

Laurent Series and z-Transform

- Geometric Series

Double Pole Examples A

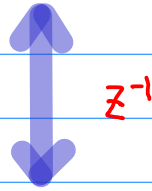
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2 formulas of z

$$\textcircled{1} \quad \frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$



$$\textcircled{2} \quad \frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

$$\frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\xrightarrow{z^{-1}} \frac{-1}{(z^{-1}-1)(z^{-1}-2)} = \left(\frac{1}{z^{-1}-1} - \frac{1}{z^{-1}-2} \right)$$

$$\begin{aligned} \frac{-1}{(z^{-1}-1)(z^{-1}-2)} &= \left(\frac{1}{z^{-1}-1} - \frac{1}{z^{-1}-2} \right) \\ &= \left(\frac{z}{1-z} - \frac{z}{1-2z} \right) \\ &= \left(\frac{-z}{z-1} + \frac{0.5z}{z-0.5} \right) \\ &= z \left(\frac{-1}{z-1} + \frac{0.5}{z-0.5} \right) \\ &= z \left(\frac{-0.5z}{(z-1)(z-0.5)} \right) \\ &= \frac{-0.5z^2}{(z-1)(z-0.5)} \end{aligned}$$

$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

Ⓐ $f(z)$

Ⓑ $X(z)$

①
$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

①-Ⓐ $|z| < 1$ $f(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$ $-|^{n+1} + (\frac{1}{2})^{n+1}$ ($n \geq 0$)

$f(z)$ $|z| > 2$ $f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $+|^{n+1} - (\frac{1}{2})^{n+1}$ ($n < 0$)

①-Ⓑ $|z| < 1$ $X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$ $-|^{n-1} + 2^{n-1}$ ($n < 1$)

$X(z)$ $|z| > 2$ $X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $+|^{n-1} - 2^{n-1}$ ($n \geq 1$)

②
$$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

②-Ⓐ $|z| < 0.5$ $f(z) = +\frac{z}{1-z} - \frac{z}{1-2z}$ $|^{n-1} - 2^{n-1}$ ($n \geq 1$)

$f(z)$ $|z| > 1$ $f(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $-|^{n-1} + 2^{n-1}$ ($n < 1$)

②-Ⓑ $|z| < 0.5$ $X(z) = +\frac{z}{1-z} - \frac{z}{1-2z}$ $+|^{n+1} - (\frac{1}{2})^{n+1}$ ($n < 0$)

$X(z)$ $|z| > 1$ $X(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $-|^{n+1} + (\frac{1}{2})^{n+1}$ ($n \geq 0$)

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
Ⓐ	$ z < p$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$
	$ z > q$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$
Ⓑ	$ z < p$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$
	$ z > q$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$
	$X(z)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$
$ z > q$	$f(z)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$
	$X(z)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$	causal ($n \geq 0$)	causal ($n \geq 1$)
$ z > q$	$f(z)$	anticausal ($n < 0$)	anticausal ($n < 1$)
$ z < p$	$X(z)$	anticausal ($n < 1$)	anticausal ($n < 0$)
$ z > q$	$X(z)$	causal ($n \geq 1$)	causal ($n \geq 0$)

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$	causal ($n \geq 0$)	causal ($n \geq 1$)
$ z < p$	$X(z)$	anticausal ($n < 1$)	anticausal ($n < 0$)
$ z > q$	$f(z)$	anticausal ($n < 0$)	anticausal ($n < 1$)
$ z > q$	$X(z)$	causal ($n \geq 1$)	causal ($n \geq 0$)

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \quad \longleftrightarrow \quad \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

$$\boxed{z} \quad - \frac{1}{| -z |} + \frac{0.5}{| -0.5z |}$$

$$\boxed{z} \quad + \frac{z}{| -z |} - \frac{z}{| -2z |}$$

$$\boxed{|z| < 1} \quad |z| < 1 \quad |0.5z| < 1$$

$$\boxed{|z| < 0.5} \quad |z| < 1 \quad |2z| < 1$$

$$\boxed{z^{-1}} \quad - \frac{z^{-1}}{| -z^{-1} |} - \frac{z^{-1}}{| -2z^{-1} |}$$

$$\boxed{z^{-1}} \quad - \frac{1}{| -z^{-1} |} + \frac{0.5}{| -0.5z^{-1} |}$$

$$\boxed{|z| > 2} \quad |z^{-1}| < 1 \quad |2z^{-1}| < 1$$

$$\boxed{|z| > 1} \quad |z^{-1}| < 1 \quad |0.5z^{-1}| < 1$$

$- \frac{1}{ -z } + \frac{0.5}{ -0.5z }$	$+ \frac{z}{ -z } - \frac{z}{ -2z }$
$\cdot \frac{1}{z} \quad \cdot z \quad \cdot \frac{z}{z} \quad \cdot \frac{z}{z}$	$\cdot \frac{1}{z} \quad \cdot z \quad \cdot \frac{1}{2z} \quad \cdot 2z$
$\frac{z^{-1}}{ -z^{-1} } - \frac{z^{-1}}{ -2z^{-1} }$	$- \frac{1}{ -z^{-1} } + \frac{0.5}{ -0.5z^{-1} }$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)} \right)$$

