

State Monad (3D)

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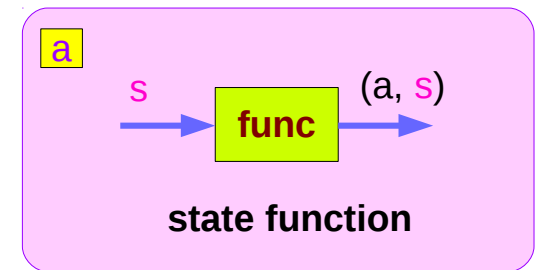
Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

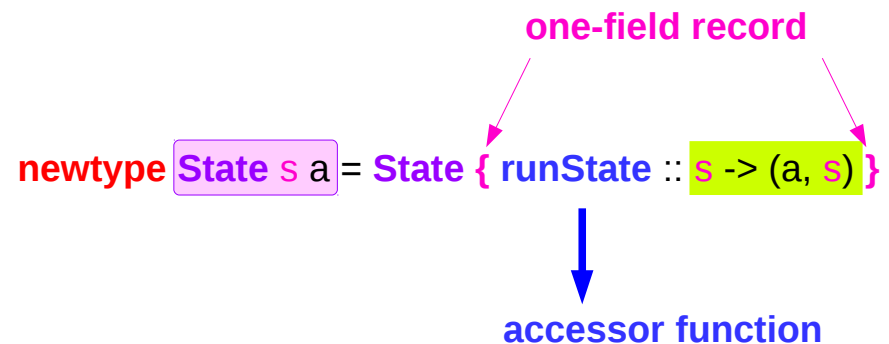
The state function

The Haskell type **State** describes **functions** that take a **state** and return both a **result** and an **updated state**, which are given back in a **tuple**.

The **state function** is wrapped by a **data type** definition which comes along with a **runState accessor** no need for pattern matching



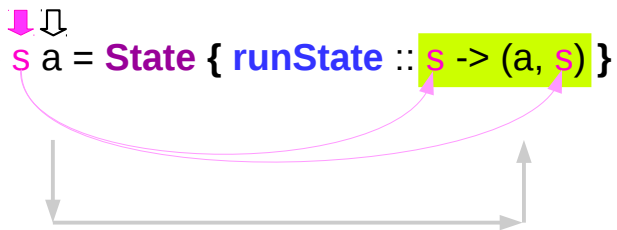
$p :: \text{State } s \ a$



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Type State

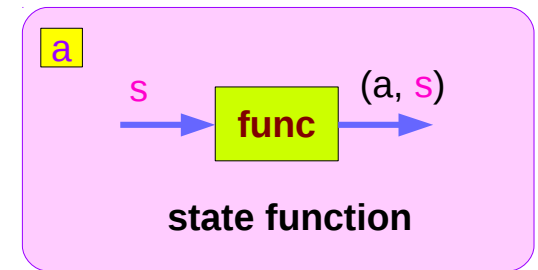
```
newtype State s a = State { runState :: s -> (a, s) }
```



s : the type of the state,
 a : the type of the produced result
 $s \rightarrow (a, s)$: function type

`State String`,
`State Int`,
`State SomeLargeDataStructure`,
and so forth.

Calling the type `State` is arguably a bit of a misnomer because the wrapped value is not the state itself but a **state processor** (accessor function: `runState`)



$p :: \text{State } s \ a$

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

State Packages

Control.Monad.**Trans.State**, **transformers** package. (focused here)

Control.Monad.**State**, **mtl** package.

Control.Monad.**State.Lazy**, **mtl** package.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

The “state” function

Control.Monad.Trans.State, transformers package. (focused here)

no State constructor

but a “state” function

```
state :: (s -> (a, s)) -> State s a
```

Control.Monad.State, mtl package

Implements the State in somewhat different way

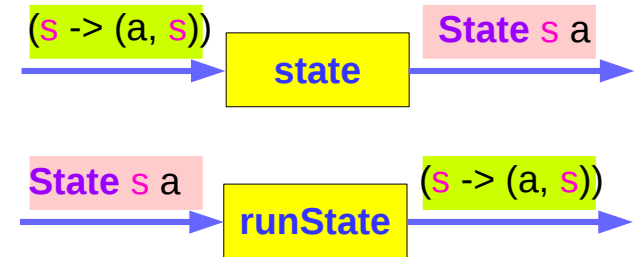


https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

runState function

State is a record with only one element,
whose type is a function ($:: s \rightarrow (a, s)$)

runState converts a value of type **State s a**
to a function of this type ($:: s \rightarrow (a, s)$)



```
ghci> :t runState
```

```
runState :: State s a -> s -> (a, s)
```

Every time you apply **runState** to the value of type **State s a**,
the result is a function of type $s \rightarrow (a, s)$.

```
newtype State s a = State { runState :: s -> (a, s) }
```

<https://stackoverflow.com/questions/3240947/understanding-haskell-accessor-functions>

state & runState function



`runState :: State s a -> s -> (a, s)`

`runState :: State s a -> s -> (a, s)`

`newtype State s a = State { runState :: s -> (a, s) }`

<https://stackoverflow.com/questions/3240947/understanding-haskell-accessor-functions>

Instantiating a State Monad

wrap a function type and give it a name.

$s \rightarrow (a, s)$

State s can be made a *Monad instance*, for every type s

the *Monad instance* is **State** s , and not just **State**

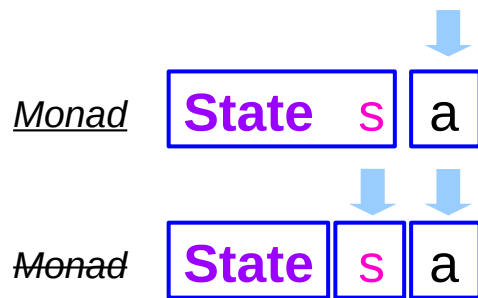
(**State** can't be made an instance of *Monad*,
as it takes two type parameters, rather than one.)

State String ,

State Int ,

State $\text{SomeLargeDataStructure}$,

and so forth.



newtype **State** s a = **State** { **runState** :: $s \rightarrow (a, s)$ }

instance **Monad** (**State** s) where

return implementation

(**>>=**) implementation

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Common implementation of **return** and **>>=**

instance Monad (**State s**) where

many different **State** monads,
one for each possible type of state -

State String,

State Int,

State SomeLargeDataStructure,

and so forth.

one implementation of

return and
(>>=);

can handle these different (**State s**) monads
according to different choices of **s**.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

return method

```
instance Monad (State s) where
```

```
return :: a -> State s a
```

```
return x = state (\s -> (x, s))
```

→ State s a



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

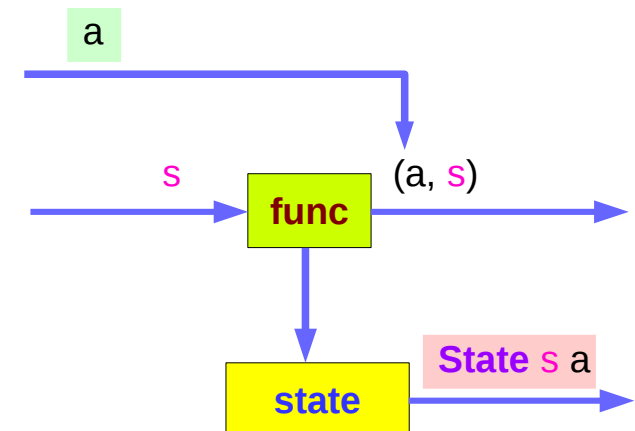
return method

instance Monad (State s) where

return :: a -> State s a

return x = **state** (\s -> (x, s)) \longrightarrow State s a

giving a value (x) to **return** results in a **state processor** function which takes a state (s) and returns it unchanged (s), together with value x we want to be returned. Finally, the function is wrapped up by **state**.



