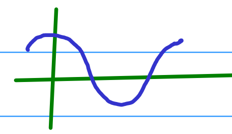
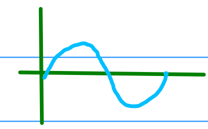
Second Order ODEs (H.1)

20150626

Taylor Series

$f(x) = \cos x$

$$\begin{aligned}
f'(x) &= -\sin x & f'(0) &= -\sin 0 = 0 \\
f^{(2)}(x) &= -\cos x & f^{(2)}(0) &= -\cos 0 = -1 \\
f^{(3)}(x) &= +\sin x & f^{(3)}(0) &= +\sin 0 = 0 \\
f^{(4)}(x) &= +\cos x & f^{(4)}(0) &= +\cos 0 = +1 \\
f^{(5)}(x) &= -\sin x & f^{(5)}(0) &= -\sin 0 = 0 \\
f^{(6)}(x) &= -\cos x & f^{(6)}(0) &= -\cos 0 = -1 \\
f^{(7)}(x) &= +\sin x & f^{(7)}(0) &= +\sin 0 = 0 \\
f^{(8)}(x) &= +\cos x & f^{(8)}(0) &= +\cos 0 = +1 \\
&\vdots & &\vdots
\end{aligned}$$

$x=0$ 근방에서 Taylor series

$$\boxed{f(x) = f(0) + \frac{f^{(1)}(0)}{1!}x^1 + \frac{f^{(2)}(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4}$$

$$\cos x = +1 + \frac{0}{1!}x^1 + \frac{-1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{-1}{4!}x^4$$

$$\begin{aligned}
f'(0) &= -\sin 0 = 0 \\
f^{(2)}(0) &= -\cos 0 = -1 \\
f^{(3)}(0) &= +\sin 0 = 0 \\
f^{(4)}(0) &= +\cos 0 = +1
\end{aligned}$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \frac{x^{12}}{12!}$$

0 의 근방에서

Series (급수)

Advanced Eng Math in plain view

ODE

2nd Order ODE : 2P.pdf Complex numbers
: Linear Equation (1A.pdf)
: Reduction of Order (....)

Zill & Wright Sec 3.3, 3.2

(%i1) `f1(x) := 1;`

(%o1) $\mathrm{f1}(x):=1$

(%i2) `f2(x) := f1(x) - x^2/factorial(2);`

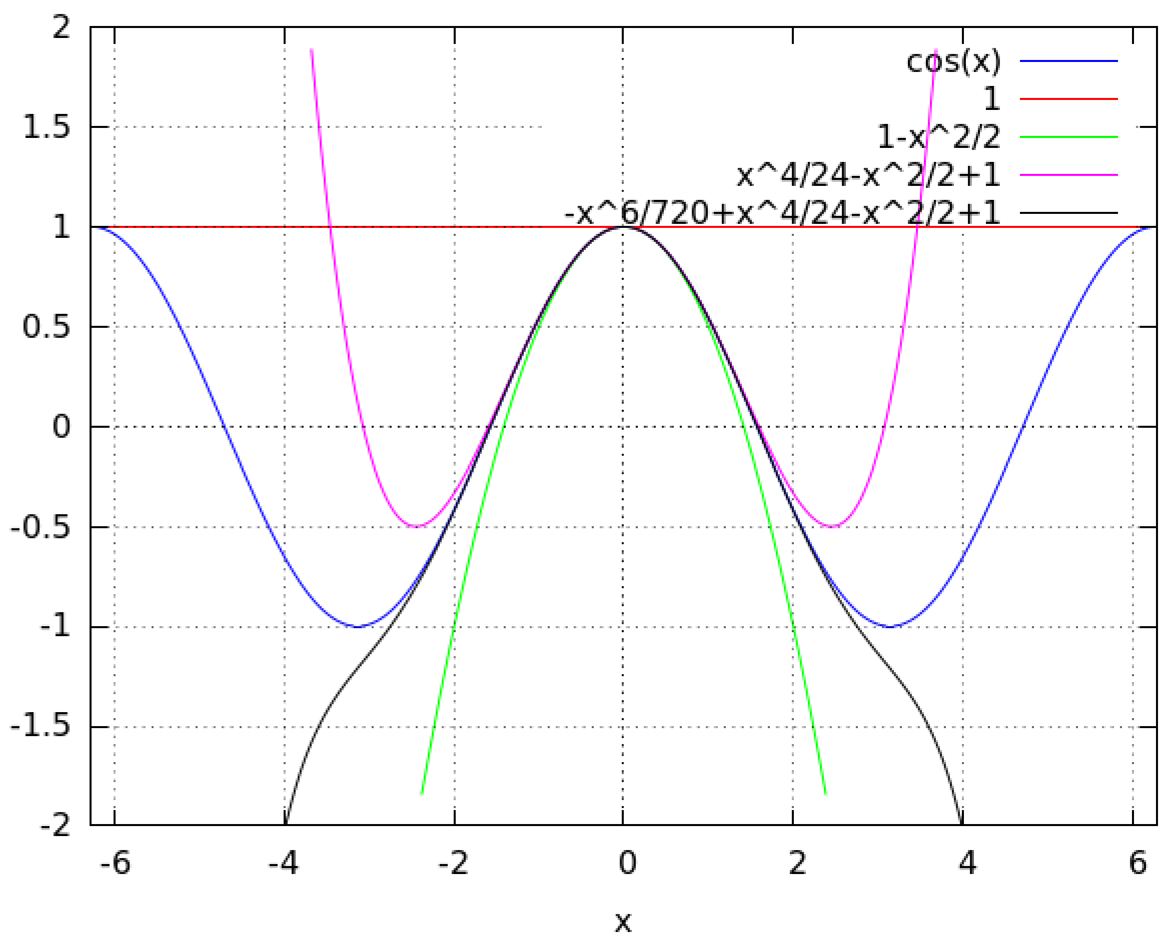
(%o2) $\mathrm{f2}(x):=\mathrm{f1}(x)-\frac{x^2}{2\,!}$

