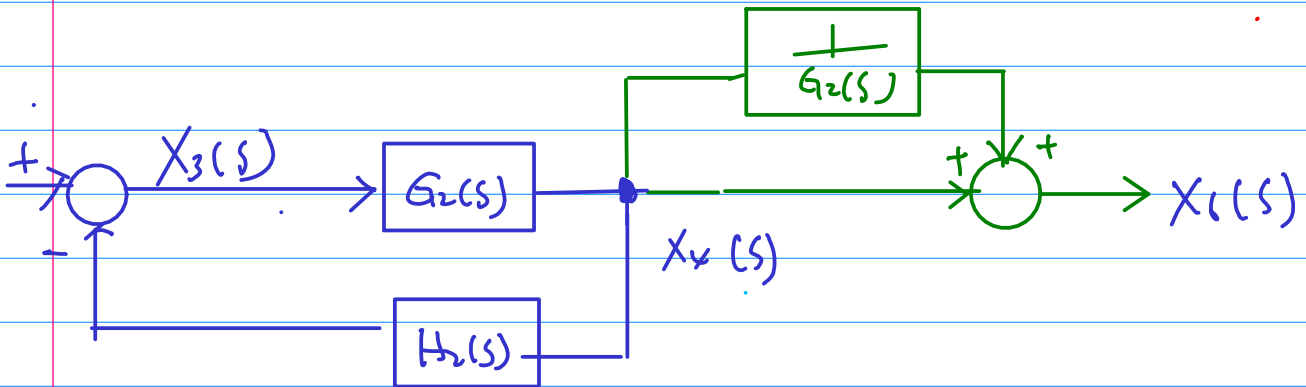




Ex 1)



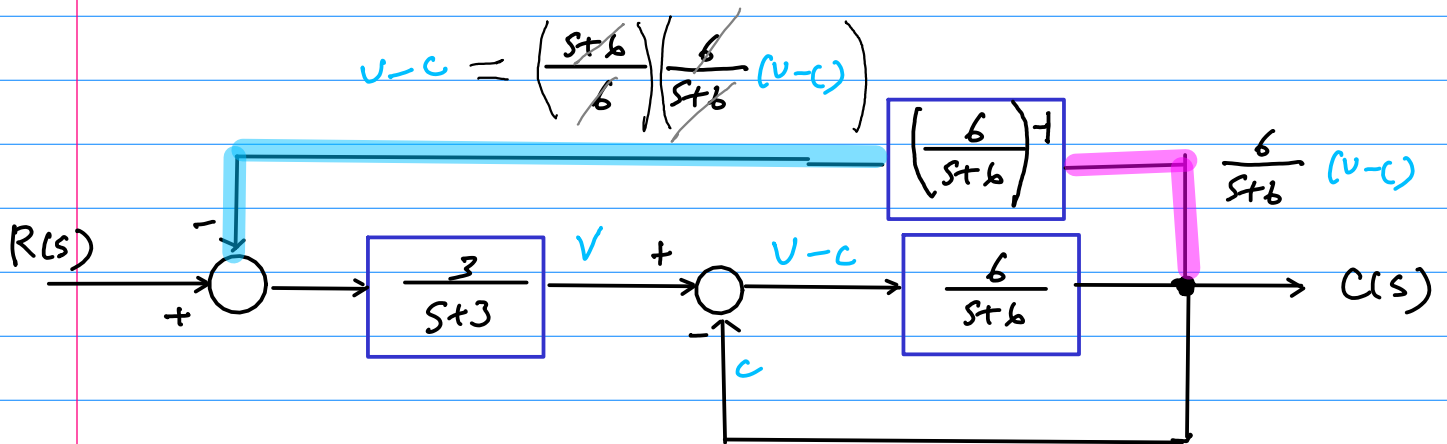
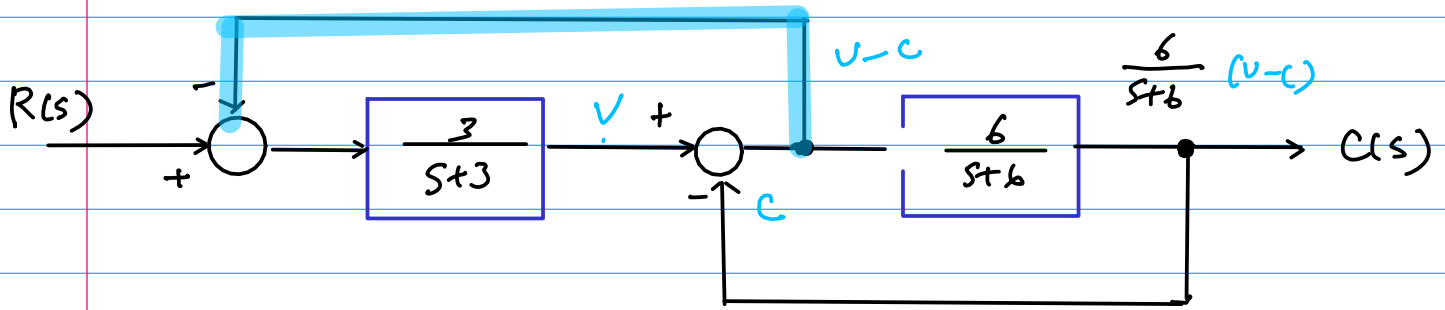
$$\frac{X_4(s)}{G_2(s)} = \frac{G_2(s) X_3(s)}{G_2(s)} = X_3(s)$$



Ex 2)