$$\boxed{z} \quad -\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$\boxed{z} \quad +\frac{z}{1-z} - \frac{z}{1-2z}$$

$$\boxed{|z| < 1} \quad f(z) \text{ causal} \quad (n \geq 0)$$

$$\boxed{|z| < 0.5} \quad f(z) \text{ causal} \quad (n \geq 1)$$

$$\boxed{z^{-1}} \quad \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\boxed{z^{-1}} \quad -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$\boxed{|z| > 2}$$

$$X(z) \text{ causal} \quad (n \geq 1)$$

$$\boxed{|z| > 1}$$

$$X(z) \text{ causal} \quad (n \geq 0)$$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$\left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

$$\boxed{z} - \frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$\boxed{z} + \frac{z}{1-z} - \frac{z}{1-2z}$$

$|z| < 1$ $f(z)$ causal ($n \geq 0$)
 $X(z)$ anticausal ($n \leq 0$)

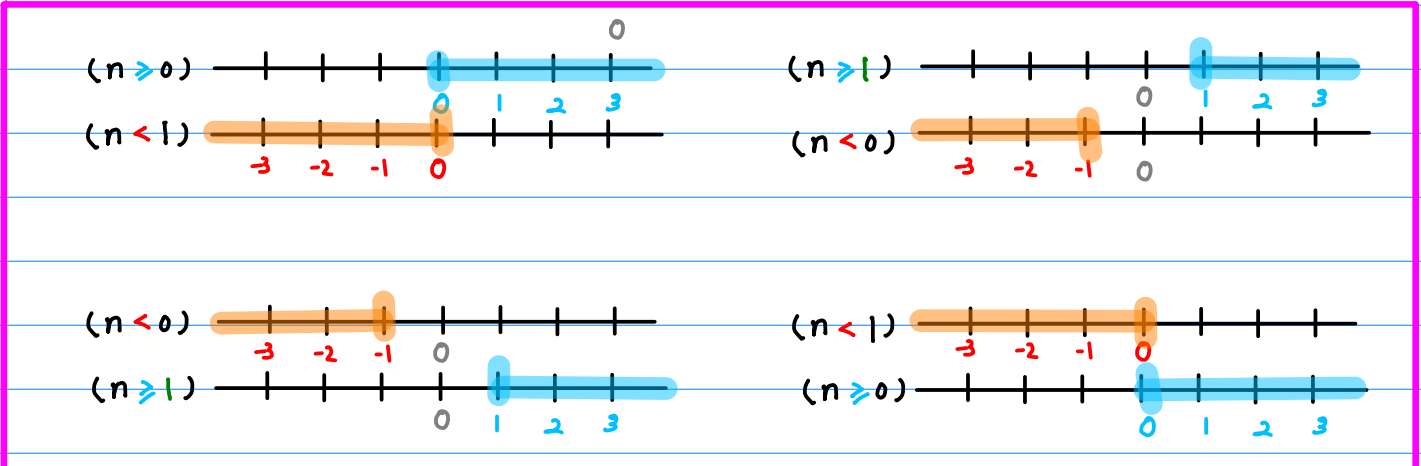
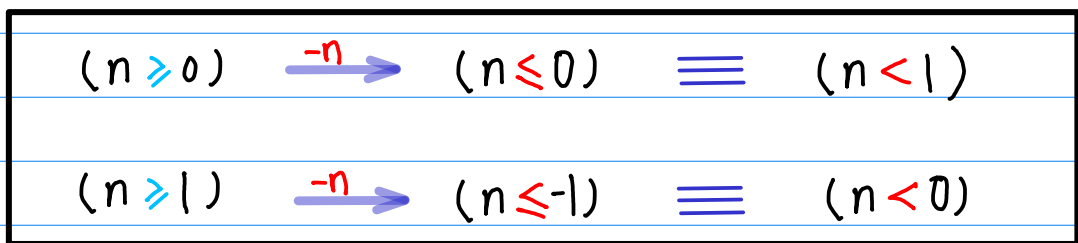
$|z| < 0.5$ $f(z)$ causal ($n \geq 1$)
 $X(z)$ anticausal ($n \leq -1$)

$$\boxed{z^{-1}} - \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\boxed{z^{-1}} - \frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$|z| > 2$ $f(z)$ anticausal ($n \leq -1$)
 $X(z)$ causal ($n \geq 1$)

