

Fourier Integral (H.1)

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Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(nx) + b_n \sin(nx))$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx) dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx) dx$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)$$

$$a_0 = \frac{1}{p} \int_{-p}^{+p} f(x) dx$$

$$a_n = \frac{1}{p} \int_{-p}^{+p} f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

$$b_n = \frac{1}{p} \int_{-p}^{+p} f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right) \right)$$

$$= \frac{1}{2} \left[\frac{L}{p} \int_{-p}^{+p} f(x) dx \right] = a_0$$

$$+ \sum_{n=1}^{\infty} \left\{ \left[\frac{1}{p} \int_{-p}^{+p} f(x) \cos\left[\frac{n\pi}{p} x\right] dx \right] \cos\left(\frac{n\pi}{p} x\right) + \left[\frac{1}{p} \int_{-p}^{+p} f(x) \sin\left[\frac{n\pi}{p} x\right] dx \right] \sin\left(\frac{n\pi}{p} x\right) \right\}$$

a_n
 b_n

$$\alpha_n = \frac{n\pi}{p} \quad \Delta\alpha = \alpha_{n+1} - \alpha_n = \frac{(n+1)\pi}{p} - \frac{n\pi}{p} = \frac{\pi}{p}$$

$$\frac{1}{p} = \frac{\Delta\alpha}{\pi}$$

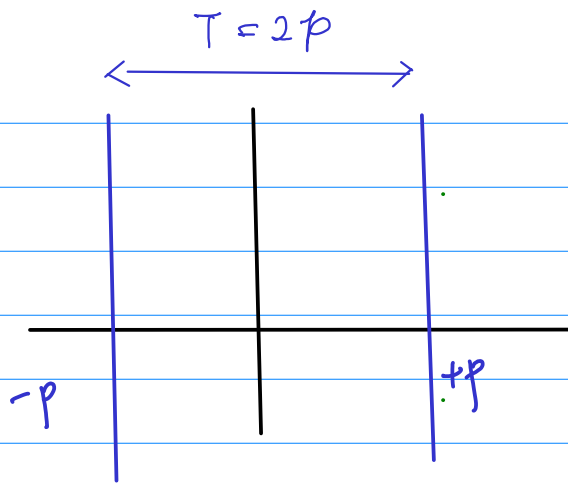
$$\alpha_n = \frac{n\pi}{p} \quad \Delta\alpha = \alpha_{n+1} - \alpha_n = \frac{(n+1)\pi}{p} - \frac{n\pi}{p} = \frac{\pi}{p}$$

$$\frac{1}{p} = \frac{\Delta\alpha}{\pi}$$

$$f(x) = \frac{1}{2} \left[\frac{1}{\pi} \int_{-p}^{+p} f(x) dx \right] \Delta\alpha$$

$$+ \sum_{n=1}^{\infty} \left\{ \left[\frac{1}{\pi} \int_{-p}^{+p} f(x) \cos(\alpha_n x) dx \right] \cos(\alpha_n x) + \left[\frac{1}{\pi} \int_{-p}^{+p} f(x) \sin(\alpha_n x) dx \right] \sin(\alpha_n x) \right\} \Delta\alpha$$

$$\omega_n = 2\pi f n \\ = \frac{2\pi}{T} n$$



$$p \rightarrow \infty$$

a periodic signal

$$p \rightarrow 0$$

$$\frac{1}{p} = \frac{\Delta\alpha}{\pi}$$

$$\Delta\alpha \rightarrow 0$$

$$\Delta\alpha = \alpha_{n+1} - \alpha_n$$

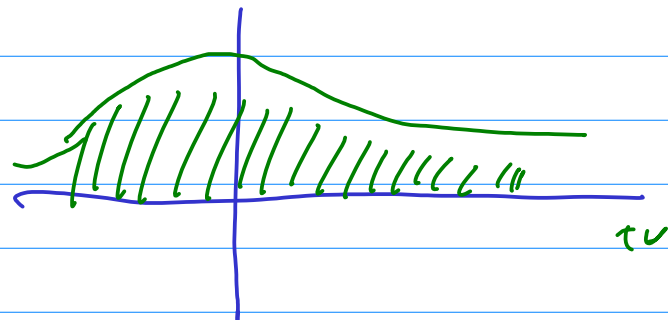
$$= \frac{(n+1)\pi}{p} - \frac{n\pi}{p} = \frac{\pi}{p}$$

