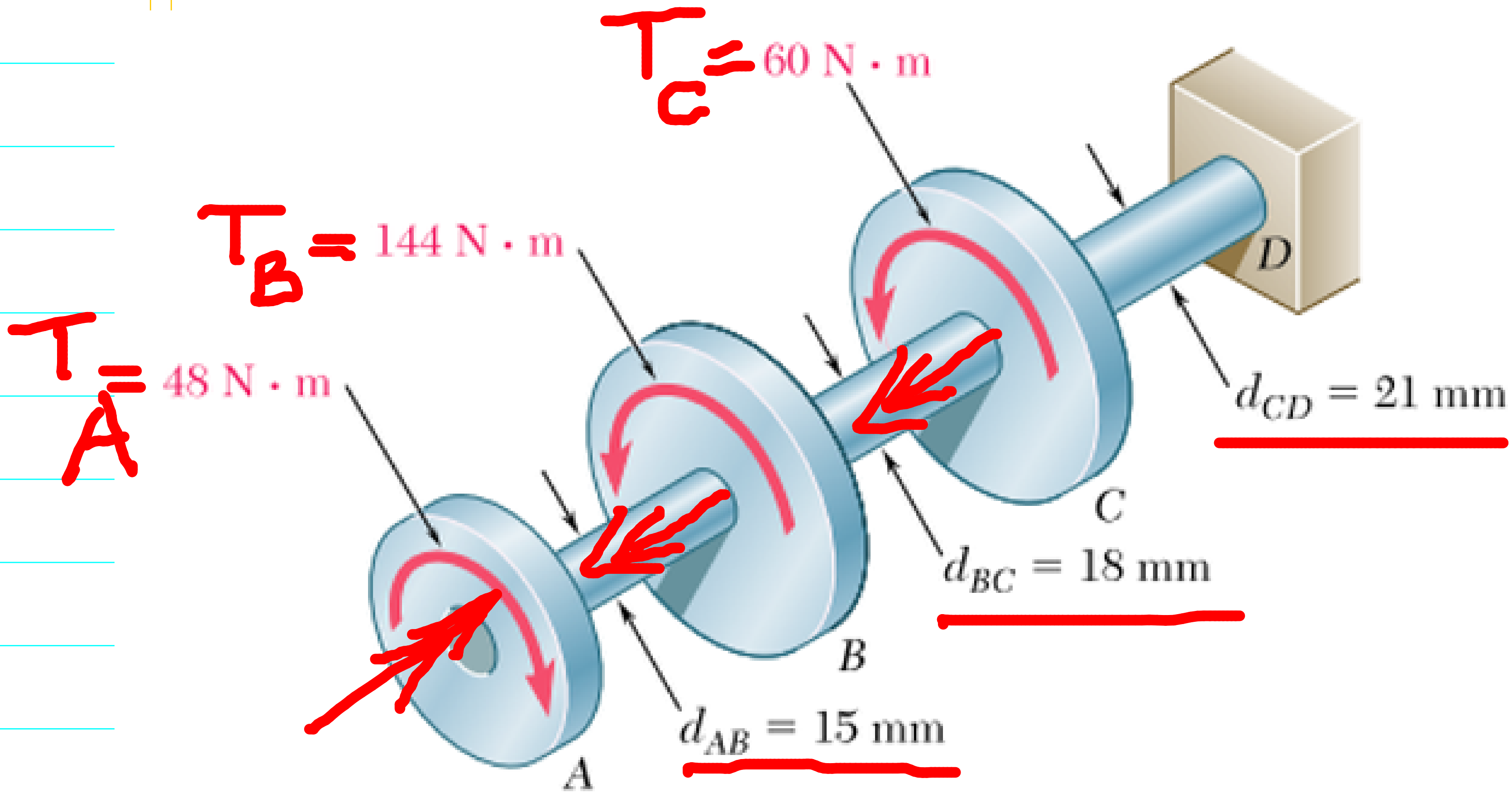
Sec.11

EGM 3520 Mechanics of Materials (MoM)

Beer et al. 2012, Mechanics of Materials, McGraw-Hill.

P3.11, p.155

P3.11, p.155



Knowing that each of the shafts *AB*, *BC*, and *CD* consists of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

Pause video NOW !

