

# DTFT of Periodic Pulse Functions (3B)

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- DTFT of Periodic Pulse Functions

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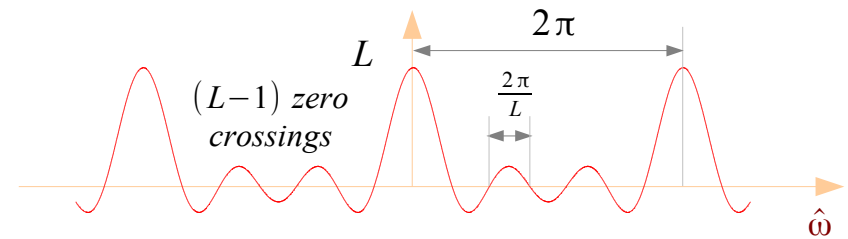
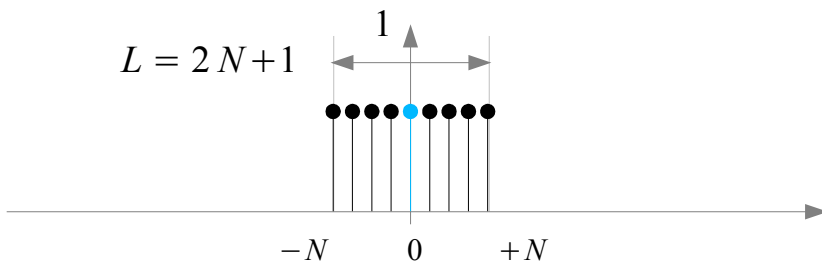
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- DTFT of a Rectangular Pulse
  - DTFT of a Shifted Rectangular Pulse
  - Spectrum Plots of the DTFT of a Rectangular Pulse
  - Spectrum Plots of the DTFT of a Shifted Rectangular Pulse

# DTFT

## Discrete Time Fourier Transform

## DTFT

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega}n} \quad \longleftrightarrow \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\hat{\omega}}) e^{+j\hat{\omega}n}$$



## DTFT (Discrete Time Fourier Transform)

$$\begin{aligned} X(e^{j\hat{\omega}}) &= \frac{\sin(\hat{\omega} L/2)}{\sin(\hat{\omega}/2)} = L D_L(e^{j\hat{\omega}}) \\ &= L \cdot \text{diric}(\hat{\omega}, L) \end{aligned}$$

# DTFT of Rect<sub>N</sub>[n]

## Discrete Time Fourier Transform

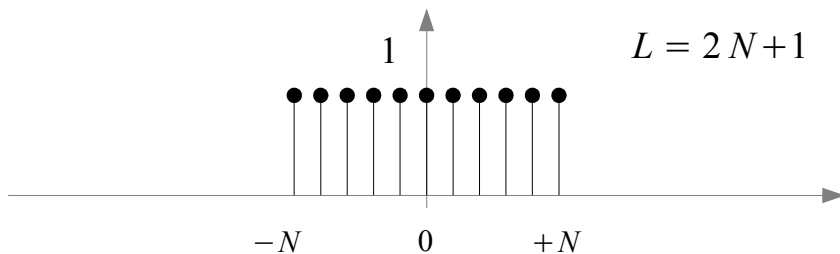
## DTFT

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega}n} \quad \longleftrightarrow \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\hat{\omega}}) e^{+j\hat{\omega}n}$$

$$\begin{aligned} X(e^{j\hat{\omega}}) &= \sum_{n=-N}^{+N} e^{-j\hat{\omega}n} x[n] \\ &= \{e^{+j\hat{\omega}N} + \dots + e^{-j\hat{\omega}N}\} \\ &= e^{+j\hat{\omega}N} \{1 + \dots + e^{-j\hat{\omega}2N}\} \\ &= e^{+j\hat{\omega}N} \frac{1 - e^{-j\hat{\omega}(2N+1)}}{1 - e^{-j\hat{\omega}}} \end{aligned}$$

$$\begin{aligned} &= e^{+j\hat{\omega}N} \frac{e^{-j\hat{\omega}(2N+1)/2} e^{+j\hat{\omega}(2N+1)/2} - e^{-j\hat{\omega}(2N+1)/2}}{e^{-j\hat{\omega}/2} e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} \\ &= \frac{e^{+j\hat{\omega}(2N+1)/2} - e^{-j\hat{\omega}(2N+1)/2}}{e^{+j\hat{\omega}/2} - e^{-j\hat{\omega}/2}} = \frac{\sin(\hat{\omega}(2N+1)/2)}{\sin(\hat{\omega}/2)} \end{aligned}$$