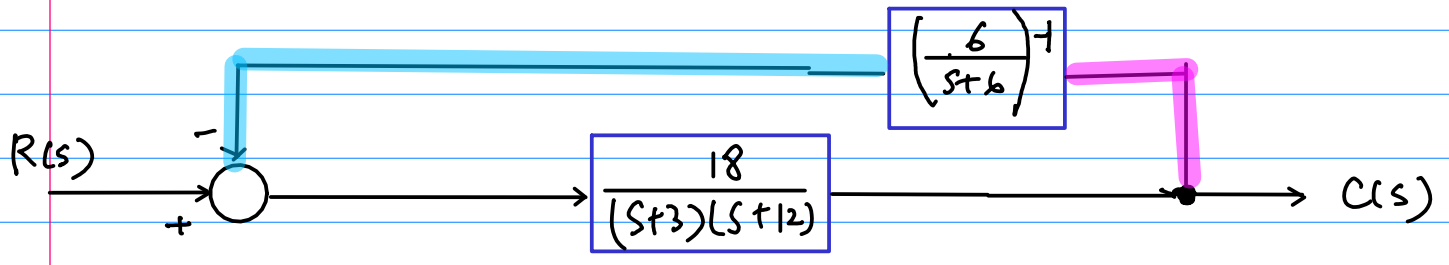
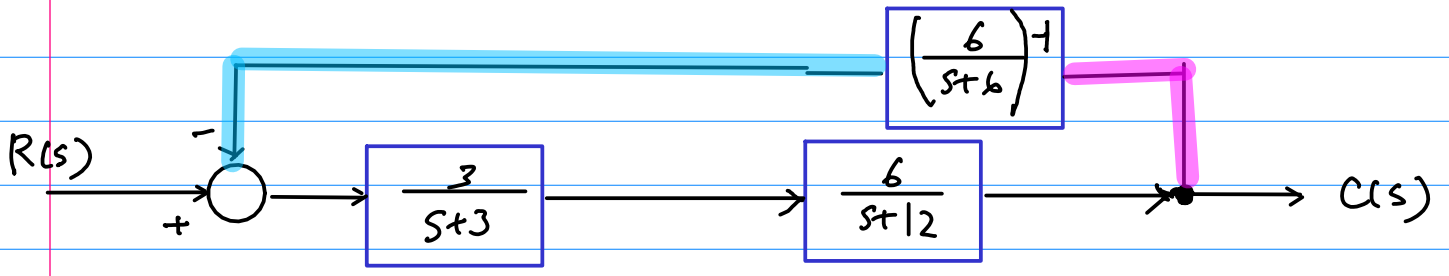
①