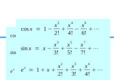
(%i3) `f2(x);`

(%o3) $1-\frac{x^2}{2}$

(%i4) `f3(x) := f2(x) + x^4 / factorial(4);`

(%o4) $\mathrm{f3}(x):=\mathrm{f2}(x)+\frac{x^4}{4\,!}$



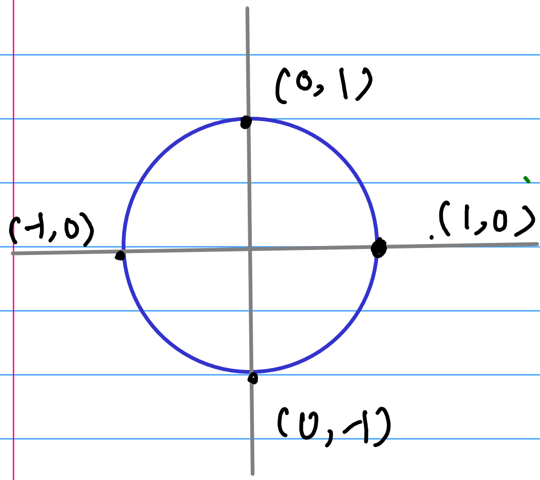


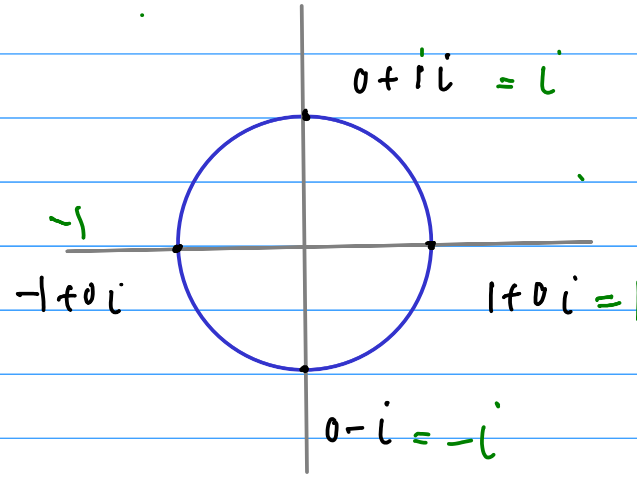
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

