Laurent Series and z-Transform - Geometric Series Double Pole Examples B

20180523

2 formulas of z

① $$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\Updownarrow\; z^{-1}$$

② $$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$\boxed{\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5}-\frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1}-\frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z}-\frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2}+\frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2}+\frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{\text{Laurent}}$ $f(z), g(z)$: causal , $f(z^{-1}), g(z^{-1})$: anti-causal

$\boxed{\text{z- Trans}}$ $X(z), Y(z)$: causal, $X(z^{-1}), Y(z^{-1})$: anti-causal

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift right, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[l, shift left] \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[u, shift right] &
\end{tikzcd}
$$

$$= f(z) \qquad Y(z^{-1}) \longleftrightarrow -(p^{-1})^{n+1}$$

$$\frac{z^{-1}}{1-pz^{-1}} = g(z^{-1}) \qquad X(z) \longleftrightarrow (p)^{n-1}$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift right, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[l, shift left] \\
\boxed{\dfrac{z}{1-pz}} \arrow[u, shift right] &
\end{tikzcd}
$$

$$= f(z^{-1}) \qquad Y(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$\frac{z}{1-pz} = g(z) \longleftrightarrow (p)^{n-1} \qquad X(z^{-1})$$

2 formulas of z : $f(z)$, $g(z)$

2 representations : $f(z^{-1})$, $g(z^{-1})$

* Simple Pole Forms

$$
\begin{tikzcd}
p & \boxed{\dfrac{1}{z-p}} \arrow[rr, leftrightarrow, "z^{-1}"] \arrow[d, shift left, "\frac{z^{-1}}{z^{-1}}\cdot"'] & & \boxed{\dfrac{1}{z^{-1}-p}} \arrow[d, shift left, "\frac{z}{z}\cdot"'] & p^{-1} \\
p & \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[rr, leftrightarrow, "z^{-1}"] \arrow[u, shift left, "\cdot\frac{z}{z}"'] & & \boxed{\dfrac{z}{1-pz}} \arrow[u, shift left, "\cdot\frac{z^{-1}}{z^{-1}}"'] & p^{-1}
\end{tikzcd}
$$

* Geometric Series Forms

$$
\begin{tikzcd}
\overset{n \geq 0}{\boxed{\begin{array}{c} -(p^{-1})^{n+1} \\ \text{causal } f(z) \end{array}}} & \overset{|z|<p}{\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}} \arrow[rr, leftrightarrow, "z^{-1}"] \arrow[d, equal, "\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}}"] & & \overset{|z|>p^{-1}}{\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}} \arrow[d, equal, "\cdot\frac{p}{p}\frac{z}{z}"'] & \overset{n \geq 0}{\boxed{\begin{array}{c} \text{causal } X(z) \\ -(p^{-1})^{n+1} \end{array}}} \\
\underset{n \geq 1}{\boxed{\begin{array}{c} \text{causal } X(z) \\ (p)^{n-1} \end{array}}} & \underset{|z|>p}{\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}} \arrow[rr, leftrightarrow, "z^{-1}"] & & \underset{|z|<p^{-1}}{\boxed{\dfrac{z}{1-pz}}} & \underset{n \geq 1}{\boxed{\begin{array}{c} (p)^{n-1} \\ \text{causal } f(z) \end{array}}}
\end{tikzcd}
$$

(A) $f(z)$ for $|z|<p$, $g(z)$ for $|z|<p^{-1}$ Laurent S

Geometric Series Forms

p $\boxed{f(z)} = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}$ ($|z|<p$) $\quad \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}$

$\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \quad \boxed{\dfrac{z}{1-pz}} = \boxed{g(z)}$ p^{-1} ($|z|<p^{-1}$)

