

Radar

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

False Alarm

N Gaussian random variables

Definition

$$P_{fa} = \int_{W_T}^{\infty} f_0(w)dw$$

$$P_d = \int_{W_T}^{\infty} f_1(w)dw$$

False Alarm Probability and Threshold (1)

N Gaussian random variables

Definition

$$f_R(r) = \frac{u(r)}{\sigma^2} r e^{-r^2/2\sigma^2}$$

$$W_t = g(R_r)$$

$$R_T = g^{-1}(W_T)$$

False Alarm Probability and Threshold (2)

N Gaussian random variables

Definition

$$\begin{aligned} P_{fa} &= \int_{W_f}^{\infty} f_0(w) dw = \int_{R_T}^{\infty} f_R(r) dr \\ &= \int_{R_T}^{\infty} \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} dr = e^{-R_T^2/2\sigma^2} \end{aligned}$$

$$R_T = \left[2\sigma^2 \ln \left(\frac{1}{P_{fa}} \right) \right]^{1/2}$$

$$W_T = g \left(\left[2\sigma^2 \ln \left(\frac{1}{P_{fa}} \right) \right]^{1/2} \right)$$

Detection Probability (1)

N Gaussian random variables

Definition

$$\begin{aligned} P_d &= \int_{W_f}^{\infty} f_1(w) dw = \int_{R_T}^{\infty} f_R(r) dr \\ &= \int_{\left[2\sigma^2 \ln\left(\frac{1}{P_{fa}}\right)\right]^{1/2}}^{\infty} \frac{r}{\sigma^2} I_0\left(\frac{r^2 A_0}{\sigma^2}\right) e^{-(r^2 + A_0^2)/2\sigma^2} dr \\ &= Q\left[\sqrt{\frac{A_0^2}{\sigma^2}}, \sqrt{2 \ln\left(\frac{1}{P_{fa}}\right)}\right] \\ Q(\alpha, \beta) &= \int_{\beta}^{\infty} \xi I_0(\alpha \xi) e^{-(\xi^2 + \alpha^2)/2} d\xi \end{aligned}$$

Detection Probability (2)

N Gaussian random variables

Definition

$$P_d = Q \left[\sqrt{\frac{A_0^2}{\sigma^2}}, \sqrt{2 \ln \left(\frac{1}{P_{fa}} \right)} \right]$$

$$Q(\alpha, \beta) = \int_{\beta}^{\infty} \xi I_0(\alpha \xi) e^{-(\xi^2 + \alpha^2)/2} d\xi$$

$$P_d \approx F \left[\frac{A_0^2}{\sigma^2} - \sqrt{2 \ln \left(\frac{1}{P_{fa}} \right)} \right]$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\xi^2/2} d\xi$$

