Sec.12

Example of non-pure bending

- Bending due to both bending moment and shear
- Cantilever beam, combined loading
  - Transverse tip force
  - Inclined force
    - Superposition
  - Axial force
  - Torsional load

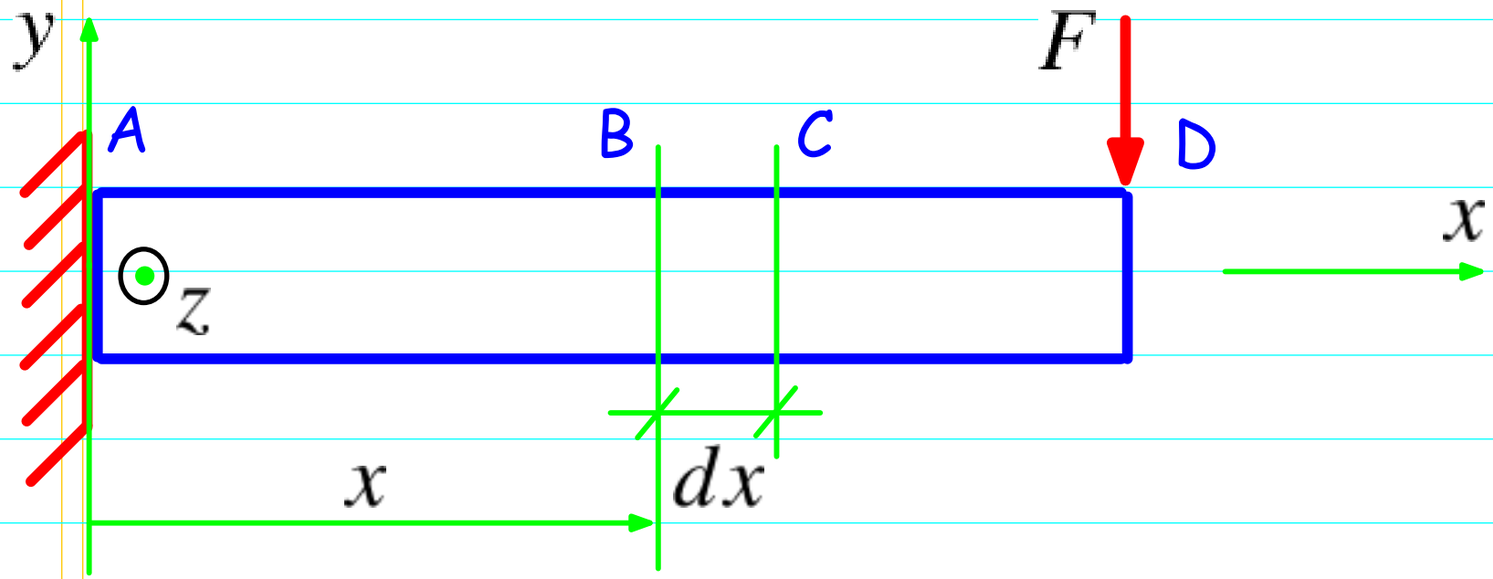
Two sign conventions for shear and bending moment

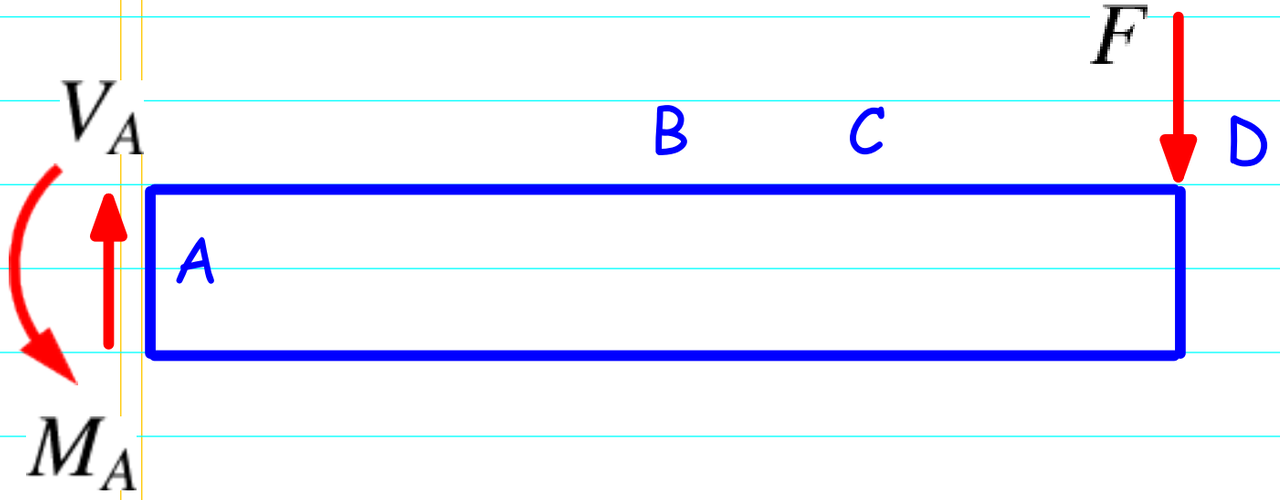
- The "up" convention
- The "down" convention