(B) $f(z^{-1})$ for $|z|>p^{-1}$, $g(z^{-1})$ for $|z|>p$

Geometric Series Forms

$$f(z) = \underbrace{\boxed{-\frac{p^{-1}}{1-p^{-1}z}}}_{|z|<p} \xleftrightarrow{z^{-1}} \underbrace{\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}}_{|z|>p^{-1}} = \boxed{f(z^{-1})}$$

$$\boxed{g(z^{-1})} = \underbrace{\boxed{\frac{z^{-1}}{1-pz^{-1}}}}_{|z|>p} \xleftrightarrow{z^{-1}} \underbrace{\boxed{\frac{z}{1-pz}}}_{|z|<p^{-1}} = g(z)$$

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

Geometric Series Forms

$$|z|<p \qquad\qquad\qquad |z|>p^{-1}$$

$$f(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \xleftrightarrow{\;z^{-1}\;} \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1})$$

$$\cdot\,\frac{p}{p}\,\frac{z^{-1}}{z^{-1}} \qquad\qquad\qquad \cdot\,\frac{p}{p}\,\frac{z}{z}$$

$$g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}} \xleftrightarrow{\;z^{-1}\;} \boxed{\frac{z}{1-pz}} = g(z)$$

$$|z|>p \qquad\qquad\qquad |z|<p^{-1}$$

$\boxed{\begin{array}{ll} f(z) & (|z|<p) \\ a_n & (n \geq 0) \end{array}} \xleftrightarrow{\;z^{-1}\;} \boxed{\begin{array}{ll} f(z^{-1}) & (|z^{-1}|<p) \\ a_{-n} & (n<1) \end{array}}\ \boxed{\begin{array}{ll} g(z) & (|z|>p^{-1}) \\ -b_n & (n<1) \end{array}}$

$\boxed{\begin{array}{ll} f(z) & (|z|>p) \\ -a_n & (n<0) \end{array}}\ \boxed{\begin{array}{ll} g(z^{-1}) & (|z^{-1}|<p^{-1}) \\ b_{-n} & (n<0) \end{array}} \xleftrightarrow{\;z^{-1}\;} \boxed{\begin{array}{ll} g(z) & (|z|<p^{-1}) \\ b_n & (n \geq 1) \end{array}}$

Laurent Series using only $\boxed{a_n} \leftrightarrow \boxed{f(z)}$

$$
\begin{tikzcd}
 & \boxed{\begin{matrix} f(z) & (|z|<p) \\ a_n & (n \geq 0) \end{matrix}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{matrix} f(z^{-1}) & (|z^{-1}|<p) \\ a_{-n} & (n<1) \end{matrix}} \arrow[d, equal] & \boxed{\begin{matrix} g(z) & (|z|>p^{-1}) \\ -b_n & (n<1) \end{matrix}} \\
\boxed{\begin{matrix} f(z) & (|z|>p) \\ -a_n & (n<0) \end{matrix}} & \boxed{\begin{matrix} g(z^{-1}) & (|z^{-1}|<p^{-1}) \\ b_{-n} & (n<0) \end{matrix}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{matrix} g(z) & (|z|<p^{-1}) \\ b_n & (n \geq 1) \end{matrix}} &
\end{tikzcd}
$$

$$a_{-n} = -b_n \qquad\qquad -a_{-n} = b_n$$

$$
\begin{tikzcd}
\boxed{\begin{matrix} f(z) & (|z|<p) \\ a_n & (n \geq 0) \end{matrix}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{\begin{matrix} f(z^{-1}) & (|z|>p^{-1}) \\ a_{-n} & (n<1) \end{matrix}} \arrow[d, equal] \\
\boxed{\begin{matrix} f(z) & (|z|>p) \\ -a_n & (n<0) \end{matrix}} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{\begin{matrix} f(z^{-1}) & (|z|<p^{-1}) \\ -a_{-n} & (n \geq 1) \end{matrix}}
\end{tikzcd}
$$