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

State Monad Examples – return

```
runState (return 'X') 1
```

```
('X',1)
```

return

set the result value but leave the state unchanged.

```
return 'X' :: State Int Char
```

```
runState (return 'X') :: Int -> (Char, Int)
```

```
initial state = 1 :: Int
```

```
final value = 'X' :: Char
```

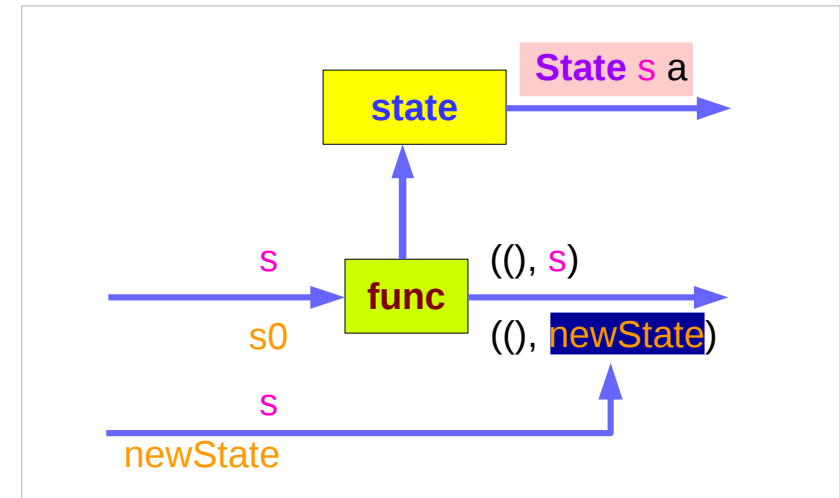
```
final state = 1 :: Int
```

```
result = ('X', 1) :: (Char, Int)
```

https://wiki.haskell.org/State_Monad

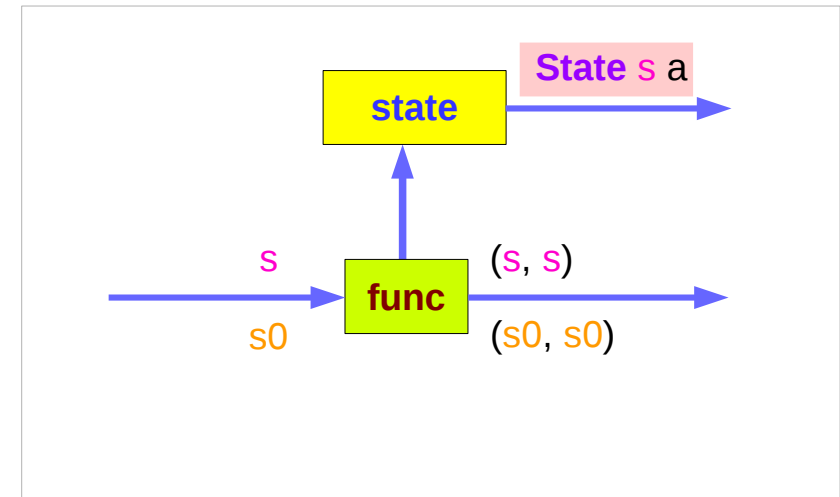
Setting and Getting the State

```
put :: s -> State s a
put newState = state $ \_ -> ((), newState)
```



```
get :: State s s
get = state $ \s -> (s, s)
```

-- getting s

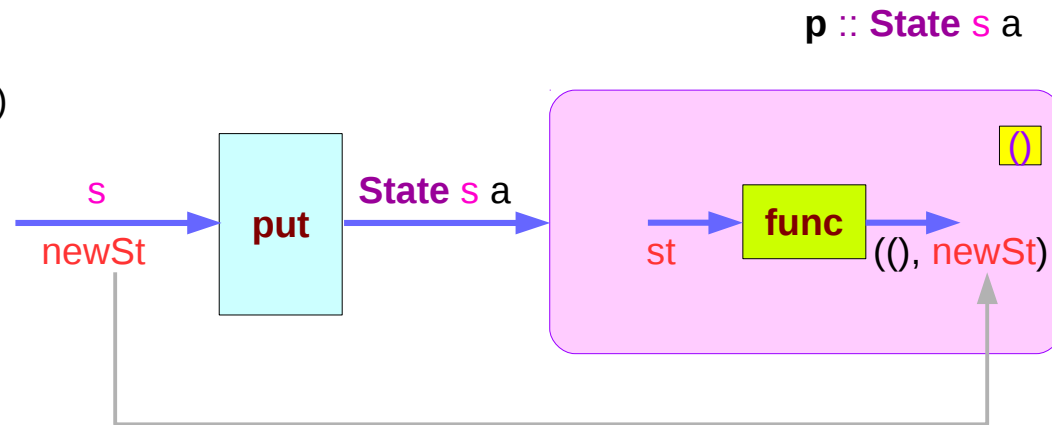


https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

put and get

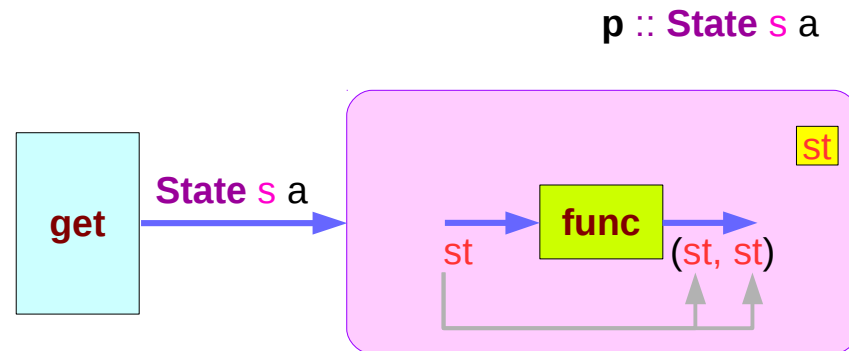
`put :: s -> State s a`

`put newSt = state $ _ -> ((), newSt)`



`get :: State s s`

`get = state $ \s -> (s, s)`



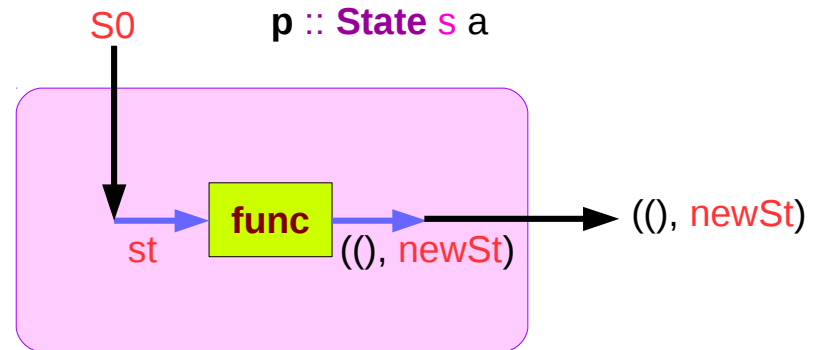
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runState put and runState get

```
put :: s -> State s a
put newSt = state $ \_ -> ((), newSt)

runState (put newSt) S0

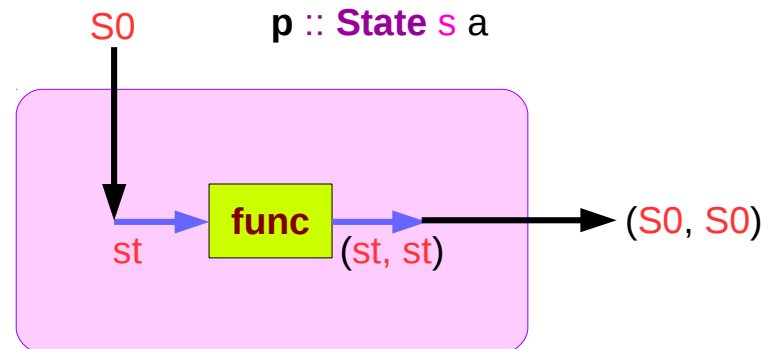
((), newSt)
```



```
get :: State s s
get = state $ \s -> (s, s)

runState (get) S0

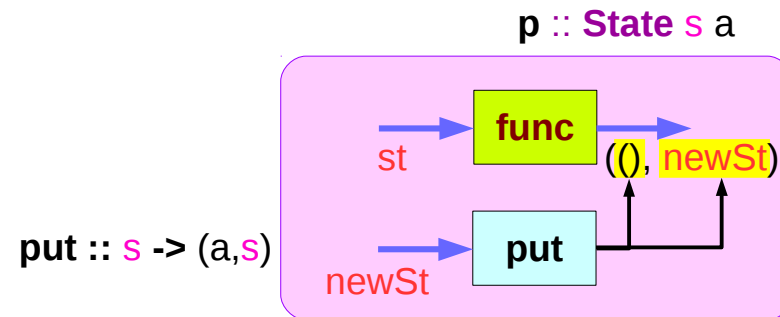
(S0, S0)
```



