First Order Logic – Semantics (3A)

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

A Signature and a Language

First specify a **signature**

Constant Symbols	$\{c_1, c_2, \ldots c_n\} = D$
Predicate Symbols	$\{\mathbf{P}_1, \mathbf{P}_2, \ldots \mathbf{P}_m\}$
Function Symbols	$\{\mathbf{f}_1, \mathbf{f}_2, \ldots \mathbf{f}_l\}$

Determines the **language**

Given a language

A **model** is specified

A **domain of discourse**

a set of entities $\{entity_1, entity_2, \ldots entity_n\}$

An **interpretation**

constant assignments	$\{c_1, c_2, \ldots c_n\} = D$
function assignments	$\mathbf{f}_1(\,), \mathbf{f}_2(\,), \ldots \mathbf{f}_l(\,)$
truth value assignments	$\mathbf{P}_1(\,), \mathbf{P}_2(\,), \ldots \mathbf{P}_m(\,)$

Model – domain of discourse

1. a nonempty set D of **entities** called a **domain of discourse**
   - this domain is a set
   - each element in the set : entity
   - each constant symbol : one entity in the domain

If we considering all individuals in a class,
The constant symbols might be
‘Mary’, - an entity
‘Fred’, - an entity
‘John’, - an entity
‘Tom’ - an entity

Model – interpretation

2. an **interpretation**

(a) an entity in D is assigned to each of the constant symbols.

Normally, every entity is assigned to a constant symbol.

(b) for each **function**,

an entity is assigned to each possible input of entities to the **function**

(c) the predicate '**True**' is always assigned the value T

The predicate '**False**' is always assigned the value F

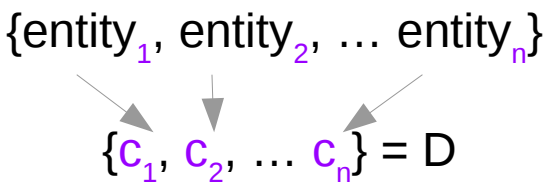
(d) for every other **predicate**,

the value T or F is assigned

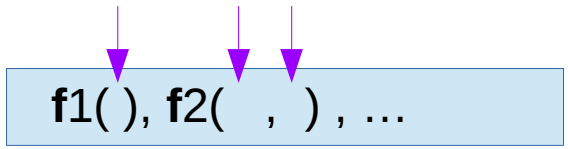
to each possible input of entities to the **predicate**

Interpretation

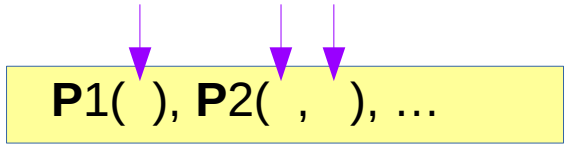
Constant assignments



Function assignments



Truth value assignments



always return **T** / **F**

Interpretation

Propositional Logic

	A	B
Interpretation I_1	T	T
Interpretation I_2	T	F
Interpretation I_3	F	T
Interpretation I_4	F	F

First Order Logic

Sentences

	P1( )	P2( ) …	S1 S2
Interpretation I_1	T	T	
Interpretation I_2	T	F	
Interpretation I_3	F	T	
Interpretation I_4	F	F	

$\{entity_1, entity_2, \ldots entity_n\}$

$\{c_1, c_2, \ldots c_n\} = D$

f1( ), **f**2(,) , …

P1( ), **P**2(,), …

always return **T** / **F**

Interpretation

Constant assignments

(a) an entity → the constant symbols.

Function assignments

(b) an entity → each possible input of entities to the **function**

Truth value assignments

(c) the value T →the predicate '**True**'
the value F → the predicate '**False**'

(d) for every other **predicate**,
the value T or F is assigned → every other predicate
to each possible input of entities to the **predicate**

Signature Model Examples A – (1)

Signature

1. constant symbols = { Mary, Fred, Sam }
2. predicate symbols = { married, young }
   married(x, y) : arity two
   young(x) : arity one

Model

1. domain of discourse D : the set of three particular *individuals*

   - this domain is a set
   - each element in the set : entity (= individuals)
   - each constant symbol : one entity in the domain (= one individual)

2. interpretation
   (a) a different individual is assigned to each of the constant symbols

   (a) an entity in D is assigned to each of the constant symbols.
   Normally, every entity is assigned to a constant symbol.