$|z| > 1$ $f(z)$ anticausal ($n \leq 0$)
 $X(z)$ causal ($n \geq 0$)



$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$|z| < 1$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^0 + 1^1 z^1 + 1^2 z^2 + \dots\right] - 1^{n+1} \\ + \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

$$|z| < 0.5$$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +\left[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots\right] + 1^{n+1} \\ - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

$$|z| > 2$$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \dots\right] + 1^{n+1} \\ - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots\right] - 2^{n+1}$$

$$* n = \quad 1 \quad 2 \quad 3$$

$$|z| > 1$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$X(z) = -\left[1^1 z^0 + 1^2 z^1 + 1^3 z^2 + \dots\right] - 1^{n+1} \\ + \left[\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots\right] + \left(\frac{1}{2}\right)^{n+1}$$

$$* n = \quad 0 \quad 1 \quad 2$$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -[1^0 + 1^1 z^1 + 1^2 z^2 + \dots] - 1^{n+1} + [(\frac{1}{2})^0 + (\frac{1}{2})^1 z^1 + (\frac{1}{2})^2 z^2 + \dots] + (\frac{1}{2})^{n+1}$$

$$\begin{aligned} 2 &= (\frac{1}{2})^{-1} \\ (\frac{1}{2}) &= 2^{-1} \end{aligned}$$

$$X(z) = -\left[\binom{-1}{n} + \binom{-1}{n-1} z^1 + \binom{-1}{n-2} z^2 + \dots \right] - 1^{n+1} + \left[\binom{-1}{n} + \binom{-1}{n-1} z^1 + \binom{-1}{n-2} z^2 + \dots \right] + 2^{n+1}$$

$n = \quad 0 \quad -1 \quad -2$

$|z| < 0.5$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots] + 1^{n+1} - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots] - 2^{n+1}$$

$$X(z) = +\left[\binom{-1}{n} z^1 + \binom{-1}{n-1} z^2 + \binom{-1}{n-2} z^3 + \dots \right] + 1^{n+1} - \left[\binom{-1}{n} z^1 + \binom{-1}{n-1} z^2 + \binom{-1}{n-2} z^3 + \dots \right] - (\frac{1}{2})^{n+1}$$

$n = \quad -1 \quad -2 \quad -3$

$|z| > 2$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$\begin{aligned} 2 &= (\frac{1}{2})^{-1} \\ (\frac{1}{2}) &= 2^{-1} \end{aligned}$$

$$f(z) = +\left[\binom{-1}{n} z^{-1} + \binom{-1}{n-1} z^{-2} + \binom{-1}{n-2} z^{-3} + \dots \right] + 1^{n+1} - \left[\binom{-1}{n} z^{-1} + \binom{-1}{n-1} z^{-2} + \binom{-1}{n-2} z^{-3} + \dots \right] - (\frac{1}{2})^{n+1}$$

$$X(z) = +\left[\binom{-1}{n} z^{-1} + \binom{-1}{n-1} z^{-2} + \binom{-1}{n-2} z^{-3} + \dots \right] + 1^{n+1} - \left[\binom{-1}{n} z^{-1} + \binom{-1}{n-1} z^{-2} + \binom{-1}{n-2} z^{-3} + \dots \right] - 2^{n+1}$$

$n = \quad 1 \quad 2 \quad 3$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\binom{-1}{n} z^0 + \binom{-1}{n-1} z^1 + \binom{-1}{n-2} z^2 + \dots \right] - 1^{n+1} + \left[2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \dots \right] + 2^{n+1}$$

$$X(z) = -\left[\binom{-1}{n} z^0 + \binom{-1}{n-1} z^1 + \binom{-1}{n-2} z^2 + \dots \right] - 1^{n+1} + \left[\binom{-1}{n} z^0 + \binom{-1}{n-1} z^1 + \binom{-1}{n-2} z^2 + \dots \right] + (\frac{1}{2})^{n+1}$$

$n = \quad 0 \quad 1 \quad 2$

$$\textcircled{1} \frac{-1}{(z-1)(z-2)} \xleftrightarrow{z^{-1}} \textcircled{2} \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$|z| < 1$$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -[1^0 + 1^1 z^1 + 1^2 z^2 + \dots] + \left[\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right]$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$X(z) = -\left[\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-1} + \left(\frac{1}{2}\right)^{-3} z^{-2} + \dots\right] + [2^0 + 2^1 z^1 + 2^2 z^2 + \dots]$$

$$a_n = -1^{n+1} + 2^{n+1} \quad (n < 1)$$

$$|z| < 0.5$$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$$f(z) = +[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots]$$

$$a_n = +1^{n+1} - 2^{n+1} \quad (n \geq 1)$$

$$X(z) = +\left[\left(\frac{1}{2}\right)^{-1} z^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-2} + \left(\frac{1}{2}\right)^{-3} z^{-3} + \dots\right] - \left[\left(\frac{1}{2}\right)^0 z^1 + \left(\frac{1}{2}\right)^1 z^2 + \left(\frac{1}{2}\right)^2 z^3 + \dots\right]$$

