DTFT of Periodic Pulse Functions (3B)

- DTFT of Periodic Pulse Functions

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8/5/13

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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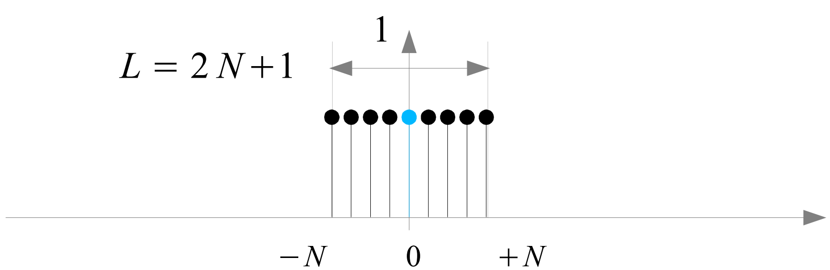
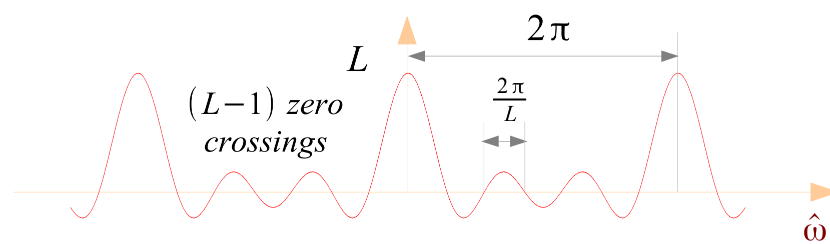
- DTFT of a Rectangular Pulse
- DTFT of a Shifted Rectangular Pulse
- Spectrum Plots of the DTFT of a Rectangular Pulse
- Spectrum Plots of the DTFT of a Shifted Rectangular Pulse

DTFT

Discrete Time Fourier Transform DTFT

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega}n} \quad \Longleftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\hat{\omega}})\, e^{+j\hat{\omega}n}$$





DTFT (Discrete Time Fourier Transform)

$$X(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)} = L\,D_L(e^{j\hat{\omega}})$$

$$= L \cdot \textbf{diric}(\hat{\omega}, L)$$

Discrete Time Fourier Transform

DTFT

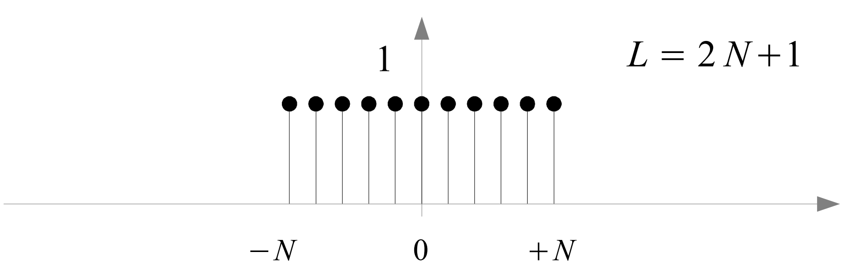
$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n]\, e^{-j\hat{\omega}n} \quad \Longleftrightarrow \quad x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(e^{j\hat{\omega}})\, e^{+j\hat{\omega}n}$$

$$\begin{aligned}
X(e^{j\hat{\omega}}) &= \sum_{n=-N}^{+N} e^{-j\hat{\omega}n}\, x[n] \\
&= \left\{e^{+j\hat{\omega}N} + \cdots + e^{-j\hat{\omega}N}\right\} \\
&= e^{+j\hat{\omega}N}\left\{1 + \cdots + e^{-j\hat{\omega}2N}\right\} \\
&= e^{+j\hat{\omega}N}\,\frac{1 - e^{-j\hat{\omega}(2N+1)}}{1 - e^{-j\hat{\omega}}}
\end{aligned}$$

$$\begin{aligned}
&= e^{+j\hat{\omega}N}\,\frac{e^{-j\hat{\omega}(2N+1)/2}}{e^{-j\hat{\omega}/2}}\,\frac{e^{+j\hat{\omega}(2N+1)/2} - e^{-j\hat{\omega}(2N+1)/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \\
&= \frac{e^{+j\hat{\omega}(2N+1)/2} - e^{-j\hat{\omega}(2N+1)/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} = \frac{\sin(\hat{\omega}(2N+1)/2)}{\sin(\hat{\omega}/2)}
\end{aligned}$$