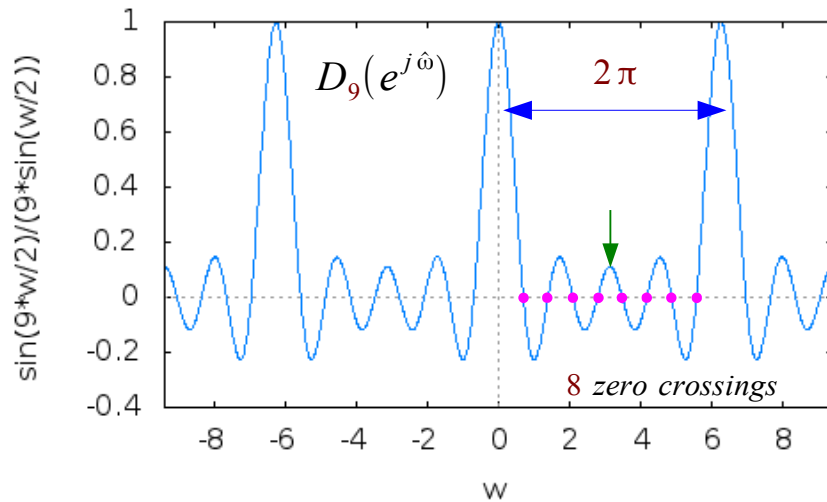
$$\begin{aligned} \Rightarrow X(e^{j\hat{\omega}}) &= \frac{\sin(\hat{\omega}L/2)}{\sin(\hat{\omega}/2)} = L D_L(e^{j\hat{\omega}}) \\ &= L \cdot \text{diric}(\hat{\omega}, L) \end{aligned}$$



## Dirichlet Function

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L \sin(\hat{\omega}/2)}$$

# Dirichlet Functions



$$D_9(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}9/2)}{9\sin(\hat{\omega}/2)}$$

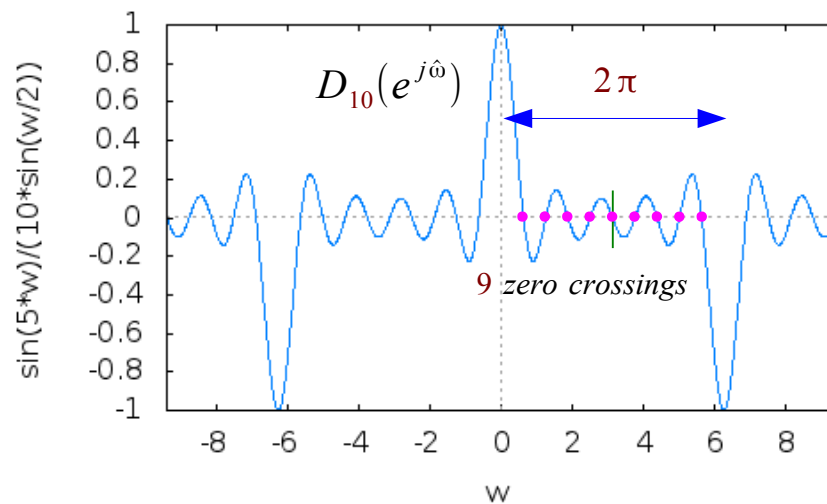
8 zero crossings

$$D_{11}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}11/2)}{11\sin(\hat{\omega}/2)}$$

10 zero crossings

$$D_{13}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}13/2)}{13\sin(\hat{\omega}/2)}$$

12 zero crossings



$$D_{10}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}10/2)}{10\sin(\hat{\omega}/2)}$$