make the same pickoff point \rightarrow parallel subsystems



$$\frac{\frac{6}{s+6}}{1 + \frac{6}{s+6}} = \frac{6}{s+12}$$

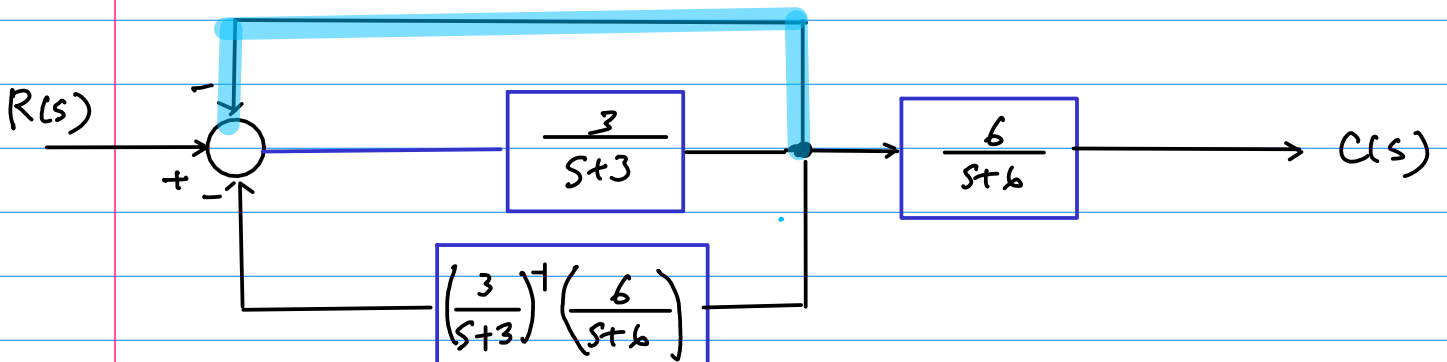
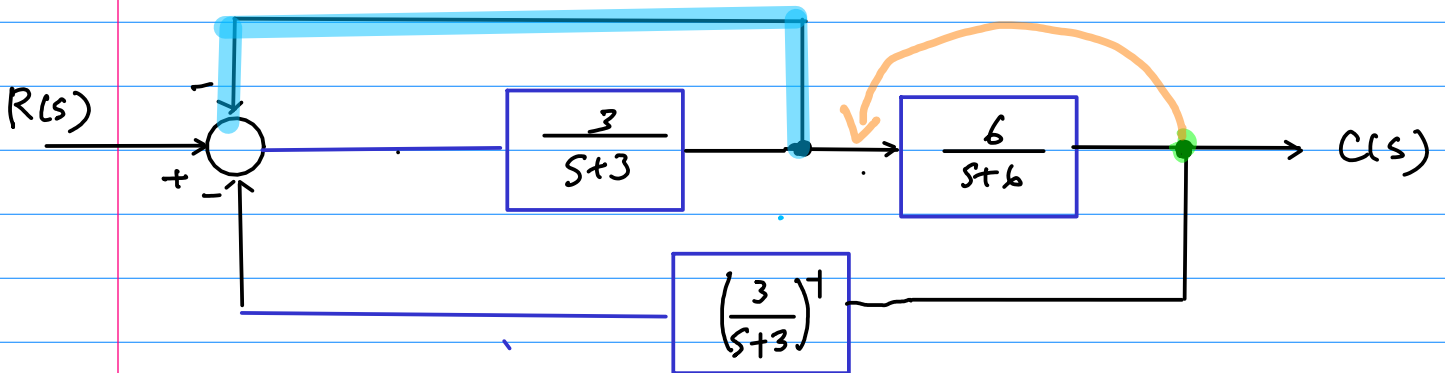
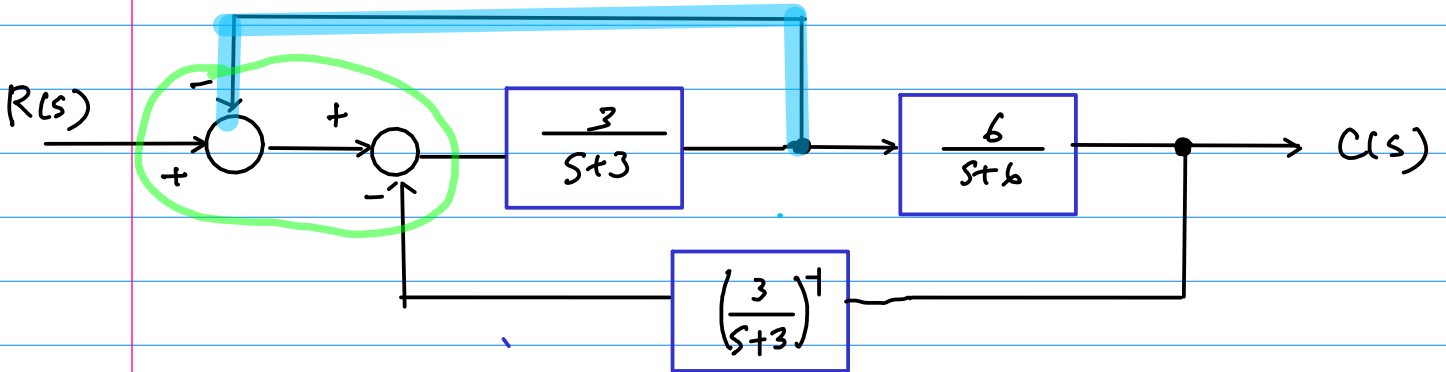
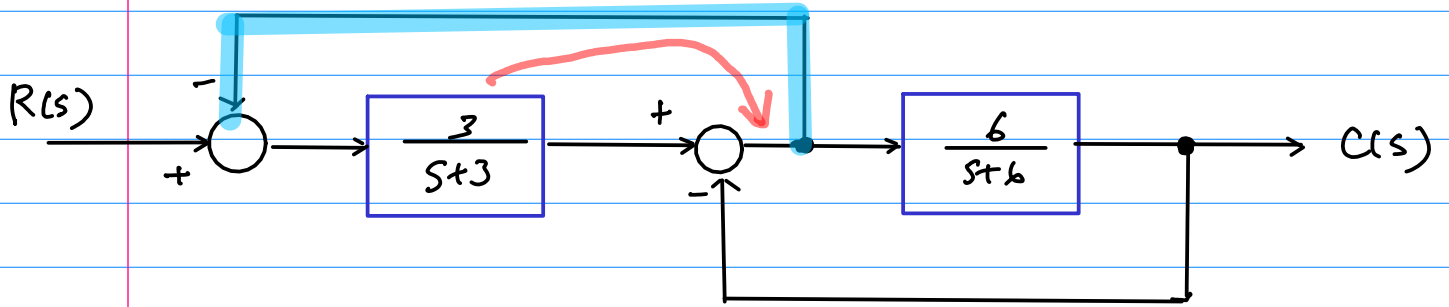
find equivalent for the inner feedback system



$$\frac{\frac{18}{(s+3)(s+12)}}{1 + \frac{18 \cdot 3}{(s+3)(s+12)} \cdot \frac{s+6}{6}} = \frac{18}{(s+3)(s+12) + 3(s+6)}$$
$$= \frac{18}{s^2 + 15s + 36 + 3s + 18}$$
$$= \frac{18}{s^2 + 18s + 54}$$

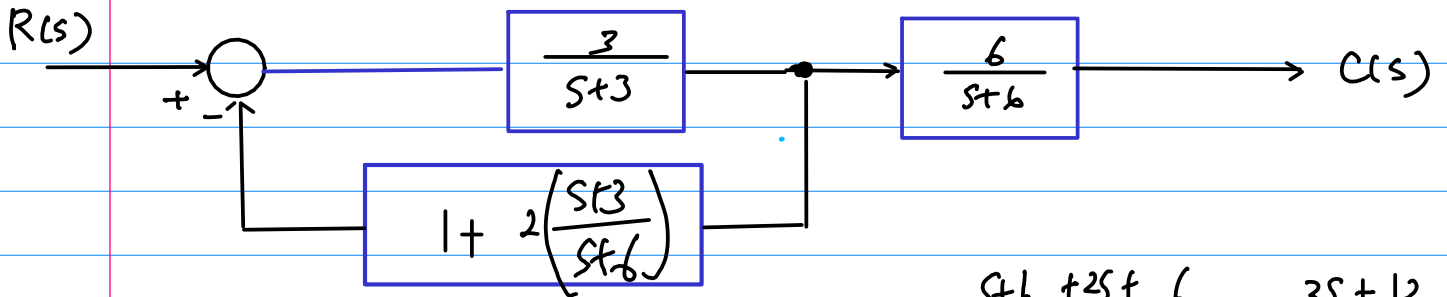
Other approach

(3)



$$\frac{s+3}{3} \cdot \frac{6}{s+6} = 2 \left(\frac{s+3}{s+6} \right)$$

(4)