Work out the next step

→ on your own first

→ discuss with teammates

if you get stuck

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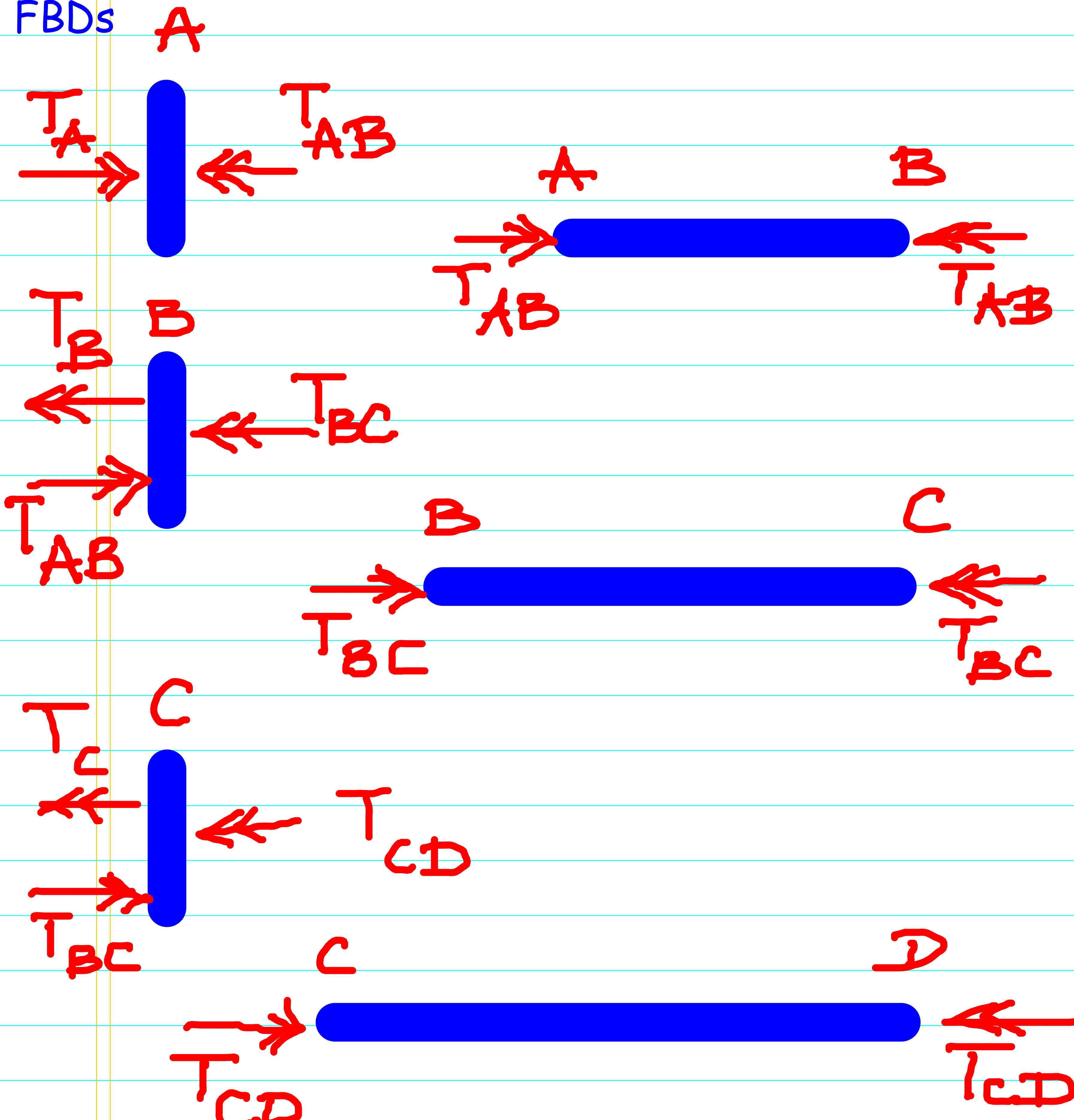
"Intelligence consists of this; that we recognize the similarity between different things, and the difference between similar things."

Baron de la Brède et de Montesquieu (1689-1755)

quoted in [Quantum field theory, E. Zeidler, 2008, p.175]

