Characteristics of Multiple Random Variables

Young W Lim

June 24, 2019



Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on Probability, Random Variables and Random Signal Principles, P.Z. Peebles, Jr. and B. Shi

Outline

1 Joint Guassian Random Variables

Bivariate Gaussian Density

two random variables

Definition

The two random variables X and Y are said to be jointly Gaussian, if their joint density function is

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}$$

$$exp\left\{\frac{-1}{2(1-\rho^2)}\cdot\left[\frac{(x-\overline{X})^2}{\sigma_X^2}-\frac{2\rho(x-\overline{X})(y-\overline{Y})}{\sigma_X\sigma_Y}+\frac{(y-\overline{Y})^2}{\sigma_Y^2}\right]\right\}$$

$$\overline{X} = E[X], Y = E[Y], \sigma_X^2 = E[(X - \overline{X})^2], \sigma_Y^2 = E[(Y - \overline{Y})^2],$$

 $\rho = E[(X - \overline{X})(Y - \overline{Y})]/\sigma_X\sigma_Y$

Bivariate Gaussian Density - Maximum value two random variables

$$f_{X,Y}(x,y) \le f_{X,Y}(\overline{X},\overline{Y}) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}$$

Multi-variate Gaussian Density

 $f_{X,Y}(x,y) = f_X(x)f_Y(x)$ is sufficient to guarantee that X and Y are statistically independent. Any uncorrelated Guassian random variables are also statistically independent a coordinate rotation (linear transformation of X and Y) through the angle

$$\theta = \frac{1}{2} tan^{-1} \left[\frac{2\rho \sigma_X \sigma_Y}{\sigma_X^2 \sigma_Y^2} \right]$$

is sufficient to convert correlated random variables X and Y having σ_X^2 and σ_Y^2 , respectively, correlation coefficient ρ , and the joint densityof $f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\cdot \exp\left[\cdots\right]$ into two statistically independent Gaussian random variables



Bivariate Gaussian Density - Uncorrelated Nrandom variables

Nrandom variables $X_1, X_2, ..., X_N$ are called jointly Gaussian if their joint density function can be written as

$$f_{X_1,\dots,X_N}(x_1,\dots,x_N) = \frac{\left| [C_X|^{-1} \right|^{1/2}}{(2\pi)^{N/2}} exp \left\{ -\frac{[x-\overline{X}]^t [C_X][x-\overline{X}]}{2} \right\}$$

$$[x - \overline{X}] = \begin{bmatrix} x_1 - \overline{X}_1 \\ x_2 - \overline{X}_2 \\ x_N - \overline{X}_N \end{bmatrix}, \quad [C_X] = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{22} & \cdots & C_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ C_{N1} & C_{N2} & \cdots & C_{NN} \end{bmatrix}$$