Pure bending, 4-point bending

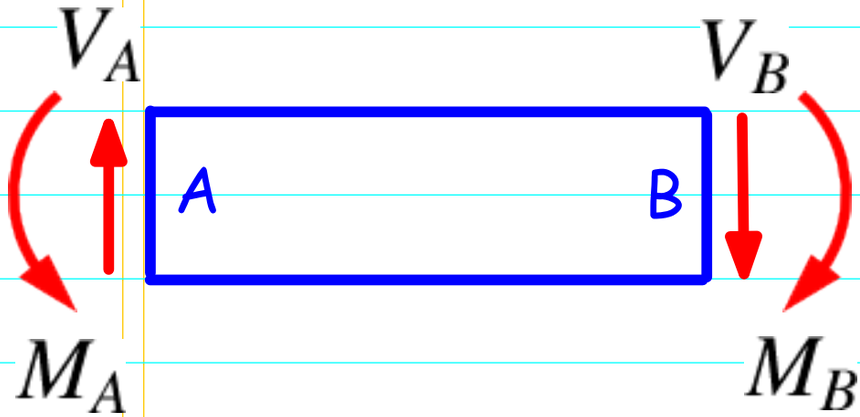
- Shear diagram
  - The "up" convention
  - The "down" convention
- Bending moment diagram
  - The "up" convention
  - The "down" convention

Cantilever beam under transverse tip force





FBD 1



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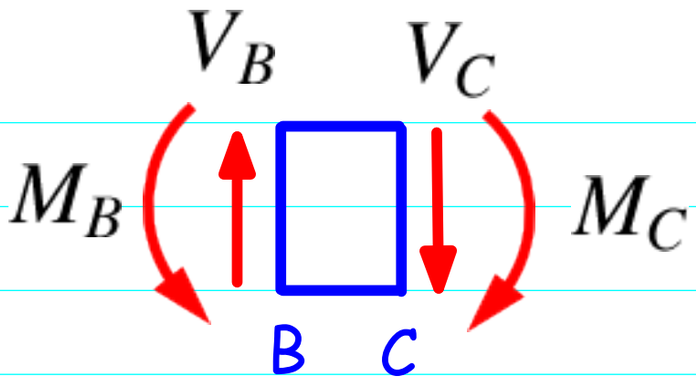
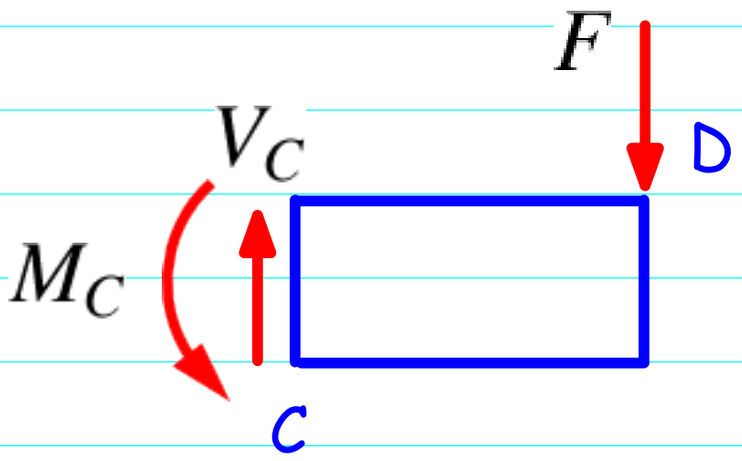
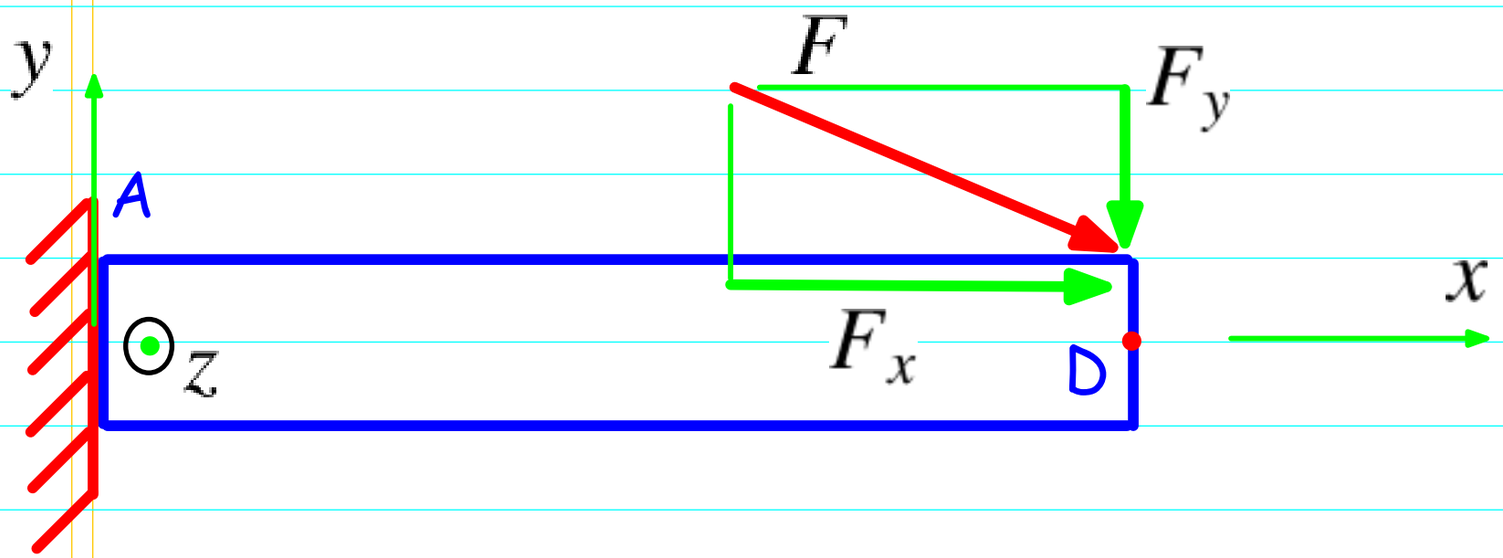