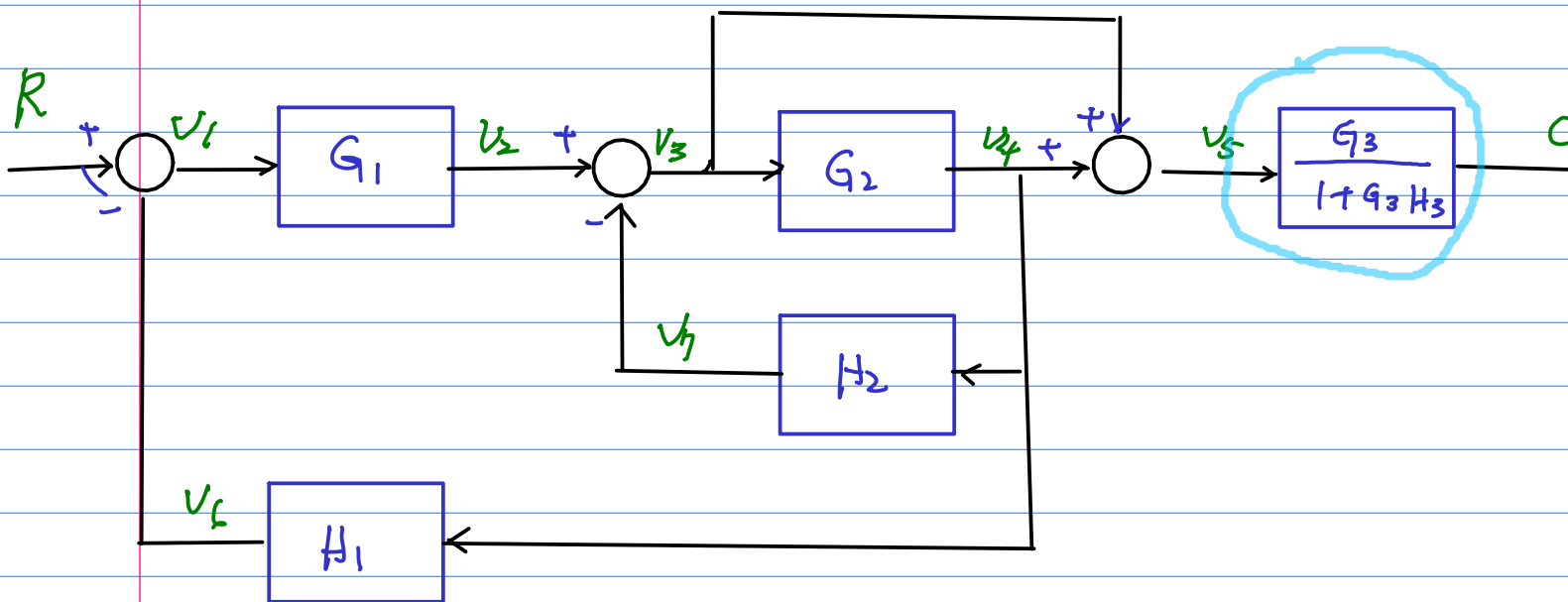
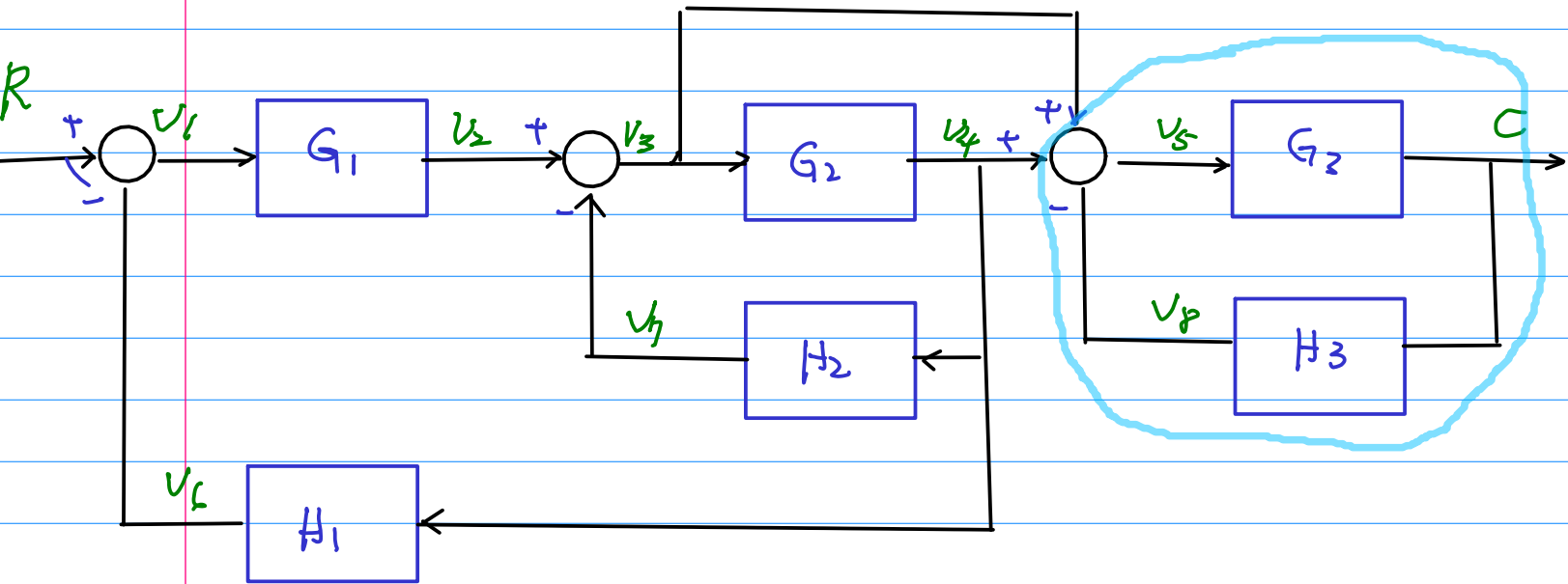
$$\frac{s+6 + 2s+6}{s+6} = \frac{3s+12}{s+6}$$