9 zero crossings

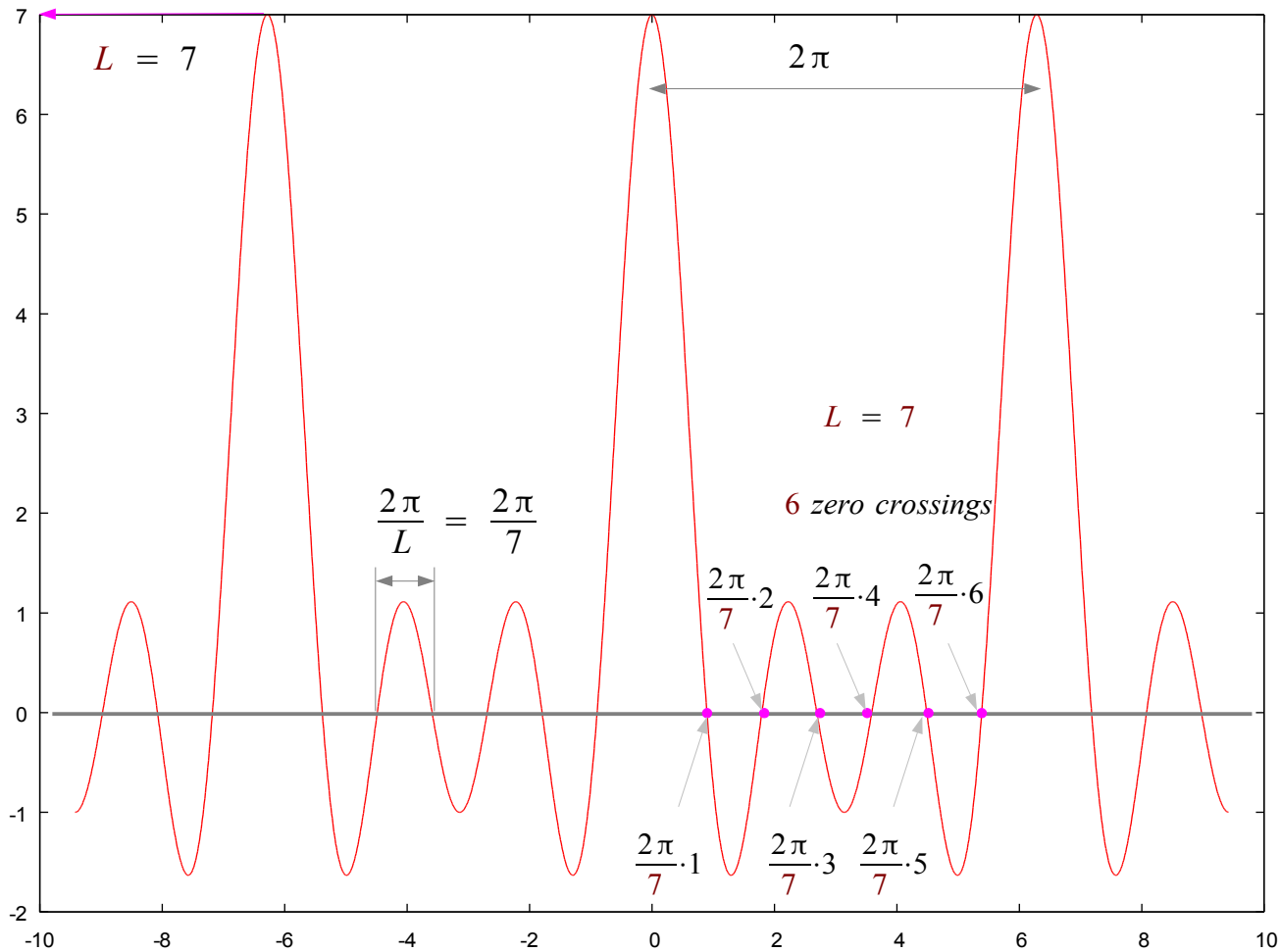
$$D_{12}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}12/2)}{12\sin(\hat{\omega}/2)}$$

11 zero crossings

$$D_{14}(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}14/2)}{14\sin(\hat{\omega}/2)}$$