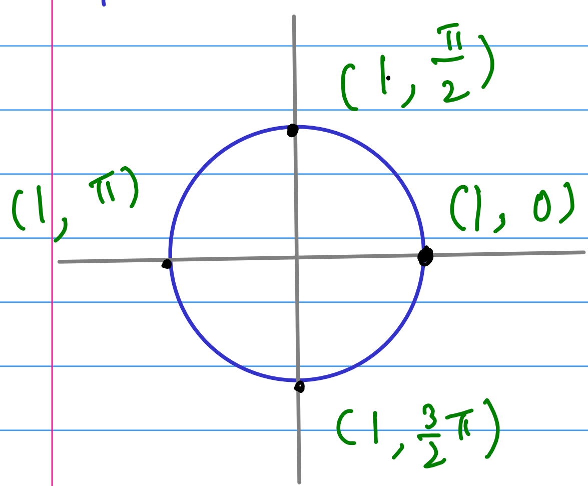
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

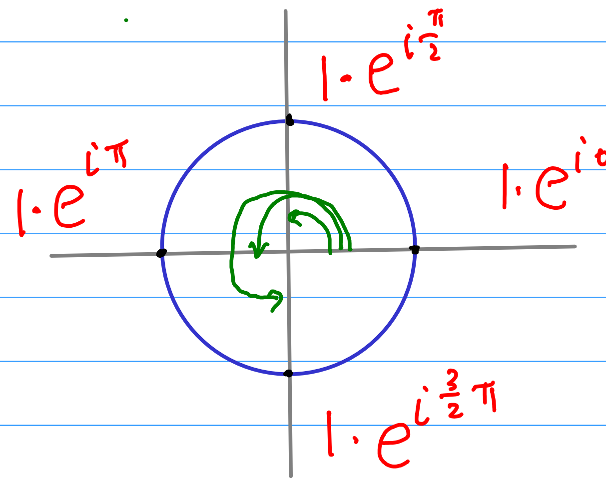
직교좌표





극좌표





$$x + yi$$
$$= re^{i\theta}$$

$$e^{i0} = \cos 0 + i\cancel{\sin 0}$$
$$e^{i\frac{\pi}{2}} = \cancel{\cos\frac{\pi}{2}} + i\sin\frac{\pi}{2}$$
$$e^{i\pi} = \cos\pi + \cancel{i\sin\pi}$$
$$e^{i\frac{3}{2}\pi} - \cancel{\cos\frac{3}{2}\pi} + i\sin\frac{3}{2}\pi$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

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Paul's online math note

Differential Equation
Second Order :
- Two distinct real roots
- Complex conjugate roots
- repeated roots
- Reduction of Orders
- Undetermined Coefficients

$$a y'' + b y' + c y = 0$$

homogeneous eq

y_h : homogeneous solution

$$a y'' + b y' + c y = \underbrace{g(x)}_{\text{x의 함수}}$$

y_p : particular solution

① 3.4 Undetermined Coefficients

$(e^x)' = e^x$

$(e^x)'' = e^x$

$(e^x)'' - 2(e^x)' + (e^x) = 0$ for all x

$y'' - 2y' + y = 0$

$m^2 - 2m + 1 = 0$ $(m-1)^2 = 0$

$m = 1$ $y_h = c_1 e^x + c_2 x e^x$

$y'' - 2y' + y = e^{3x}$

$y = Ae^{3x}$ $y' = 3Ae^{3x}$ $y'' = 9Ae^{3x}$

$9Ae^{3x} - 2 \cdot 3Ae^{3x} + Ae^{3x} = e^{3x}$

$(\underline{9A - 6A + A})e^{3x} = e^{3x}$

$4A = 1$ $A = \frac{1}{4}$

$y_p = \frac{1}{4} e^{3x}$

When the multiplication rule must be considered

$$ay'' + by' + cy = \boxed{e^{kt}}$$

$$am^2 + bm + c = 0$$

real distinct m_1, m_2

$$e^{m_1 t} \qquad e^{m_2 t} \qquad e^{kt}$$

$k = m_1$, $k = m_2$

$$ay'' + by' + cy = e^{\alpha t}\cos(kt)$$

$$e^{\alpha t}\sin(kt)$$

complex conjugate roots

$$e^{\alpha + i\beta t}, \quad e^{\alpha - i\beta t}$$

$$e^{\alpha t}\cos(\beta t), \quad e^{\alpha t}\sin(\beta t)$$

$\beta = k$

$$4y'' + 16y' + 17y = e^{-2t} \sin\left(\frac{t}{2}\right) + 6t\cos\left(\frac{t}{2}\right)$$