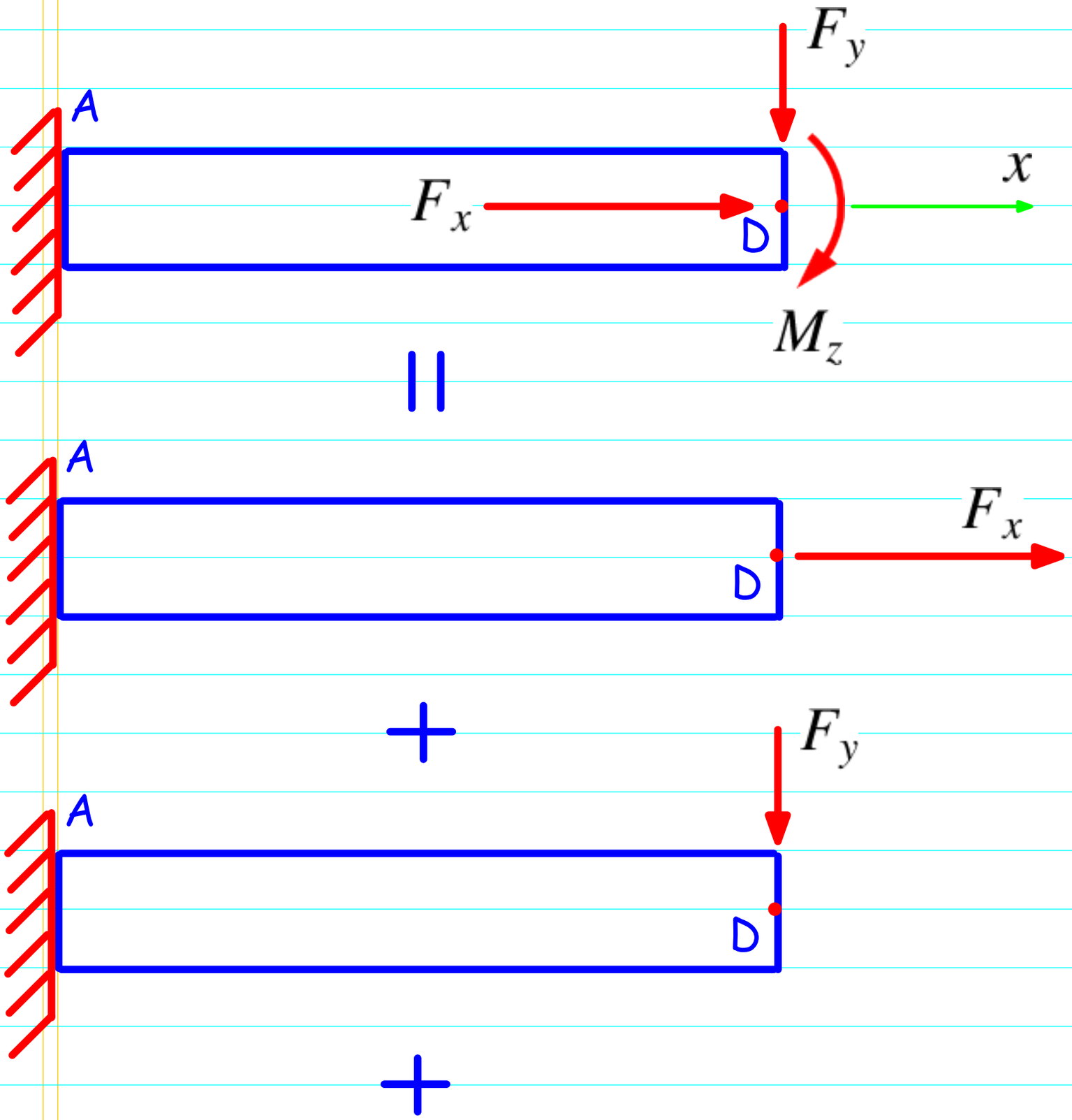
FBD 3

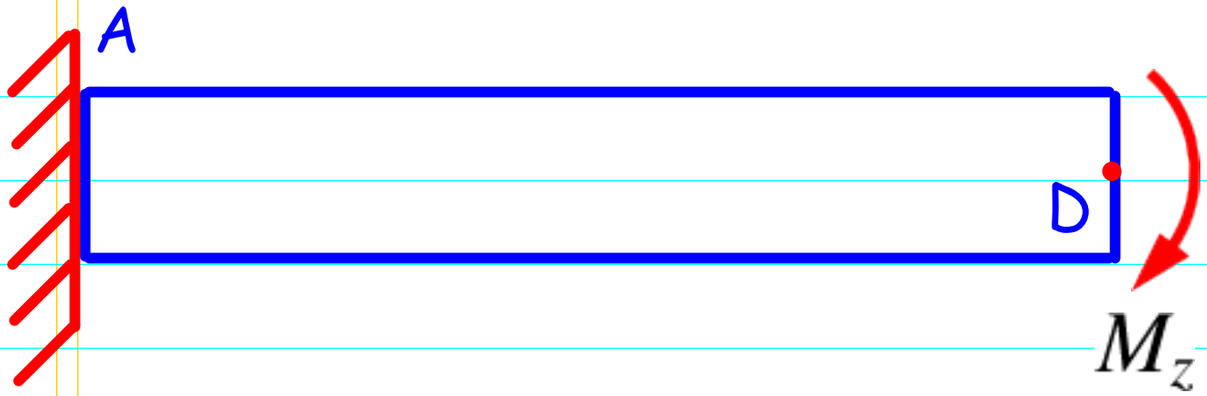


FBD 4

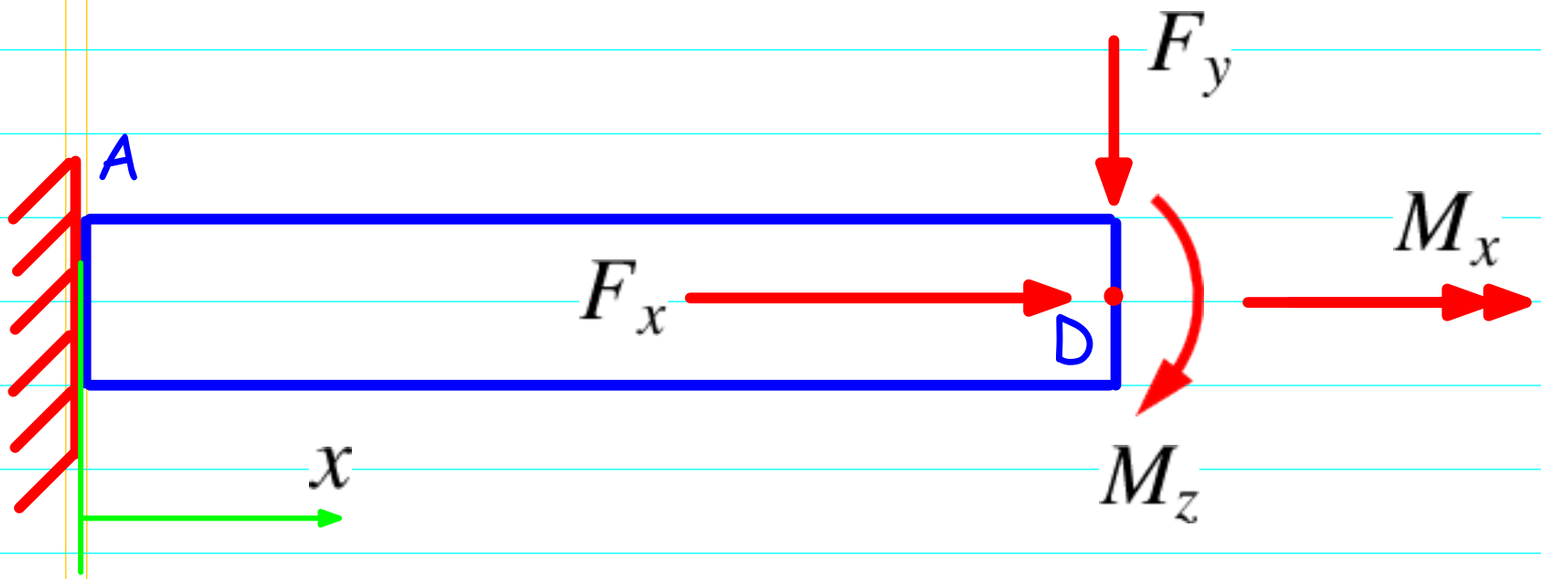
Cantilever beam under inclined force: Superposition