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$|z| > 2$$

$$\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right] - \left[\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \dots\right]$$

$$a_n = +1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$X(z) = +[1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \dots] - [2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots]$$

$$a_n = +1^{n+1} - 2^{n+1} \quad (n \geq 1)$$

$$|z| > 1$$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right] + [2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \dots]$$

$$a_n = -1^{n+1} + 2^{n+1} \quad (n < 1)$$

$$X(z) = -[1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \dots] + \left[\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots\right]$$

$$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

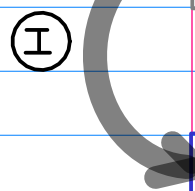
$$\textcircled{\text{III}} = \textcircled{\text{I}} + \textcircled{\text{II}}$$

$\textcircled{\text{III}}$ $\text{ROC}(z)$ $f(z^{-1})$ \longleftrightarrow $-a_{-n}$ $\ll \text{RNG}(n) \gg$ $\textcircled{\text{I}} + \textcircled{\text{II}}$

$|z| < p$ $n \geq 1$

$\text{ROC}(z)$ $f(z)$ \longleftrightarrow a_n $\text{RNG}(n)$

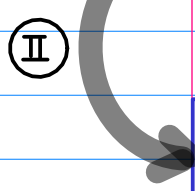
$|z| < p$ $n \geq 0$



$\text{ROC}(z^{-1})$ $f(z)$ \longleftrightarrow $-a_n$ $\overline{\text{RNG}(n)}$

$|z| > \frac{1}{p}$ $n < 0$

$n \leq -1$



$\text{ROC}(z)$ $f(z^{-1})$ \longleftrightarrow $-a_{-n}$ $\overline{\text{RNG}(-n)}$

$|z| < p$ $n \geq 1$

Compare ① with ④

$$\begin{array}{cccc} \text{ROC}(z) & f(z) & \longleftrightarrow & a_n \\ |z| < p & & & n \geq 0 \end{array}$$

①	$\text{ROC}(z^{-1})$ $ z > \frac{1}{p}$	$f(z)$	\longleftrightarrow	$-a_n$	$\overline{\text{RNG}(n)}$ $n < 0$
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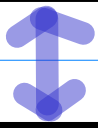

- | - n
complement



④	$\text{ROC}(z)$	$X(z)$	\longleftrightarrow	a_{-n}	$\text{RNG}(-n)$ $n < 1$
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- n - n
symmetrical

Ⓘ

$ROC(z^{-1})$	$f(z)$	$\longleftrightarrow -a_n$	$RNG(n)$
$ z > \frac{1}{p}$			$n < 0$

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
z^{-1}	$ z < p$	$f(z)$ $-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$	$f(z)$ $+1^{n-1} - 2^{n-1} \quad (n \geq 1)$
			
	$ z > q$	$f(z)$ $+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$	$f(z)$ $-1^{n-1} + 2^{n-1} \quad (n < 1)$

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
z^{-1}	$ z < p$	$X(z)$ $-1^{n-1} + 2^{n-1} \quad (n < 1)$	$X(z)$ $+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$
			
	$ z > q$	$X(z)$ $+1^{n-1} - 2^{n-1} \quad (n \geq 1)$	$X(z)$ $-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$

II

ROC(z^{-1})
 $|z| > \frac{1}{p}$

$f(z^{-1})$

a_{-n}

RNG($-n$)
 $n < 1$

		z^{-1}	
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$
		$-n$	$-n$
$ z > q$	$f(z)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$
		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$

		z^{-1}	
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$X(z)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$
		$-n$	$-n$
$ z > q$	$X(z)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$
		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$

$$2^{-n+1} = (\frac{1}{2})^n \cdot 2 = (\frac{1}{2})^{n-1} \quad (\frac{1}{2})^{-n-1} = 2^n \cdot 2 = 2^{n+1}$$

$$(\frac{1}{2})^{-n+1} = 2^n \cdot \frac{1}{2} = 2^{n-1} \quad 2^{-n-1} = (\frac{1}{2})^n \cdot \frac{1}{2} = (\frac{1}{2})^{n+1}$$

III

$ROC(z)$ $f(z^{-1})$ \longleftrightarrow $-a^{-n}$ $\overline{RNG}(-n)$
 $|z| < p$ $n \geq 1$



