Angle Recoding CORDIC
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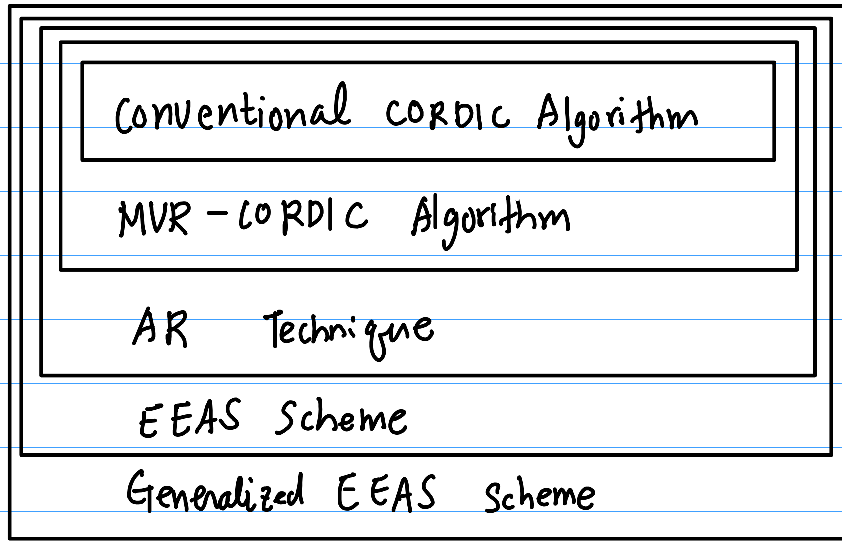
20180823 Wed

Vector Rotational CORDIC



Vector Rotation Alg	Selection of Rotation Sequence	Elementary Angle Set	Micro Rotation	Angle Quantization	
				θ_i	N_A
Conventional CORDIC	$\mu = \{-1, +1\}$	EAS S	complete	$\mu(i)\, a(i)$	W Fixed
Angle Recoding	$\mu = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	N' Variable
MVR-CORDIC	$\alpha = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	R_m Fixed
EEAS	$\alpha_1, \alpha_2 = \{-1, 0, +1\}$	EEAS S_2	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_1(i) \cdot 2^{-s_1(i)})$	R_m Fixed
Generalized EEAS	$\alpha_1, \alpha_2, \cdots, \alpha_{d-1} = \{-1, 0, +1\}$	EEAS S_d $d \geq 3$	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} \ldots + \alpha_{d-1}(i) \cdot 2^{-s_{d-1}(i)})$	R_m Fixed

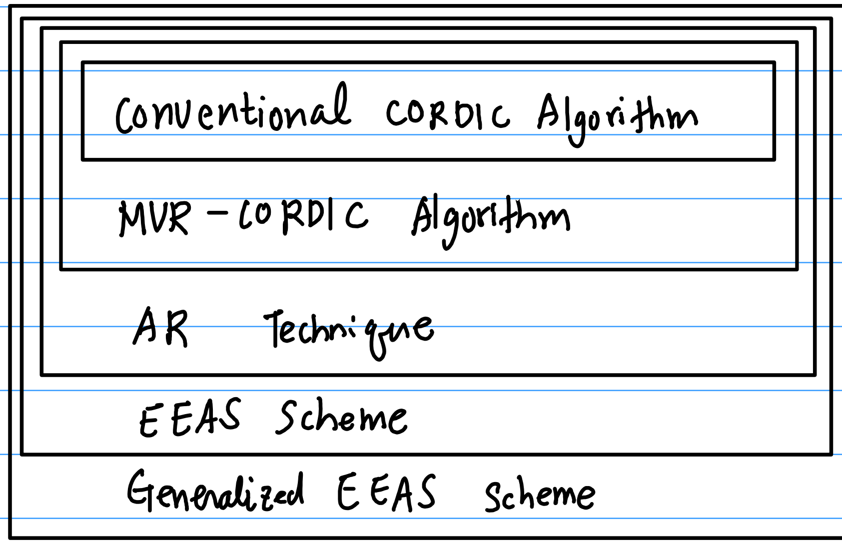
Family of Vector Rotational CORDIC

AQ process (Angle Quantization) — { CORDIC
AR (Angle Recoding)
MVR - CORDIC (Modified Vector Rotation)
EEAS (Extended Elementary Angle Set)
Generalized EEAS

AQ process with various EAS and
and suitable combinations of subangles

to decompose the target rotational angle
into several easy-to-implement subangles

minimizing the angle quantization error ξ_m
to obtain the best precision performance



EEAS covers $\begin{cases} \text{MVR-CORDIC} \\ \text{AR} \end{cases}$

a subset of EEAS $\hat{S}_2$ EAS S_1

MVR-CORDIC a subset of AR

one constraint on the iteration number

Angle Quantization

Quantization process on the rotational angle θ

decompose the original rotational angle θ
into severales θ_i's

sum up those subangles to approximate
the original angle as close as possible

Minimize the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A : the number of sub-angles

$$\theta = \theta_0 + \theta_1 + \theta_2 + \cdots + \theta_{N_A-1} + \xi_m$$

design issues in the AQ process

① need to determine the sub-angles θ_i

each θ_i needs to be easy-to-implement

② how to select and combine these sub-angles ξ_m

such that the angle quantization error ξ_m

can be minimized

Angle Quantization

Quantization process on the rotational angle θ

decompose θ into several subangles θ_i's

the angle quantization error

$$\boxed{\xi_m \triangleq \theta - \sum_{i=0}^{N_A - 1} \theta_i}$$

N_A the number of subangles
$\theta_0, \theta_1, \cdots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \cdots + \theta_{N_A-1} + \xi_m$$

data : W-bit word length		
the iteration number : N		$N \leq W$
the restricted iteration number : R_m		$R_m \ll W$