A
F_y
F_x
D
x
M_z
=
A
D
F_x
+
F_y
A
D
+

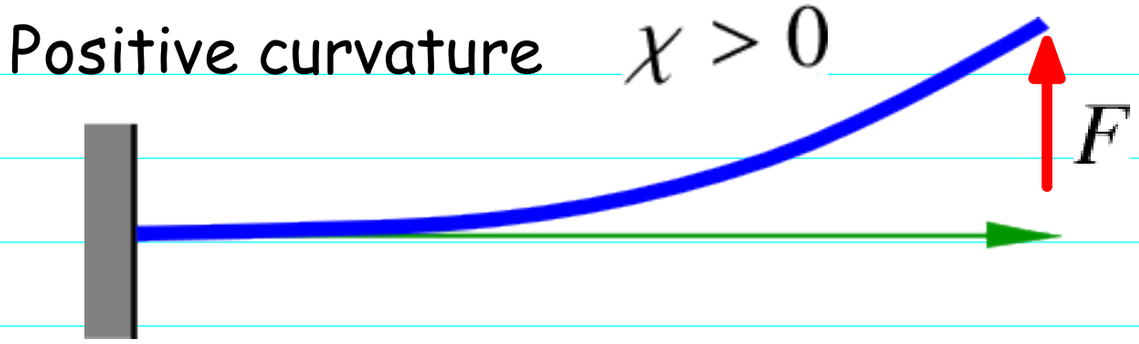


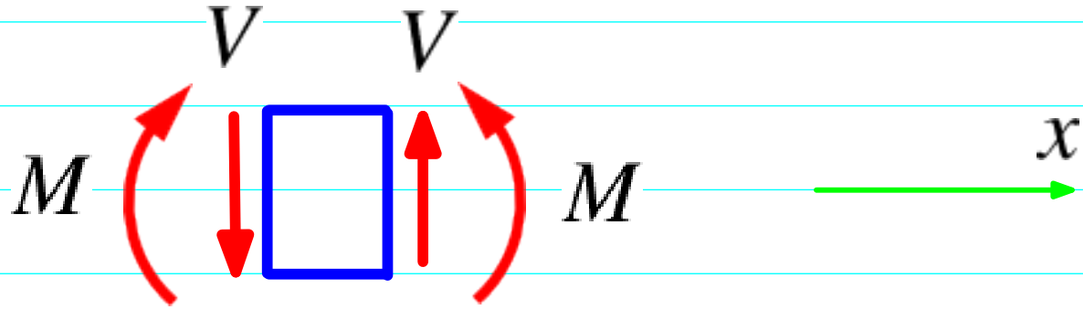
Cantilever beam under combined forces and moments