I + II



			$\xleftrightarrow{z^{-1}}$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$
		$\xleftrightarrow{-n, -1}$	$\xleftrightarrow{-n, -1}$
$ z > q$	$f(z)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$
		$\xleftrightarrow{-n, -1}$	$\xleftrightarrow{-n, -1}$
		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$

			$\xleftrightarrow{z^{-1}}$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$		$\xleftrightarrow{-n, -1}$	$\xleftrightarrow{-n, -1}$
	$X(z)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$
$ z > q$		$\xleftrightarrow{-n, -1}$	$\xleftrightarrow{-n, -1}$
	$X(z)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$
		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$

IV

$ROC(z)$	$X(z)$	\longleftrightarrow	a_{-n}	$RNG(-n)$
$ z < p$				$n < 1$

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$	$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$	
	$X(z)$	$-1^{n-1} + 2^{n-1} \quad (n < 1)$	
$ z > q$	$f(z)$	$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$	
	$X(z)$	$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$	

		$(p, q) = (1, 2)$	$(p, q) = (0.5, 1)$
		① $\frac{-1}{(z-1)(z-2)}$	② $\frac{-0.5z^2}{(z-1)(z-0.5)}$
$ z < p$	$f(z)$		$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$
	$X(z)$		$+1^{n+1} - (\frac{1}{2})^{n+1} \quad (n < 0)$
$ z > q$	$f(z)$		$-1^{n-1} + 2^{n-1} \quad (n < 1)$
	$X(z)$		$-1^{n+1} + (\frac{1}{2})^{n+1} \quad (n \geq 0)$

$$X(z) \quad |z| < 0.5 \quad |z| > 2$$

anticausal causal

$$\textcircled{1} - \textcircled{B} \quad \frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2} \right)$$

$$|z| < 0.5 \quad X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad \boxed{-1^{n-1} + 2^{n-1}} \quad (n < 1)$$

$$-\left(|^1 z^0 + |^2 z^1 + |^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots \right)$$

$$-\left(|^1 z^0 + |^2 z^1 + |^3 z^2 + \dots \right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$$|z| > 2 \quad X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{+1^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

$$+\left(|^0 z^1 + |^1 z^2 + |^2 z^3 + \dots \right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots \right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\textcircled{2} - \textcircled{B} \quad \frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{z-1} + \frac{0.5z}{z-0.5} \right)$$

$$|z| < 0.5 \quad X(z) = +\frac{z}{1-z} - \frac{z}{1-2z} \quad \boxed{+1^{n+1} - \left(\frac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$+\left(|^0 z^1 + |^1 z^2 + |^2 z^3 + \dots \right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \dots \right)$$

$$+\left(|^0 z + |^1 z^2 + |^2 z^3 + \dots \right) - \left(2^0 z + 2^1 z^2 + 2^2 z^3 + \dots \right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$$|z| > 2 \quad X(z) = -\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad \boxed{-1^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$-\left(|^1 z^0 + |^2 z^1 + |^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)^0 z^0 + \left(\frac{1}{2}\right)^1 z^1 + \left(\frac{1}{2}\right)^2 z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

Ⓘ

$ROC(z^{-1})$	$f(z)$	\longleftrightarrow	$-a_n$	$\overline{RNG}(n)$
$ z > \frac{1}{p}$				$n < 0$

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$|z| < 0.5$ $X(z)$

$a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)$

$|z| > 2$ $X(z)$

$b_n = \left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)$

$\{ |z| < 0.5 \} \cap \{ |z| > 2 \} = \emptyset \quad \longrightarrow \quad a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z| < a$

$X(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$

$a^{n+1} \quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

ROC'

$|z| > a^{-1}$

$X(z) = -\frac{z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$
 $= -\sum_{k=-1}^{\infty} a^{k+1} z^k$

$a^{n+1} \quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$\frac{a}{1-az}$	=	$\sum_{n=0}^{\infty} a^{n+1} z^n$	$\frac{z}{1-az}$	=	$\sum_{n=1}^{\infty} a^{n-1} z^n$
$\frac{z^{-1}}{1-a^{-1}z^{-1}}$	=	$-\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$	$\frac{a^{-1}}{1-a^{-1}z^{-1}}$	=	$-\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$
		$= -\sum_{k=-1}^{\infty} a^{k+1} z^k$			$= -\sum_{k=0}^{\infty} a^{k-1} z^k$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^n$$

$$-\frac{z^{-1}}{1-a^1 z^1} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$$

$$= -\sum_{k=-1}^{\infty} a^{k+1} z^k$$

$$\frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$-\frac{a^{-1}}{1-a^1 z^1} \Rightarrow -\sum_{n=0}^{\infty} a^{-n+1} z^{-n}$$

