## DT Correlation (1B)

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## Correlation of Energy Signals

## Discrete Time LTI System

Energy Signals

$$
\begin{aligned}
R_{x y}[m] & =\sum_{n=-\infty}^{+\infty} x[n] y^{*}[n+m] \\
& =\sum_{n=-\infty}^{+\infty} x[n-m] y^{*}[n]
\end{aligned}
$$

$$
R_{x y}[m]=\sum_{n=-\infty}^{+\infty} x[n] y[n+m]
$$

$$
=\sum_{n=-\infty}^{+\infty} x[n-m] y[n] \text { (real) }
$$



## Continuous Time LTI System

## Energy Signals

$$
\begin{aligned}
R_{x y}(\tau) & =\int_{-\infty}^{+\infty} x(t) y^{*}(t+\tau) d t \\
& =\int_{-\infty}^{+\infty} x(t-\tau) y^{*}(t) d t \\
R_{x y}(\tau) & =\int_{-\infty}^{+\infty} x(t) y(t+\tau) d t \quad \text { (real) } \\
& =\int_{-\infty}^{+\infty} x(t-\tau) y(t) d t \quad \text { (real) }
\end{aligned}
$$



## Correlation of Power Signals

## Discrete Time LTI System

## Power Signals

$$
\begin{aligned}
R_{x y}[m] & =\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] y^{*}[n+m] \\
& =\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n-m] y^{*}[n]
\end{aligned}
$$

$R_{x y}[m]=\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] y[n+m] \quad$ (real)
$=\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n-m] y[n]$ (real)


## Continuous Time LTI System

## Power Signals

$$
\begin{aligned}
R_{x y}(\tau) & =\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t) y^{*}(t+\tau) d t \\
& =\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t-\tau) y^{*}(\tau) d t \\
R_{x y}(\tau) & =\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t) y(t+\tau) d t \quad \text { (real) } \\
& =\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t-\tau) y(\tau) d t \quad \text { (real) }
\end{aligned}
$$



## Correlation of Periodic Power Signals

## Discrete Time LTI System

Periodic Power Signals

$$
\begin{aligned}
R_{x y}[m] & =\frac{1}{N} \sum_{n=(N)} x[n] y^{*}[n+m] \\
& =\frac{1}{N} \sum_{n=(N)} x[n-m] y^{*}[n]
\end{aligned}
$$

$$
\begin{equation*}
R_{x y}[m]=\frac{1}{N} \sum_{n=(N)} x[n] y[n+m] \tag{real}
\end{equation*}
$$

$$
\begin{equation*}
=\frac{1}{N} \sum_{n=(N)} x[n-m] y[n] \tag{real}
\end{equation*}
$$



Continuous Time LTI System

## Periodic Power Signals

$$
\begin{aligned}
R_{x y}(\tau) & =\frac{1}{T} \int_{T} x(t) y^{*}(t+\tau) d t \\
& =\frac{1}{T} \int_{T} x(t-\tau) y^{*}(\tau) d t \\
R_{x y}(\tau) & =\frac{1}{T} \int_{T} x(t) y(t+\tau) d t \\
& =\frac{1}{T} \int_{T} x(t-\tau) y(t) d t
\end{aligned}
$$

## Correlation \& Convolution : Energy Signals

## Discrete Time LTI System

## Energy Signals

$$
\begin{aligned}
& R_{x y}[m]=\quad \sum_{n=-\infty}^{+\infty} x[n] y[n+m] \text { (real) } \\
& x[n] * y[n]=\sum_{m=-\infty}^{+\infty} x[n-m] y[m] \\
& R_{x y}[m]=\quad x[-m] * y[m] \\
& R_{x y}[m] \quad \stackrel{\text { DTFT }}{\triangleleft} X^{*}(F) Y(F)
\end{aligned}
$$

$x[-n] * y[n]=\sum_{m=-\infty}^{+\infty} x[-n+m] y[m]$
$x[-m] * y[m]=\sum_{n=-\infty}^{+\infty} x[n-m] y[n]$
$x[-m] * y[m]=\sum_{n=-\infty}^{+\infty} x[n] y[n+m]$