Laurent Series $\boxed{a_n} \leftrightarrow f(z)$

$$\boxed{a_n \quad (n \geq 0)} \xleftrightarrow{z^{-1}} \boxed{a_{-n} \quad (n < 1)}$$

$$\boxed{b_{-n} \quad (n < 0)} \xleftrightarrow{z^{-1}} \boxed{b_n \quad (n \geq 1)}$$

$$\begin{tikzcd}
\boxed{a_n \quad (n \geq 0)} \arrow[d, equal] & \boxed{-b_n \quad (n < 1)} \arrow[d, equal] \\
\boxed{-a_n \quad (n < 0)} & \boxed{b_n \quad (n \geq 1)}
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{a_n \quad (n \geq 0)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{a_{-n} \quad (n < 1)} \arrow[d, equal] \\
\boxed{-a_n \quad (n < 0)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{-a_{-n} \quad (n \geq 1)}
\end{tikzcd}$$

Laurent Series $a_n \leftrightarrow \boxed{f(z)}$

$$\begin{tikzcd}
\boxed{f(z) \quad (|z|<p)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{f(z^{-1}) \quad (|z^{-1}|<p)}
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{g(z^{-1}) \quad (|z^{-1}|<p^{-1})} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{g(z) \quad (|z|<p^{-1})}
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{f(z) \quad (|z|<p)} \arrow[d, equal] & \boxed{f(z^{-1}) \quad (|z|>p^{-1})} \arrow[d, equal] \\
\boxed{g(z^{-1}) \quad (|z|>p)} & \boxed{g(z) \quad (|z|<p^{-1})}
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{f(z) \quad (|z|<p)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{f(z^{-1}) \quad (|z|>p^{-1})} \arrow[d, equal] \\
\boxed{f(z) \quad (|z|>p)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{f(z^{-1}) \quad (|z|<p^{-1})}
\end{tikzcd}$$

Z-Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

Geometric Series Forms

$$
\begin{tikzcd}[column sep=large, row sep=huge]
Y(z^{-1}) = \underset{|z|<p}{\boxed{-\dfrac{p^{-1}}{1-p^{-1}z}}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal, "\cdot\frac{p}{p}\cdot\frac{z^{-1}}{z^{-1}}"] & \underset{|z|>p^{-1}}{\boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}}} = Y(z) \arrow[d, equal, "\cdot\frac{p}{p}\cdot\frac{z}{z}"] \\
X(z) = \underset{|z|>p}{\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}} \arrow[r, leftrightarrow, "z^{-1}"] & \underset{|z|<p^{-1}}{\boxed{\dfrac{z}{1-pz}}} = X(z^{-1})
\end{tikzcd}
$$

causal $X(z),\ Y(z)$

anti-causal $X(z^{-1}),\ Y(z^{-1})$

$g(z^{-1})$

$$
\begin{tikzcd}[column sep=large, row sep=huge]
\underset{n<1}{\boxed{\text{anti-causal } -x_n}} & \underset{|z|<p}{\boxed{Y(z^{-1})}} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal, "\cdot\frac{p}{p}\cdot\frac{z^{-1}}{z^{-1}}"] & \underset{|z|>p^{-1}}{\boxed{Y(z)}} \arrow[d, equal, "\cdot\frac{p}{p}\cdot\frac{z}{z}"] & \underset{n\geq 0}{\boxed{\text{causal } y_n}} \\
\underset{n\geq 1}{\boxed{\text{causal } x_n}} & \underset{|z|>p}{\boxed{X(z)}} \arrow[r, leftrightarrow, "z^{-1}"] & \underset{|z|<p^{-1}}{\boxed{X(z^{-1})}} & \underset{n<0}{\boxed{\text{anti-causal } -y_n}}
\end{tikzcd}
$$

Ⓐ $X(z)$ for $|z|>p$, $Y(z)$ for $|z|>p^{-1}$ z-Transform

Geometric Series Forms

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad \overset{|z|>p^{-1}}{\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}} = \boxed{Y(z)} \;\; p^{-1}$$

$$p \;\; \boxed{X(z)} = \overset{|z|>p}{\boxed{\frac{z^{-1}}{1-pz^{-1}}}} \qquad \boxed{\frac{z}{1-pz}}$$