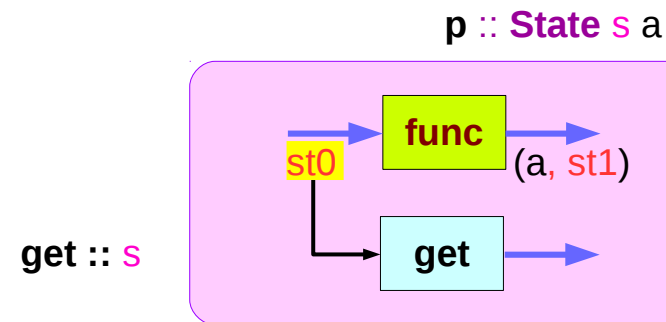
https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

put and get viewed as inside functions

```
put :: s -> State s a
put newSt = state $ \_ -> ((), newSt)
```



```
get :: State s s
get = state $ \s -> (s, s)
```



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Inside the state monad

Whenever **sc** is a **stateful computation**
sc can be directly assigned to **x**, **inside** the state monad,

```
x <- sc
```

the result of the stateful computation **sc** is assigned to **x**
(like **evalState** is called with an initial state).

In order to check the current state, you can do

```
s <- get
```

and **s** will have the value of the current state.

<https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell>

Inside Functions and runState Functions

Most monads are equipped with some "run" functions such as **runState**, **execState**, and so forth.

But, frequent calling such functions inside the monad shows that the functionality of the monad does not fully exploited

```
s0 <- get
let (a,s') = runState s s0
put s'
```

```
-- Read state
-- Pass state to 's', get new state
-- Save new state
```



```
a <- s
```

<https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell>

Redundant computation examples

```
s0 <- get
```

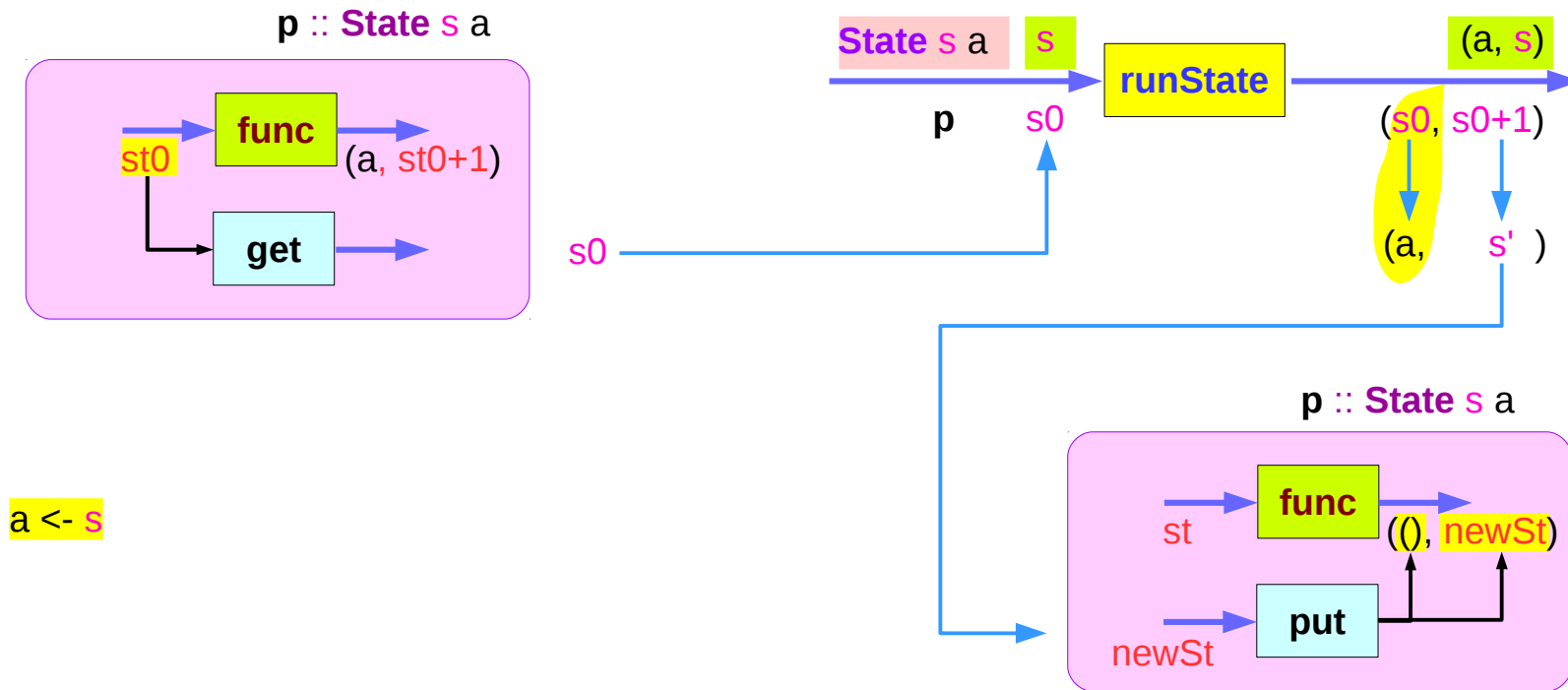
```
-- Read state
```

```
let (a,s') = runState p s0
```

- Pass state to **p**, get new state

```
put s'
```

```
-- Save new state
```



<https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell>

Inside function examples

```
collectUntil :: (s -> Bool) -> State s a -> State s [a]
collectUntil f s = step
  where
    step = do a <- s
            liftM (a:) continue
    continue = do s' <- get
                if f s' then return [] else step
```

```
simpleState = state (\x -> (x,x+1))
```

```
*Main> evalState (collectUntil (>10) simpleState) 0
[0,1,2,3,4,5,6,7,8,9,10]
```

<https://stackoverflow.com/questions/11250328/working-with-the-state-monad-in-haskell>

liftM

```
liftM :: (Monad m) => (a -> b) -> m a -> m b
mapM  :: (Monad m) => (a -> m b) -> [a] -> m [b]
```

liftM lifts a function of type `a -> b` to a monadic counterpart.

mapM applies a function which yields a monadic value to a list of values, yielding list of results embedded in the monad.

```
> liftM (map toUpper) getLine
```

```
Hallo
```

```
"HALLO"
```

```
> :t mapM return "monad"
```

```
mapM return "monad" :: (Monad m) => m [Char]
```

<https://stackoverflow.com/questions/5856709/what-is-the-difference-between-liftm-and-mapm-in-haskell>

mapM

```
> :t mapM return "monad"  
mapM return "monad" :: (Monad m) => m [Char]
```

```
> map (x -> [x+1]) [1,2,3]  
[[2],[3],[4]]
```

```
> mapM (x -> [x+1]) [1,2,3]  
[[2,3,4]]
```

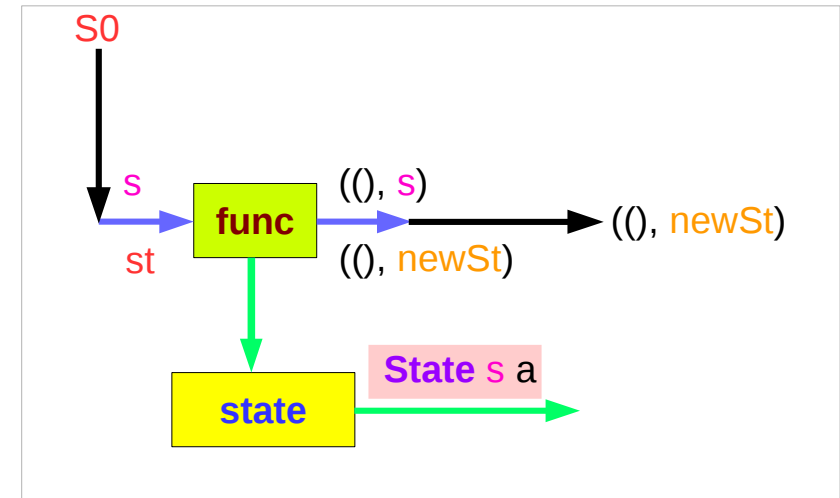
<https://stackoverflow.com/questions/5856709/what-is-the-difference-between-liftm-and-mapm-in-haskell>

Setting the State

```
put :: s -> State s a
put newSt = state $ \_ -> ((), newSt)
```

Given a wanted `state newState`,
`put` generates a **state processor**
which ignores whatever the `state` it receives,
and gives back the `state` we originally provided to `put`.
the same `state`

Since we don't care about the **result** (`a`) of this processor
(all we want to do is to change the `state`),
the first element of the tuple will be `()`,
the **universal placeholder value**.