FBDs



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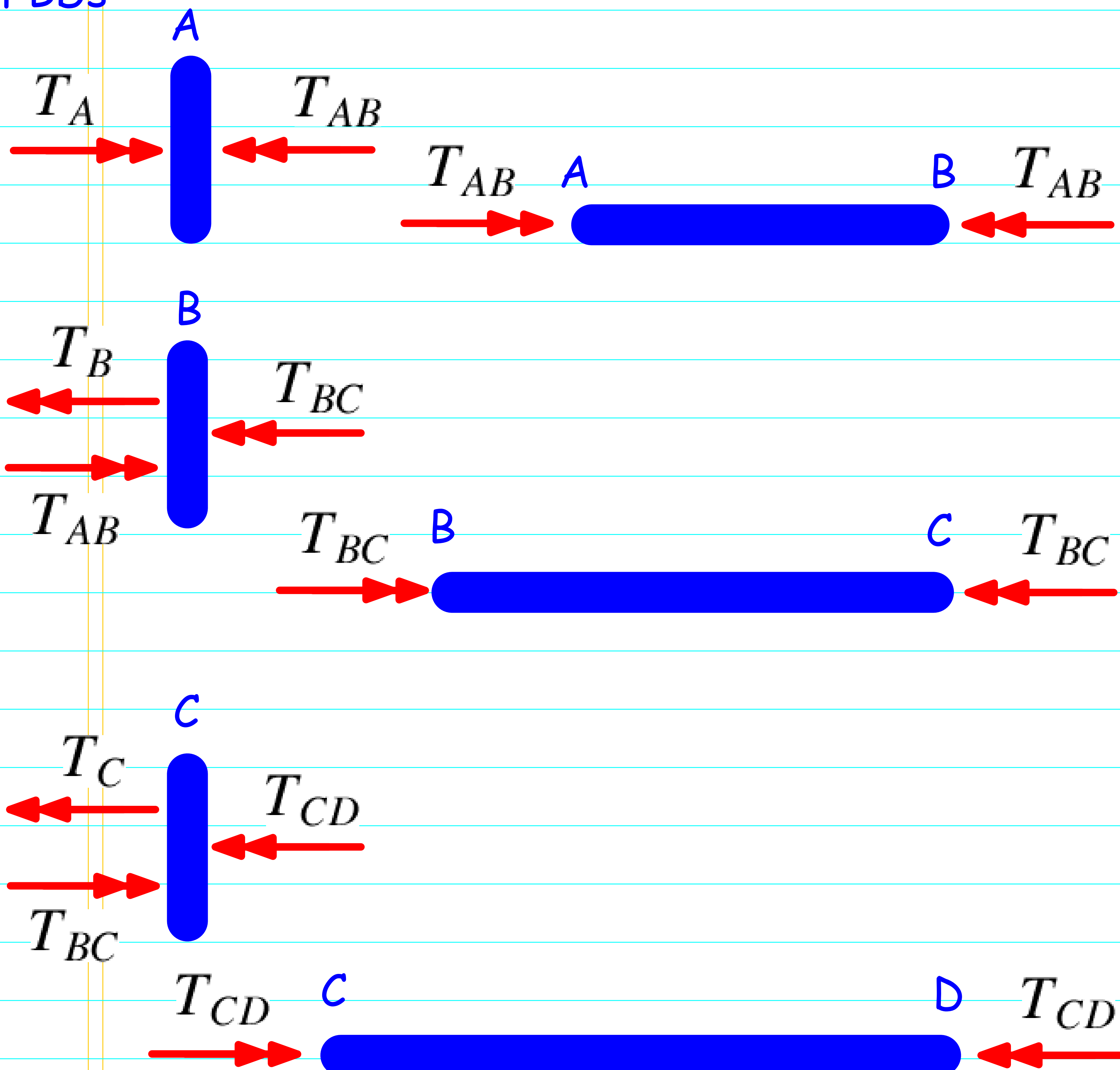

Method

FBD of disk A

deduce T_{AB} in terms of T_A

FBD of disk B

deduce T_{BC} in terms of T_A , T_B

FBD of disk C

deduce T_{CD} in terms of T_A , T_B , T_C

Knowing the internal torque applied on each shaft, and knowing the diameter of each shaft, you can now compute the maximum shear stress on each shaft, using the elastic torsion formulas (or better yet re-formulate these formulas).

Does the length of each shaft matter ?

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Computation

FBD of disk A $T_{AB} = T_A$ (1)