Ⓑ $X(z^{-1})$ for $|z|<p^{-1}$, $Y(z^{-1})$ for $|z|<p$

Geometric Series Forms

$$\boxed{X(z^{-1})} = \overset{|z|<p}{\boxed{-\frac{p^{-1}}{1-p^{-1}z}}} \xleftrightarrow{z^{-1}} \overset{|z|>p^{-1}}{\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}} = Y(z)$$

$$X(z) = \underset{|z|>p}{\boxed{\frac{z^{-1}}{1-pz^{-1}}}} \xleftrightarrow{z^{-1}} \underset{|z|<p^{-1}}{\boxed{\frac{z}{1-pz}}} = \boxed{Y(z^{-1})}$$

Z- Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

Geometric Series Forms

$$\underset{|z|<p}{Y(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}}} \quad \xleftrightarrow{z^{-1}} \quad \underset{|z|>p^{-1}}{\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = Y(z)}$$

$$\cdot\frac{p}{p}\frac{z^{-1}}{z^{-1}} \qquad\qquad \cdot\frac{p}{p}\frac{z}{z}$$

$$\underset{|z|>p}{X(z) = \boxed{\frac{z^{-1}}{1-pz^{-1}}}} \quad \xleftrightarrow{z^{-1}} \quad \underset{|z|<p^{-1}}{\boxed{\frac{z}{1-pz}} = X(z^{-1})}$$

$X(z)$ $(\lvert z\rvert < p)$	$Y(z^{-1})$ $(\lvert z^{-1}\rvert > p^{-1})$	$\xleftrightarrow{z^{-1}}$	$Y(z)$ $(\lvert z\rvert > p^{-1})$
$-x_n$ $(n<1)$	y_{-n} $(n<1)$		y_n $(n \geq 0)$

$X(z)$ $(\lvert z\rvert > p)$	$\xleftrightarrow{z^{-1}}$	$X(z^{-1})$ $(\lvert z^{-1}\rvert > p)$	$Y(z)$ $(\lvert z\rvert < p^{-1})$
x_n $(n \geq 1)$		x_{-n} $(n<0)$	$-y_n$ $(n<0)$

z-Transform using only $\boxed{x_n} \leftrightarrow \boxed{X(z)}$

$$\boxed{\begin{array}{l} X(z)\ (|z|<p) \\ -x_n\ (n<1) \end{array}} \ \boxed{\begin{array}{l} Y(z^{-1})\ (|z^{-1}|>p^{-1}) \\ y_{-n}\ (n<1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{l} Y(z)\ (|z|>p^{-1}) \\ y_n\ (n\geq 0) \end{array}}$$

$$\boxed{\begin{array}{l} X(z)\ (|z|>p) \\ x_n\ (n\geq 1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{l} X(z^{-1})\ (|z^{-1}|>p) \\ x_{-n}\ (n<0) \end{array}} \ \boxed{\begin{array}{l} Y(z)\ (|z|<p^{-1}) \\ -y_n\ (n<0) \end{array}}$$