$$4y'' + 16y' + 17y = 0$$

$$4m^2 + 2 \cdot 8\, m + 17 = 0 \qquad m = \frac{-8 \pm \sqrt{64 - 68}}{4}$$

$$m = -2 \pm \frac{1}{2} i$$

$$y_c = c_1 e^{(-2 + \frac{1}{2} i)t} + c_2 e^{(-2 - \frac{1}{2} i)t}$$

$$= c_3 e^{-2t} \cos\left(\frac{1}{2} t\right) + c_4 e^{-2t} \sin\left(\frac{1}{2} t\right)$$

$$4y'' + 16y' + 17y = e^{-2t}\sin\left(\frac{t}{2}\right) + 6t\cos\left(\frac{t}{2}\right)$$

$$e^{-2t}\left(A\cos\left(\frac{t}{2}\right) + B\sin\left(\frac{t}{2}\right)\right) + (Ct+D)\cos\left(\frac{t}{2}\right) \quad (Et+F)\sin\left(\frac{t}{2}\right)$$

$$y_c = c_3\, e^{-2t}\cos\left(\frac{1}{2}t\right) + c_4\, e^{-2t}\sin\left(\frac{1}{2}t\right)$$

$$t\, e^{-2t}\left(A\cos\left(\frac{t}{2}\right) + B\sin\left(\frac{t}{2}\right)\right) + (Ct+D)\cos\left(\frac{t}{2}\right) \quad (Et+F)\sin\left(\frac{t}{2}\right)$$

$$y_c = c_1 e^{(-2+\frac{1}{2}i)t} + c_2 e^{(-2-\frac{1}{2}i)t}$$

$c_1 = \frac{1}{2}$, $c_2 = \frac{1}{2}$

$$y_3 = \frac{1}{2} e^{(-2+\frac{1}{2}i)t} + \frac{1}{2} c_2 e^{(-2-\frac{1}{2}i)t}$$ a solution

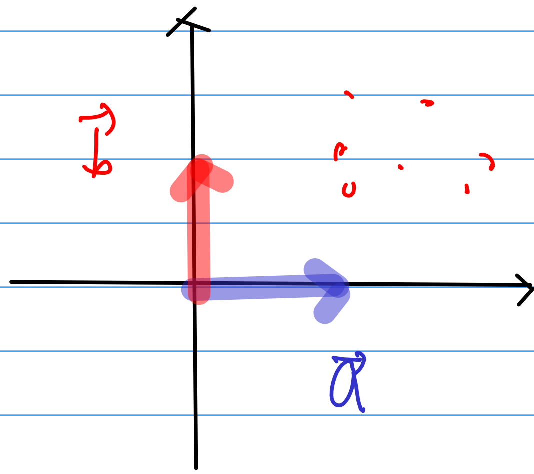
$c_1 = \frac{1}{2i}$ $c_2 = -\frac{1}{2i}$

$$y_4 = \frac{1}{2i} e^{(-2+\frac{1}{2}i)t} - \frac{1}{2i} c_2 e^{(-2-\frac{1}{2}i)t}$$ a solution

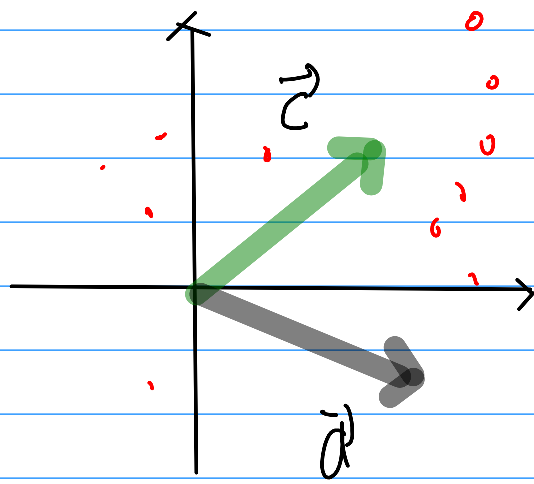
$$y_3 = e^{2t}\left(\frac{e^{+\frac{1}{2}it} + e^{-\frac{1}{2}it}}{2}\right) = \underline{e^{2t}\cos\left(\frac{t}{2}\right)}$$ a solution

$$y_4 = e^{2t}\left(\frac{e^{+\frac{1}{2}it} - e^{-\frac{1}{2}it}}{2i}\right) = \underline{e^{2t}\sin\left(\frac{t}{2}\right)}$$ a solution

$$c_3 y_3 + c_4 y_4$$



$c_1 \vec{a} + c_2 \vec{b}$



$c_3 \vec{c} + c_4 \vec{d}$