T_{AB} = T_A

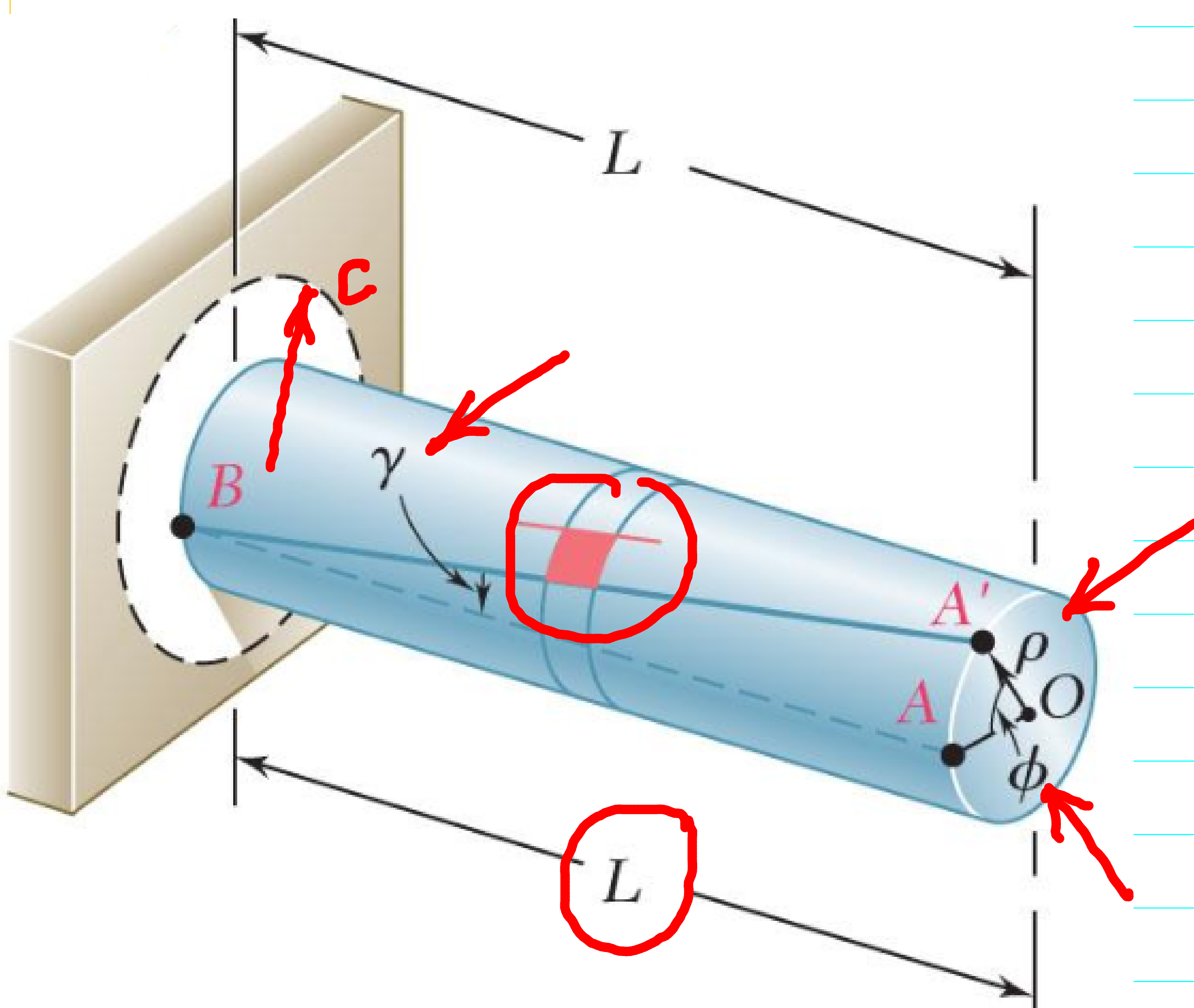
FBD of disk B $T_{BC} = T_{AB} - T_B = T_A - T_B$ (2)

T_{BC} = T_{AB} - T_B = T_A - T_B

FBD of disk C $T_{CD} = T_{BC} - T_C = T_A - T_B - T_C$

T_{CD} = T_{BC} - T_C = T_A - T_B - T_C (3)

Formulation of torsion problem:



Shear in terms of angle of twist:

$$L\gamma = \rho\phi \Rightarrow \gamma = \frac{\rho\phi}{L}\ ,\ \text{with } 0 \le \rho \le c \qquad (1)$$

L \gamma = \rho \phi \Rightarrow \gamma = \frac{\rho \phi}{L} \ , \text{ with } 0 \le \rho \le c

$$\gamma_{max} = \frac{c\phi}{L} \Rightarrow \frac{\phi}{L} = \frac{\gamma_{max}}{c} \qquad (2)$$

\gamma_{max} = \frac{c \phi}{L} \Rightarrow \frac{\phi}{L} = \frac{\gamma_{max}}{c}

(1)-(2): $$\gamma = \frac{\rho\phi}{L} = \frac{\rho\gamma_{max}}{c} \qquad (3)$$

\gamma = \frac{\rho \phi}{L} = \frac{\rho \gamma_{max}}{c}

Hooke's law: $$\tau = G\gamma = G\frac{\rho\gamma_{max}}{c} \qquad (4)$$

\tau = G \gamma = G \frac{\rho \gamma_{max}}{c}

Resultant torque:

$$T = \int_A \rho\, dF = \int_A \rho\, \tau dA = G\frac{\gamma_{max}}{c}\int_A \rho^2\, dA \qquad (5)$$