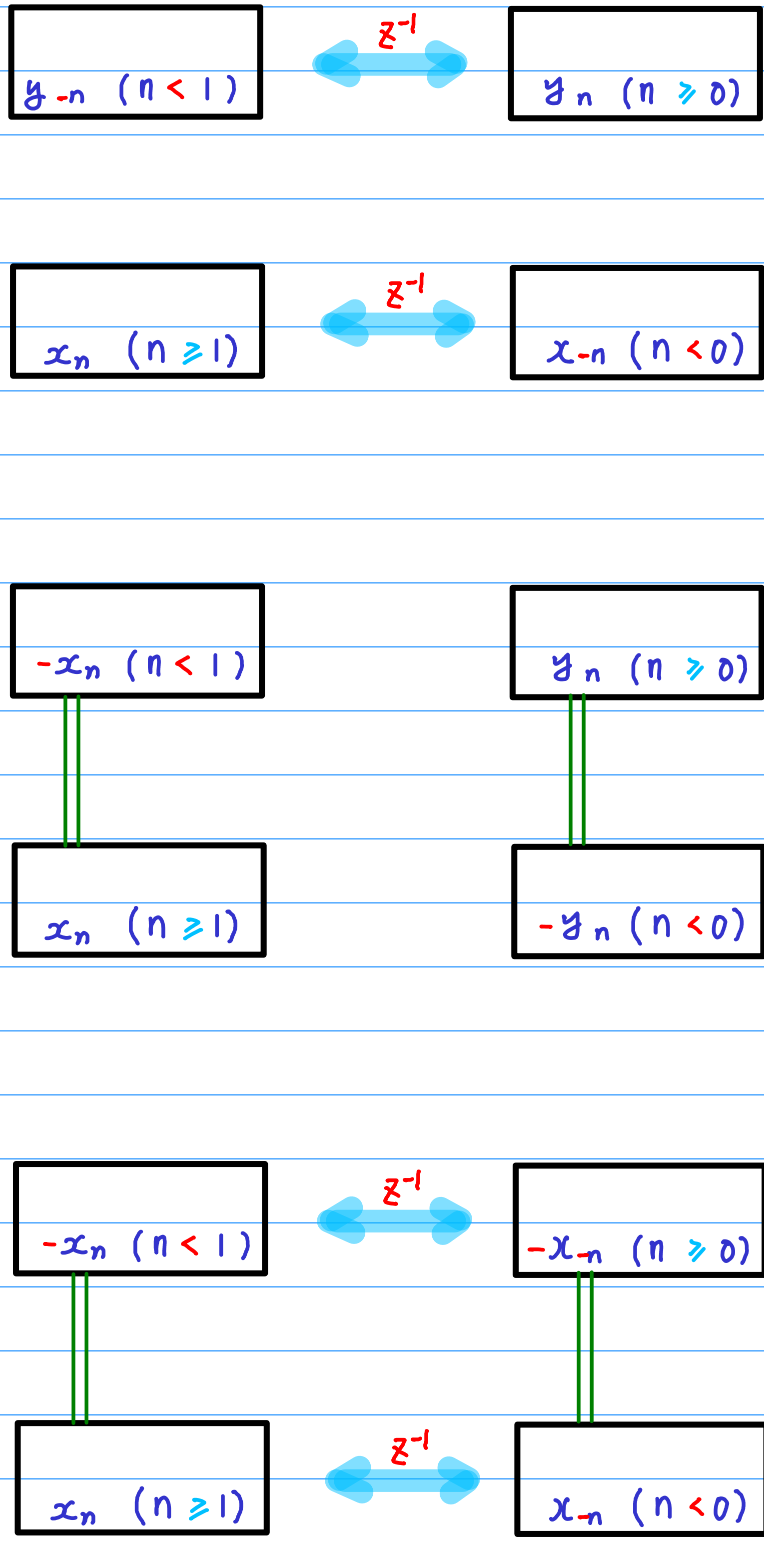
$$x_{-n} = -y_n \qquad -x_{-n} = y_n$$

$$\boxed{\begin{array}{l} X(z)\ (|z|<p) \\ -x_n\ (n<1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{l} X(z^{-1})\ (|z|>p^{-1}) \\ -x_{-n}\ (n\geq 0) \end{array}}$$

y_n

$$\boxed{\begin{array}{l} X(z)\ (|z|>p) \\ x_n\ (n\geq 1) \end{array}} \xleftrightarrow{z^{-1}} \boxed{\begin{array}{l} X(z^{-1})\ (|z|<p^{-1}) \\ x_{-n}\ (n<0) \end{array}}$$

z - Transform $\boxed{x_n} \leftrightarrow X(z)$



z - Transform $\quad x_n \leftrightarrow \boxed{X(z)}$

$$\begin{tikzcd}
\boxed{Y(z^{-1})\ (|z^{-1}| > p^{-1})} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{Y(z)\ (|z| > p^{-1})}
\end{tikzcd}$$