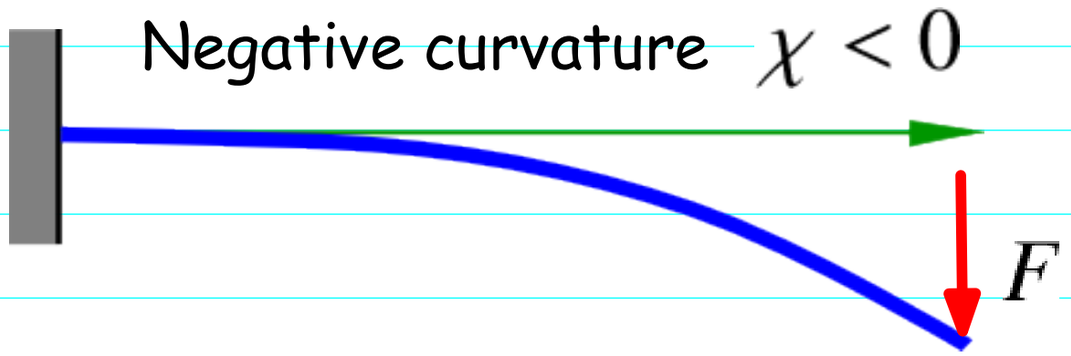
Two sign conventions for shear and bending moment

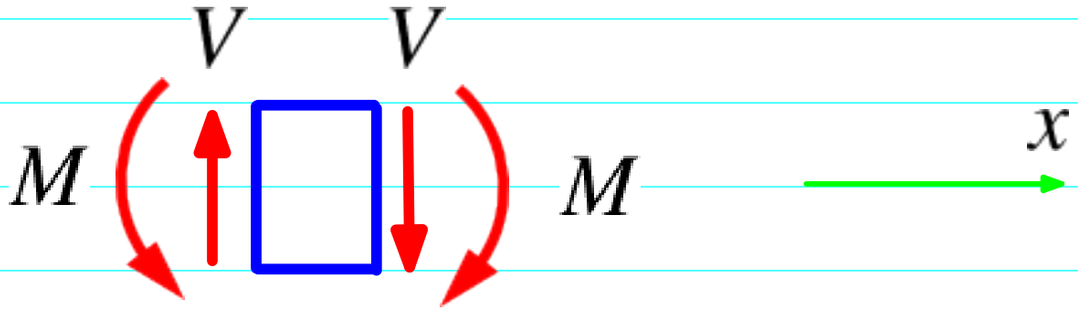
The "up" (positive curvature) convention