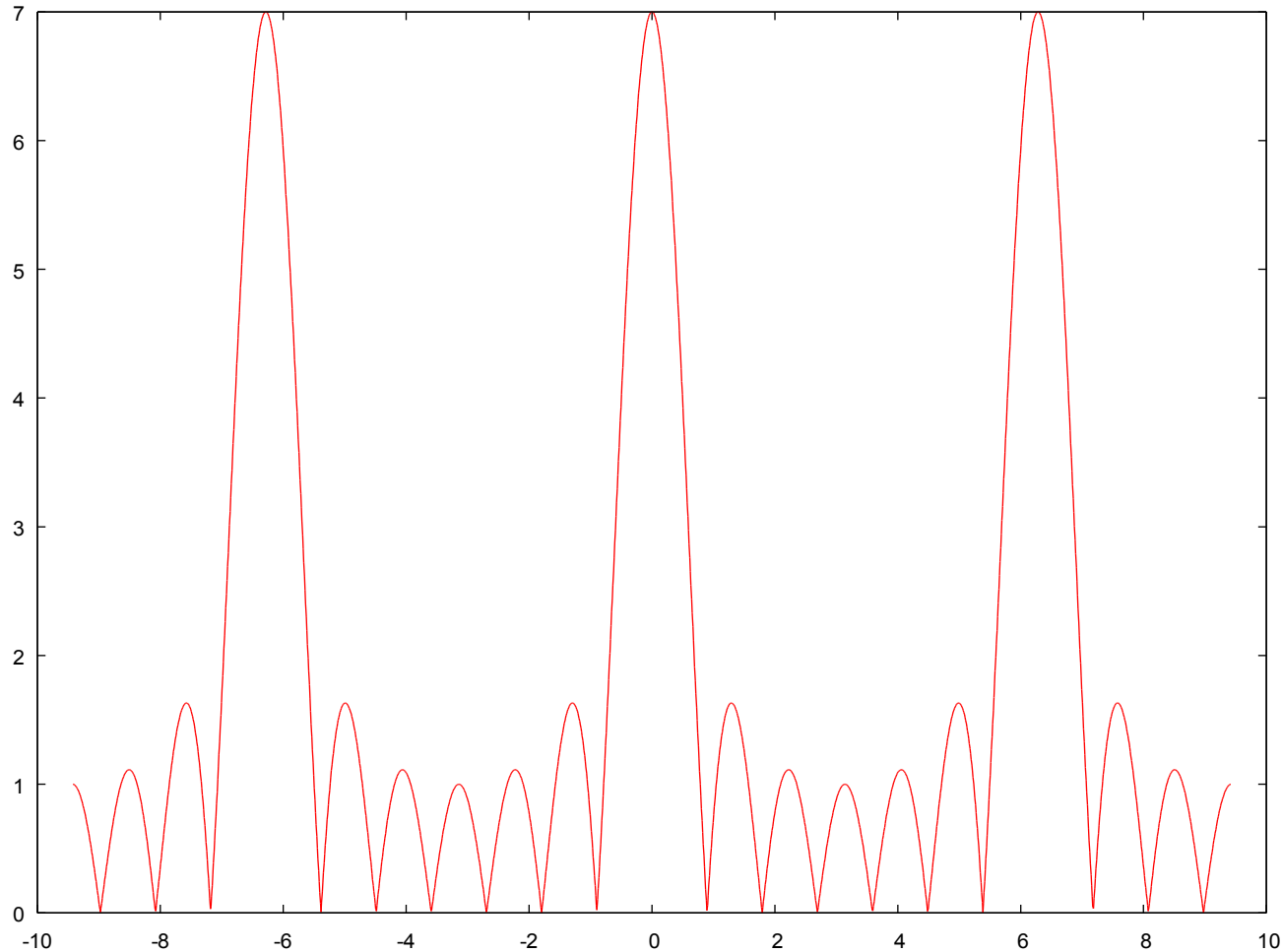
13 zero crossings

# $L D_L(e^{j\hat{\omega}})$ (1)



$$\begin{aligned}
 X(e^{j\hat{\omega}}) &= \frac{\sin(\hat{\omega} L/2)}{\sin(\hat{\omega}/2)} \\
 &= L D_L(e^{j\hat{\omega}}) \\
 &= L \cdot \text{diric}(\hat{\omega}, L)
 \end{aligned}$$