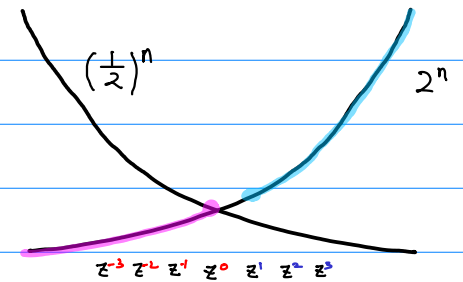
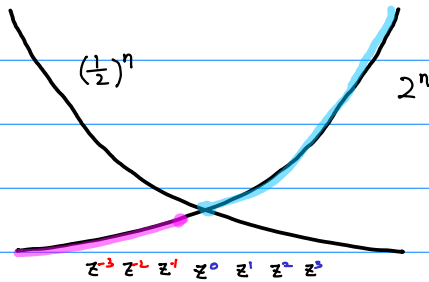
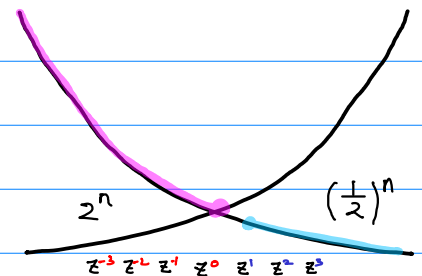
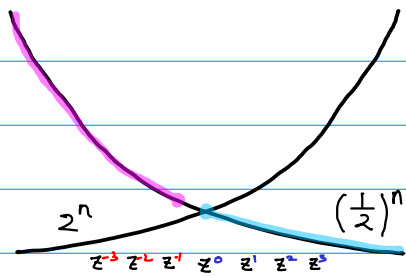
$$= -\sum_{k=0}^{\infty} a^{k-1} z^k$$

$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \dots$$

$$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \dots$$

$$z + a z^2 + a^2 z^3 + a^3 z^4 + \dots$$

$$a^{-1} + a^{-2} z^1 + a^{-3} z^2 + a^{-4} z^3 + \dots$$



IV

$\text{ROC}(z)$ $ z < p$	$X(z)$	\longleftrightarrow	a_{-n}	$\text{RNG}(n)$ $n \leq 0$
------------------------------	--------	-----------------------	----------	-------------------------------

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$$|z| < 0.5 \quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$



$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots \right)$$

$n=0 \quad n=1 \quad n=2$
 $n=0 \quad n=1 \quad n=2$

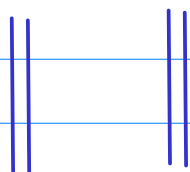
$$|z| < 0.5 \quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}} \quad (n \leq 0)$$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \dots \right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots \right)$$

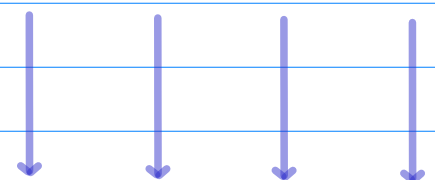
$$-\left(\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \dots \right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \dots \right)$$

$n=0 \quad n=-1 \quad n=-2$
 $n=0 \quad n=-1 \quad n=-2$

$$\text{ROC} \quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \quad a^{n+1} \quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$



$$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{n+1} z^n$$



$$\text{ROC} \quad X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{a}\right)^{k+1} z^{-k}$$

$$a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$$

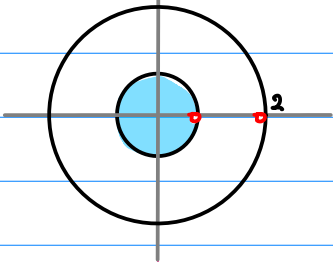
$$n \leq 0 \quad n \leq 1 \quad n > 0 \quad n > 1$$

ROC	$f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$	a^{n+1}	$n \geq 0$	$n \geq 1$	$n < 0$	$n < 1$
$\uparrow z^{-1}$	$\uparrow z^{-1}$	$\downarrow -n$	\downarrow	\downarrow	\downarrow	\downarrow
ROC	$X(z) = \sum_{k=0}^{\infty} (a)^{k+1} z^{-k}$	$(\frac{1}{a})^{-n+1}$	$n < 0$	$n < 1$	$n \geq 0$	$n \geq 1$
		$= a^{-n+1}$				

