

# Complex Random Processes

Young W Lim

September 28, 2019

Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



## Definition

$$Z(t) = X(t) + jY(t)$$

$$E[Z(t)] = E[X(t)] + jE[Y(t)]$$

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}^*$$

# Pseudo-correlation and covariance functions

$N$  Gaussian random variables

## Definition

$$R_{ZZ}(t, t + \tau) = E[Z(t)Z^*(t + \tau)]$$

$$C_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}^*$$

$$\tilde{R}_{ZZ}(t, t + \tau) = E[Z(t)Z(t + \tau)]$$

$$\tilde{C}_{ZZ}(t, t + \tau) = E[Z(t) - E[Z(t)]] \{E[Z(t + \tau) - E[Z(t + \tau)]]\}$$

# Proper Random Processes

$N$  Gaussian random variables

## Definition

A complex random process  $Z(t)$  is said to be proper if the pseudo-autocovariance function is identically zero.

If  $Z(t)$  is at least wide-sense stationary, the mean value becomes a constant

$$\bar{Z} = \bar{X} + j\bar{Y}$$

the correlation and pseudo-correlation functions are independent of absolute time

$$R_{ZZ}(t, t + \tau) = R_{ZZ}(\tau) \quad \tilde{R}_{ZZ}(t, t + \tau) = \tilde{R}_{ZZ}(\tau)$$

$$C_{ZZ}(t, t + \tau) = C_{ZZ}(\tau) \quad \tilde{C}_{ZZ}(t, t + \tau) = \tilde{C}_{ZZ}(\tau)$$