$$\Rightarrow \quad X(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)} = L\,D_L(e^{j\hat{\omega}})$$

$$= L \cdot \textbf{diric}(\hat{\omega}, L)$$

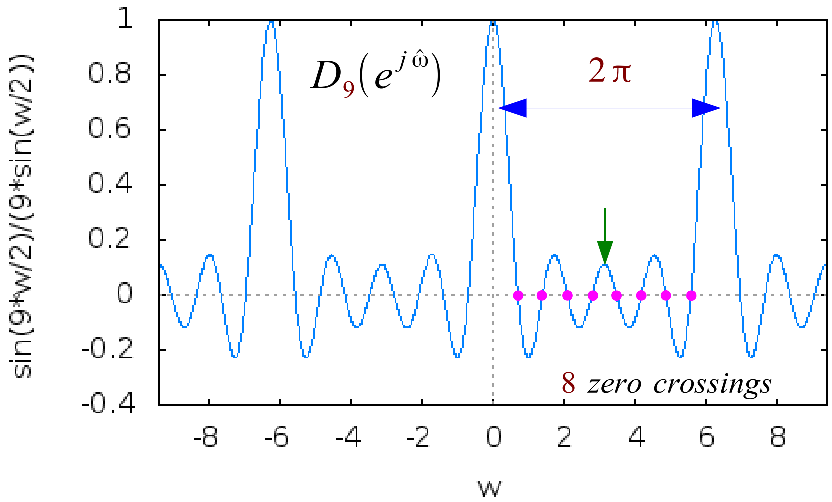


$L = 2N+1$

Dirichlet Function

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L\sin(\hat{\omega}/2)}$$

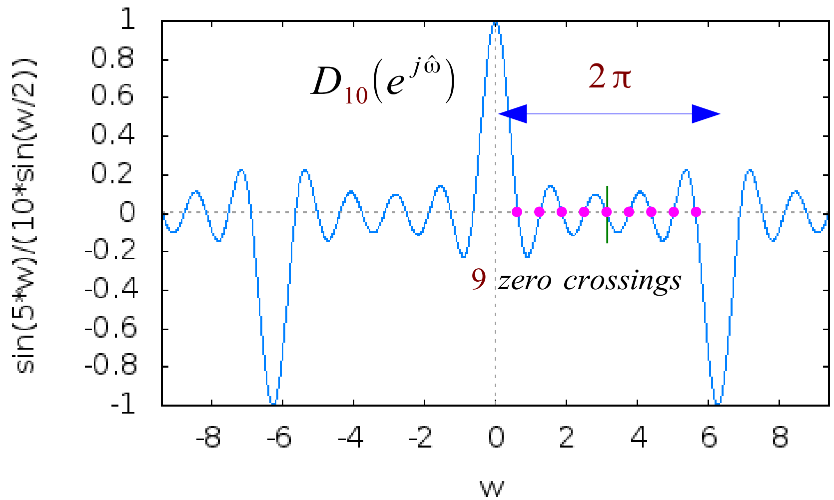
Dirichlet Functions



$$D_9(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\,9/2)}{9\sin(\hat{\omega}/2)}$$ 8 *zero crossings*

$$D_{11}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\,11/2)}{11\sin(\hat{\omega}/2)}$$ 10 *zero crossings*

$$D_{13}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\,13/2)}{13\sin(\hat{\omega}/2)}$$ 12 *zero crossings*