$$\lim_{\Delta\alpha \rightarrow 0} \sum_{n=1}^{\infty} \{ \quad \} \Delta\alpha \rightarrow \int_0^{\infty} \{ \quad \} dx$$

$$\lim_{p \rightarrow \infty} \frac{1}{2} \left[\frac{1}{p} \int_{-p}^{+p} f(x) dx \right] < \infty$$

$$\int_{-\infty}^{+\infty} f(x) dx < \infty$$

$$\lim_{p \rightarrow \infty} \frac{\int_{-p}^{+p} f(x) dx}{p} =$$



$$\lim_{p \rightarrow \infty} \frac{M_{L^{\infty}}}{p} = 0$$

$$\lim_{p \rightarrow \infty}$$

$$f(x) = \frac{1}{2} \left[\frac{1}{\pi} \int_{-p}^{+p} f(x) dx \right] \Delta \alpha$$

$$+ \sum_{n=1}^{\infty} \left\{ \left[\frac{1}{\pi} \int_{-p}^{+p} f(x) \cos[\alpha_n x] dx \right] \cos(\alpha_n x) + \left[\frac{1}{\pi} \int_{-p}^{+p} f(x) \sin[\alpha_n x] dx \right] \sin(\alpha_n x) \right\} \Delta \alpha$$

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} f(t) \cos(\alpha t) dt \cos(\alpha x) + \int_{-\infty}^{+\infty} f(t) \sin(\alpha t) dt \sin(\alpha x) \right\} d\alpha$$

Fourier Integral.

$$\int_0^{\infty} d\alpha$$

positive
freq only

$$\int_{-\infty}^{+\infty} dt$$

both pos & neg
time.

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \begin{array}{l} \int_{-\infty}^{+\infty} f(t) \cos(\alpha t) dt \cos(\alpha x) + \\ \int_{-\infty}^{\infty} f(t) \sin(\alpha t) dt \sin(\alpha x) \end{array} \right\} d\alpha$$

$$f(x) = \frac{1}{\pi} \int_0^{\infty} [A(\alpha) \cos(\alpha x) + B(\alpha) \sin(\alpha x)] d\alpha$$

α 의 의미는 $\rightarrow \alpha : \text{freq.}$

$$A(\alpha) = \int_{-b}^{+b} f(t) \cos(\alpha t) dt$$

즉 $-p \sim +p$ $\int_{+p}^{-p} dt$
 2π $p \rightarrow \infty$ $\Rightarrow \int_{-\infty}^{\infty} dt$

$$B(\alpha) = \int_{-b}^{+b} f(t) \sin(\alpha t) dt$$

\uparrow
 즉 α 의 의미

When $f(x)$: **even**

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} f(t) \cos(\alpha t) dt \cos(\alpha x) + \int_{-\infty}^{+\infty} \underbrace{f(t)}_{\text{even}} \underbrace{\sin(\alpha t)}_{\text{odd}} dt \sin(\alpha x) \right\} d\alpha$$

odd \rightarrow should go to 0

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} f(t) \cos(\alpha t) dt \cos(\alpha x) \right\} d\alpha$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \left\{ \int_0^{+\infty} f(t) \cos(\alpha t) dt \cos(\alpha x) \right\} d\alpha$$

$$A(\alpha) = \int_0^{+\infty} f(t) \cos(\alpha t) dt$$

Fourier Cosine Integral

When $f(x)$: **Odd**

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} \overbrace{f(t) \cos(\alpha t) dt}^{\text{odd}} \cos(\alpha x) + \int_{-\infty}^{\infty} f(t) \sin(\alpha t) dt \sin(\alpha x) \right\} d\alpha$$