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

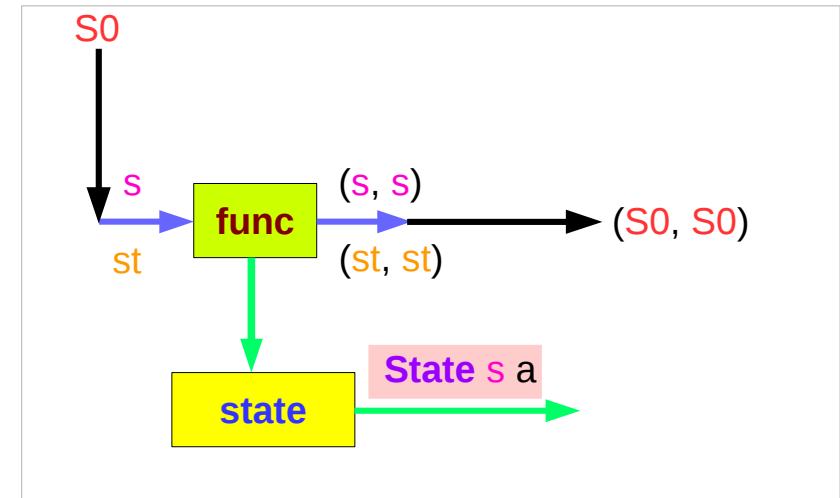
Getting the State

```
get :: State s s
```

```
get = state $ \s -> (s, s)
```

The resulting **state processor** gives back the **state st** it is given in both as a **result** and as a **state**.

That means the **state** will remain unchanged, and that a copy of it will be made available for us to manipulate.



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

evalState and execState

runState

unwrap the **State s a** value
to get the actual **state processing function**
which is then applied to some **initial state**.

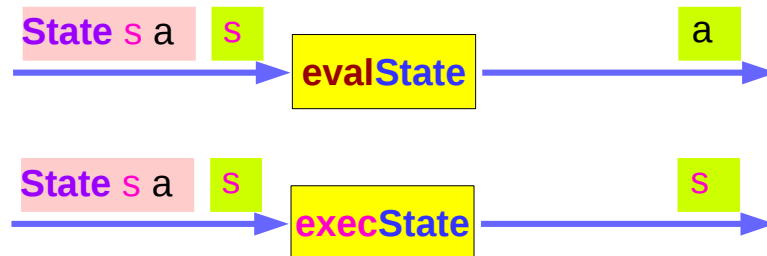
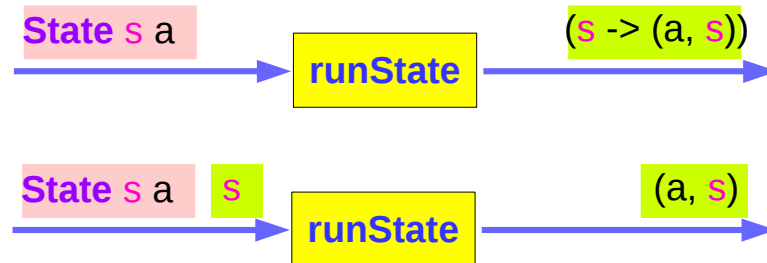
Given a **State s a** and an **initial state s**,

evalState only the result value

execState just the new state.

evalState :: **State s a** -> **s** -> **a**
evalState p s = fst (**runState p s**)

execState :: **State s a** -> **s** -> **s**
execState p s = snd (**runState p s**)



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

State Monad Examples – **get**

```
runState get 1
```

```
(1,1)
```

get

set the result value to the state and leave the state unchanged.

Comments:

```
get :: State Int Int
runState get :: Int -> (Int, Int)
initial state = 1 :: Int
final value = 1 :: Int
final state = 1 :: Int
```

```
get :: State s s
get = state $ \s -> (s, s)
```

https://wiki.haskell.org/State_Monad

State Monad Examples – put

```
runState (put 5) 1
```

```
((),5)
```

put

set the result value to () and set the state value.

Comments:

```
put 5 :: State Int ()
```

```
runState (put 5) :: Int -> ((),Int)
```

```
initial state = 1 :: Int
```

```
final value = () :: ()
```

```
final state = 5 :: Int
```

```
put :: s -> State s a
```

```
put newState = state $ \_ -> ((), newState)
```

https://wiki.haskell.org/State_Monad

Put and get in mtl packages

Return leaves the state unchanged and sets the result:

```
-- ie: (return 5) 1 -> (5,1)
```

```
return :: a -> State s a
```

```
return x s = (x,s)
```

Get leaves state unchanged and sets the result to the state:

```
-- ie: get 1 -> (1,1)
```

```
get :: State s s
```

```
get s = (s,s)
```

Put sets the result to () and sets the state:

```
-- ie: (put 5) 1 -> ((),5)
```

```
put :: s -> State s ()
```

```
put x s = ((),x)
```

https://wiki.haskell.org/State_Monad

Unwrapped Implementation Examples (1)

Return leaves the state unchanged and sets the result:

```
-- ie: (return 5) 1 -> (5,1)
```

```
return :: a -> State s a
```

```
return x s = (x,s)
```

Get leaves state unchanged and sets the result to the state:

```
-- ie: get 1 -> (1,1)
```

```
get :: State s s
```

```
get s = (s,s)
```

Put sets the result to () and sets the state:

```
-- ie: (put 5) 1 -> ((),5)
```

```
put :: s -> State s ()
```

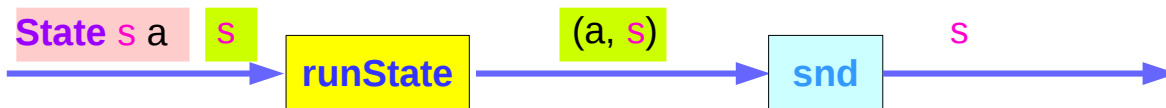
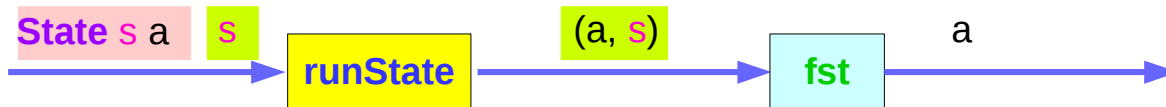
```
put x s = ((),x)
```

https://wiki.haskell.org/State_Monad

Unwrapped Implementation Examples (1)

```
evalState :: State s a -> s -> a
evalState act = fst . runState act

execState :: State s a -> s -> s
execState act = snd . runState act
```



https://wiki.haskell.org/State_Monad

Unwrapped Implementation Examples (2)

```
modify :: (s -> s) -> State s ()  
modify f = do { x <- get; put (f x) }
```

```
gets :: (s -> a) -> State s a  
gets f = do { x <- get; return (f x) }
```

```
runState (modify (+1)) 1  
  ((), 2)
```

```
runState (gets (+1)) 1  
  (2, 1)
```

```
evalState (gets (+1)) 1  
  2
```

```
execState (gets (+1)) 1  
  1
```

get & put :

