

# CMOS Delay-7 (H.8) Delay Model

20170202

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# References

Some Figures from the following sites

[1] <http://pages.hmc.edu/harris/cmosvlsi/4e/index.html>  
Weste & Harris Book Site

[2] [en.wikipedia.org](http://en.wikipedia.org)

$\beta$  : Device Transconductance Parameter

$k$  : Process Transconductance Parameter

$\mu$  : Electron / Hole Mobility

$$\text{PMOS} \quad \beta_p = k'_p \left(\frac{W}{L}\right)_p \quad k'_p = \mu_p C_{ox} \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$\text{nMOS} \quad \beta_n = k'_n \left(\frac{W}{L}\right)_n \quad k'_n = \mu_n C_{ox} \quad C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$\text{PMOS} \quad \beta_p = \mu_p \frac{\epsilon_{ox}}{t_{ox}} \left(\frac{W}{L}\right)_p$$

$$\text{nMOS} \quad \beta_n = \mu_n \frac{\epsilon_{ox}}{t_{ox}} \left(\frac{W}{L}\right)_n$$

Saturation Current

$$I_{d_p} = \frac{\beta_p}{2} (V_{GS_n} - |V_{TP}|)^2 \quad V_{TP} < 0$$

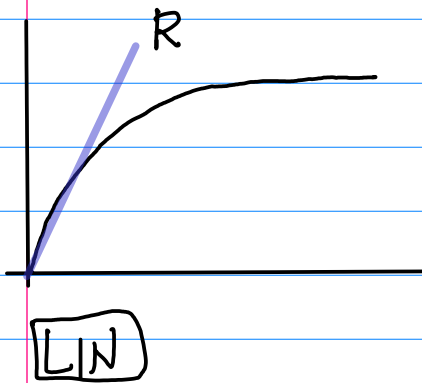
$$I_{d_n} = \frac{\beta_n}{2} (V_{GS_n} - V_{TN})^2 \quad V_{TN} > 0$$

$$\frac{\beta_n}{\beta_p} = \frac{k'_n \left(\frac{W}{L}\right)_n}{k'_p \left(\frac{W}{L}\right)_p}$$

$$\frac{k'_n}{k'_p} = 2 \sim 3$$

$$\frac{k'_n}{k'_p} = \frac{\mu_n}{\mu_p} = \gamma$$

$$\frac{\beta_n}{\beta_p} = \frac{k'_n \left(\frac{W}{L}\right)_n}{k'_p \left(\frac{W}{L}\right)_p}$$



$$R_n = \frac{1}{\beta_n (V_{DD} - V_{Tn})}$$

$$R_p = \frac{1}{\beta_p (V_{DD} - V_{Tp})}$$

fall time  $t_f$        $\tau_n = R_n C_{out}$

rise time  $t_r$        $\tau_p = R_p C_{out}$

$$C_{out} = C_{para} + C_L$$

fall time	$t_f = 2.2 \tau_n = \ln 9 \tau_n$	$0.9 V_{DD} \rightarrow 0.1 V_{DD}$
rise time	$t_r = 2.2 \tau_p = \ln 9 \tau_p$	$0.1 V_{DD} \rightarrow 0.9 V_{DD}$
propagation delay time	$t_p = \frac{1}{2} (t_{pf} + t_{pr})$ $= 0.35 (t_{pf} + t_{pr})$	$0.5 V_{DD} \rightarrow 0.5 V_{DD}$
propagation fall time	$t_{pf} = 0.7 \tau_n = \ln 2 \tau_n$	$V_{DD} \rightarrow 0.5 V_{DD}$
propagation rise time	$t_{pr} = 0.7 \tau_p = \ln 2 \tau_p$	$0 \rightarrow 0.5 V_{DD}$

$$\tau_n = R_n (C_{para} + C_L)$$

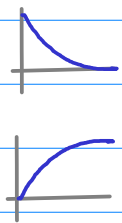
$$\tau_p = R_p (C_{para} + C_L)$$

$$C_{out} = C_{para} + C_L$$

$$\left(\frac{W}{L}\right)_p = r \left(\frac{W}{L}\right)_n$$

$$r = \frac{\mu_n}{\mu_p} = \frac{k'_n}{k'_p} > 1$$

$$R_n = R_p = R = \frac{1}{\beta(V_{DD} - V_T)}$$

$$\begin{cases} V_{out}(t) = V_{DD} (1 - e^{-t/\tau}) \\ V_{out}(t) = V_{DD} e^{-t/\tau} \end{cases}$$


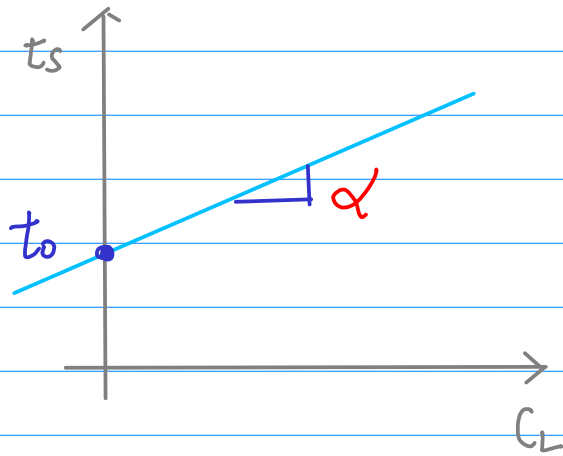
$$\tau = RC_{out} = R(C_{par} + C_L)$$

### Generic Switching Delay

$$t_s = t_0 + \alpha C_L \Rightarrow t_s = t_r = t_f$$

## Generic Switching Delay

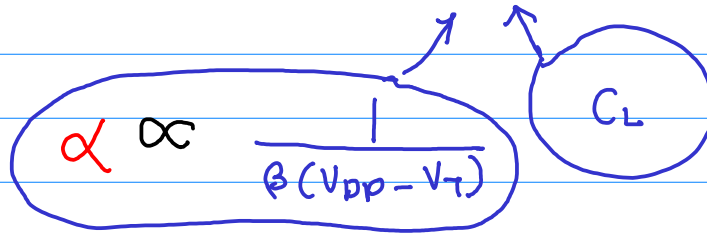
$$t_s = t_0 + \alpha C_L$$



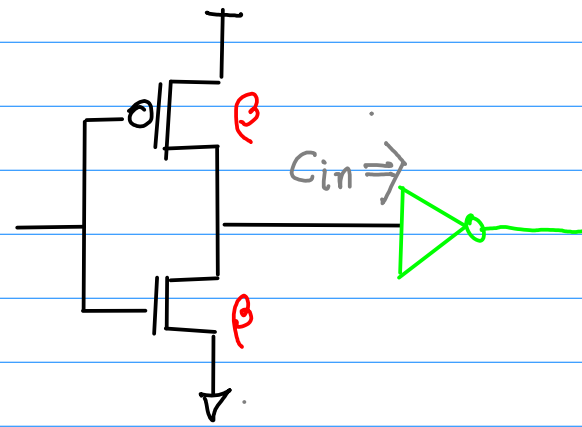
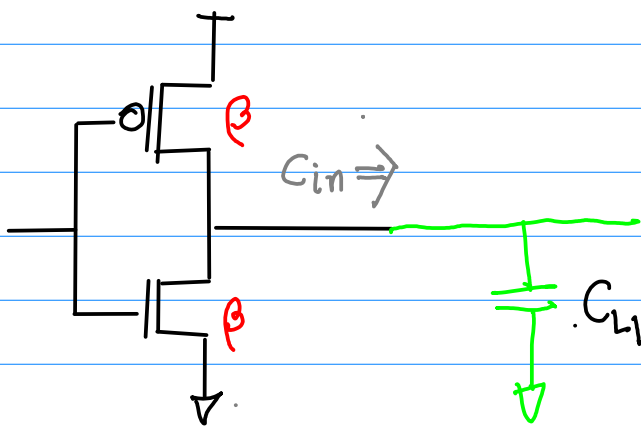
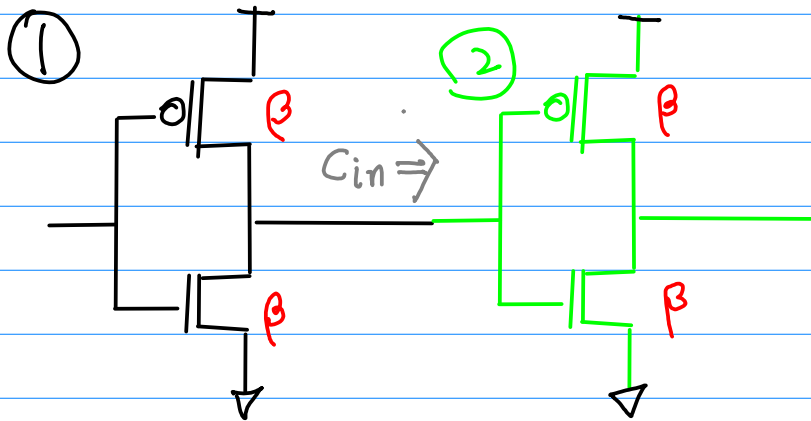
$t_0$  : zero delay