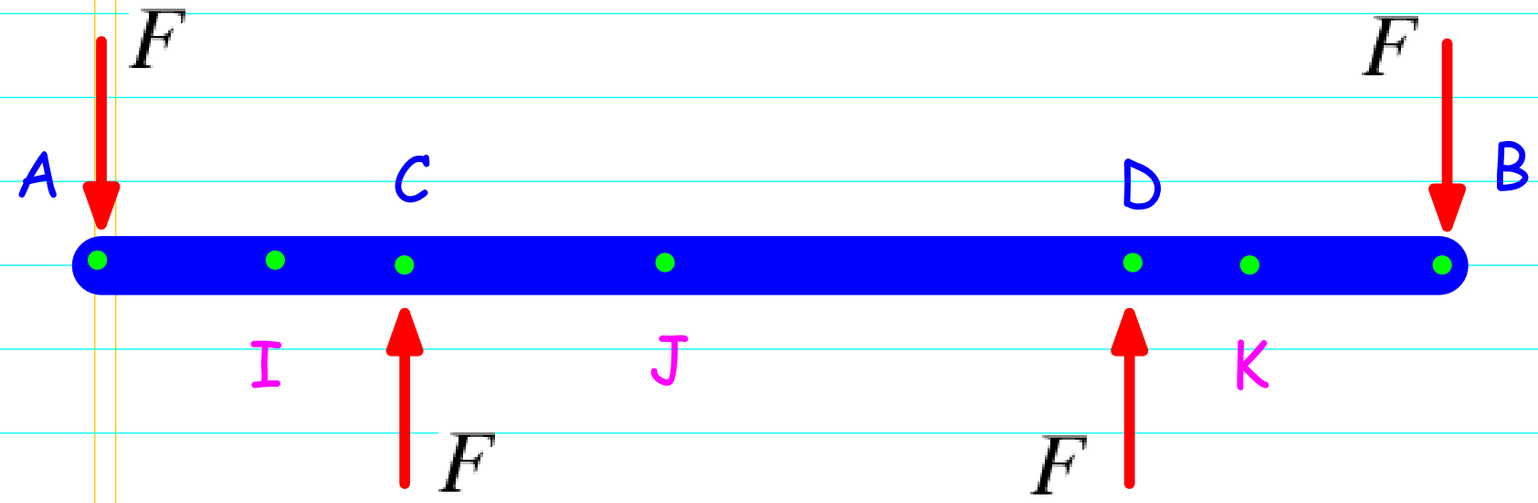


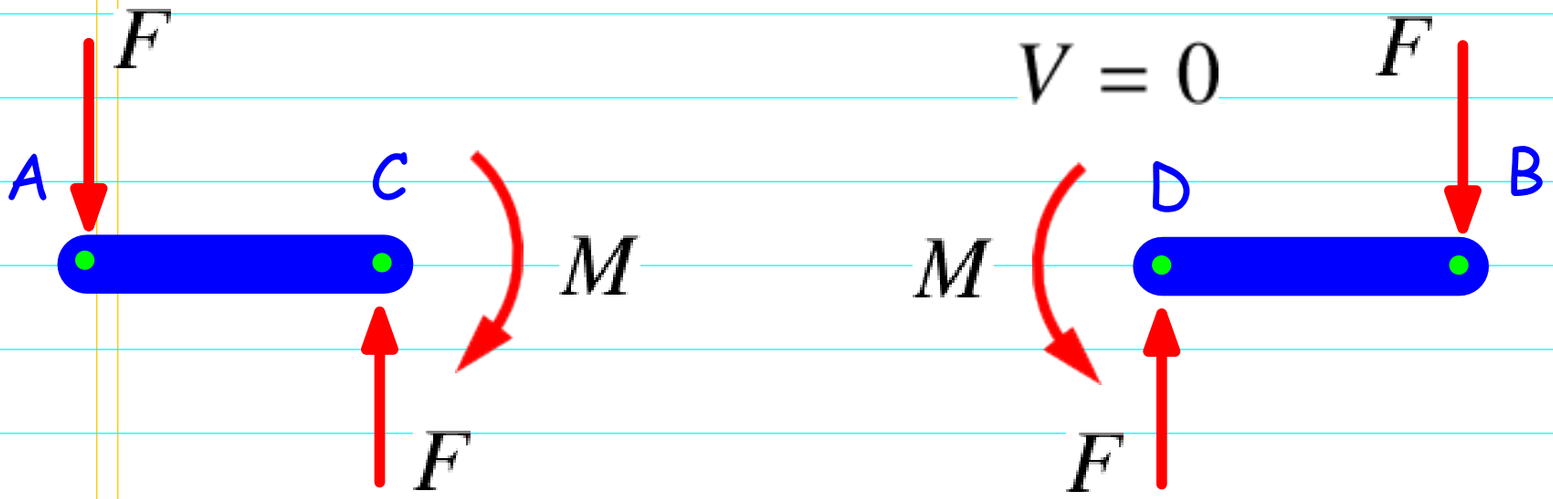
The "down" (negative curvature) convention





Pure bending, 4-point bending



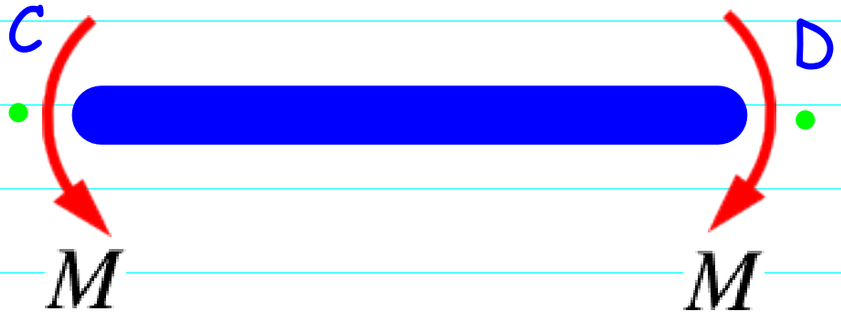


V_{C^+} = 0

$V_{C^+} = 0$

$V_{D^-} = 0$

V_{D^-} = 0



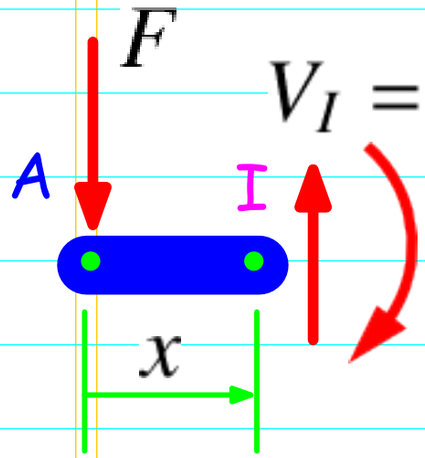
$C^+ = C + \epsilon$ a point a little to the right of point D