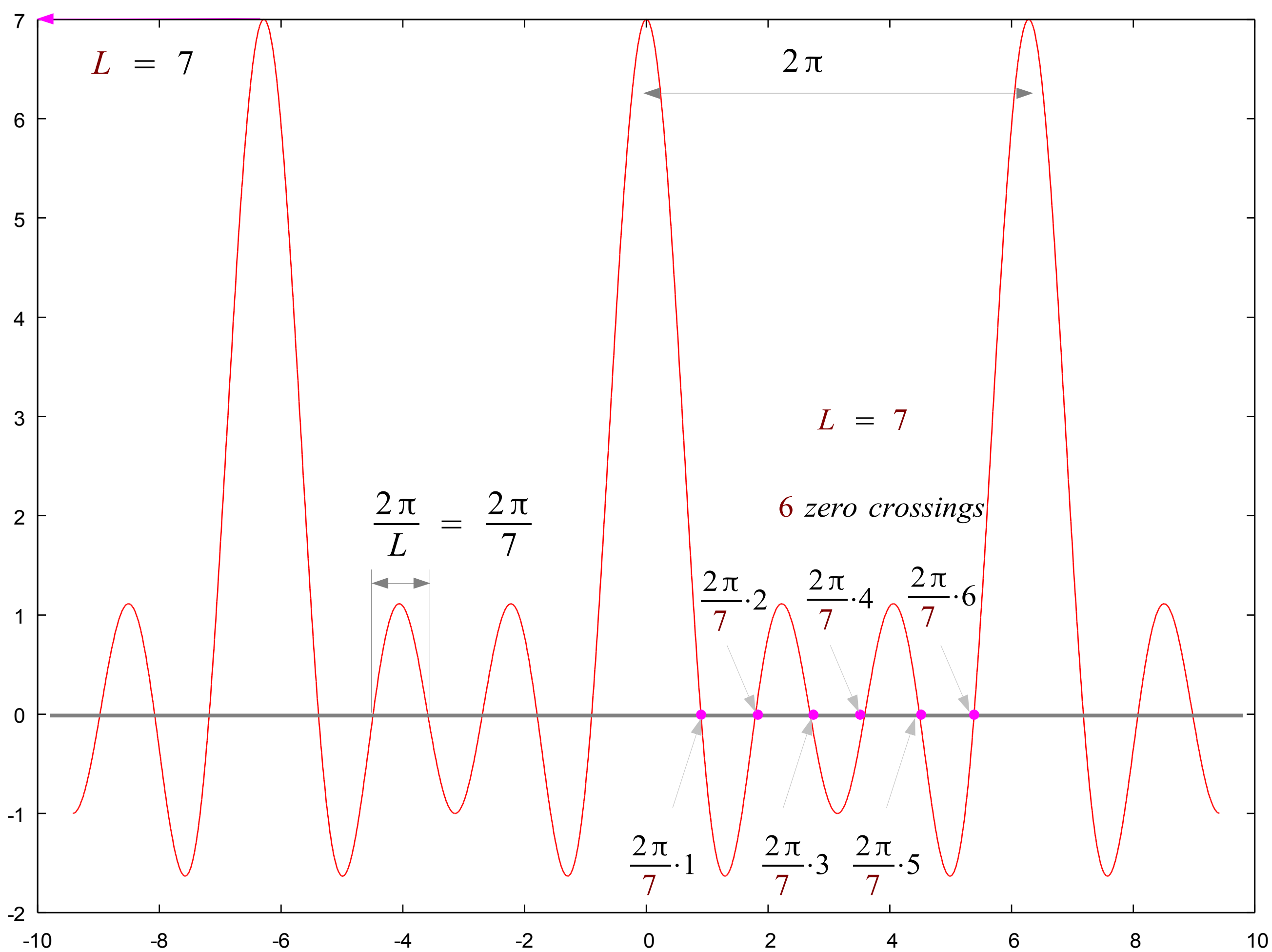


$$D_{10}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\,10/2)}{10\sin(\hat{\omega}/2)}$$ 9 *zero crossings*

$$D_{12}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\,12/2)}{12\sin(\hat{\omega}/2)}$$ 11 *zero crossings*

$$D_{14}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}\,14/2)}{14\sin(\hat{\omega}/2)}$$ 13 *zero crossings*

$L\,D_L(e^{j\hat{\omega}})$ (1)