$\alpha$  : slope

$$\tau \approx RC$$







reference case

$$C_{in} = C_{L1}$$

Generic Switching Delay of ①

$$t_{s1} = t_0 + \alpha C_{L1}$$

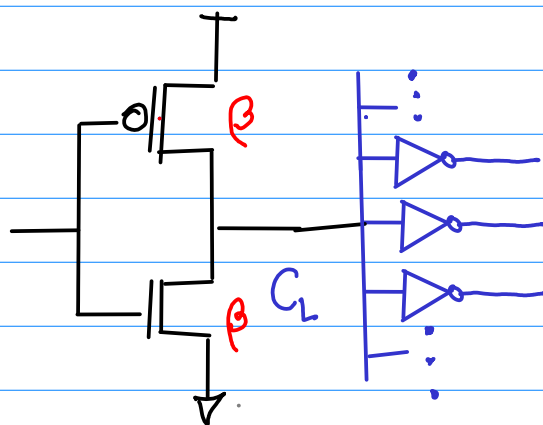
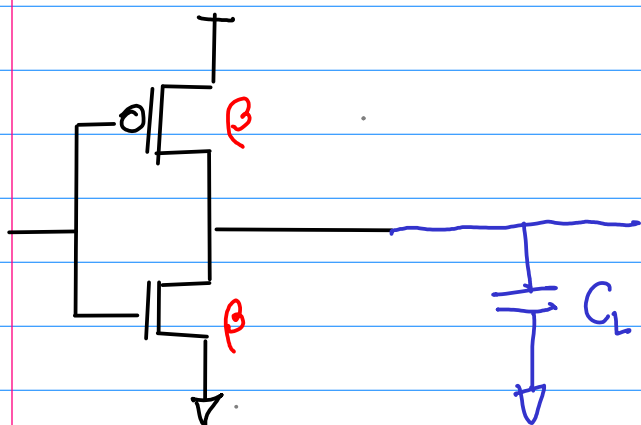
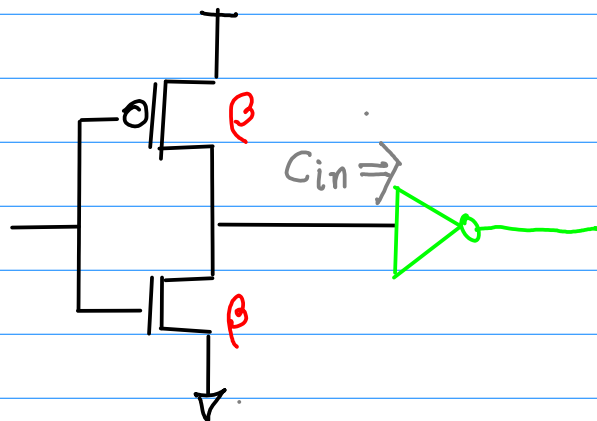
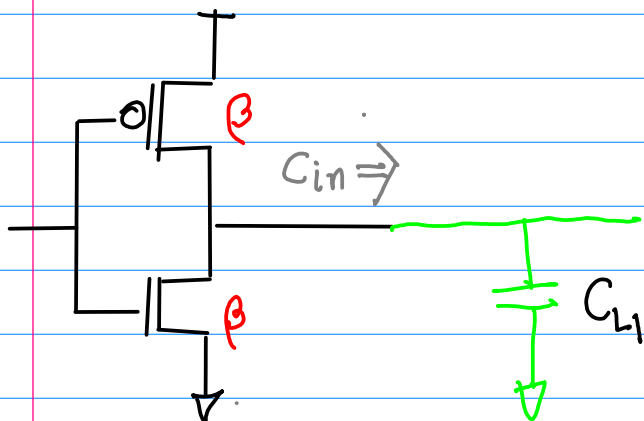
$$= t_0 + \alpha C_{in}$$

$$\begin{aligned} C_{in} &= C_{Gn} + C_{Gp} \\ &= C_{ox} (A_{Gn} + A_{Gp}) \end{aligned} \quad A_i: \text{ gate area}$$

the channel length  $L$  assumed

$$\begin{aligned} C_{in} &= C_{ox} L (W_n + W_p) \\ &= C_{ox} L (W_n + r W_p) \\ &= C_{ox} L W_n \cdot (1 + r) \\ &= C_{Gn} (1 + r) \end{aligned}$$

When  $C_L \gg C_{in}$



to minimize  $t_s$

$\alpha \downarrow \Rightarrow R \downarrow \Rightarrow \beta \uparrow \Rightarrow$  bigger size

speed v.s. area tradeoff

$$t_s = t_0 + \alpha C_L \quad t \approx RC$$

$$\alpha \propto \frac{1}{\beta(V_{DD} - V_T)}$$

Diagram showing the relationship between  $\alpha$  and  $C_L$ . The gain factor  $\alpha$  is proportional to  $\frac{1}{\beta(V_{DD} - V_T)}$ . The load capacitor  $C_L$  is shown in a circle, with arrows pointing to the  $C_L$  term in the equation above and the  $\alpha$  term in the equation below.

to minimize  $t_s$

$$\alpha \downarrow \Rightarrow R \downarrow \Rightarrow \beta \uparrow \Rightarrow \text{bigger size}$$

Speed v.s. Area tradeoff

Scaling Factor  $S$

$$\beta' = S \beta$$

$$R' = \frac{R}{S}$$

$$\alpha' = \frac{\alpha}{S^2}$$

$$t_s = t_0 + \left( \frac{\alpha}{S} \right) C_L$$

Compensation Factor  $\left( \frac{1}{S} \right)$

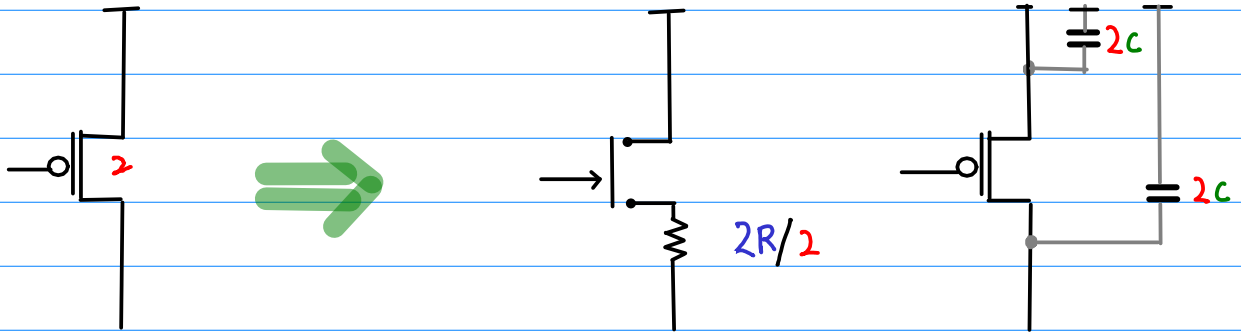
enables a NOT gate drive larger values of  $C_L$

If  $C_L = S C_{in}$  (increased by the scaling factor  $S$ )