C^+ = C + \epsilon

$D^- = D - \epsilon$ a point a little to the left of point D

D^- = D - \epsilon

Shear and bending moment at I, J, K

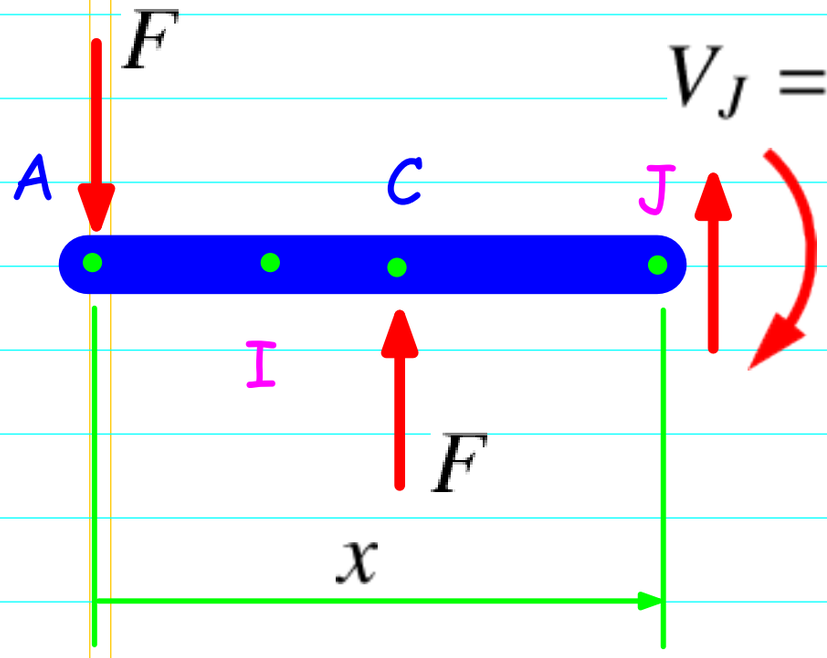


$$V_I = V(x) = F \quad (1)$$

V_I = V(x) = F

$$M_I = M(x) = F\,x \quad (2)$$

M_I = M(x) = F \, x



$$V_J = V(x)$$

$$M_J = M(x)$$

$$V_J = V(x) = 0 \text{ for } x_{C^+} \le x \le x_{D^-} \quad (3)$$

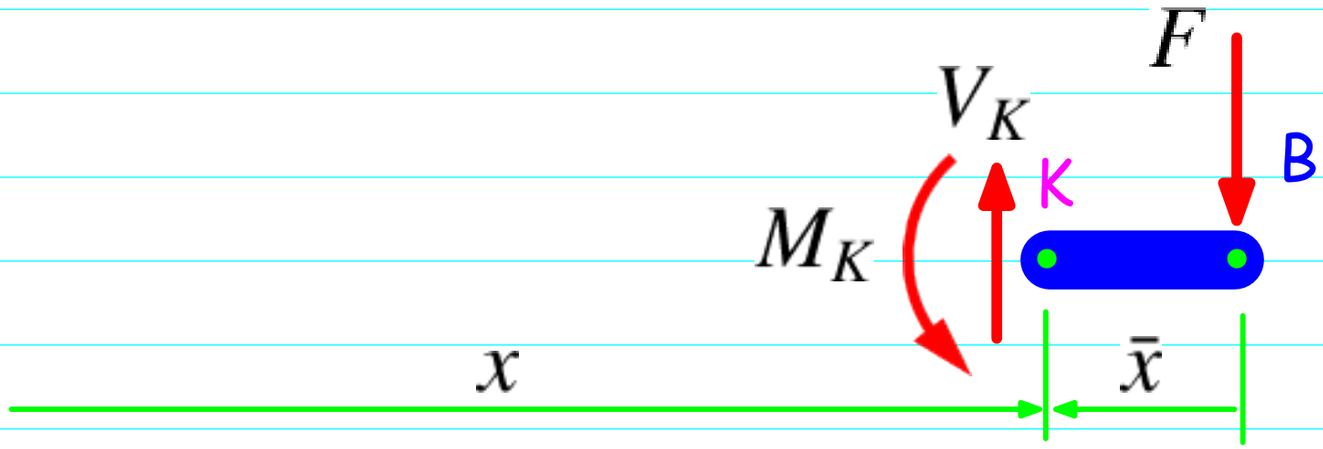
V_J = V(x) = 0 \text{ for } x_{C^+} \le x \le x_{D^-}

$$M_J = M(x) = Fx - F(x - x_C) = Fx_C \qquad (1)$$

M_J = M(x) = Fx - F (x - x_C) = F x_C

$$\text{for } x_{C^+} \le x \le x_{D^-}$$

\text{for } x_{C^+} \le x \le x_{D^-}



$$\bar{x} = \overline{AB} - x \qquad (2)$$

V_K = V (x) = V (\bar x) = F

$$V_K = V(x) = V(\bar{x}) = F \qquad (3)$$

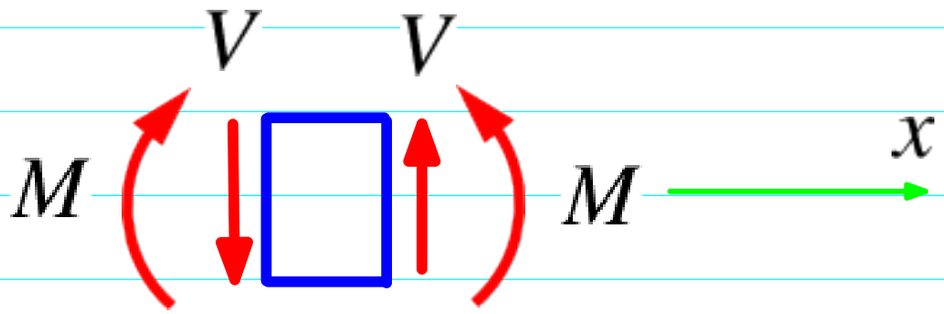
V_K = V (x) = V (\bar x) = F

$$M_K = M(\bar{x}) = F\bar{x} = F(\overline{AB} - x) \qquad (4)$$

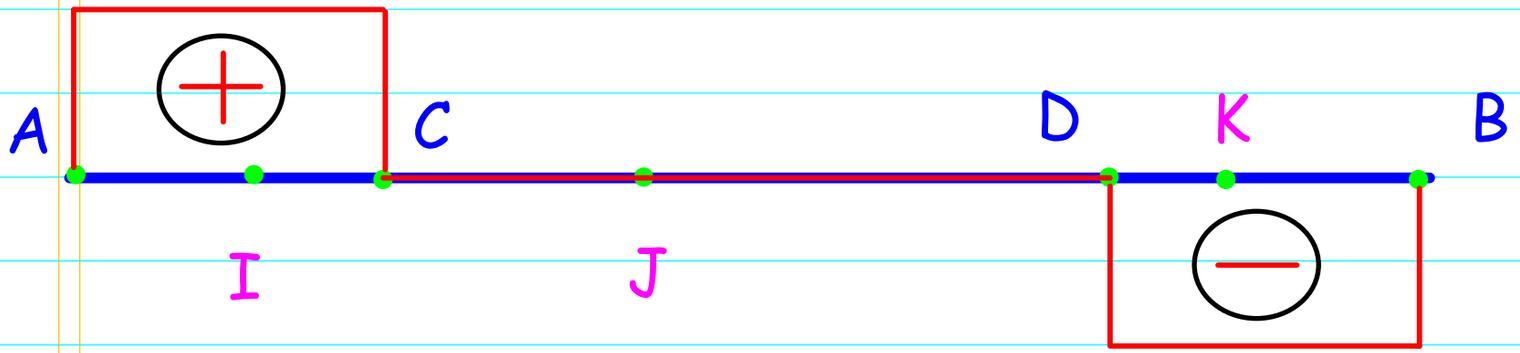
M_K = M(\bar x) = F \bar x = F (\overline{AB} - x)