→ should go to 0

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \int_{-\infty}^{+\infty} f(t) \sin(\alpha t) dt \sin(\alpha x) \right\} d\alpha$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \left\{ \int_0^{+\infty} f(t) \sin(\alpha t) dt \sin(\alpha x) \right\} d\alpha$$

$$B(\alpha) = \int_0^{+\infty} f(t) \sin(\alpha t) dt$$

Fourier Sine Integral

Fourier Transform

$$F(\omega) = \int_{-\infty}^{+\infty} f(x) e^{i\omega x} dx$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) e^{-i\omega x} d\omega$$

$$F(\omega) = \int_{-\infty}^{+\infty} f(x) e^{i\omega x} dx$$

$$= \int_{-\infty}^{+\infty} f(x) [\cos(\omega x) + i \sin(\omega x)] dx$$

$$= \int_{-\infty}^{+\infty} f(x) \cos(\omega x) dx + i \int_{-\infty}^{+\infty} f(x) \sin(\omega x) dx$$

$$= \int_{-\infty}^{+\infty} f(x) \cos(\omega x) dx + i \int_{-\infty}^{+\infty} f(x) \sin(\omega x) dx$$

(15.3)

Fourier Integral

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left[A(\alpha) \underline{\cos(\alpha x)} + B(\alpha) \underline{\sin(\alpha x)} \right] d\alpha$$

$$A(\alpha) = \int_{-b}^{+b} f(x) \cos(\alpha x) dx$$

$$B(\alpha) = \int_{-b}^{+b} f(x) \sin(\alpha x) dx$$

Fourier Transform

Laplace Transform

$$\mathcal{L}^{-1}\{F(s)\}$$

$F(s) = \int_0^{\infty} f(x) e^{-sx} dx$

$\mathcal{L}\{f(x)\}$

$s = \underline{\sigma} + i\underline{\omega}$

$$\mathcal{L}^{-1}\{F(\omega)\}$$

$F(\omega) = \int_{-\infty}^{+\infty} f(x) e^{i\omega x} dx$

$\tilde{\mathcal{F}}\{f(x)\}$

Fourier Sine, Cosine Transform

$$x \in [0, \infty)$$

$$F_s(\alpha) = \int_0^{\infty} f(x) \sin \alpha x \, dx$$

Fourier Sine Transform

$$F_c(\alpha) = \int_0^{\infty} f(x) \cos \alpha x \, dx$$

Fourier Cosine Transform

$$f(x) = \frac{2}{\pi} \int_0^{\infty} F_s(\alpha) \sin \alpha x \, d\alpha$$

$$f(x) = \frac{2}{\pi} \int_0^{\infty} F_c(\alpha) \cos \alpha x \, d\alpha$$

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left[A(\alpha) \cos(\alpha x) + B(\alpha) \sin(\alpha x) \right] d\alpha$$

$$A(\alpha) = \int_{-\infty}^{+\infty} f(x) \cos(\alpha x) \, dx$$

$$B(\alpha) = \int_{-\infty}^{+\infty} f(x) \sin(\alpha x) \, dx$$

Fourier Integral

$$f(x) = \frac{1}{\pi} \int_0^{\infty} \left[A(\alpha) \cos(\alpha x) + B(\alpha) \sin(\alpha x) \right] d\alpha$$

$$\alpha \in [0, \infty]$$

$$A(\alpha) = \int_{-b}^{+b} f(t) \cos(\alpha t) dt$$

$$t \in [-\infty, \infty]$$

$$B(\alpha) = \int_{-b}^{+b} f(t) \sin(\alpha t) dt$$