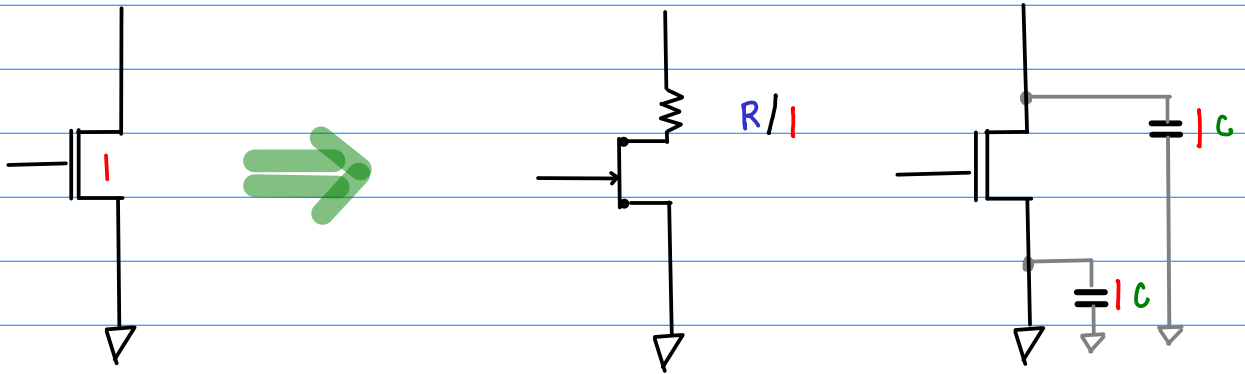
then the switching time is the same

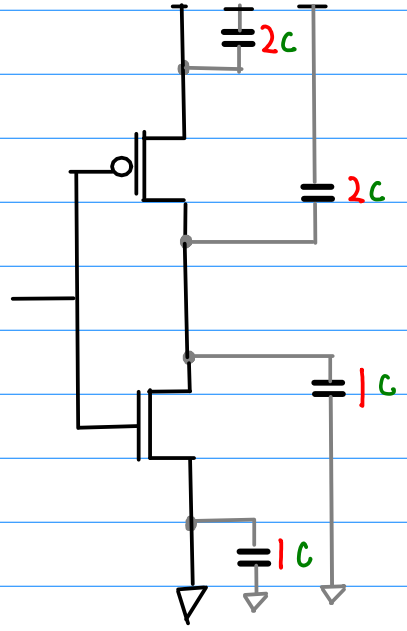
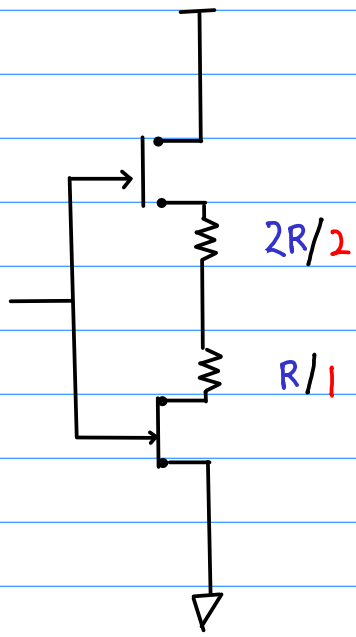
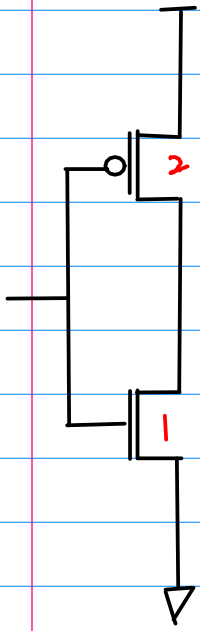
# RC Delay Model

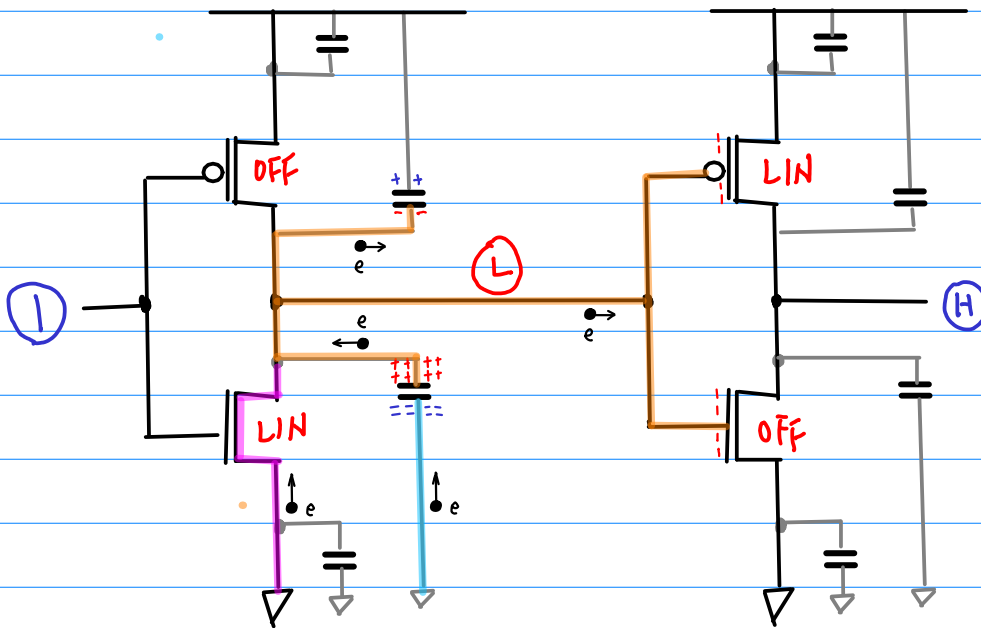
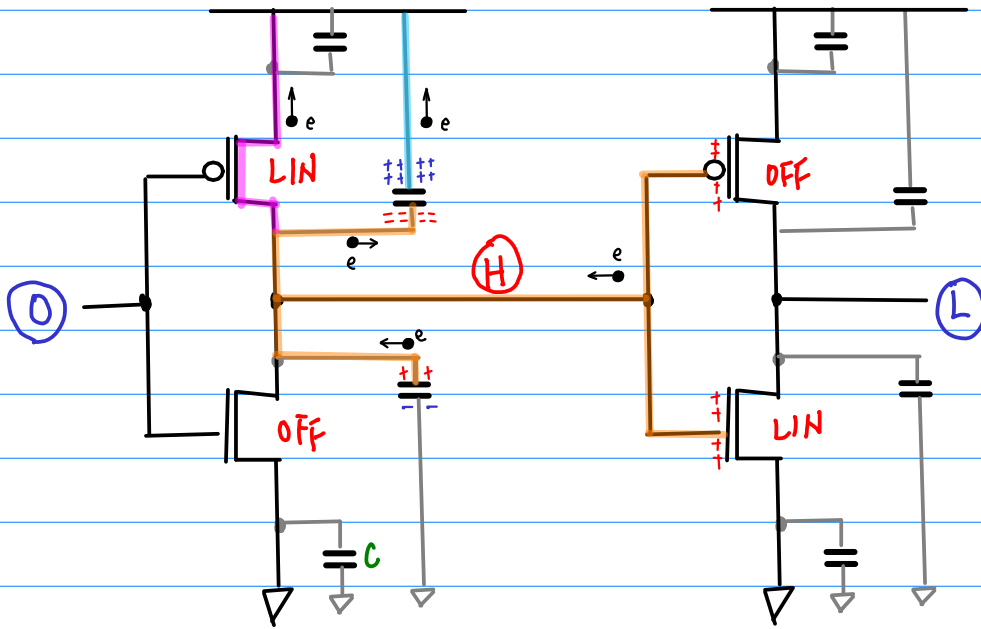
pMOS