Signature Model Examples A – (2)

(b) for each **function**,
an entity is assigned to each possible input of entities to the **function**

(c) the predicate '**True**' is always assigned the value T
The predicate '**False**' is always assigned the value F

(d) the truth value assignments for every predicate

young(Mary) = F, young(Fred) = F, young(Sam) = T

married(Mary, Mary) = F, married(Mary, Fred) = T, married(Mary, Sam) = F
married(Fred, Mary) = T, married(Fred, Fred) = F, married(Fred, Sam) = F
married(Sam, Mary) = F, married(Sam, Fred) = F, married(Sam, Sam) = F

(d) for every other **predicate**,
the value T or F is assigned
to each possible input of entities to the **predicate**

(Mary, Mary), (Mary, Fred), (Mary, Sam)
(Fred, Mary), (Fred, Fred), (Fred, Sam)
(Sam, Mary), (Sam, Fred), (Sam, Sam)

Signature Model Examples B – (1)

Signature

1. constant symbols = { Fred, Mary, Sam }
2. predicate symbols = { love } love(x, y) : arity two
3. function symbols = { mother } mother(x) : arity one

Model

1. domain of discourse D : the set of three particular individuals
2. interpretation
   (a) a different individual is assigned to each of the constant symbols

   (b) the truth value assignments for every predicate
   love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
   love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
   love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F

   (c) the function assignments
   mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

Signature Model Examples B – (2)

2. interpretation

(a) a different individual is assigned to each of the constant symbols

(a) an entity in D is assigned to each of the constant symbols.
Normally, every entity is assigned to a constant symbol.

(b) the truth value assignments

(b) for each **function**,
an entity is assigned to each possible input of entities to the **function**

love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F

(c) the function assignments

(d) for every other **predicate**,
the value T or F is assigned
to each possible input of entities to the **predicate**

mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

The truth value of sentences

The truth values of **all sentences** are assigned :

1. the truth values for **sentences** developed with the symbols ¬, ∧, ∨, ⇒, ⇔
are assigned as in propositional logic.

2. the truth values for two terms connected by the **=** symbol is **T**
if both terms refer to the same entity; otherwise it is **F**

3. the truth values for ∀x p(x) has value **T**
if p(x) has value **T** for **every assignment** to x of an entity in the domain D;
otherwise it has value **F**

4. the truth values for ∃x p(x) has value **T**
if p(x) has value **T** for **at least one assignment** to x of an entity in the domain D;
otherwise it has value **F**

5. the operator **precedence** is as follows ¬, =, ∧, ∨, ⇒, ⇔

6. the **quantifiers** have precedence over the operators

7. **parentheses** change the order of the precedence

Formulas and Sentences

An **formula**

- A **atomic formula**
- The operator ¬ followed by a **formula**
- Two formulas separated by $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$
- A quantifier following by a variable followed by a formula

A **sentence**

- A **formula** with no free variables

$\forall x$ love(x,y)	: free variable y	: not a sentence
$\forall x$ tall(x)	: no free variable	: a sentence

Finding the truth value

Find the truth values of **all sentences**

1. ¬, ∧, ∨, ⇒, ⇔
2. = symbol
3. ∀x p(x)
4. ∃x p(x)
5. the **operator precedence** is as follows ¬, =, ∧, ∨, ⇒, ⇔
6. the **quantifiers** (∀, ∃) have precedence over the **operators**
7. **parentheses** change the order of the precedence

Sentence Examples (1)

Signature

Constant Symbols = {Socrates, Plato, Zeus, Fido}
Predicate Symbols = {human, mortal, legs} all arity one

Model
D: the set of these four particular individuals

Interpretation

(a) a different individual is assigned to each of the constant symbols

(b) the truth value assignment

human(Socrates)=T, human(Plato)=T, human(Zeus)=F, human(Fido)=F
mortal(Socrates)=T, mortal(Plato)=T, mortal(Zeus)=F, mortal(Fido)=T
legs(Socrates)=T, legs(Plato)=T, legs(Zeus)=T, legs(Fido)=T

Sentence Examples (2)

Sentence 1: human(Zeus) ∧ human(Fido) ∨ human(Socrates) = T
F ∧ F ∨ T

Sentence 2: human(Zeus) ∧ (human(Fido) ∨ human(Socrates)) = F
F ∧ (F ∨ T)

Sentence 3: ∀x human(x) = F
human(Zeus)=F, human(Fido)=F

Sentence 4: ∀x mortal(x) = F
mortal(Zeus)=F

Sentence 5: ∀x legs(x) = T
legs(Socrates)=T, legs(Plato)=T, legs(Zeus)=T, legs(Fido)=T

Sentence 6: ∃x human(x) = T
human(Socrates)=T, human(Plato)=T

Sentence 7: ∀x (human(x) ⇒ mortal(x)) = T

Sentence Examples (3)

Sentence 7: $\forall x$ (human(x) $\Rightarrow$ mortal(x)) = T

human(Socrates)=T,	mortal(Socrates)=T,	T $\Rightarrow$ T : T
human(Plato)=T,	mortal(Plato)=T,	T $\Rightarrow$ T : T
human(Zeus)=F,	mortal(Zeus)=F,	F $\Rightarrow$ F : T
human(Fido)=F	mortal(Fido)=T	F $\Rightarrow$ T : T

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