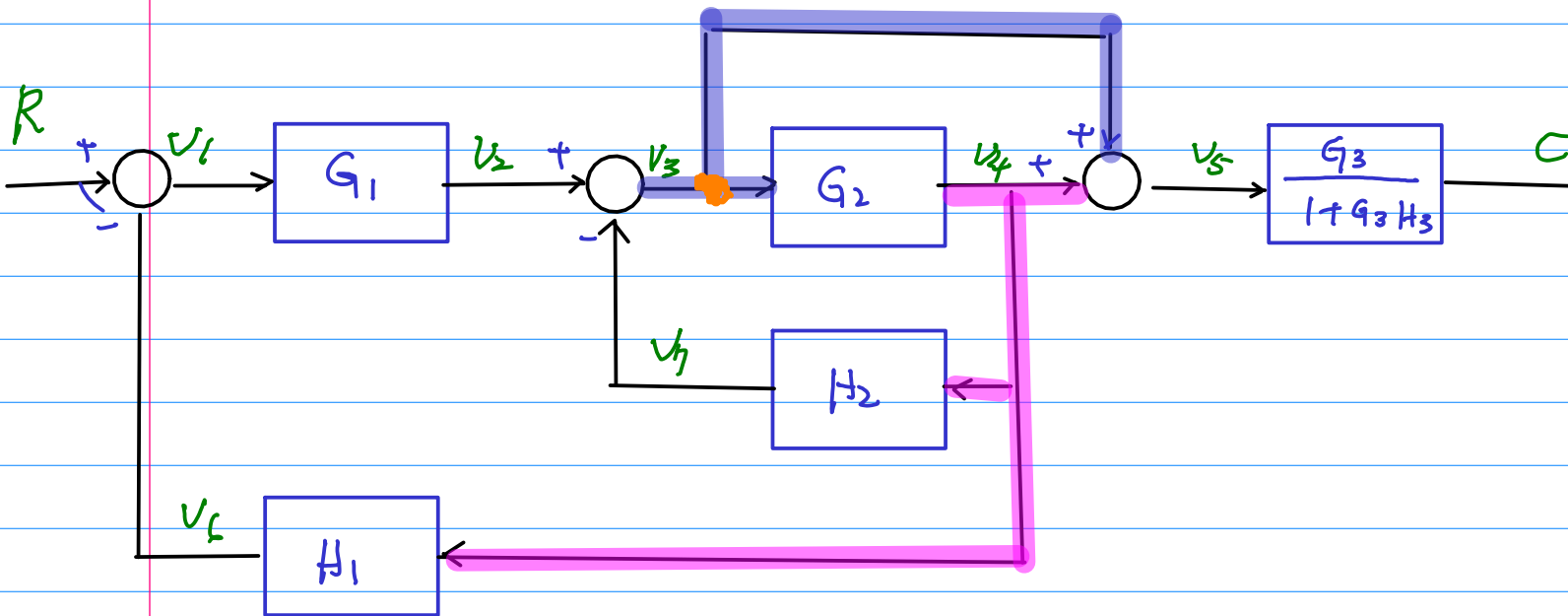
$$\frac{\frac{3}{s+3}}{1 + \frac{3}{s+3} \cdot \frac{3(s+4)}{s+6}} \cdot \frac{6}{s+6} = \frac{3}{(s+3) + \frac{9(s+4)}{s+6}} \cdot \frac{6}{s+6}$$