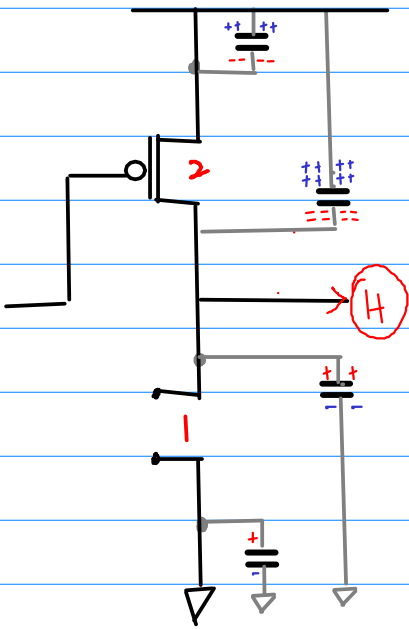


nMOS



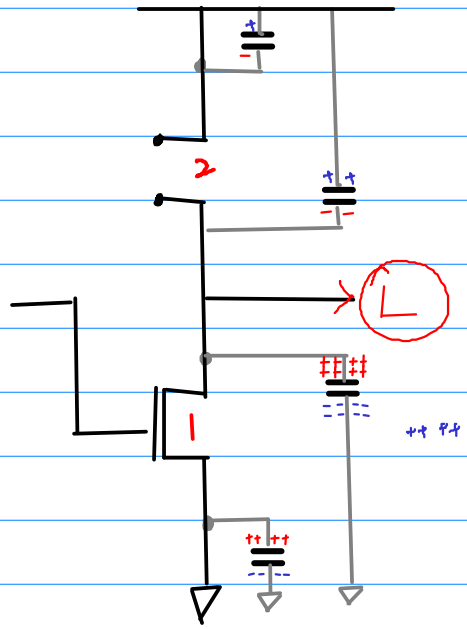




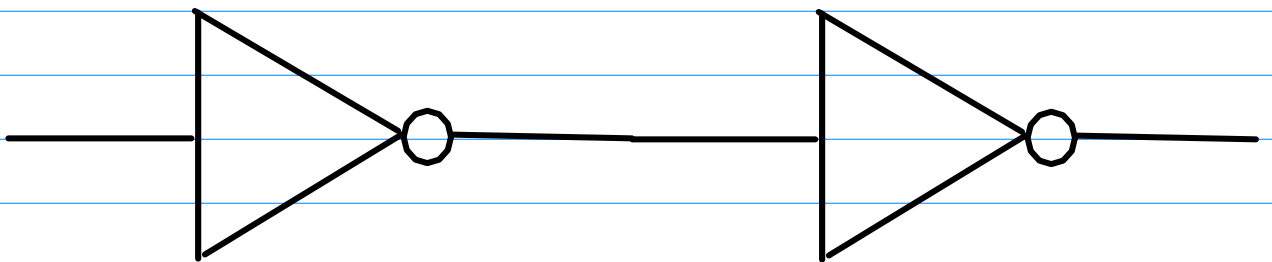
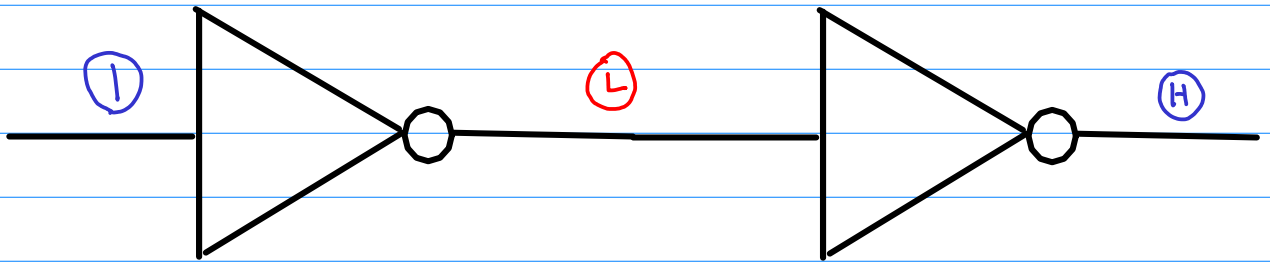
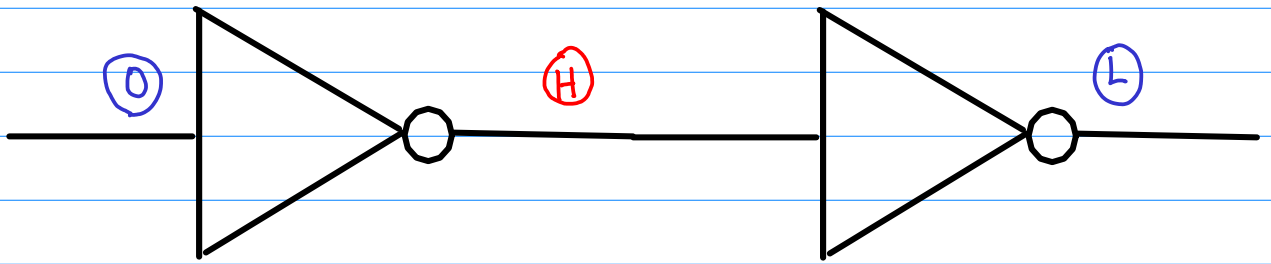


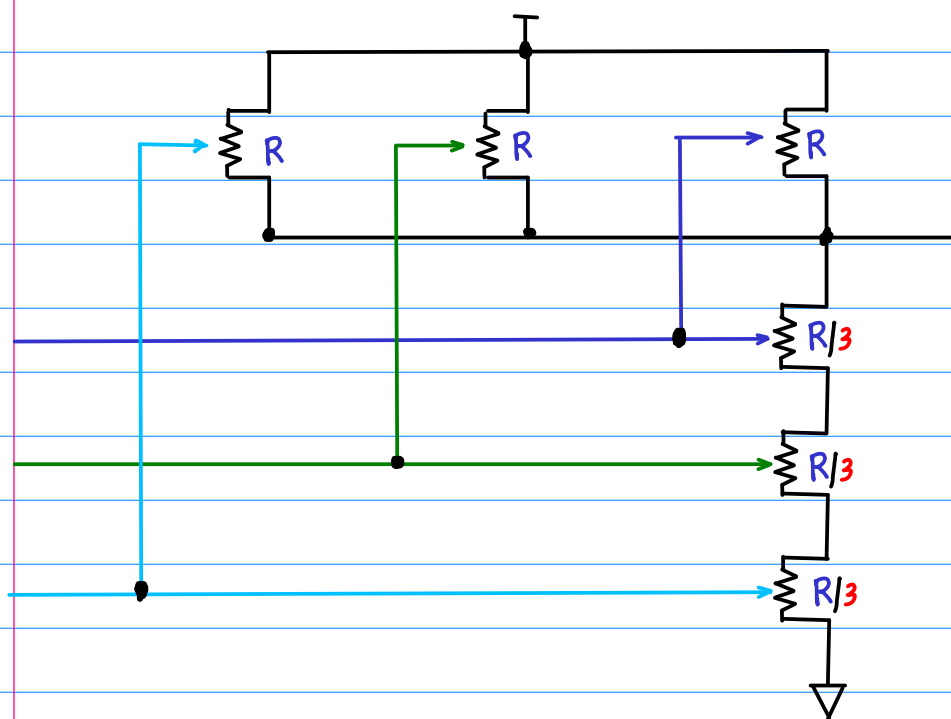
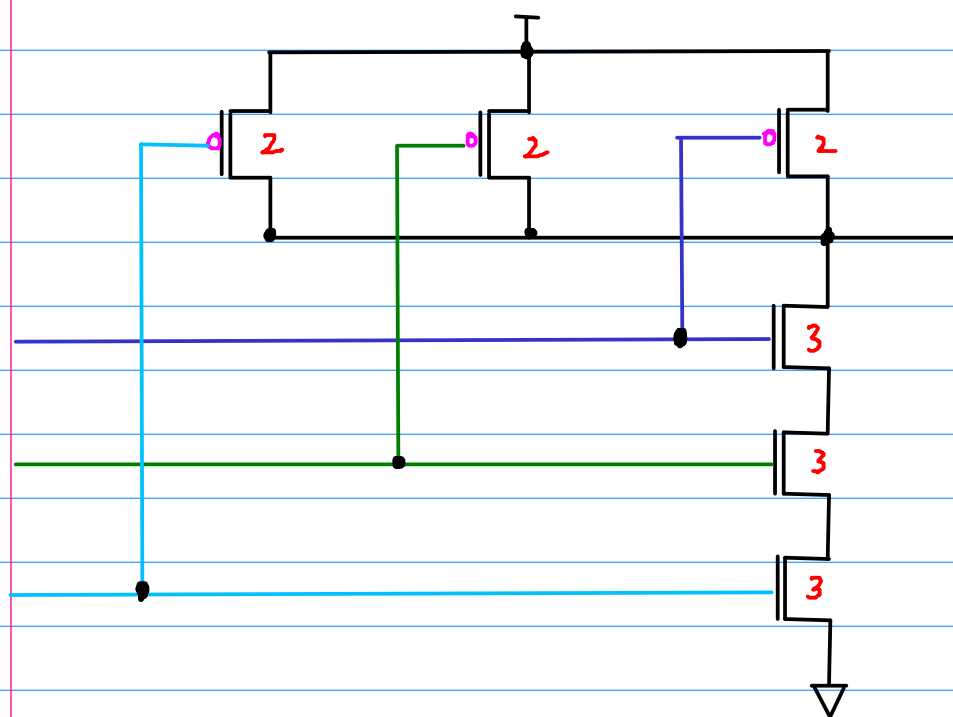
charge +  
discharge -