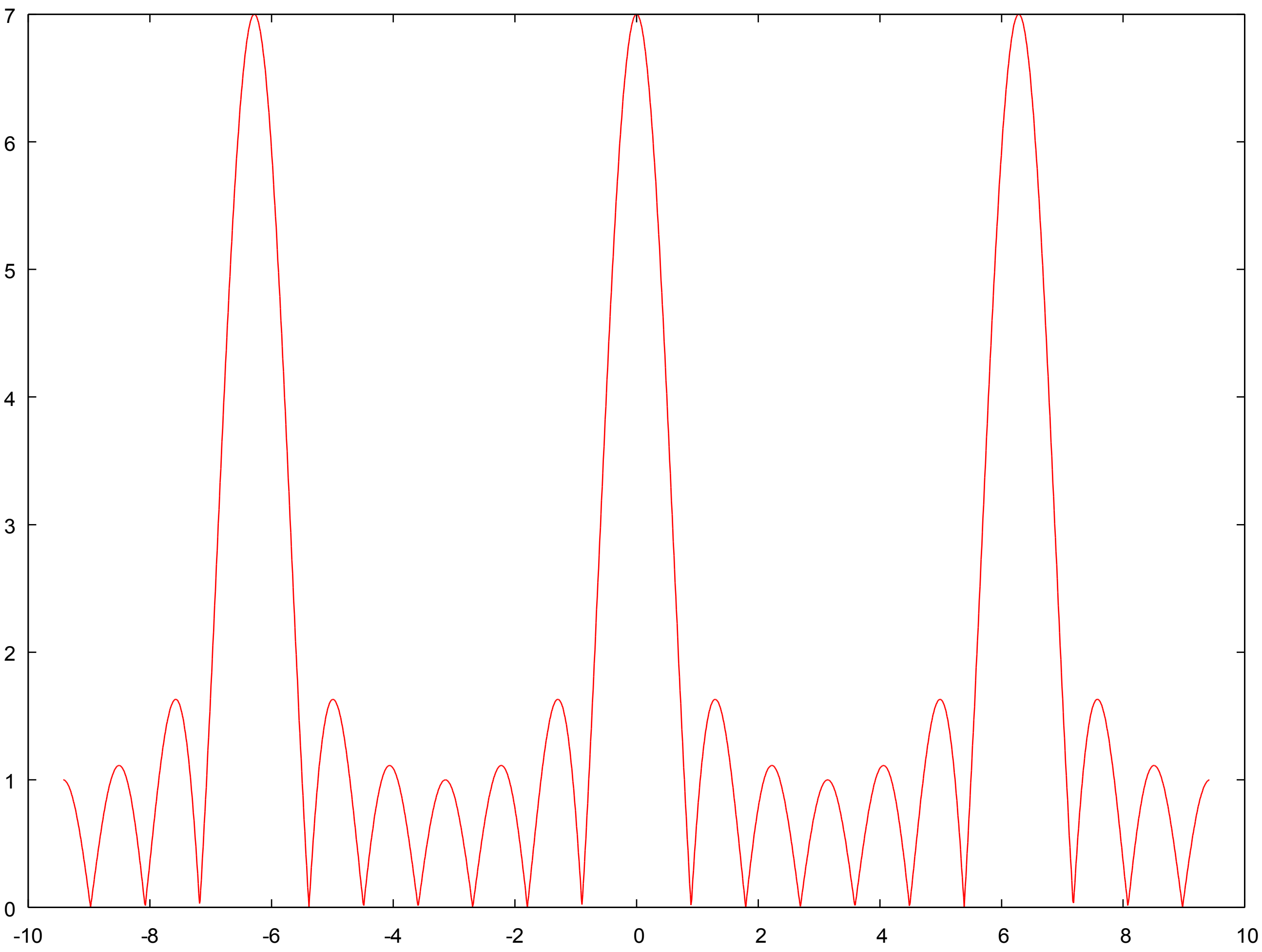


$$X(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega} L/2)}{\sin(\hat{\omega}/2)}$$