## Continuous Time LTI System

## Energy Signals

$$
\begin{aligned}
& R_{x y}(\tau)=\int_{-\infty}^{+\infty} x(t) y(t+\tau) d t \\
& x(t) * y(t)=\int_{-\infty}^{+\infty} x(t-\tau) y(\tau) d \tau \\
& R_{x y}(\tau)=\quad x(-\tau) * y(\tau) \\
& R_{x y}(\tau) \quad \underbrace{}_{\underbrace{\text { CTFT }}} X^{*}(f) Y(f)
\end{aligned}
$$

$$
\begin{aligned}
& x(-t) * y(t)=\int_{-\infty}^{+\infty} x(-t+\tau) y(\tau) d \tau \\
& x(-\tau) * y(\tau)=\int_{-\infty}^{+\infty} x(t-\tau) y(t) d t \\
& x(-\tau) * y(\tau)=\int_{-\infty}^{+\infty} x(t) y(t+\tau) d t
\end{aligned}
$$

## Time Reversal Fourier Transforms

Discrete Time LTI System

$$
\begin{array}{ll}
x[m]= & \int_{1} X(F) e^{+j 2 \pi F n} d F \\
x[-m]= & \int_{1}^{1} X(F) e^{-j 22 F F n} d F \\
x[-m]= & -\int^{1} X(-v) e^{+j 22 v v n} d v \\
x[-m] & \stackrel{\text { DTFT }}{ }{ }^{2} X(-F)
\end{array}
$$

$$
-m \quad-F
$$

Continuous Time LTI System

$$
\begin{array}{cc}
x(t)= & \int_{-\infty}^{+\infty} X(f) e^{+j 2 \pi f t} d f \\
x(-t)= & \int_{-\infty}^{+\infty} X(f) e^{-j 2 \pi f t} d f \\
x(-t)= & -\int_{+\infty}^{-\infty} X(-v) e^{+j 2 \pi v t} d v \\
x(-t) & \underbrace{\text { CTFT }} X(-f) \\
-t & -f
\end{array}
$$

## Conjugate Fourier Transforms

## Discrete Time LTI System

$$
x[m]=\quad \int_{1} X(F) e^{+j 2 \pi F n} d F
$$

Continuous Time LTI System

$$
x^{*}[m]=\int_{1} x^{*}(F) e^{-j 2 \pi F m} d F
$$

$$
x^{*}[m]=\quad-\int x^{*}(-v) e^{+j 2 \pi v m} d v
$$

$$
x^{*}[m] \stackrel{\text { DTFT }}{\Leftrightarrow} X^{*}(-F)
$$

$$
* m \quad *-F
$$

$$
\begin{array}{cc}
x(t)= & \int_{-\infty}^{+\infty} X(f) e^{+j 2 \pi f t} d f \\
x^{*}(t)= & \int_{-\infty}^{+\infty} X^{*}(f) e^{-j 2 \pi f t} d f \\
x^{*}(t)= & -\int_{+\infty}^{+\infty} X^{*}(-v) e^{+j 2 \pi v t} d v \\
x^{*}(t) & { }^{c T F T} X^{*}(-f) \\
* t & *-f
\end{array}
$$

$$
\begin{array}{rll}
\hline x^{*}(-t) & \stackrel{\text { CTFT }}{ } & X^{*}(f) \\
*(-t) & & *-(-f)
\end{array}
$$

## Fourier Transforms of Real Signals

## Discrete Time LTI System

$$
x[m]=\quad \int_{1} X(F) e^{+j 2 \pi F n} d F
$$

| $\chi^{*}[m]$ | $\stackrel{\text { DTFT }}{\\|}$ | $X^{*}(-F)$ |
| :---: | :---: | :---: |
| $x^{*}[-m]$ | $\stackrel{\text { DTFT }}{ }$ | $X^{*}(F)$ |
| \\| |  | II |
| $x[-m]$ | $\stackrel{\text { DTET }}{\Rightarrow}$ | $X(-F)$ |