Causal $f(z)$ $X(z)$
 $|z| < 0.5$ $|z| > 2$

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

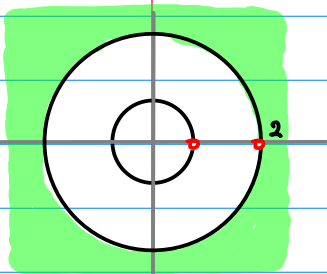
$|z| < 2$ $|z| < 0.5$



$$f(z) = (-2) \frac{0.5}{0.5-z} + (0.5) \frac{2}{2-z} \quad (|z| < 0.5)$$

$$a_n = (-2) \begin{matrix} \downarrow \\ 2^n \\ -2^{n+1} \end{matrix} + (0.5) \begin{matrix} \downarrow \\ (\frac{1}{2})^n \\ (\frac{1}{2})^{n+1} \end{matrix} \quad (n \geq 0)$$

$$\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2} \right) \quad |z| > 2$$



$$X(z) = 0.5 \frac{z}{z-0.5} - 2 \frac{z}{z-2} \quad (|z| > 2)$$

$|z| > 2$ $|z| > 0.5$

$$a_n = (0.5) \begin{matrix} \downarrow \\ (\frac{1}{2})^n \\ (\frac{1}{2})^{n+1} \end{matrix} - 2 \cdot \begin{matrix} \downarrow \\ 2^n \\ 2^{n+1} \end{matrix} \quad (n \geq 0)$$

Anti-causal

$$f(z)$$

$$|z| > 2$$

$$X(z)$$

$$|z| < 0.5$$

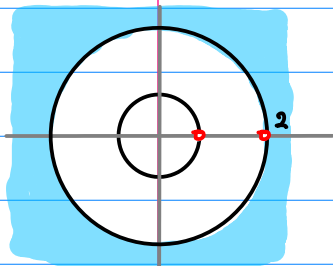
$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right)$$

$$|z| > 2$$

$$|z| > 0.5$$

$$f(z) = (-2) \frac{-0.5}{0.5-z} + (0.5) \frac{-2}{2-z} \quad (|z| > 0.5)$$

$$a_n = (+2) 2^n - (0.5) \left(\frac{1}{2}\right)^n \quad (n < 0)$$
$$+ 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$$



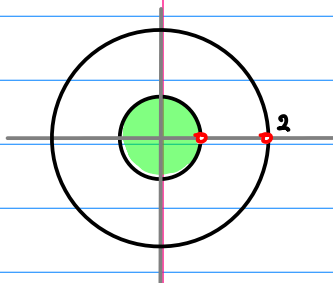
$$\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{z-0.5} - \frac{2z}{z-2} \right)$$

$$|z| < 2$$

$$|z| < 0.5$$

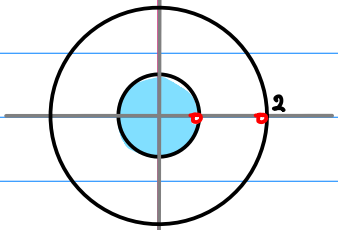
$$X(z) = 0.5 \frac{-z}{z-0.5} - 2 \frac{-z}{z-2} \quad (|z| < 2)$$

$$a_n = -(0.5) \left(\frac{1}{2}\right)^n + 2 \cdot 2^n \quad (n < 0)$$
$$- \left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$$



①-Ⓐ $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)} \quad |z| < 0.5 \quad |z| > 2$
causal anticausal

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



$|z| < 0.5$

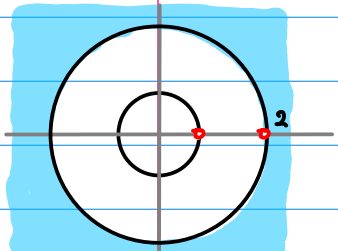
$$f(z) = \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})}$$

$$= -\sum_{n=0}^{\infty} (2)^{n+1} (z)^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} (z)^n$$

$$= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n$$

$(n \geq 0) \quad a_n = -2^{n+1} + (\frac{1}{2})^{n+1}$

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

$$f(z) = \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{z}{2})} \neq$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^n (\frac{1}{z})^{n+1} - \sum_{n=0}^{\infty} (2)^n (\frac{1}{z})^{n+1}$$

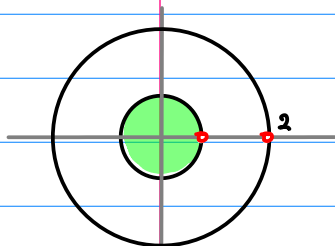
$$= \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n}$$

$$= \sum_{n=-1}^{-\infty} (2)^{n+1} z^n - \sum_{n=-1}^{-\infty} (\frac{1}{2})^{n+1} z^n$$

$(n < 0) \quad a_n = 2^{n+1} - (\frac{1}{2})^{n+1}$

① - ③ $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$ $|z| < 0.5$ $|z| > 2$
anticausal *causal*