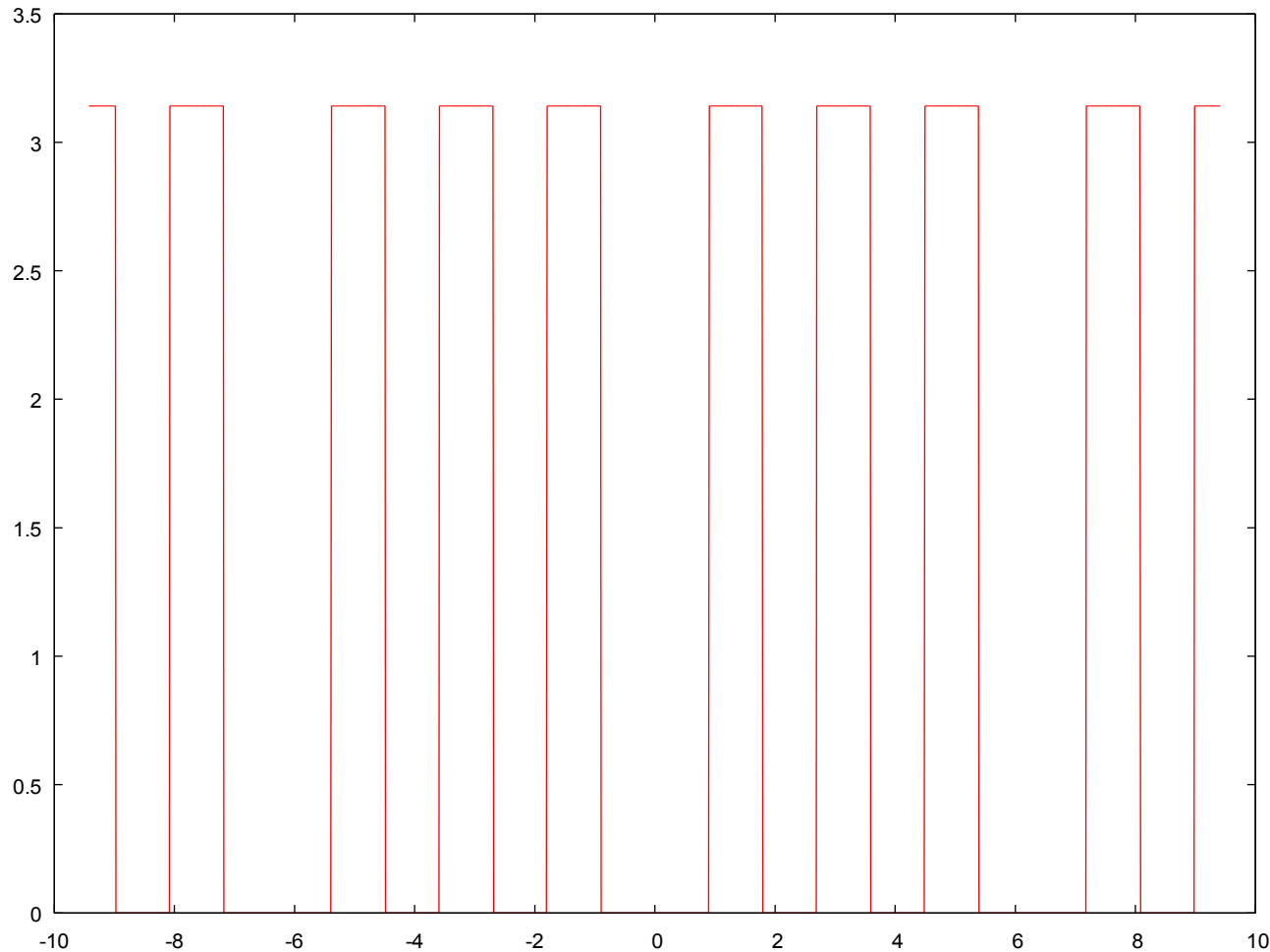
# Magnitude Response of $L D_L(e^{j\hat{\omega}})$



$$\begin{aligned} X(e^{j\hat{\omega}}) &= \frac{\sin(\hat{\omega} L/2)}{\sin(\hat{\omega}/2)} \\ &= L D_L(e^{j\hat{\omega}}) \\ &= L \cdot \text{diric}(\hat{\omega}, L) \end{aligned}$$



# Phase Response of $L D_L(e^{j\hat{\omega}})$



$$\begin{aligned} X(e^{j\hat{\omega}}) &= \frac{\sin(\hat{\omega} L/2)}{\sin(\hat{\omega}/2)} \\ &= L D_L(e^{j\hat{\omega}}) \\ &= L \cdot \mathit{diric}(\hat{\omega}, L) \end{aligned}$$

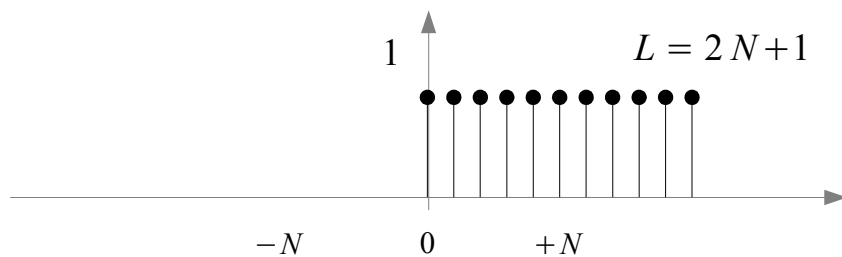


# Rect<sub>N</sub>[n-N] DTFT

## Discrete Time Fourier Transform

## DTFT

$$X(e^{j\hat{\omega}}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega}n} \quad \longleftrightarrow \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\hat{\omega}}) e^{+j\hat{\omega}n}$$



## Dirichlet Function

$$D_L(e^{j\hat{\omega}}) = \frac{\sin(\hat{\omega}L/2)}{L \sin(\hat{\omega}/2)}$$



## References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] G. Beale, [http://teal.gmu.edu/~gbeale/ece\\_220/fourier\\_series\\_02.html](http://teal.gmu.edu/~gbeale/ece_220/fourier_series_02.html)
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- [5] M. J. Roberts, Fundamentals of Signals and Systems