A real
signal
$x[-m] \quad X^{*}(F)$
Hermitian
Symmetry

Continuous Time LTI System

$$
x(t)=\int_{-\infty}^{\pi x} x(f) e^{+f R z t} d f
$$

| $\chi^{*}(t)$ | $\stackrel{\text { cime }}{ }$ | $X^{*}(-f)$ |
| :---: | :---: | :---: |
| $x^{*}(-t)$ | (1) | $X^{*}(f)$ |
| II |  | II |
| $x(-t)$ | $\stackrel{\text { gIr }}{\sim}$ | $X(-f)$ |
| $\underset{\substack{\text { Areal } \\ \text { signal }}}{ }$ |  | $\underset{\substack{\text { Hemmitian } \\ \text { Symetry }}}{ }$ |
| $x(-t)$ |  | $X^{*}(f)$ |

## Correlation \& Convolution : Energy Signals

## Discrete Time LTI System

## Energy Signals

Correlation Definition A

$$
R_{x y}[m]=\quad \sum_{n=-\infty}^{+\infty} x[n] y^{*}[n+m]
$$

conjugate the second

## Correlation Definition B

$$
R_{x y}[m]=\quad \sum_{n=-\infty}^{+\infty} x^{*}[n] y[n+m]
$$

conjugate the first


Continuous Time LTI System

## Energy Signals

Correlation Definition A

$$
R_{x y}(\tau)=\quad \int_{-\infty}^{+\infty} \chi(t) y^{*}(t+\tau) d t
$$

## Correlation Definition B

$$
R_{x y}(\tau)=\int_{-\infty}^{+\infty} \chi^{*}(t) y(t+\tau) d t
$$



## Correlation \& Convolution : Energy Signals

## Discrete Time LTI System

## Energy Signals

## Correlation Definition A

$$
R_{x y}[m]=\quad \sum_{n=-\infty}^{+\infty} x[n] y^{*}[n+m]
$$

## Convolution

$$
\begin{aligned}
& x[n] * y^{*}[n]=\sum_{m=-\infty}^{+\infty} x[n-m] y^{*}[m] \\
& x[-m] * y^{*}[m]=\sum_{n=-\infty}^{+\infty} x[n-m] y^{*}[n]
\end{aligned}
$$

$$
R_{x y}[m]=x[-m] * y^{*}[m]
$$

$$
R_{x y}[m] \stackrel{\text { DTFT }}{\underbrace{\text { P/ }}} X(-F) Y^{*}(-F)
$$

$$
R_{x y}[m]
$$

DTFT

$$
X(F) Y^{*}(F)
$$

(even) (even)

Continuous Time LTI System

## Energy Signals

## Correlation Definition A

$$
R_{x y}(\tau)=\quad \int_{-\infty}^{+\infty} x(t) y^{*}(t+\tau) d t
$$

## Convolution

$$
\begin{aligned}
& x(t) * y^{*}(t)=\int_{-\infty}^{+\infty} x(t-\tau) y^{*}(\tau) d \tau \\
& x(-\tau) * y^{*}(\tau)=\int_{-\infty}^{+\infty} x(t-\tau) y^{*}(t) d t
\end{aligned}
$$

$$
R_{x y}(\tau)=x(-\tau) * y^{*}(\tau)
$$

$$
R_{x y}(\tau) \stackrel{\text { CTFT }}{\overbrace{}^{*}} X(-f) Y^{*}(-f)
$$

$$
R_{x y}(\tau)
$$

$$
\underbrace{\text { CTF }}
$$

$$
X(f) Y^{*}(f)
$$

## Correlation \& Convolution : Energy Signals

## Discrete Time LTI System

## Energy Signals

## Correlation Definition B

$$
R_{x y}[m]=\quad \sum_{n=-\infty}^{+\infty} x^{*}[n] y[n+m]
$$

## Convolution

$$
\begin{aligned}
& x^{*}[n] * y[n]=\sum_{m=-\infty}^{+\infty} x^{*}[n-m] y[m] \\
& x^{*}[-m] * y[m]=\sum_{n=-\infty}^{+\infty} x^{*}[n-m] y[n]
\end{aligned}
$$