Vector Rotation CORDIC Family

⓪ Conventional CORDIC

① AR

② MVR

③ EEAS

Angle Quantization

Quantization process on the rotational angle θ

decompose θ into several subangles θ_i's

the angle quantization error

$$\boxed{\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i}$$

N_A the number of subangles

$\theta_0, \theta_1, \cdots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \cdots + \theta_{N_A-1} + \xi_m$$

data : W-bit word length		
the iteration number : N		$N \leqslant W$
the restricted iteration number : R_m		$R_m \ll W$

CSD (Canonical Signed Digit) Quantization

digital filter designs

coefficients are recoded

in terms of SPT (Signed Power of Two) terms

multiplication can be easily realized

with shift-and-add operations

$h_2 = (-0.156249)_{10} \Rightarrow (0.0\bar{1}011)_2$

$W=8$, 3 non-zero digits

① CSD quantization decomposes

coefficients into several SPT terms

(sub-coefficients)

② the multiplication of a coefficient

can be reformed

through the combination of

the non-zero SPT sub-coefficients

quantize the rotation angle θ

decompose the rotation angle θ
into several sub-angles θ_i's

the rotational operation of each θ_i
should be easily realized

If each θ_i can be realized
using only shift-and-add operations

the rotation of θ can be performed
through successive applications of
sub-angle rotations
in a cost-effective way

approximation target	Coefficient h_i	Rotation angle θ
Basic Element	Non-zero digit 2^{-i}	Sub-angle $a(i) = \tan^{-1}(2^{-i})$
Basic Operation	shift-and-add operation	2 shift-and-add operations
Approximation Equation	$h_i \approx \sum_{j=0}^{N_D-1} g_j \cdot 2^{-d_j}$	$\theta \approx \sum_{j=0}^{R_m-1} \alpha(j) \cdot a(s(j))$
	$g_j \in \{-1, 0, +1\}$ $d_j \in \{0, 1, \cdots, W-1\}$ N_D = the number of non-zero digits	N_A = the number of sub-angles

try to approach the target rotation angle θ
step by step

decisions are made in each step
by choosing the best combination of $\alpha(i)\, a(s(i))$

So as to minimize $|\xi_m|$

$\alpha(i)$, $a(i)$ are determined such that
the error function is minimized

$$J(i) = |\theta(i) - \alpha(i)\, a(s(i))|$$

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m)\, a(s(m))$$

terminated if no further improvement can be found

$$J(i) \geq J(i-1)$$

or $\alpha(R_m - 1)$ and $s(R_m - 1)$
are determined at the end

Rotation Angle $\theta = \frac{13\pi}{64}$

Conventional CORDIC

$\bar{\mu} = [1\ \ -1\ \ 1\ \ 1\ \ -1\ \ 1\ \ -1\ \ -1\ \ 1\ \ -1\ \ -1\ \ 1\ \ 1\ \ -1\ \ 1\ \ 1]$

Angle Reiodng – Greedy

$\bar{\mu} = [1\ \ 0\ \ 0\ \ -1\ \ 0\ \ 0\ \ -1\ \ -1\ \ 0\ \ 0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 1]$

MVR – CORDIC – Greedy

$\bar{\alpha} = [1\ \ -1\ \ -1\ \ -1]$

$\bar{s} = [0\ \ 3\ \ 6\ \ 7]$

MVR – CORDIC – Semi Greedy (P = 2)

$\bar{\alpha} = [1\ \ -1\ \ -1\ \ 1]$

$\bar{s} = [0\ \ 3\ \ 5\ \ 7]$

MVR – CORDIC – TBS

$\bar{\alpha} = [1\ \ 1\ \ -1\ \ -1]$

$\bar{s} = [1\ \ 2\ \ 4\ \ 7]$

EEAS – Greedy

$\bar{\alpha}_0 = [1\ \ -1]$ $\quad$ $\bar{\alpha}_1 = [-1\ \ -1]$

$\bar{s}_0 = [0\ \ 2]$ $\quad$ $\bar{s}_1 = [8\ \ 10]$

EEAS – TBS $\quad$ $R_m = 2$

$\bar{\alpha}_0 = [1\ \ 1]$ $\quad$ $\bar{\alpha}_1 = [-1\ \ -1]$

$\bar{s}_0 = [0\ \ 6]$ $\quad$ $\bar{s}_1 = [3\ \ 5]$

EEAS – TBS $\quad$ $R_m = 3$

$\bar{\alpha}_0 = [1\ \ -1\ \ 1]$ $\quad$ $\bar{\alpha}_1 = [-1\ \ 1\ \ -1]$

$\bar{s}_0 = [0\ \ 3\ \ 7]$ $\quad$ $\bar{s}_1 = [15\ \ 6\ \ 2]$