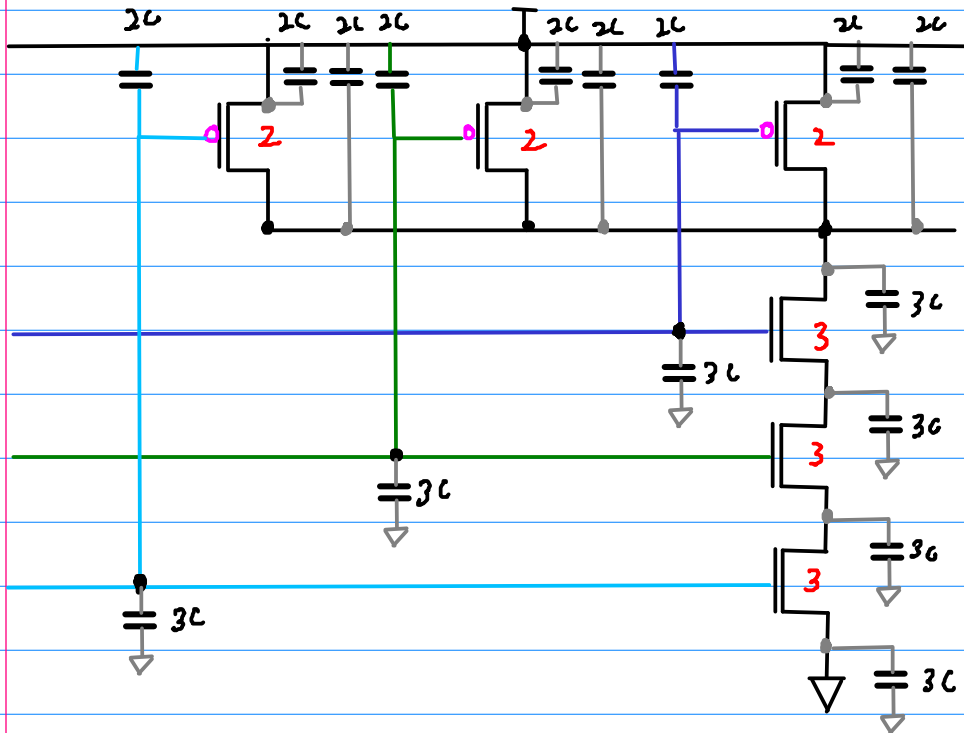
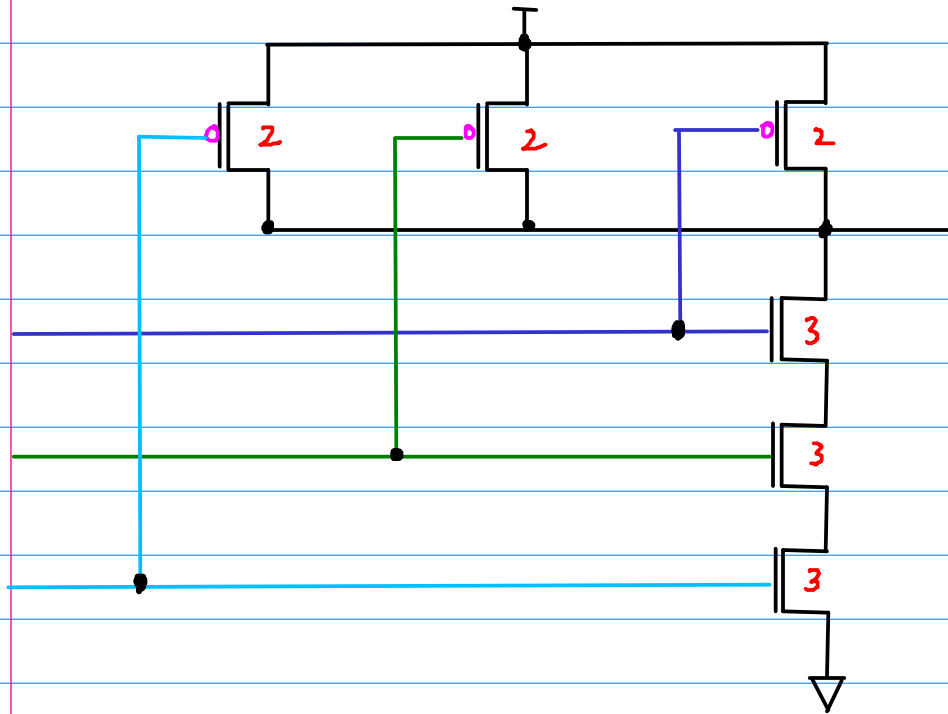
charge +

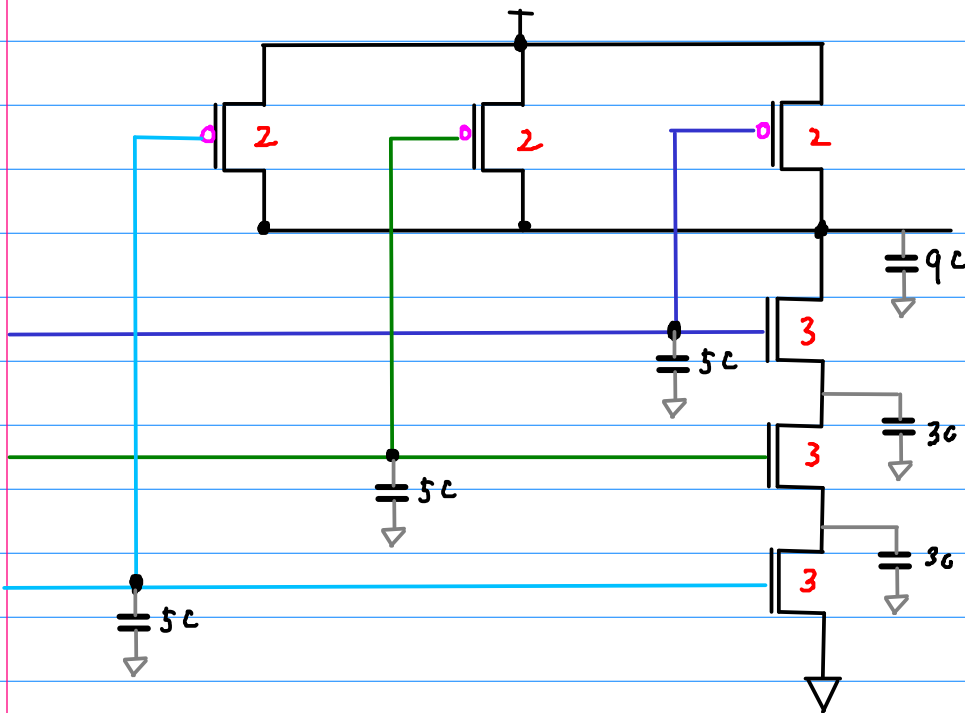
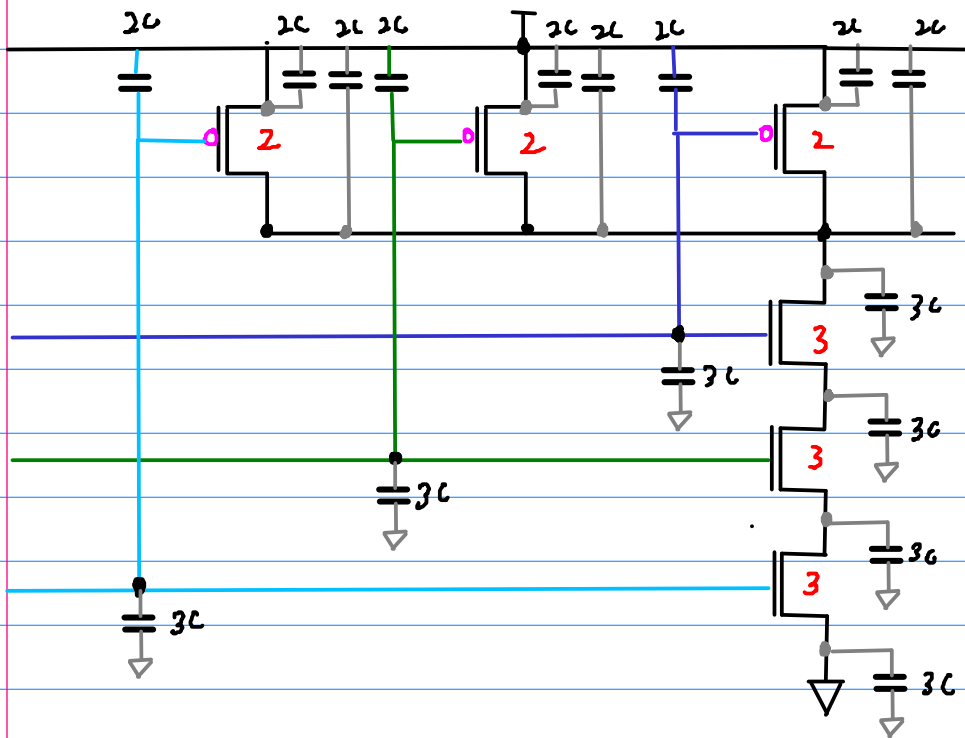