$$
R_{x y}[m]=\left[x^{*}[-m] * y[m]\right]
$$

$$
R_{x y}[m] \stackrel{\text { DTFT }}{\triangleleft} X^{*}(F) Y(F)
$$

Continuous Time LTI System

## Energy Signals

## Correlation Definition B

$$
R_{x y}(\tau)=\quad \int_{-\infty}^{+\infty} x^{*}(t) y(t+\tau) d t
$$

## Convolution

$$
\begin{aligned}
& x^{*}(t) * y(t)=\int_{-\infty}^{+\infty} x^{*}(t-\tau) y(\tau) d \tau \\
& x^{*}(-\tau) * y(\tau)=\int_{-\infty}^{+\infty} x^{*}(t-\tau) y(t) d t
\end{aligned}
$$

$$
R_{x y}(\tau)=\left[x^{*}(-\tau) * y(\tau)\right]
$$

$$
R_{x y}(\tau) \stackrel{\text { CTFT }}{\overbrace{}^{*}} X^{*}(f) Y(f)
$$

## Correlation Functions

Discrete Time LTI System

Energy Signals

$$
R_{x y}[m]=\sum_{n=-\infty}^{\infty} x[n] y[n+m]
$$

Power Signals
Power Signal + Energy Signal
$R_{x y}[m]=\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] y[n+m]$

Continuous Time LTI System

## Energy Signals

$R_{x y}(\tau)=\quad \int_{-\infty}^{+\infty} x(t) y(t+\tau) d t$

Power Signals
Power Signal + Energy Signal
$R_{x y}(\tau)=\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t) y(t+\tau) d t$

## AutoCorrelation of Energy Signals

## Discrete Time LTI System

Energy Signals

$$
\begin{aligned}
R_{x x}[m] & =\sum_{n=-\infty}^{+\infty} x[n] x^{*}[n+m] \\
& =\sum_{n=-\infty}^{+\infty} x[n-m] x^{*}[n]
\end{aligned}
$$

$R_{x x}[0]=\sum_{n=-\infty}^{+\infty} x^{2}[n]$ total energy

Continuous Time LTI System

## Energy Signals

$$
\begin{aligned}
R_{x x}(\tau) & =\int_{-\infty}^{+\infty} x(t) x^{*}(t+\tau) d t \\
& =\int_{-\infty}^{+\infty} x(t) x^{*}(t+\tau) d t \\
R_{x x}(0) & =\int_{-\infty}^{+\infty} x^{2}(t) d t \quad \text { total energy }
\end{aligned}
$$

## AutoCorrelation of Power Signals

## Discrete Time LTI System

Power Signals

$$
\begin{aligned}
R_{x x}[m] & =\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n] x^{*}[n+m] \\
& =\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x[n-m] x^{*}[n]
\end{aligned}
$$

$R_{x x}[0]=\lim _{N \rightarrow \infty} \frac{1}{N} \sum_{n=(N)} x^{2}[n]$ total energy

Continuous Time LTI System

Power Signals

$$
\begin{aligned}
R_{x x}(\tau) & =\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t) x^{*}(t+\tau) d t \\
& =\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x(t-\tau) x^{*}(\tau) d t
\end{aligned}
$$

$R_{x x}(0)=\lim _{T \rightarrow \infty} \frac{1}{T} \int_{T} x^{2}(t) d t$ total energy

## AutoCorrelation of Periodic Power Signals

Discrete Time LTI System

## Periodic Power Signals

$$
\begin{aligned}
R_{x x}[m] & =\frac{1}{N} \sum_{n=(N)} x[n] x^{*}[n+m] \\
& =\frac{1}{N} \sum_{n=(N)} x[n-m] x^{*}[n]
\end{aligned}
$$

$$
R_{x x}[m]=\quad \frac{1}{N} \sum_{n=(N)} x[n] x^{*}[n+m]
$$

Continuous Time LTI System

## Periodic Power Signals

$$
\begin{aligned}
R_{x x}(\tau) & =\frac{1}{T} \int_{T} x(t) x^{*}(t+\tau) d t \\
& =\frac{1}{T} \int_{T} x(t-\tau) x^{*}(\tau) d t \\
R_{x x}(\tau) & =\frac{1}{T} \int_{T} x(t) x^{*}(t+\tau) d t
\end{aligned}
$$

## References

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