$$= L\,D_L(e^{j\hat{\omega}})$$

$$= L \cdot \textbf{diric}(\hat{\omega},\ L)$$

Magnitude Response of L $D_L(e^{j\hat{\omega}})$

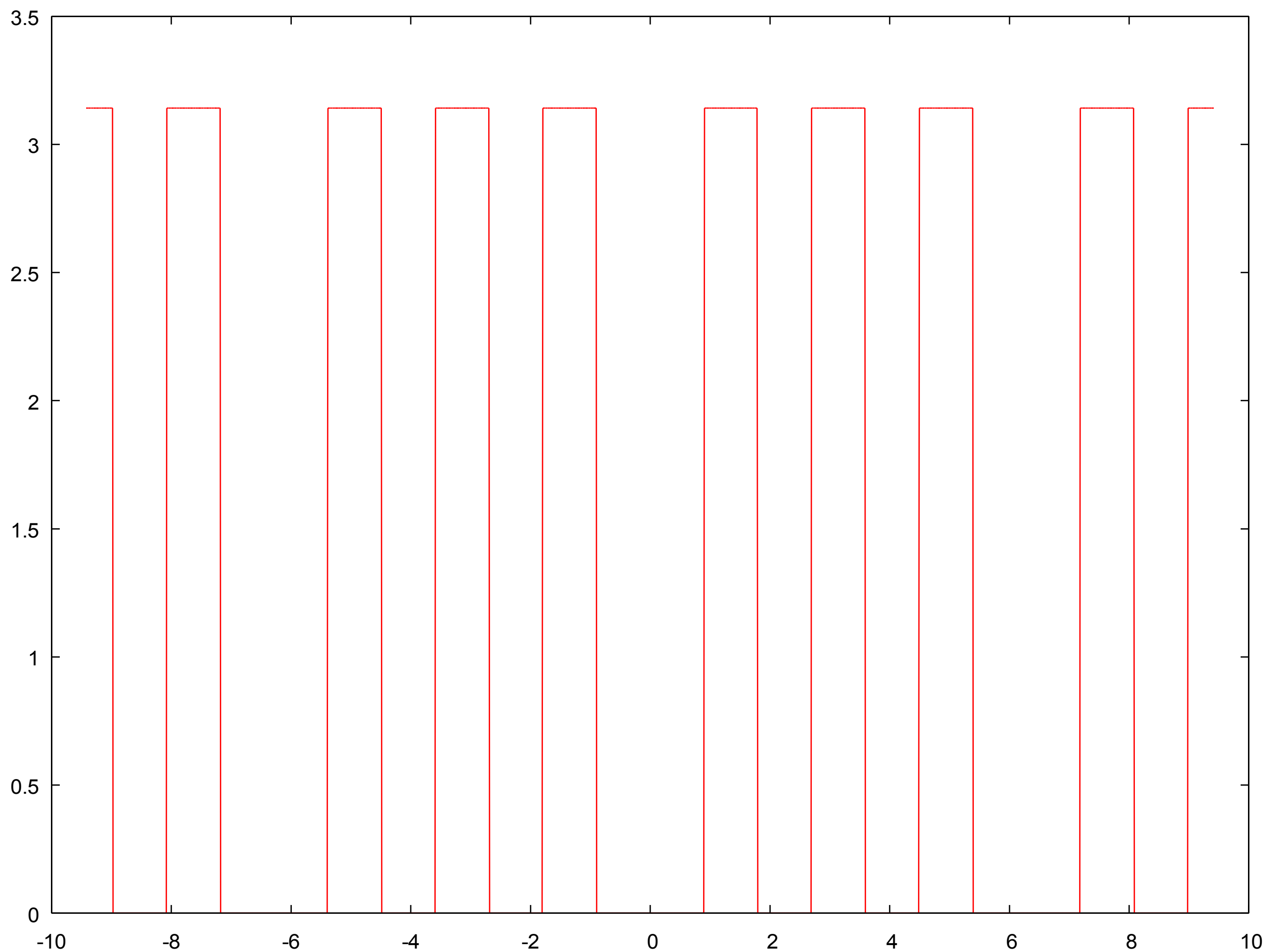


$$X(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega} L/2)}{\sin(\hat{\omega}/2)}$$

$$= L\, D_L(e^{j\hat{\omega}})$$

$$= L \cdot \boldsymbol{diric}(\hat{\omega},\ L)$$

Phase Response of L $D_L(e^{j\hat{\omega}})$



$$X(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)}$$

$$= L\,D_L(e^{j\hat{\omega}})$$

$$= L \cdot \textbf{diric}(\hat{\omega},\ L)$$

$\text{Rect}_N[n-N]$ DTFT

Discrete Time Fourier Transform DTFT

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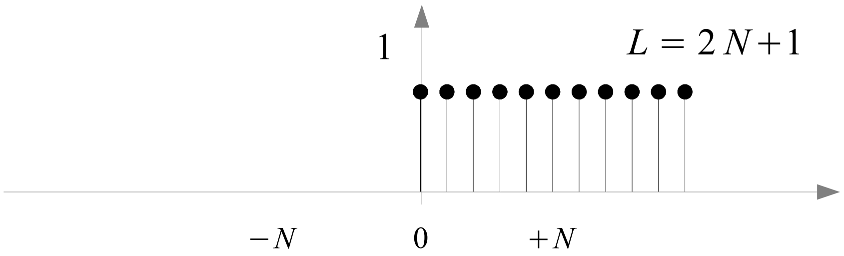

Dirichlet Function

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega} L/2)}{L\sin(\hat{\omega}/2)}$$

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References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] G. Beale, http://teal.gmu.edu/~gbeale/ece_220/fourier_series_02.html
[4] C. Langton, http://www.complextoreal.com/chapters/fft1.pdf
[5] M. J. Roberts, Fundamentals of Signals and Systems