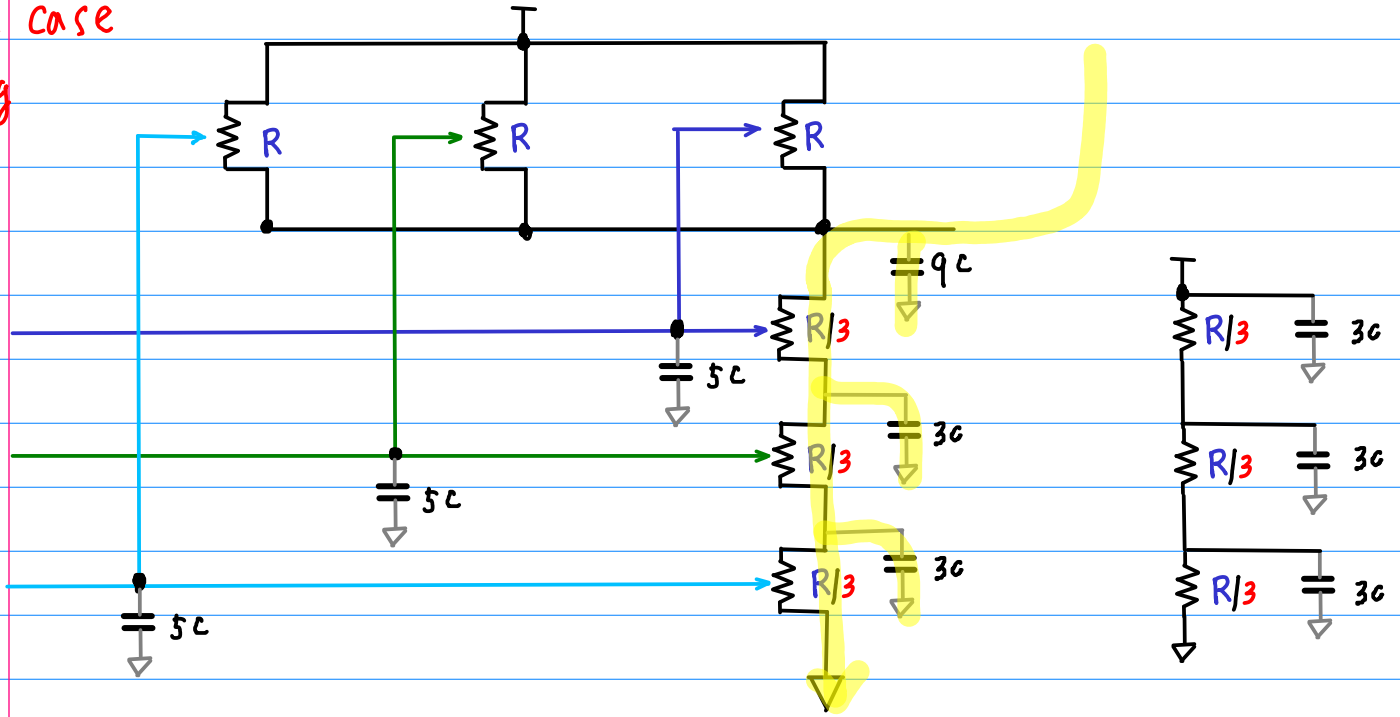




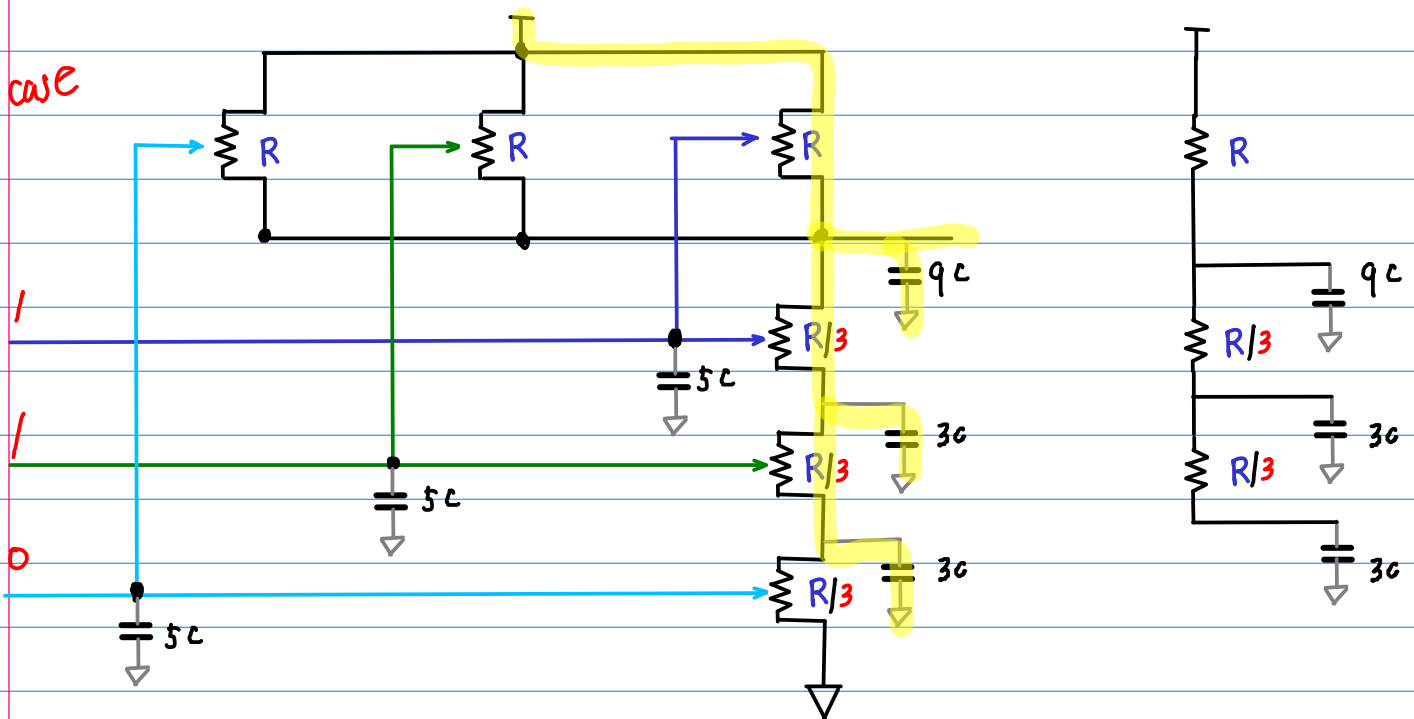




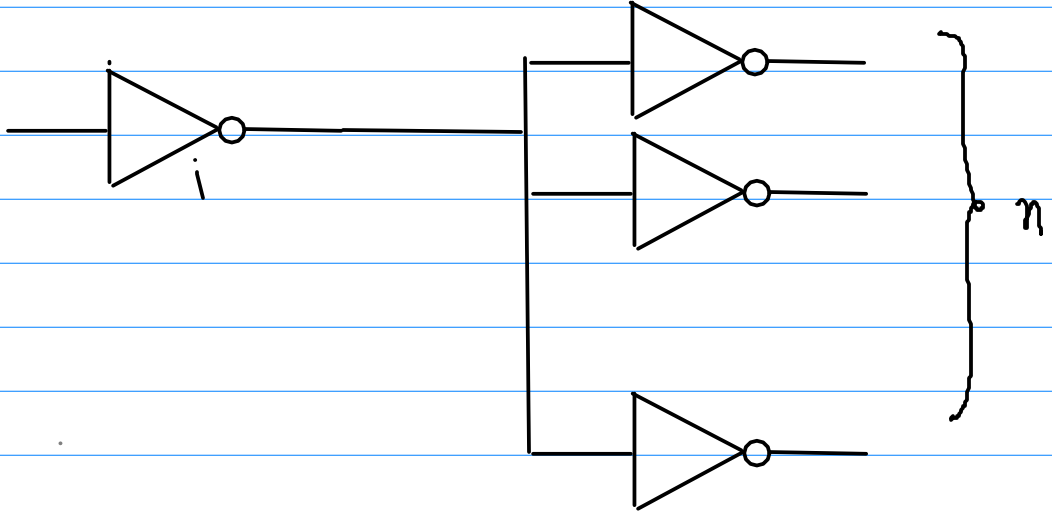
Worst case falling



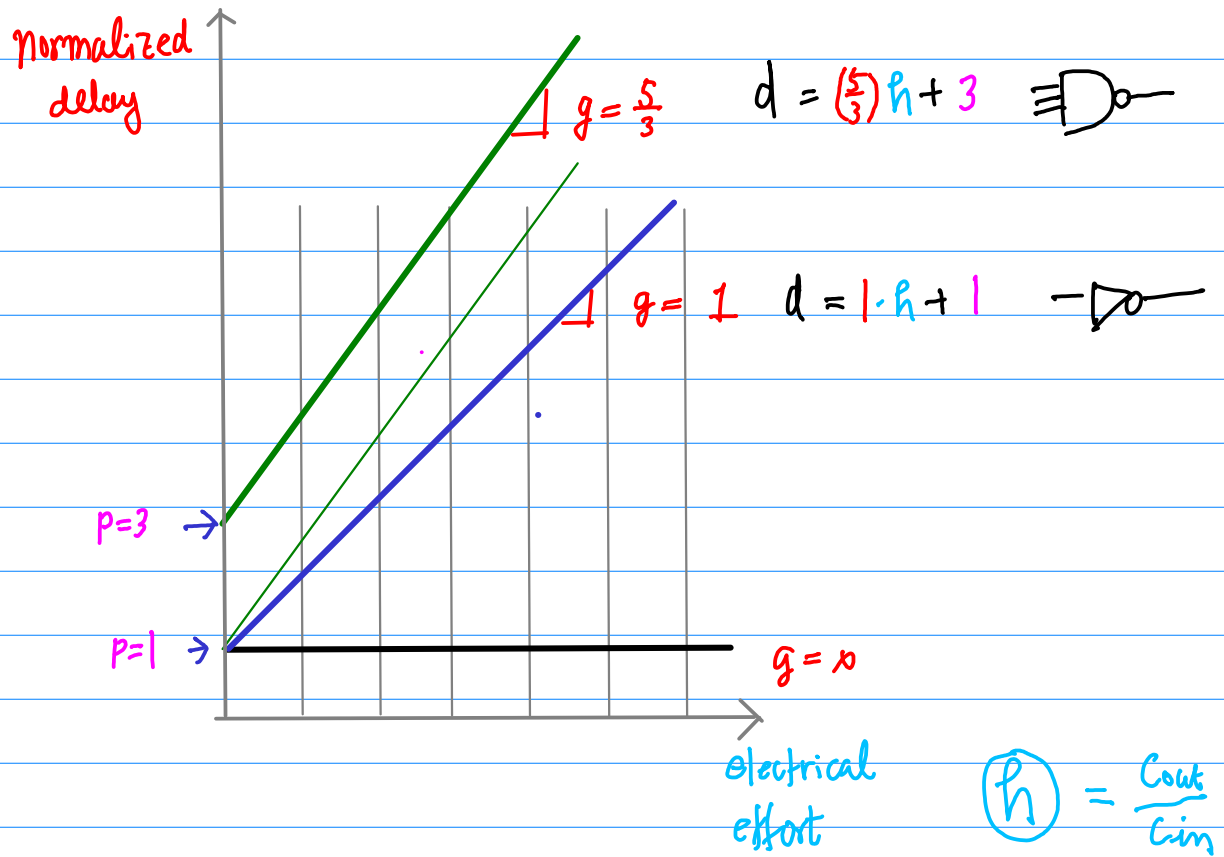
Worst case rising



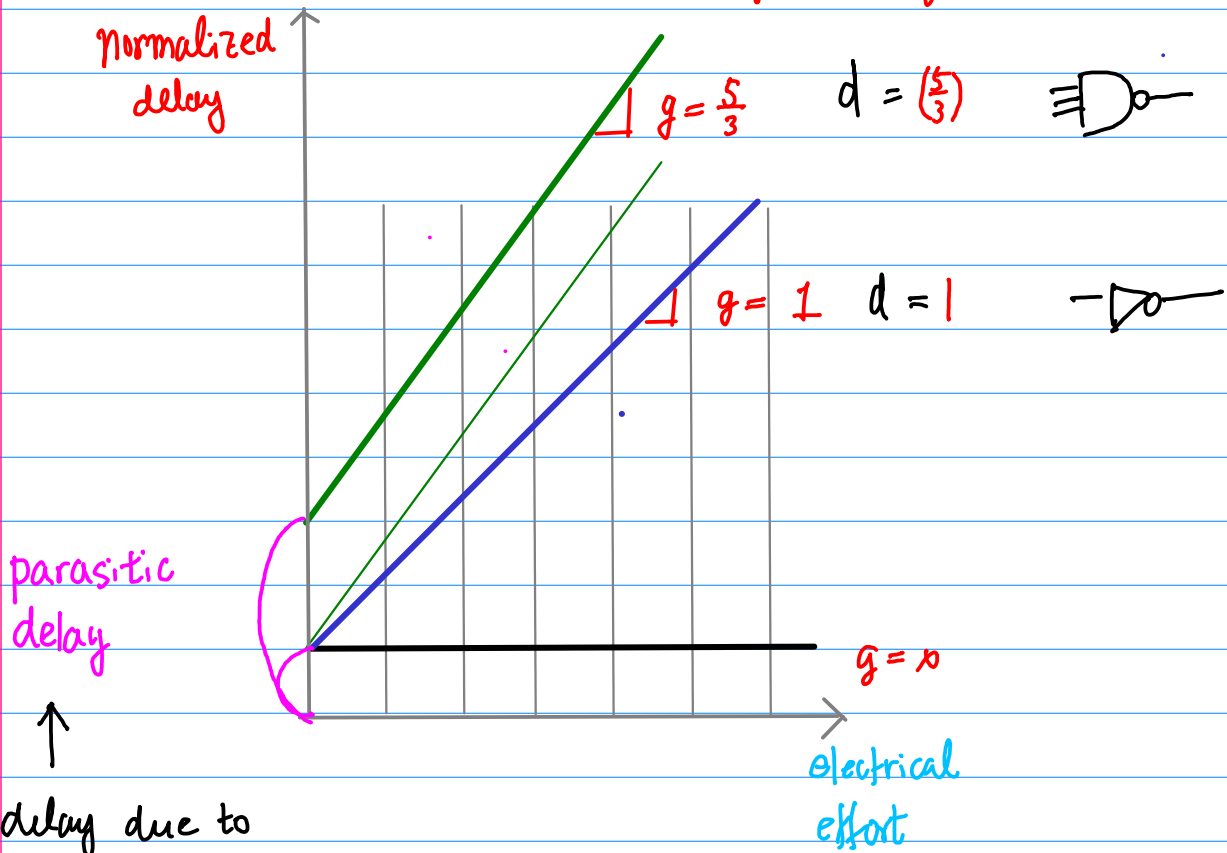
0



# Linear Delay Model



Slope : logical effort



$$d = g \cdot h + p$$

↑     ↑     ↑  
k    c    c



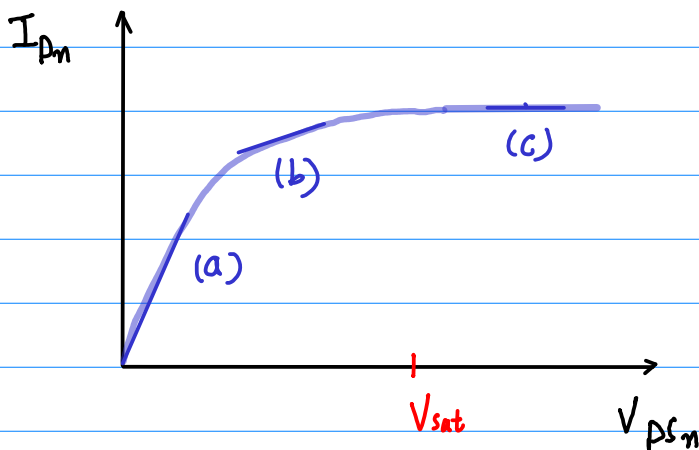


# FET RC Model

$$R_n = \frac{V_{DSn}}{I_{Dn}}$$

$$I_{Dn} \cong \beta_n (V_{GSn} - V_{Tn}) V_{DSn}$$

non-saturated	{	$R_n \cong \frac{1}{\beta_n (V_{GSn} - V_{Tn})}$	(a) small $V_{DSn}$	$V_{DSn}^2 \approx 0$
		$R_n = \frac{2}{\beta_n [2(V_{GSn} - V_{Tn}) - V_{DSn}]}$	(b)	$V_{DS} \neq 0$
saturated	{	$R_n = \frac{2V_{DSn}}{\beta_n (V_{GSn} - V_{Tn})^2}$	(c)	



$$R_n \propto \frac{1}{\beta_n} \quad \beta_n = k'_n \left(\frac{W}{L}\right)_n$$

$$R_n = \frac{\eta}{\beta_n (V_{DD} - V_{Tn})} \quad \eta = 1 \sim 6$$

↑  
largest possible  $V_{GSn}$

$$\boxed{R_n = \frac{1}{\beta_n (V_{DD} - V_{Tn})}} \quad \Omega \quad \eta = 1$$

$$C_g = C_{ox} A_g = C_{ox} W L'$$

$$C_{gs} = \frac{1}{2} C_g$$

$$C_{gd} = \frac{1}{2} C_g$$

$$C_o = C_j A_{pm}$$

$$C = \frac{C_o}{\left(1 + \frac{V_R}{\phi_o}\right)^{m_j}}$$

$$\phi_o = \left(\frac{kT}{q}\right) \ln \left[ \frac{N_d N_a}{n_i^2} \right]$$

$$A_{bot} = X W$$

$$C_{bot} = C_j X W$$