functions inside the State monad

get :: s

put :: s -> (a, s)

https://wiki.haskell.org/State_Monad

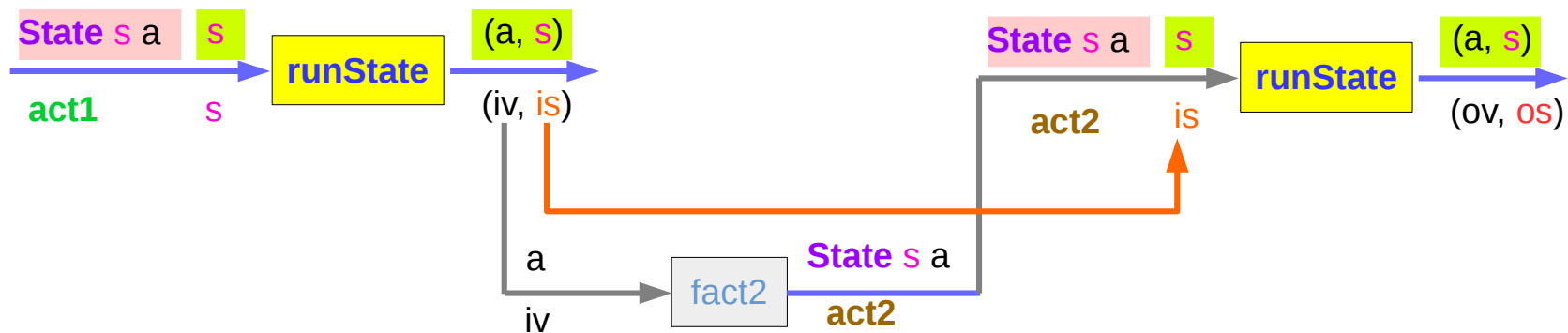
Unwrapped Implementation Examples (3)

$(\gg=) :: \text{State } s \ a \rightarrow (a \rightarrow \text{State } s \ b) \rightarrow \text{State } s \ b$

$(\text{act1} \gg= \text{fact2}) \ s = \text{runState } \text{act2} \ is$

where $(iv, is) = \text{runState } \text{act1} \ s$

$\text{act2} = \text{fact2 } iv$



https://wiki.haskell.org/State_Monad

Function type of `>>=`

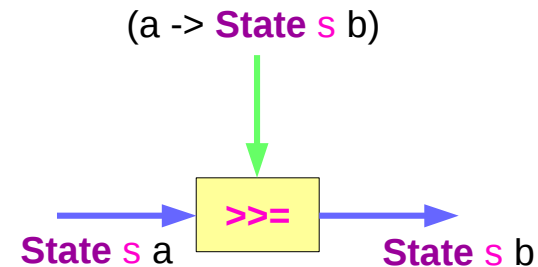
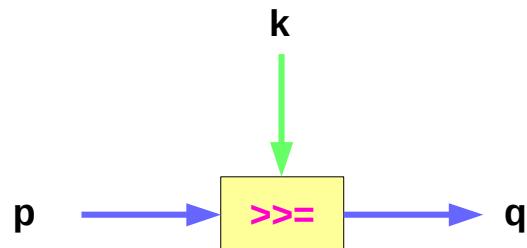
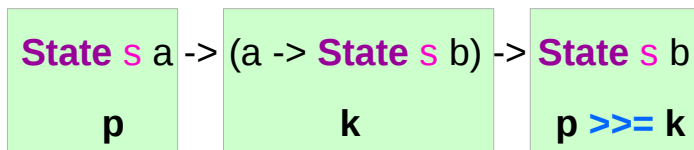
instance Monad (State s) where

`(>>=)` :: State s a -> (a -> State s b) -> State s b

`p >>= k = q` where

`p` :: State s a

`k` :: (a -> State s b)



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

1st and 2nd arguments of `>>=` :

```
instance Monad (State s) where
```

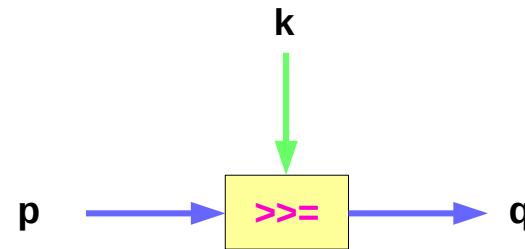
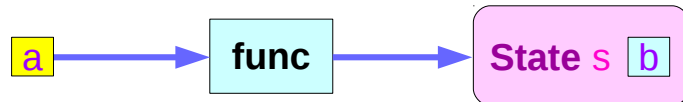
```
(>>=) :: State s a -> (a -> State s b) -> State s b
```

```
p >>= k = q where
```

`p :: State s a`

State s a

`k :: (a -> State s b)`



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Binding operator >>=

```
instance Monad (State s) where
```

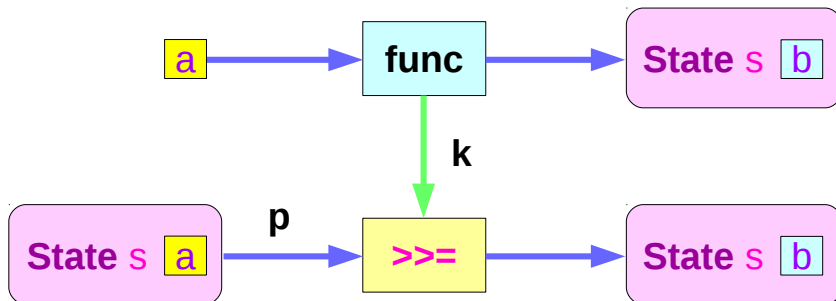
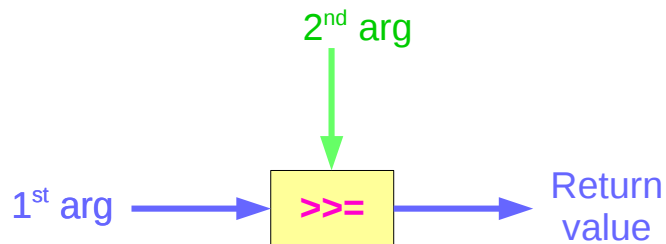
```
(>>=) :: State s a -> (a -> State s b) -> State s b
```

```
p >>= k = q where
```

$p :: \text{State } s \ a$ State Monad value

$k :: (a \rightarrow \text{State } s \ b)$ State Monad returning function

$p \gg= k = q$



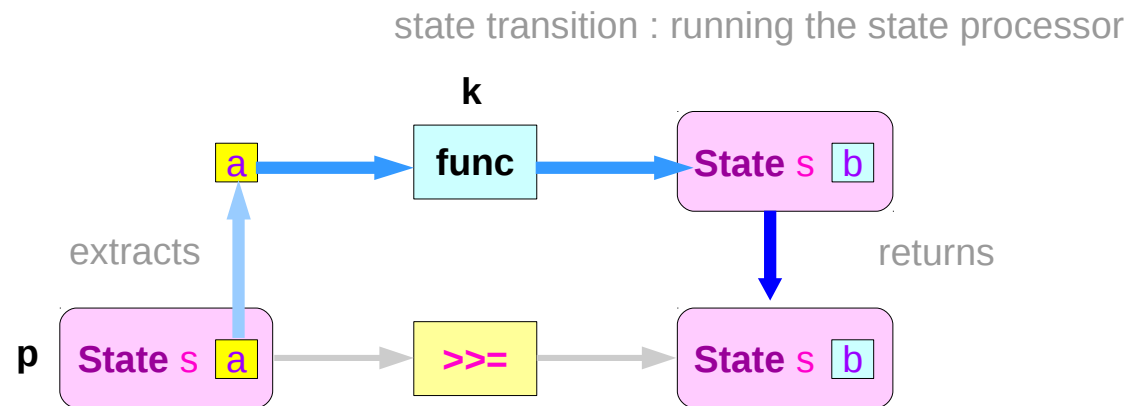
https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Conceptual computation flow of $\gg=$

instance Monad (State s) where

($\gg=$) :: State s a -> (a -> State s b) -> State s b

p $\gg=$ k = q where



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Three Orthogonal Functions

Thinking of extraction : a slightly misleading intuition.