Shear diagram

The "up" convention



$V(x) = F$

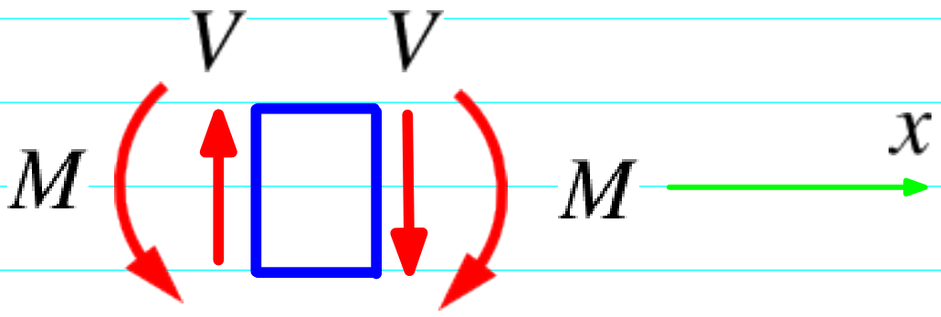


V(x) = F

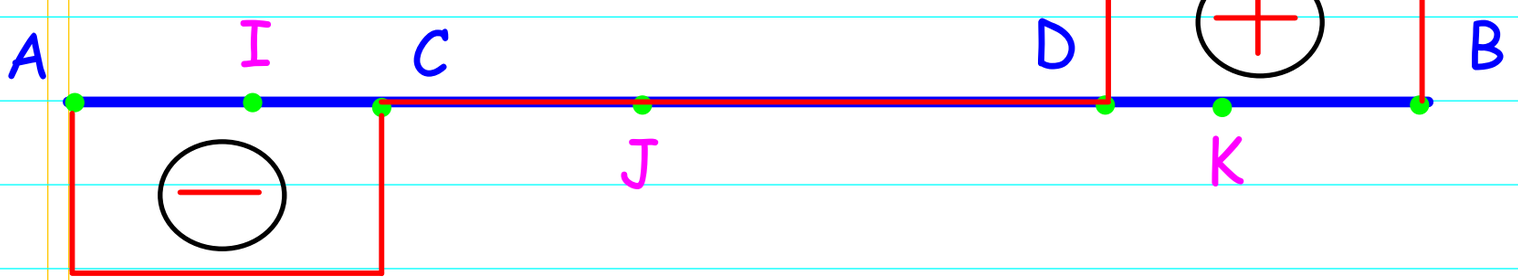
V(x) = - F

$V(x) = -F$

The "down" convention



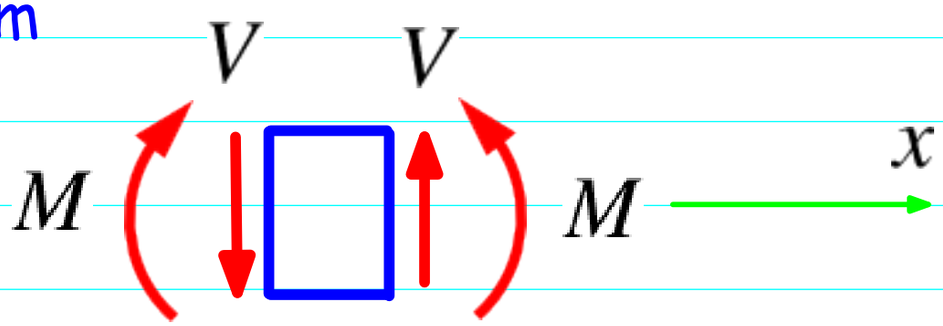
$V(x) = F$

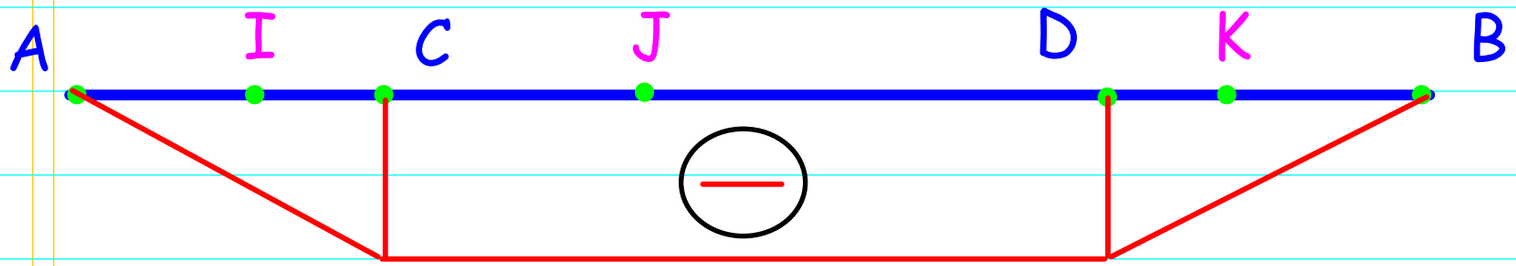


$V(x) = -F$

Bending moment diagram

The "up" convention





The "down" convention