$$A_{sw} = 2(W \times x_j) + 2(X + x_j)$$

$$= x_j P_{sw}$$

$$P_{sw} = 2(W + X)$$

$$C_{sw} = C_{jsw} P_{sw}$$

$$C_{jsw} = C_j x_j$$

$$X \rightarrow X + L_o$$

$$\begin{aligned} C_{\eta} &= C_{bot} + C_{su} \\ &= C_j A_{bot} + C_{jsu} P_{su} \end{aligned}$$

$$C_{\eta} = \frac{C_j A_{bot}}{\left(1 + \frac{v}{\phi_o}\right)^{n_j}} + \frac{C_{jsu} P_{su}}{\left(1 + \frac{v}{\phi_{su}}\right)^{m_{jsu}}}$$

$$C_s = C_{qs} + C_{s\beta}$$

$$C_D = C_{qD} + C_{DB}$$

# P FET Characteristics

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$I_{Dp} = \frac{\beta_p}{2} (V_{SGp} - |V_{Tp}|)^2$$

$$\beta_p = K'_p \left(\frac{W}{L}\right)_p$$

$$K'_p = \mu_p C_{ox}$$

$$r = \frac{\mu_n}{\mu_p} = 2 \sim 3$$

$$\beta_n = K'_n \left(\frac{W}{L}\right)_n$$

$$\beta_p = K'_p \left(\frac{W}{L}\right)_p$$

$$V_{sat} = V_{SGp} - |V_{Tp}|$$

$$I_{Dp} = \frac{\beta_p}{2} [2(V_{SGp} - |V_{Tp}|) V_{SDp} - V_{SDp}^2]$$

$$I_{Dp} = \frac{\beta_p}{2} [V_{SGp} - |V_{Tp}|]^2$$

$$R_p = \frac{1}{\beta_p (V_{DD} - |V_{Tp}|)}$$

$$R_p \propto \frac{1}{\beta_p} = \frac{1}{K'_p \left(\frac{W}{L}\right)_p}$$

$$C_{gp} = C_{ox} (W L)_p$$

$$C_{gs} = \frac{1}{2} C_{gp} \approx C_{gp}$$

$$C_p = C_j A_{bot} + C_{jsw} P$$

## Fall Time Calculation

$$i = -C_{out} \frac{dV_{out}}{dt} = \frac{V_{out}}{R_n}$$

$$V_{out}(t) = V_{DD} e^{-t/\tau_n}$$

$$\tau_n = R_n C_{out}$$