$$= \frac{18}{(s+3)(s+6) + 9(s+4)}$$

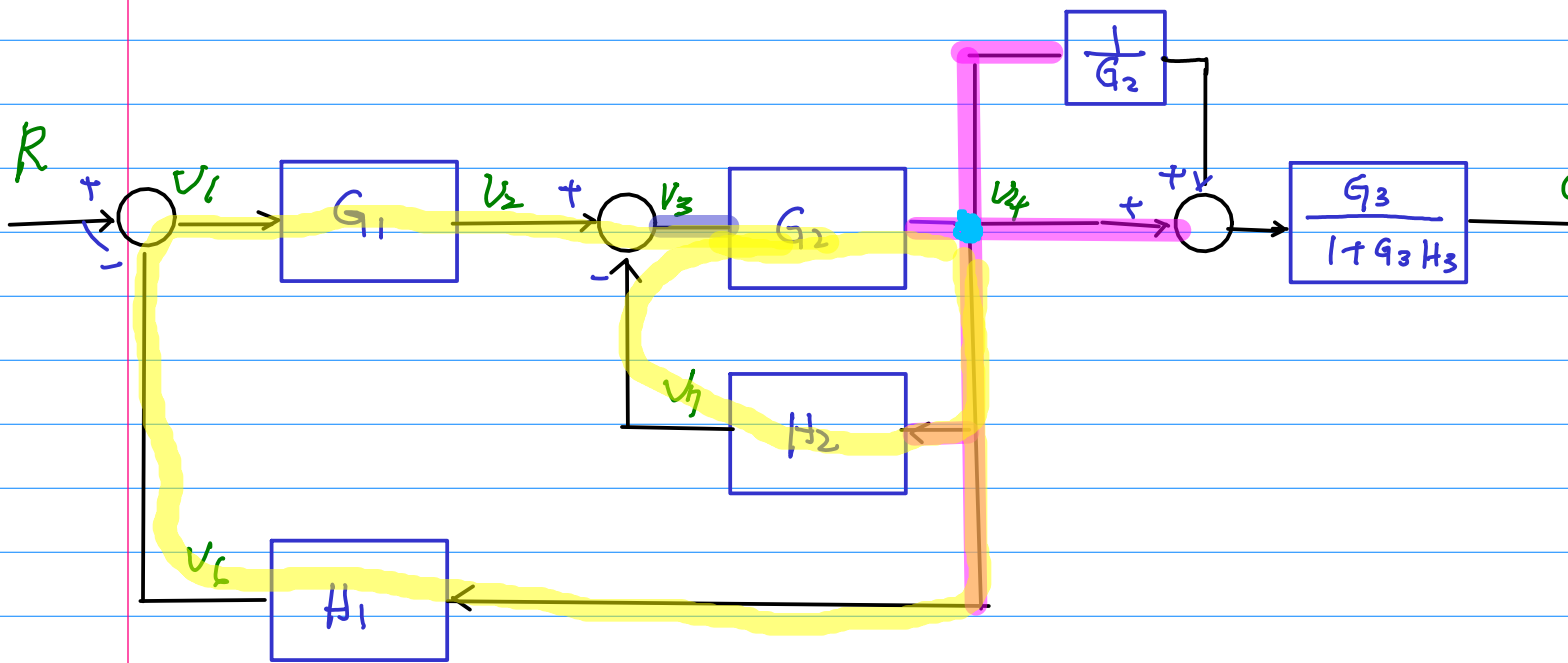
$$s^2 + 9s + 18 + 9s + 36$$