Nothing is being "extracted" from a monad.

The more *fundamental* definition of a monad can be stated by three orthogonal functions:

fmap :: (a -> b) -> (m a -> m b)

return :: a -> m a

join :: m (m a) -> m a

m is a monad.

<https://stackoverflow.com/questions/15016339/haskell-computation-in-a-monad-meaning>

Three Orthogonal Functions and $>>=$

fmap :: (a -> b) -> (m a -> m b)

return :: a -> m a

join :: m (m a) -> m a

```
(a -> b) -> (m a -> m b)
(a -> m b) -> (m a -> m (m b))
(a -> m b) -> (m a -> m b)
```

how to implement ($>>=$) with these:

starting with arguments of type m a and a -> m b,

your only option is using **fmap** to get something of type m (m b),

(a -> m b) -> (m a -> m (m b))

after which you can use **join** to flatten the nested "layers" to get just m b.

(a -> m b) -> (m a -> m b)

<https://stackoverflow.com/questions/15016339/haskell-computation-in-a-monad-meaning>

Monad Law

```
(a -> b) -> (m a -> m b)
(a -> m b) -> (m a -> m (m b))
(a -> m b) -> (m a -> m b)
```

join :: m (m a) -> m a

nothing is being taken "out" of the monad

as the computation going *deeper* into the monad,

with successive steps being *collapsed* into a single layer of the monad.

when **join** (m (m a) -> m a) is applied, it doesn't matter

as long as *the nesting order is preserved* (a form of *associativity*) and

that the *monadic layer* introduced by **return** does *nothing* (an *identity* value for **join**).

Left identity	return a >>= f	f a
Right identity	m >>= return	m
Associativity	(m >>= f) >>= g	m >>= (\x -> f x >> g)

<https://stackoverflow.com/questions/15016339/haskell-computation-in-a-monad-meaning>

Applying the state function to **p** and **r**

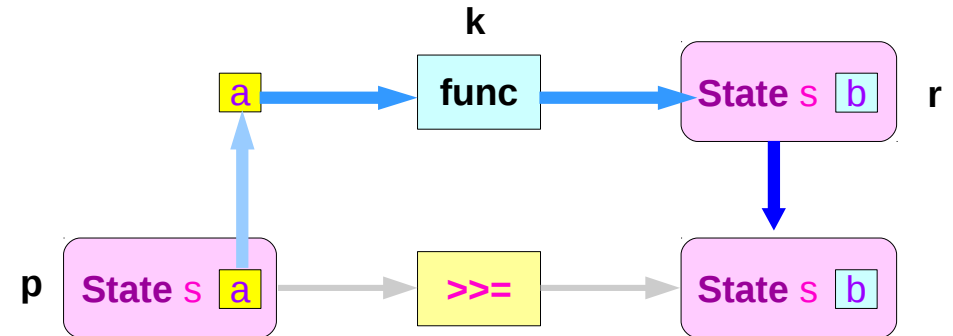
```
instance Monad (State s) where
```

```
(>>=) :: State s a -> (a -> State s b) -> State s b
```

```
p >>= k = q where
```

```
  p' = runState p           -- p' :: s -> (a, s)
```

```
  k' = runState . k         -- k' :: a -> s -> (b, s)
```

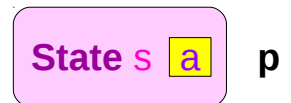


```
newtype State s a = State { runState :: s -> (a, s) }
```

```
state :: (s -> (a, s)) -> State s a
```

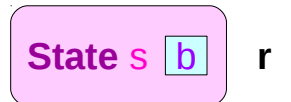
```
p' :: s -> (a, s)
```

```
p' = runState p
```



```
r' :: s -> (b, s)
```

```
r' = runState r
```



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Applying **k** and the state function

```
instance Monad (State s) where
```

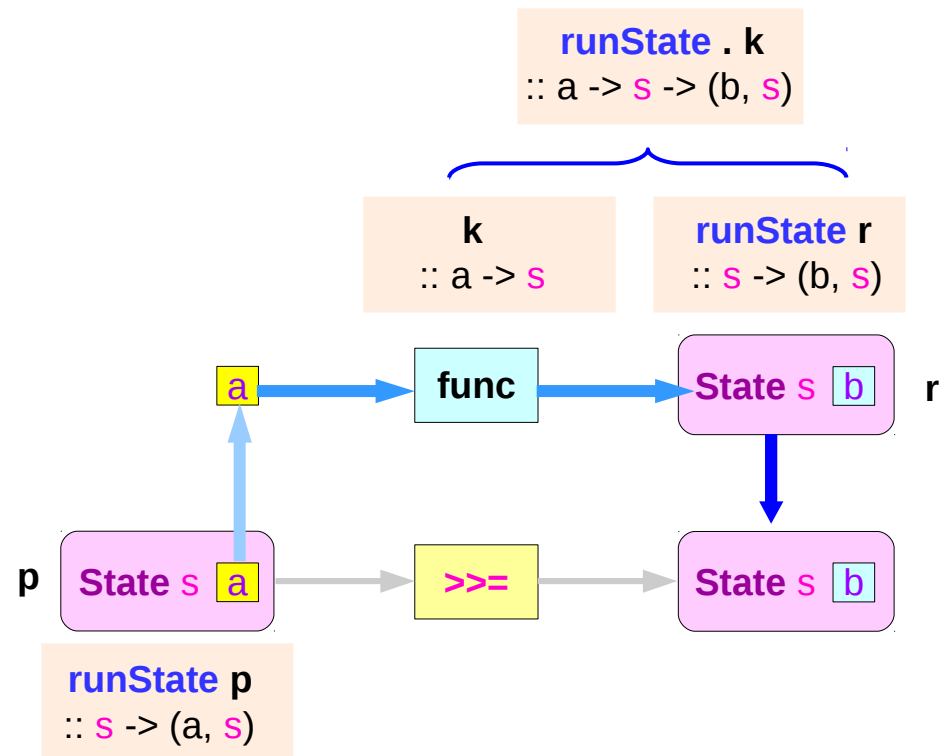
```
(>>=) :: State s a -> (a -> State s b) -> State s b
```

```
p >>= k = q where
```

```
  p' = runState p           -- p' :: s -> (a, s)
  k' = runState . k         -- k' :: a -> s -> (b, s)
```

```
newtype State s a = State { runState :: s -> (a, s) }
```

```
state :: (s -> (a, s)) -> State s a
```



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

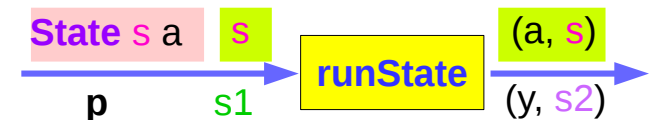
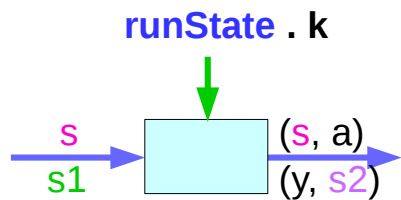
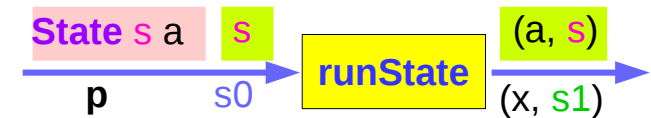
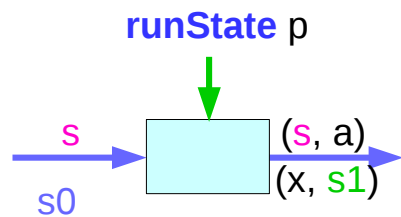
Running the state processor

```
instance Monad (State s) where
```

```
(>>=) :: State s a -> (a -> State s b) -> State s b
```

```
p >>= k = q where
```

```
  p' = runState p           -- p' :: s -> (a, s)
  k' = runState . k         -- k' :: a -> s -> (b, s)
```