$$t = \tau_n \ln \left( \frac{V_{DD}}{V_{out}} \right)$$

$$\begin{aligned} t_f = t_y - t_x &= \tau_n \ln \left( \frac{V_{DD}}{0.1 V_{DD}} \right) - \tau_n \ln \left( \frac{V_{DD}}{0.9 V_{DD}} \right) \\ &= \tau_n \ln(9) \end{aligned}$$

$$t_{HL} = t_f \cong 2.2 \tau_n$$



## Rise Time Calculation

$$i = -C_{out} \frac{dV_{out}}{dt} = \frac{V_{DD} - V_{out}}{R_p}$$

$$V_{out}(t) = V_{DD} [1 - e^{-t/\tau_p}]$$

$$\tau_p = R_p C_{out}$$

$$t = \tau_p \ln \left( \frac{V_{DD}}{V_{out}} \right)$$

$$\begin{aligned} t_f = t_r - t_u &= \tau_p \ln \left( \frac{V_{DD}}{0.1 V_{DD}} \right) - \tau_n \ln \left( \frac{V_{DD}}{0.9 V_{DD}} \right) \\ &= \tau_p \ln(9) \end{aligned}$$

$$t_{LH} = t_r \cong 2.2 \tau_p$$

# Propagation Delay

$$t_p = \frac{(t_{pf} + t_{pr})}{2}$$

$$t_{pf} = \ln(2) \cdot \tau_n$$

$$t_{pr} = \ln(2) \cdot \tau_p$$

$$t_p \cong 0.35 (\tau_n + \tau_p)$$

$$C_{out} = C_{FET} + C_L$$

$$t_r = 2.2 R_p (C_{FET} + C_L)$$

$$t_f = 2.2 R_n (C_{FET} + C_L)$$

$$t_r = t_{ro} + \alpha_p C_L$$

$$t_f = t_{ro} + \alpha_n C_L$$

$$C_L = 0 \rightarrow t_r = t_{ro} \cong 2.2 R_p C_{FET}$$

$$C_L = 0 \quad t_f = t_{ro} \cong 2.2 R_n C_{FET}$$

$$\alpha_p = 2.2 R_p = \frac{2.2}{\beta_p (V_{DD} - |V_{Tp}|)}$$

$$\alpha_n = 2.2 R_n = \frac{2.2}{\beta_n (V_{DD} - |V_{Tn}|)}$$

$$\beta_p = k'_p \left(\frac{W}{L}\right)_p$$

$$\beta_n = k'_n \left(\frac{W}{L}\right)_n$$