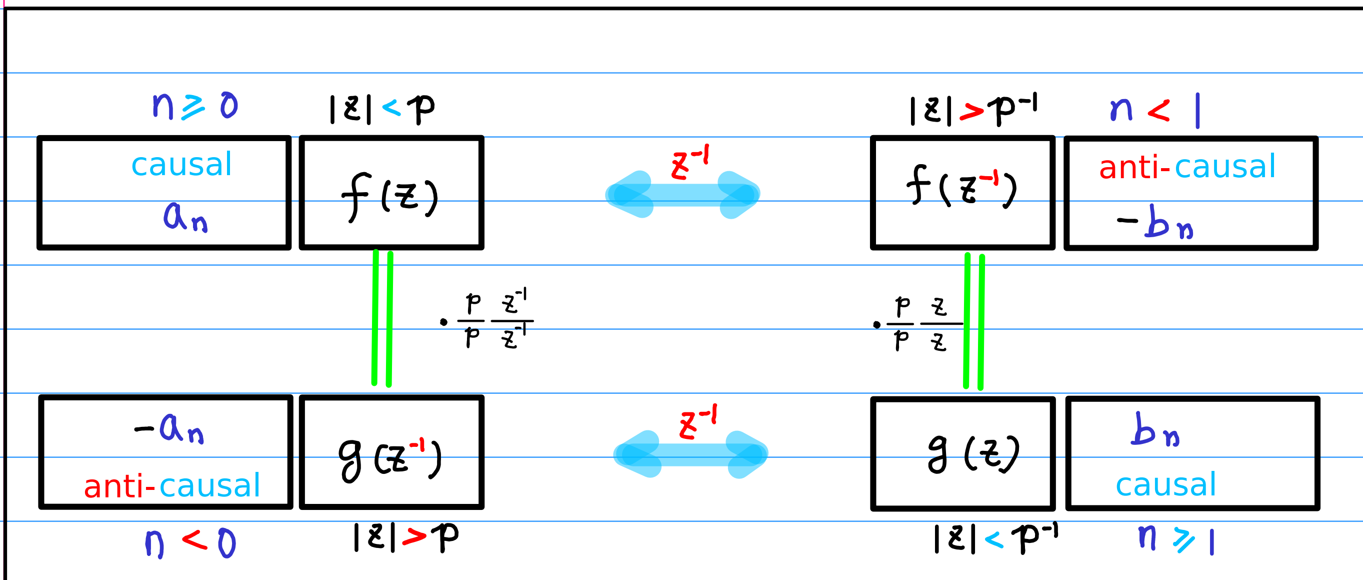
$$\begin{tikzcd}
\boxed{X(z)\ (|z| > p)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{X(z^{-1})\ (|z^{-1}| > p)}
\end{tikzcd}$$

$$\begin{tikzcd}
\boxed{X(z)\ (|z| < p)} \arrow[d, equal] & \boxed{Y(z)\ (|z| > p^{-1})} \arrow[d, equal] \\
\boxed{X(z)\ (|z| > p)} & \boxed{Y(z)\ (|z| < p)}
\end{tikzcd}$$