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

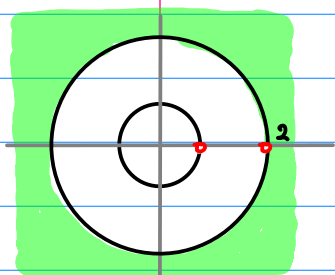


$|z| < 0.5$

$$\begin{aligned} X(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} (z)^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} (z)^n \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^n \\ &= -\sum_{n=0}^{-\infty} (\frac{1}{2})^{n-1} z^{-n} + \sum_{n=0}^{-\infty} (2)^{n-1} z^{-n} \end{aligned}$$

$$(n \leq 0) \quad a_n = -(\frac{1}{2})^{n-1} + 2^{n-1}$$

$$\frac{3}{2} \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2} \right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



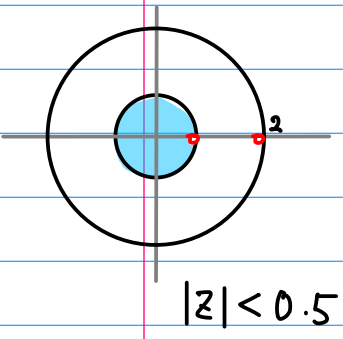
$|z| > 2$

$$\begin{aligned} X(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{z}{2})} \neq \\ &= \sum_{n=0}^{\infty} (\frac{1}{2})^n (\frac{1}{z})^{n+1} - \sum_{n=0}^{\infty} (2)^n (\frac{1}{z})^{n+1} \\ &= \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \end{aligned}$$

$$(n > 0) \quad a_n = (\frac{1}{2})^{n-1} - (2)^{n-1}$$

② - Ⓐ $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $|z| < 0.5$ *causal* $|z| > 2$ *anticausal*

$$\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



$$f(z) = -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \neq$$

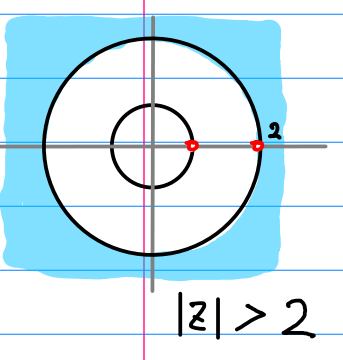
$$= -\sum_{n=0}^{\infty} (2)^n (z)^{n+1} + \sum_{n=0}^{\infty} (\frac{1}{2})^n (z)^{n+1}$$

$$= -\sum_{n=1}^{\infty} (2)^{n-1} z^n + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n$$

↓ ↓

$$(n > 0) \quad a_n = -2^{n-1} + (\frac{1}{2})^{n-1}$$

$$\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$$f(z) = \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{z}{2})}$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} (\frac{1}{z})^n - \sum_{n=0}^{\infty} (2)^{n+1} (\frac{1}{z})^n$$

$$= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} - \sum_{n=0}^{\infty} (2)^{n+1} z^{-n}$$

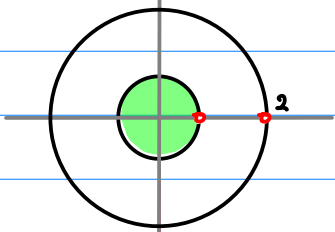
$$= \sum_{n=0}^{-\infty} (2)^{n-1} z^n - \sum_{n=0}^{-\infty} (\frac{1}{2})^{n-1} z^n$$

↓ ↓

$$(n \leq 0) \quad a_n = 2^{n-1} - (\frac{1}{2})^{n-1}$$

② - B $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ $|z| > 2$
anticausal *causal*

$$\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

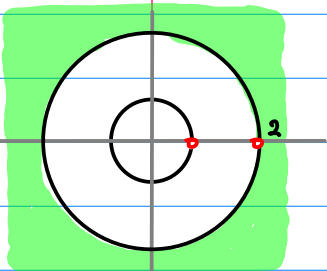


$|z| < 0.5$

$$\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (2)^n (z)^{n+1} + \sum_{n=0}^{\infty} (\frac{1}{2})^n (z)^{n+1} \\ &= -\sum_{n=1}^{\infty} (2)^{n-1} z^n + \sum_{n=1}^{\infty} (\frac{1}{2})^{n-1} z^n \\ &= -\sum_{n=-1}^{\infty} (\frac{1}{2})^{n+1} z^{-n} + \sum_{n=-1}^{\infty} (2)^{n+1} z^{-n} \end{aligned} \neq$$

$(n < 0)$ $a_n = -(\frac{1}{2})^{n+1} + 2^{n+1}$

$$\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)} \right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\begin{aligned} X(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{z}{2})} \\ &= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} (\frac{1}{z})^n - \sum_{n=0}^{\infty} (2)^{n+1} (\frac{z}{2})^n \\ &= \sum_{n=0}^{\infty} (\frac{1}{2})^{n+1} z^{-n} - \sum_{n=0}^{\infty} (2)^{n+1} z^{-n} \end{aligned}$$

$(n \geq 0)$ $a_n = (\frac{1}{2})^{n+1} - 2^{n+1}$