T = \int_A \rho \, dF = \int_A \rho \, \tau dA = G \frac{\gamma_{max}}{c} \int_A \rho^2 \, dA

$$\boxed{T = GJ\frac{\gamma_{max}}{c} = \frac{GJ}{L}\phi} \qquad (6)$$

T = GJ \frac{\gamma_{max}}{c} = \frac{GJ}{L} \phi

From the 2nd eq. in (6) p.11-6 for torsional deformation, one can observe the following similarity and difference with axial deformation (remember Montesquieu):

Axial deformation — Torsional deformation

$$\delta = \frac{PL}{EA} \leftrightarrow \phi = \frac{TL}{GJ} \quad (1)$$

\delta = \frac{PL}{EA} \leftrightarrow \phi = \frac{TL}{GJ}

$$P = \frac{EA}{L}\delta \leftrightarrow T = \frac{GJ}{L}\phi \quad (2)$$

P = \frac{EA}{L} \delta \leftrightarrow T = \frac{GJ}{L} \phi

$L/(EA)$ axial flexibility

EA/L axial stiffness

$L/(GJ)$ torsional flexibility

GJ/L torsional stiffness

(6) p.11-6:

$$\tau_{max} = G\gamma_{max} = \frac{Tc}{J} \qquad (1)$$

\tau_{max} = G \gamma_{max} = \frac{T c}{J}

Polar moment of inertia of solid circular section:

$$J = \int_A \rho^2 dA = \int_{\rho=0}^{\rho=c} \rho^2 (2\pi\rho d\rho) = \frac{1}{2}\pi c^4 \qquad (2)$$

J = \int_A \rho^2 dA = \int_{\rho = 0}^{\rho = c} \rho^2 (2 \pi \rho d\rho) = \frac12 \pi c^4

Maximum shear stress in bar AB:

$$\tau_{max,AB} = \frac{T_{AB}\, c_{AB}}{J_{AB}} = \frac{T_A\, d_{AB}/2}{\frac12 \pi (d_{AB}/2)^4} = \frac{4T_A}{\pi (d_{AB})^3} \qquad (3)$$

\tau_{max, AB} = \frac{T_{AB} \, c_{AB}}{J_{AB}} = \frac{T_{A} \, d_{AB} / 2}{\frac12 \pi (d_{AB}/2)^4} = \frac{4 T_A}{\pi (d_{AB})^3}

Maximum shear stress in bar BC:

$$\tau_{max,BC} = \frac{T_{BC}\, c_{BC}}{J_{BC}} = \frac{(T_A - T_B)\, d_{BC}/2}{\frac12 \pi (d_{BC}/2)^4}$$

$$= \frac{4(T_A - T_B)}{\pi (d_{BC})^3} \qquad (4)$$

\tau_{max, BC} = \frac{T_{BC} \, c_{BC}}{J_{BC}} = \frac{(T_{A} - T_B) \, d_{BC} / 2}{\frac12 \pi (d_{BC}/2)^4}
= \frac{4 (T_A - T_B)}{\pi (d_{BC})^3}

Maximum shear stress in bar BC:

$$\tau_{max,CD} = \frac{T_{CD}\, c_{CD}}{J_{CD}} = \frac{(T_A - T_B - T_C)\, d_{CD}/2}{\frac{1}{2}\pi(d_{CD}/2)^4}$$

$$= \frac{4(T_A - T_B - T_C)}{\pi(d_{CD})^3} \qquad (1)$$

\tau_{max, CD} = \frac{T_{CD} \, c_{CD}}{J_{CD}} = \frac{(T_{A} - T_B - T_C) \, d_{CD} / 2}{\frac12 \pi (d_{CD}/2)^4}
= \frac{4 (T_A - T_B - T_C)}{\pi (d_{CD})^3}

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Without doing any computation, we can determine which maximum shear stress is largest as follows:

$$T_A > T_A - T_B > T_A - T_B - T_C \qquad (1)$$

T_A > T_A - T_B > T_A - T_B - T_C

$$d_{AB} < d_{BC} < d_{CD} \qquad (2)$$

d_{AB} < d_{BC} < d_{CD}

(3)-(4) p.11-8 and (1) p.11-7:

$$\tau_{max,AB} > \tau_{max,BC} > \tau_{max,CD} \qquad (3)$$

\tau_{max, AB} > \tau_{max, BC} > \tau_{max, CD}

Now use (3) p.11-8 to compute $\tau_{max,AB}$