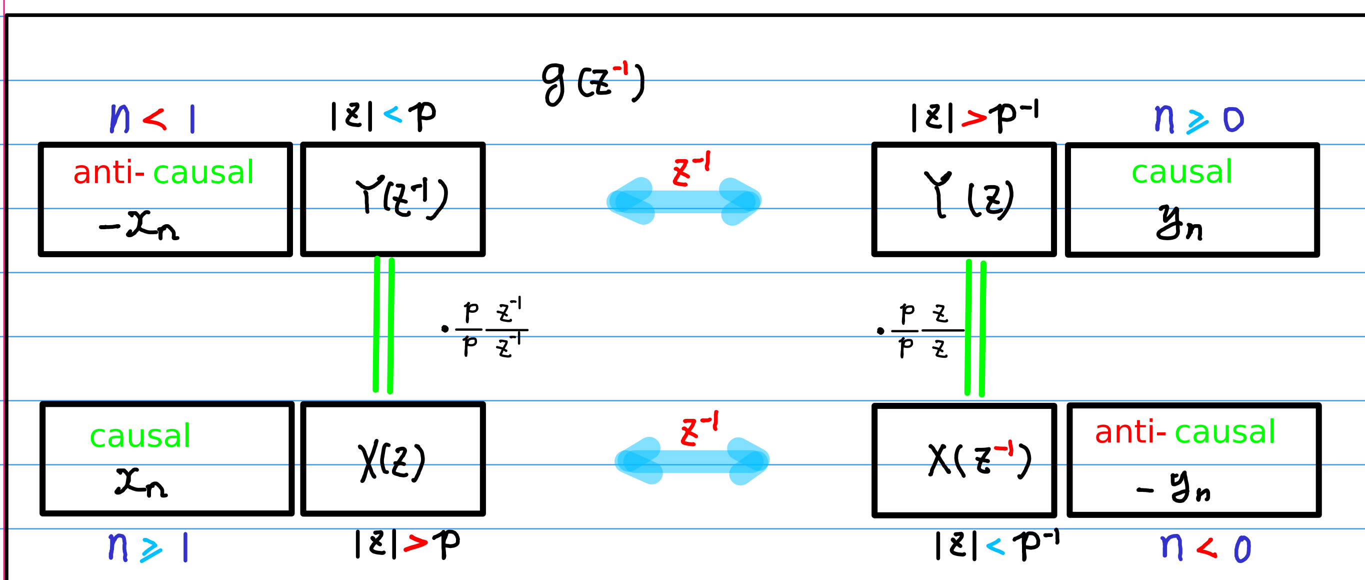
$$\begin{tikzcd}
\boxed{X(z)\ (|z| < p)} \arrow[r, leftrightarrow, "z^{-1}"] \arrow[d, equal] & \boxed{X(z^{-1})\ (|z| > p^{-1})} \arrow[d, equal] \\
\boxed{X(z)\ (|z| > p)} \arrow[r, leftrightarrow, "z^{-1}"] & \boxed{X(z^{-1})\ (|z| < p)}
\end{tikzcd}$$