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

State Transition

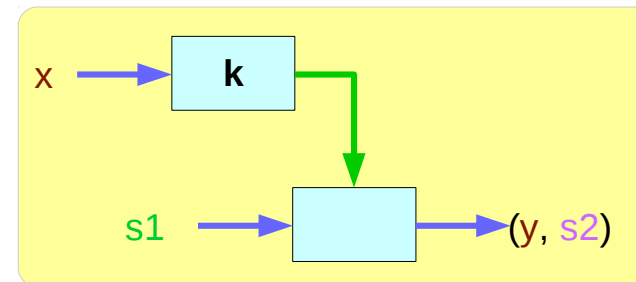
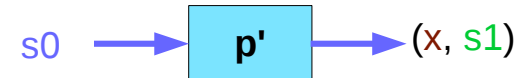
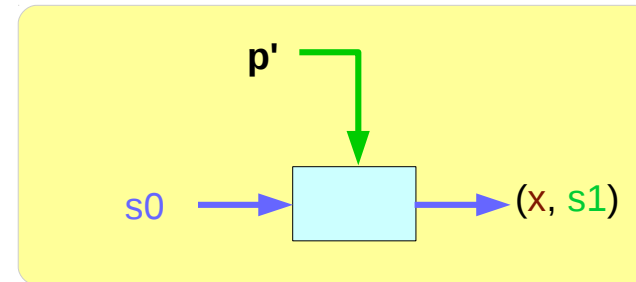
instance Monad (State s) where

(>=>) :: State s a -> (a -> State s b) -> State s b

p >=> k = q where

p' = runState p	-- p' :: s -> (a, s)
k' = runState . k	-- k' :: a -> s -> (b, s)
q' s0 = (y, s2) where	-- q' :: s -> (b, s)
(x, s1) = p' s0	-- (x, s1) :: (a, s)
(y, s2) = k' x s1	-- (y, s2) :: (b, s)
q = state q'	

$((), s0) \longrightarrow (x, s1) \longrightarrow (y, s2)$



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

State Transition from s_0 to s_2

instance Monad (State s) where

$(\gg=) :: \text{State } s \ a \rightarrow (a \rightarrow \text{State } s \ b) \rightarrow \text{State } s \ b$

$p \gg= k = q$ where

$p' = \text{runState } p$ $-- p' :: s \rightarrow (a, s)$

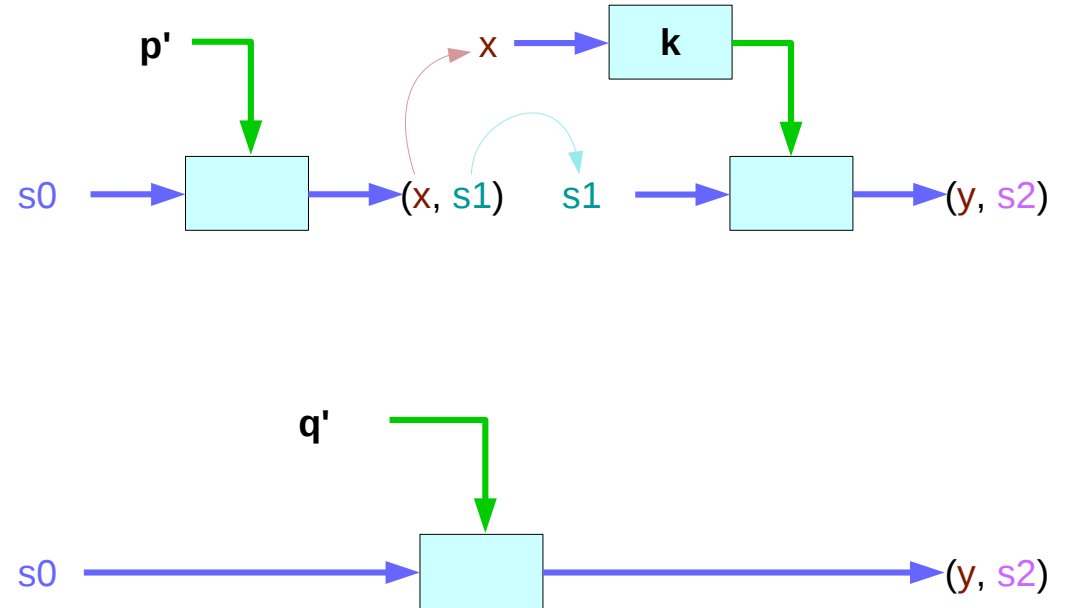
$k' = \text{runState} . k$ $-- k' :: a \rightarrow s \rightarrow (b, s)$

$q' \ s_0 = (y, s_2)$ where $-- q' :: s \rightarrow (b, s)$

$(x, s_1) = p' \ s_0$ $-- (x, s_1) :: (a, s)$

$(y, s_2) = k' \ x \ s_1$ $-- (y, s_2) :: (b, s)$

$q = \text{state } q'$

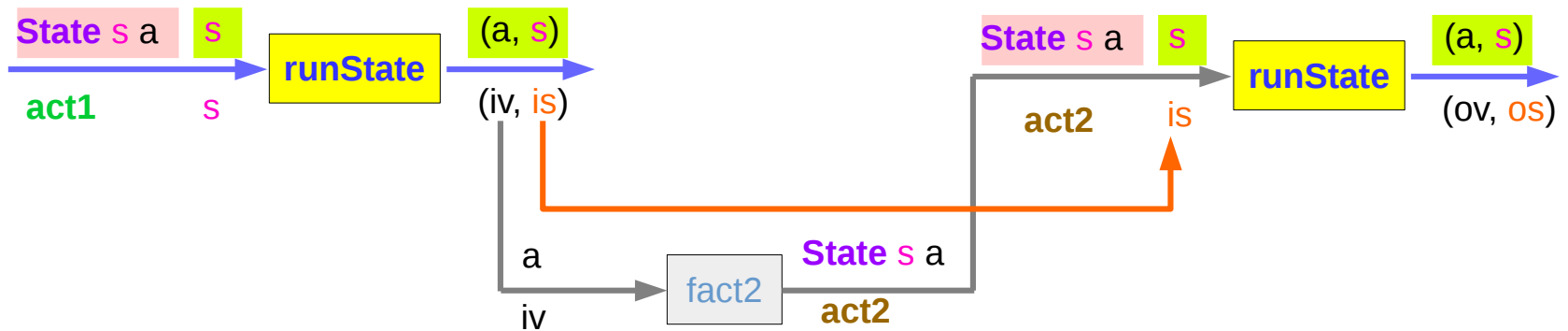
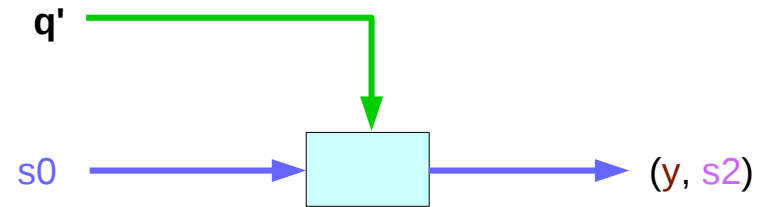
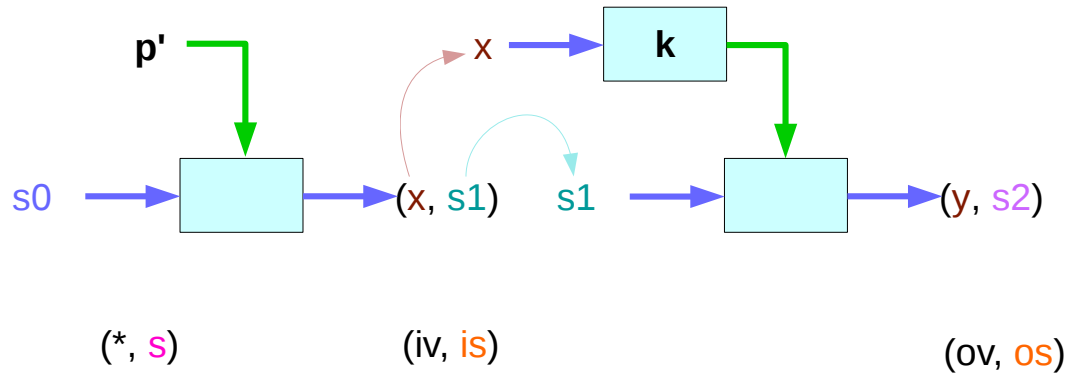


state :: $(s \rightarrow (a, s)) \rightarrow \text{State } s \ a$

newtype State s a = State { runState :: $s \rightarrow (s, a)$ }

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

State Transition from s_0 to s_2



https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Another implementation of $\gg=$

```
instance Monad (State s) where
```