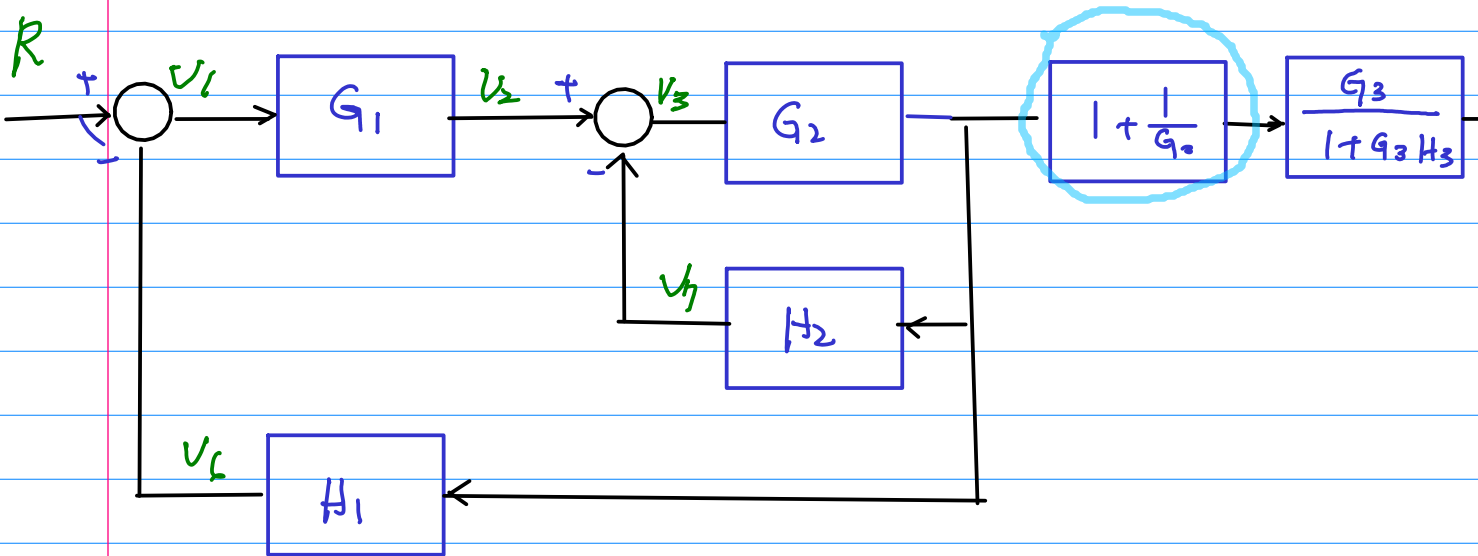
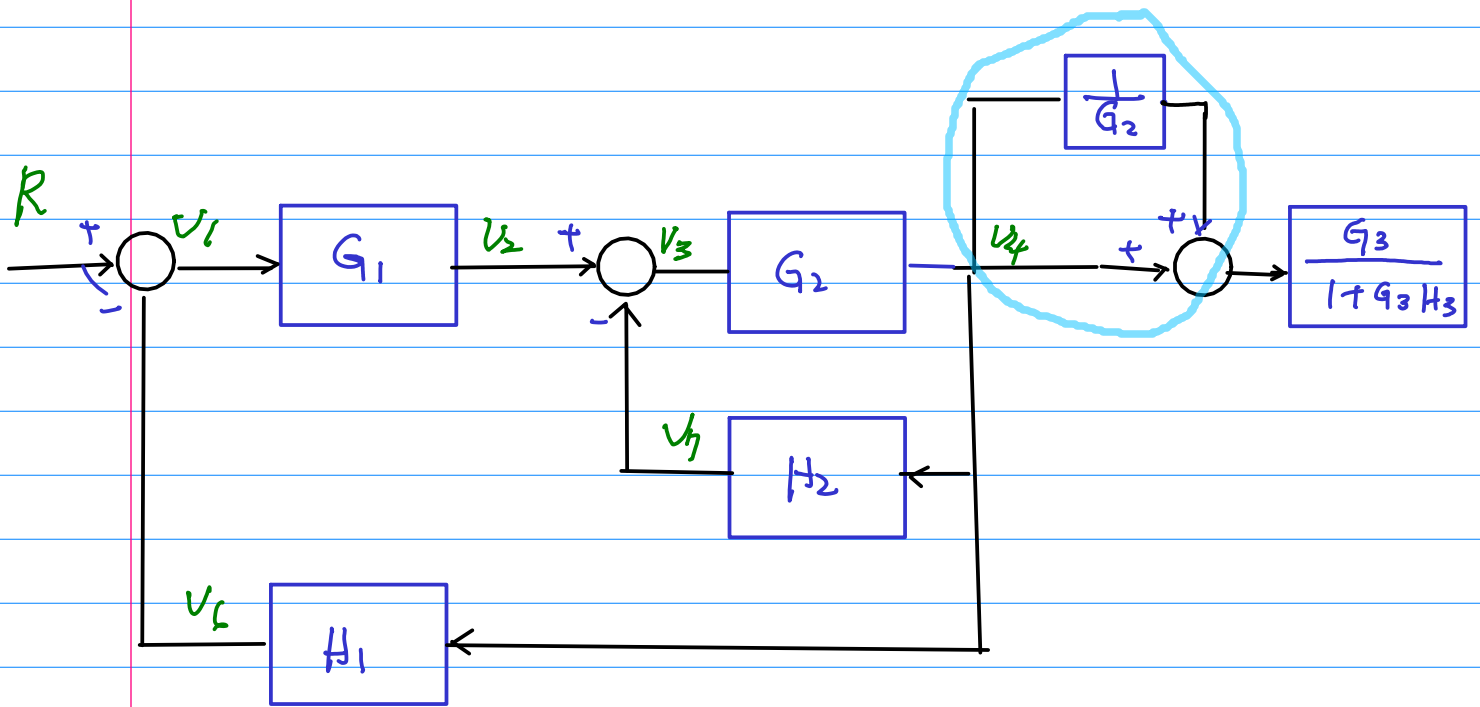
$$\frac{18}{s^2 + 18s + 54}$$

