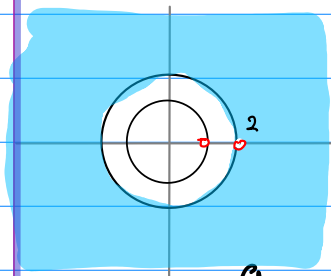


Laurent Series and z-Transform Examples case 4.A

20171007

Copyright (c) 2016 - 2017 Young W. Lim.

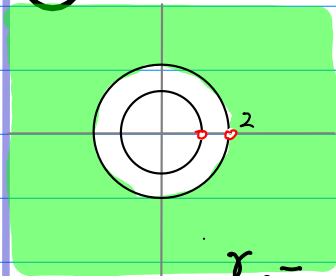
Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{cases}$$

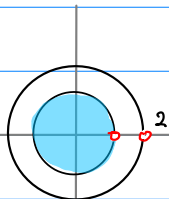
$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

Ⓘ



$$x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

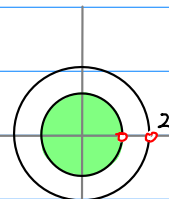
$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

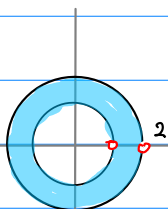
$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

Ⓛ



$$x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

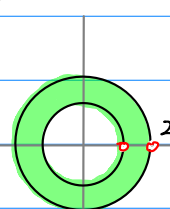
$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

Ⓜ



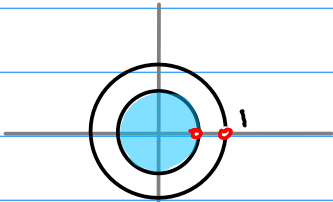
$$x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$

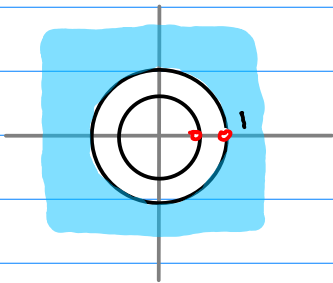
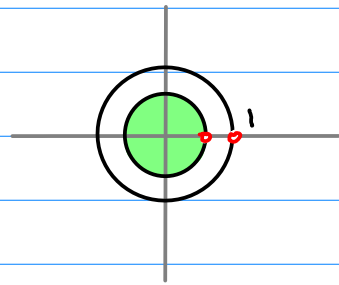


4.A

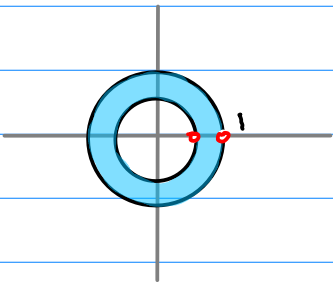
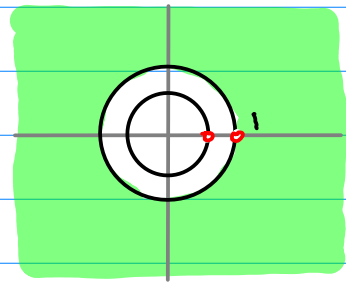
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \longrightarrow X(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)}$$



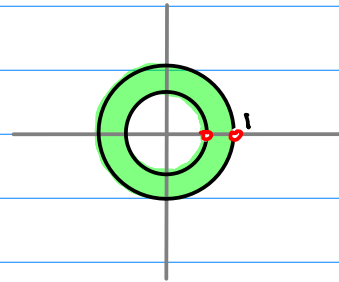
$$\sum_{n=-1}^{\infty} [1 - 2^{n-1}] z^n$$



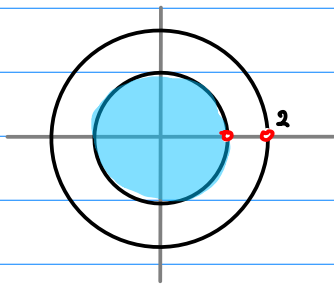
$$\sum_{n=-1}^{\infty} [2^{n-1} - 1] z^n$$



$$-\sum_{n=-1}^{\infty} z^n - \sum_{n=1}^{\infty} 2^{n-1} z^n$$



$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$



$\sum_{n=1}^{\infty}$

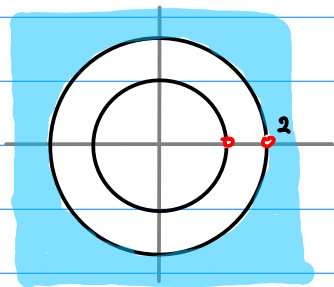
$$[1 - 2^{n-1}] z^n$$

$$+ \frac{\left(\frac{z}{1}\right)}{1 - \left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1 - \left(\frac{2z}{1}\right)}$$

$$= + \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{2z}{1}\right)^n$$

$$= \sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= \sum_{n=1}^{\infty} [1 - 2^{n-1}] z^n$$

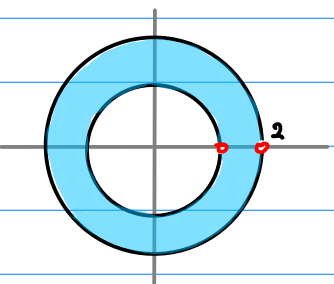


$$- \frac{(1)}{1 - \left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1 - \left(\frac{1}{2z}\right)}$$

$$= - \sum_{n=0}^{\infty} (1) \left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right) \left(\frac{1}{2z}\right)^n$$

$$= \sum_{n=0}^{\infty} [2^{-n-1} - 1] z^{-n}$$

$$= \sum_{n=-1}^{\infty} [2^{n-1} - 1] z^n$$



$$- \frac{(1)}{1 - \left(\frac{1}{z}\right)} - \frac{\left(\frac{z}{1}\right)}{1 - \left(\frac{2z}{1}\right)}$$

$$= - \sum_{n=0}^{\infty} \left(\frac{1}{z}\right) \left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty} \left(\frac{z}{1}\right) \left(\frac{2z}{1}\right)^n$$

$$= - \sum_{n=0}^{\infty} z^{-n-1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= - \sum_{n=-1}^{\infty} z^n - \sum_{n=1}^{\infty} 2^{n-1} z^n$$