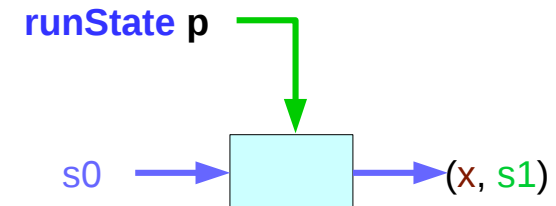
```
(>>=) :: State s a -> (a -> State s b) -> State s b
```

```
p >>= k = state $ \ s0 ->
```

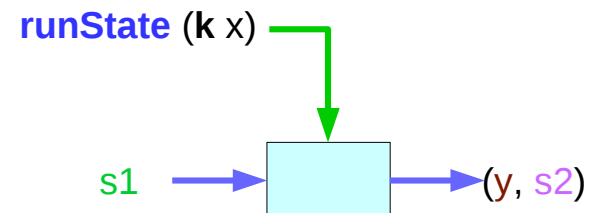
```
  let (x, s1) = runState p s0
```

```
  in runState (k x) s1
```

```
state (\ s0 -> (y, s2))
```



-- running the first processor on $s0$.



-- running the second processor on $s1$.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Examples (1)

```
type ST = State -> State
```

```
type ST a = State -> (a, State)
```

```
Char -> State -> (Int, State)
```

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Examples (1)

```
instance Monad ST where
```

```
-- return :: a -> ST a
```

```
return x = \s -> (x,s)
```

```
-- (>=) :: ST a -> (a -> ST b) -> ST b
```

```
st >= f = \s -> let (x,s') = st s in f x s'
```

>= provides a means of sequencing **state transformers**:

st >= **f** applies the **state transformer st** to an initial state **s**,
then applies the function **f** to the resulting value **x**
to give a second **state transformer (f x)**,
which is then applied to the modified state **s'** to give the final result:

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Maybe Monad

```
class Monad m where
  return :: a -> m a
  (>>=) :: m a -> (a -> m b) -> m b

instance Monad Maybe where
  -- return    :: a -> Maybe a
  return x    = Just x

  -- (>>=)    :: Maybe a -> (a -> Maybe b) -> Maybe b
  Nothing >>= _ = Nothing
  (Just x) >>= f = f x
```

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Maybe Monad

a monad is a parameterised type m
that supports `return` and `>>=` functions of the specified types

m must be a parameterised type, rather than just a type

It is because of this declaration
that the `do` notation can be used to sequence `Maybe` values.

More generally, Haskell supports the use of this notation with any monadic type.

examples of types that are monadic,
the benefits that result from recognising and exploiting this fact.

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

List Monad

The maybe monad provides a simple model of computations that can fail,

a value of type `Maybe a` is either `Nothing` (failure)
the form `Just x` for some `x` of type `a` (success)

The list monad generalises this notion,
by permitting multiple results in the case of success.

More precisely, a value of `[a]` is
either the empty list `[]` (failure)
or the form of a non-empty list `[x1,x2,...,xn]` (success)
for some `xi` of type `a`

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

List Monad

```
instance Monad [] where
```

```
-- return :: a -> [a]
```

```
return x = [x]
```

```
-- (>=>) :: [a] -> (a -> [b]) -> [b]
```

```
xs >=> f = concat (map f xs)
```

return converts a value into a *successful* result containing that value

>=> provides a means of *sequencing* computations
that may produce *multiple results*:

xs >=> **f** applies the function **f** to each of the *results* in the list **xs**
to give a *nested list of results*,
which is then concatenated to give a *single list of results*.

(Aside: in this context, [] denotes the list type [a] without its parameter.)

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

```
instance Monad ST where
```

```
-- return :: a -> ST a
```

```
return x = \s -> (x,s)
```

```
-- (>=>) :: ST a -> (a -> ST b) -> ST b
```

```
st >=> f = \s -> let (x,s') = st s in f x s'
```

Examples (1)

```
pairs :: [a] -> [b] -> [(a,b)]           do  
pairs xs ys = do x <- xs  
                y <- ys  
                return (x, y)
```

this function returns all possible ways of pairing elements from two lists

each possible value x from the list xs, and
each value y from the list ys, and
return the pair (x,y).

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Examples (1)

```
pairs :: [a] -> [b] -> [(a,b)]           do  
pairs xs ys = do x <- xs  
                y <- ys  
                return (x, y)
```

```
pairs xs ys = [(x,y) | x <- xs, y <- ys]   comprehension
```

In fact, there is a formal connection
between the do notation and the comprehension notation.
Both are simply different shorthands
for repeated use of the `>>=` operator for lists.

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Simple Examples (1)

```
(>>) :: Monad m => m a -> m b -> m b;
```

`a1 >> a2` takes the actions `a1` and `a2` and returns the mega action which is `a1-then-a2-returning-the-value-returned-by-a2`.

```
> type State = Int
```

```
> fresh :: ST0 Int
```

```
> fresh = S0 (\n -> (n, n+1))
```

```
> wtf1 = fresh >>
```

```
>   fresh >>
```

```
>   fresh >>
```

```
>   fresh
```

```
ghci> apply0 wtf1 0
```

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Simple Examples (2)

```
return :: a -> ST0 a
```

```
> wtf2 = fresh >>= \n1 ->
```

```
>   fresh >>= \n2 ->
```

```
>   fresh >>
```

```
>   fresh >>
```

```
>   return [n1, n2]
```

```
> wtf2' = do { n1 <- fresh;
```

```
>           n2 <- fresh;
```

```
>           fresh ;
```

```
>           fresh ;
```

```
>           return [n1, n2];
```

```
>           }
```

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Simple Examples (3)

```
ghci> apply0 wtf2 0
```

```
> wtf3 = do n1 <- fresh
```

```
>     fresh
```

```
>     fresh
```

```
>     fresh
```

```
>     return n1
```

<https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html>

Dice Examples

to generate `Int` dice - result : a number between 1 and 6
throw results from a pseudo-random generator of type `StdGen`.

the type of the **state processors** will be

```
State StdGen Int
```

```
StdGen -> (Int, StdGen)
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

randomR

the StdGen type : an instance of **RandomGen**

randomR :: (Random a, RandomGen g) => (a, a) -> g -> (a, g)

assume a is Int and g is StdGen

the type of **randomR**

randomR (1, 6) :: StdGen -> (Int, StdGen)

already have a **state processing function**

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

randomR

```
randomR (1, 6) :: StdGen -> (Int, StdGen)
```

```
rollDie :: State StdGen Int
```

```
rollDie = state $ randomR (1, 6)
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Some Examples (1)

module StateGame where

import Control.Monad.State

```
-- Example use of State monad
-- Passes a string of dictionary {a,b,c}
-- Game is to produce a number from the string.
-- By default the game is off, a C toggles the
-- game on and off. A 'a' gives +1 and a b gives -1.
-- E.g
-- 'ab'   = 0
-- 'ca'   = 1
-- 'cabca' = 0
-- State = game is on or off & current score
--       = (Bool, Int)
```

https://wiki.haskell.org/State_Monad

Some Examples (2)

```
type GameValue = Int
type GameState = (Bool, Int)

playGame :: String -> State GameState GameValue
playGame [] = do
  (_, score) <- get
  return score
```

https://wiki.haskell.org/State_Monad

Some Examples (3)

```
playGame (x:xs) = do
  (on, score) <- get
  case x of
    'a' | on -> put (on, score + 1)
    'b' | on -> put (on, score - 1)
    'c'   -> put (not on, score)
    _     -> put (on, score)
  playGame xs

startState = (False, 0)

main = print $ evalState (playGame "abcaaacbbcabab") startState
```

https://wiki.haskell.org/State_Monad

References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>