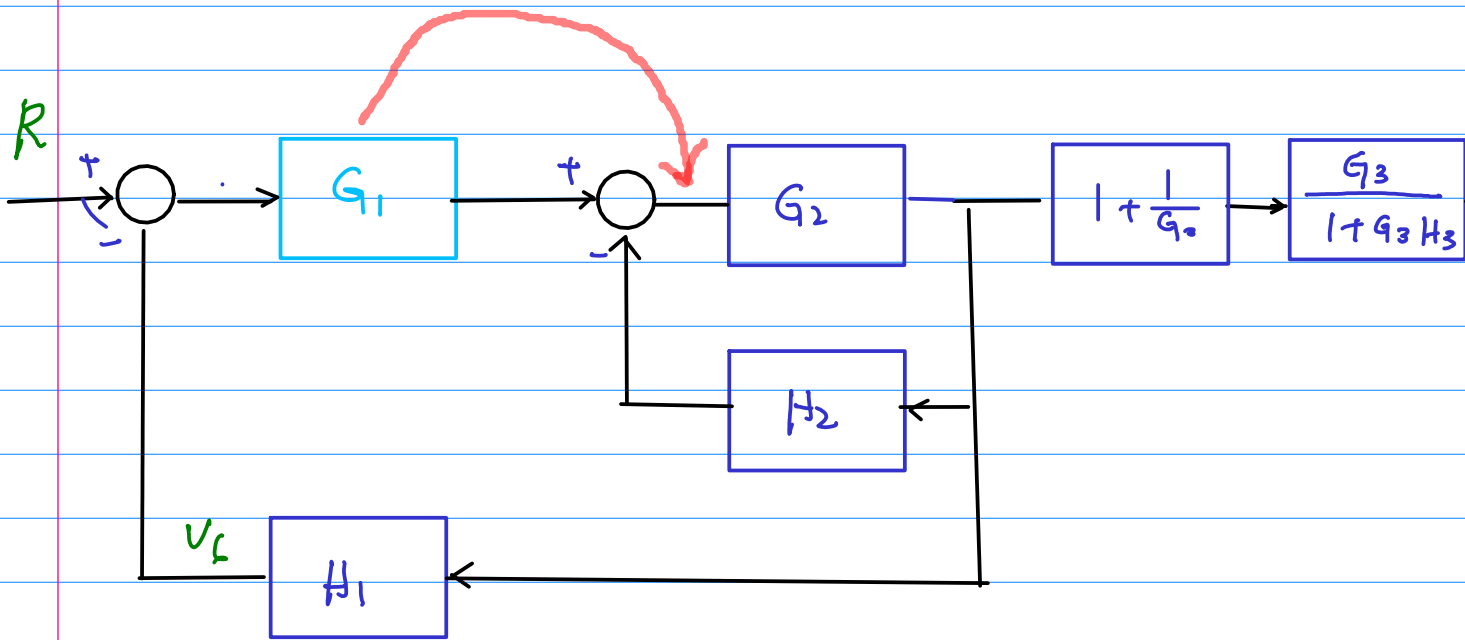




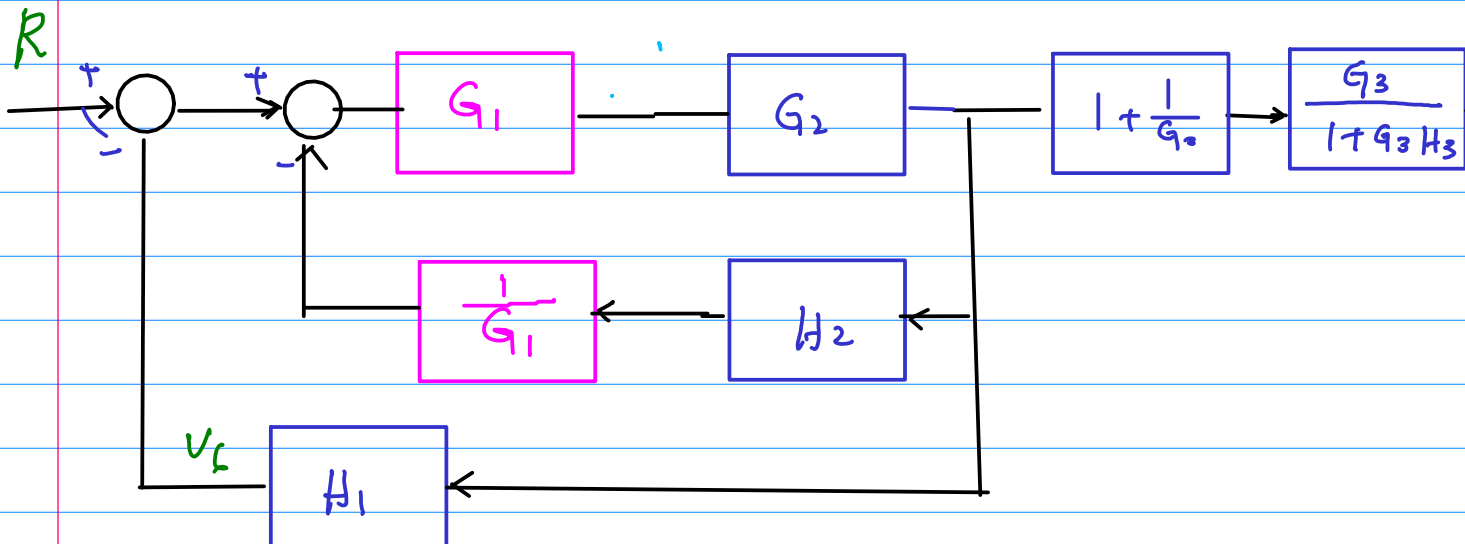
move the pickoff point to make parallel subsystems

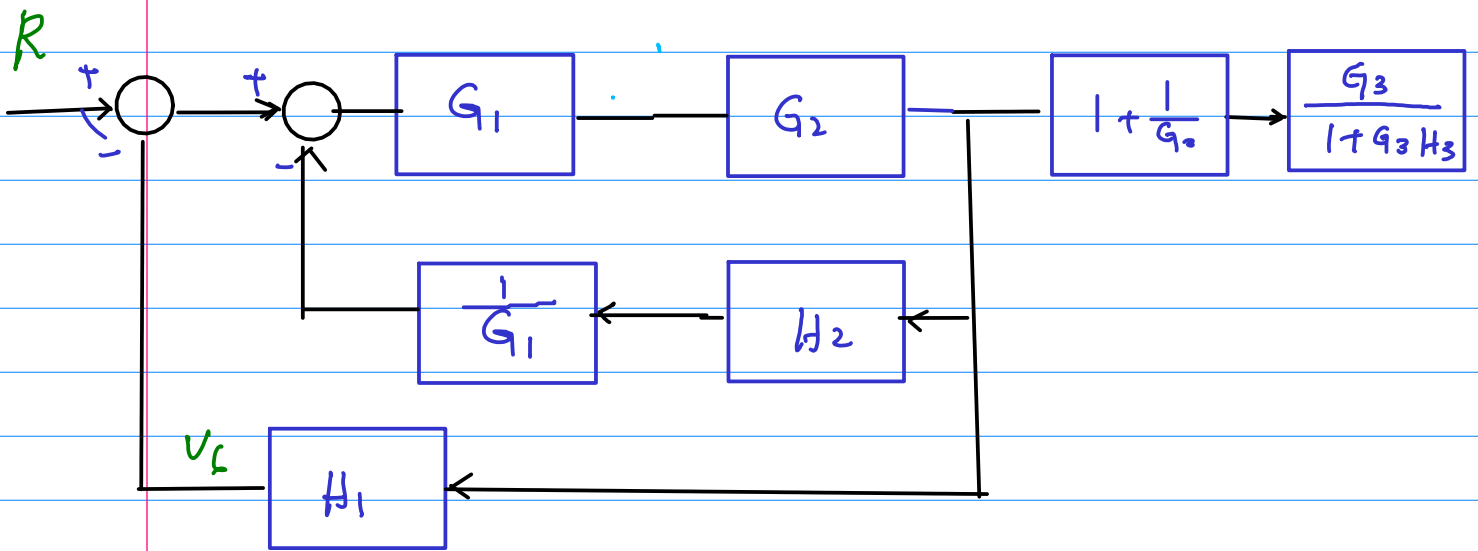






move G_1 to the right
to create parallel subsystems





Collapse the two junction points

