Multiple Random Variables

Young W Lim

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

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Statistically independent

Definition

two events A and B are statistically independent iff

$$P(A \cap B) = P(A)P(B)$$

Let $A = \{X \leq x\}$and $B = \{Y \leq y\}$
the two random variables X and Y are statistically independent iff

$$P\{X \leq x, Y \leq y\} = P\{X \leq x\}P\{Y \leq y\}$$

$$F_{X,Y}(x,y) = F_X(x)F_Y(y)$$

$$f_{X,Y}(x,y) = f_X(x)f_Y(y)$$

Conditional distribution function
for 2 random variables X and Y

Definition

the conditional distribution function
of random variables X and Y

$$F_X(x|B) = F_X(x|Y \leq y) = \frac{P\{X \leq x, Y \leq y\}}{P\{Y \leq y\}} = \frac{F_{X,Y}(x,y)}{F_Y(y)}$$

if X and Y are statisitcally independent

$$F_X(x|B) = F_X(x|Y \leq y) = F_X(x)$$

$$F_Y(y|A) = F_Y(y|X \leq x) = F_Y(y)$$

$$f_X(x|B) = f_X(x|Y \leq y) = f_X(x)$$

$$f_Y(y|A) = f_Y(y|X \leq x) = f_Y(y)$$

Statistical independence
of N random variables $X_1, X_2, ..., X_N$ and Y

Definition

events $A_i = \{X_i \leq x_i\}$ $\quad\quad i = 1, 2, ..., N$
where x_i are real numbers
the random variables $X_1, X_2, ..., X_N$
are said to be statistically independent iff

$$P(A_1 \cap A_2 \cap ... \cap A_N) = P(A_1)P(A_2)...P(A_N)$$

Statistical independence
of N random variables $X_1, X_2, ..., X_N$ and Y

Definition

It can be shown that
if $X_1, X_2, ..., X_N$ are statistically independent
then any set that consists of X_i's is independent of any other sets

for example, consider 4 random variables X_1, X_2, X_3, X_4
$X_4$is statistically independent of $X_1 + X_2 + X_3$
$X_3$is statistically independent of $X_1 + X_2$