Laurent Series & Z-Transform ①

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

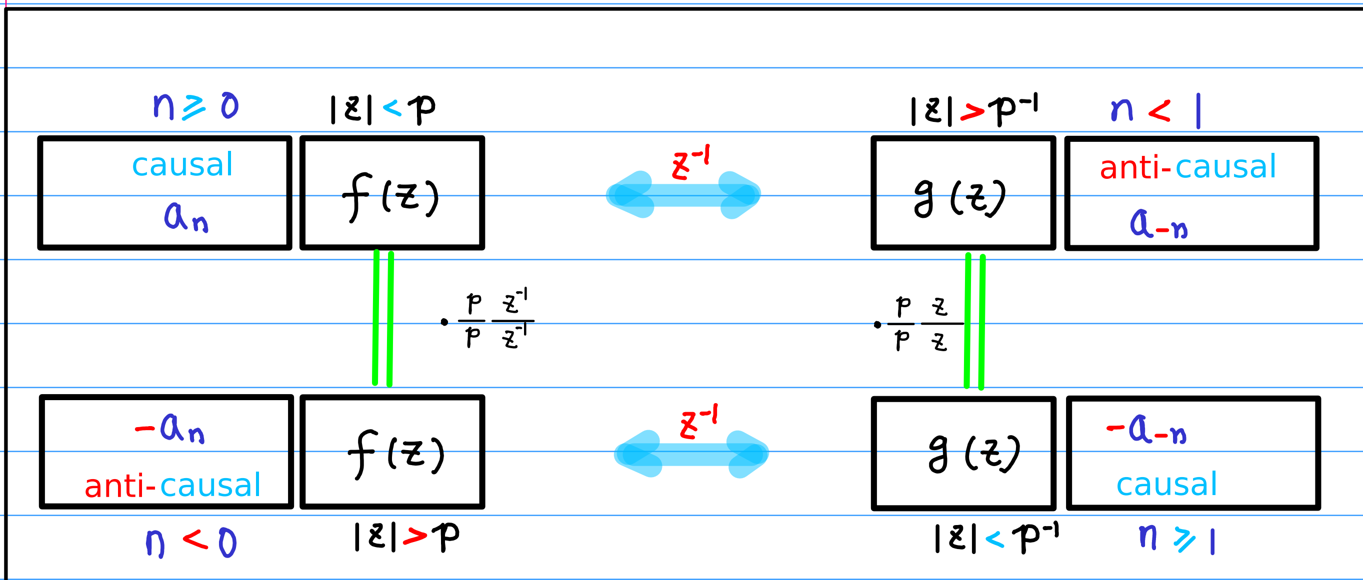


Z-Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

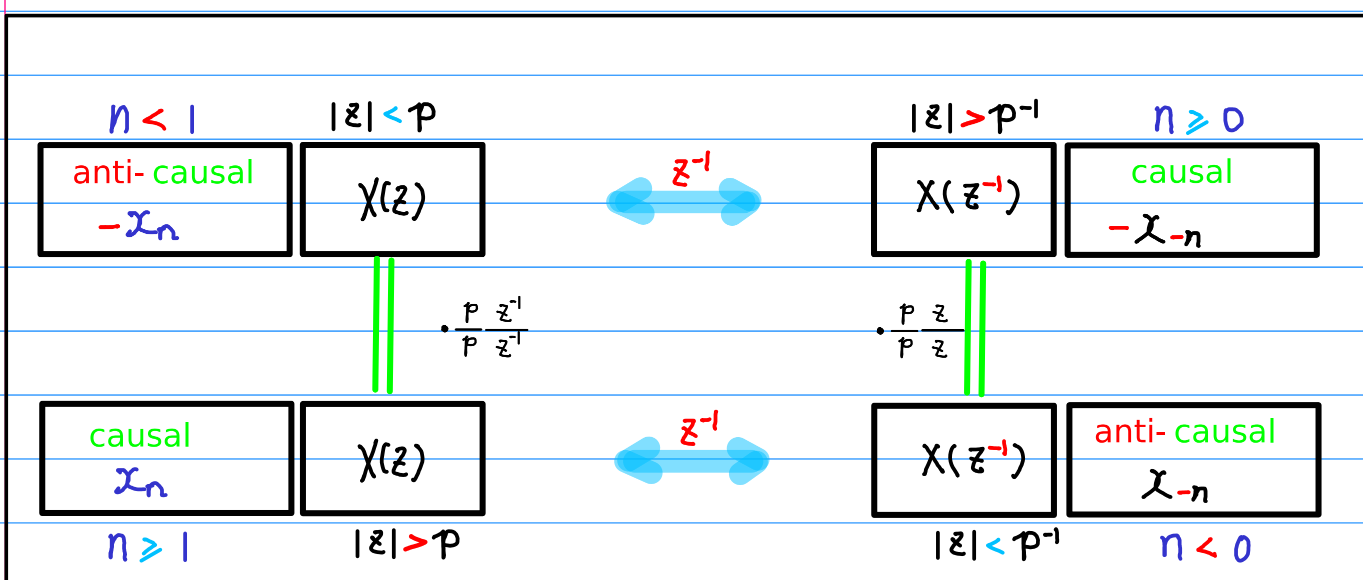


Laurent Series & Z-Transform ②

Laurent Series $a_n \leftrightarrow f(z)$



Z-Transform $X(z) \leftrightarrow x_n$



causal $f(z)$ $(|z| < p)$ anti-causal $f(z)$ $(|z| > p)$

$f(z) \leftrightarrow a_n$ $(n \geq 0)$ $g(z^{-1}) \leftrightarrow -a_n$ $(n < 0)$

$n \geq 0$ $|z| < p$

causal f(z)	
$-(p^{-1})^{n+1}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$

$$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{n} \qquad n \geq 0$$

$\cdot(-1)$ $\cdot \dfrac{p}{p}\,\dfrac{z^{-1}}{z^{-1}}$

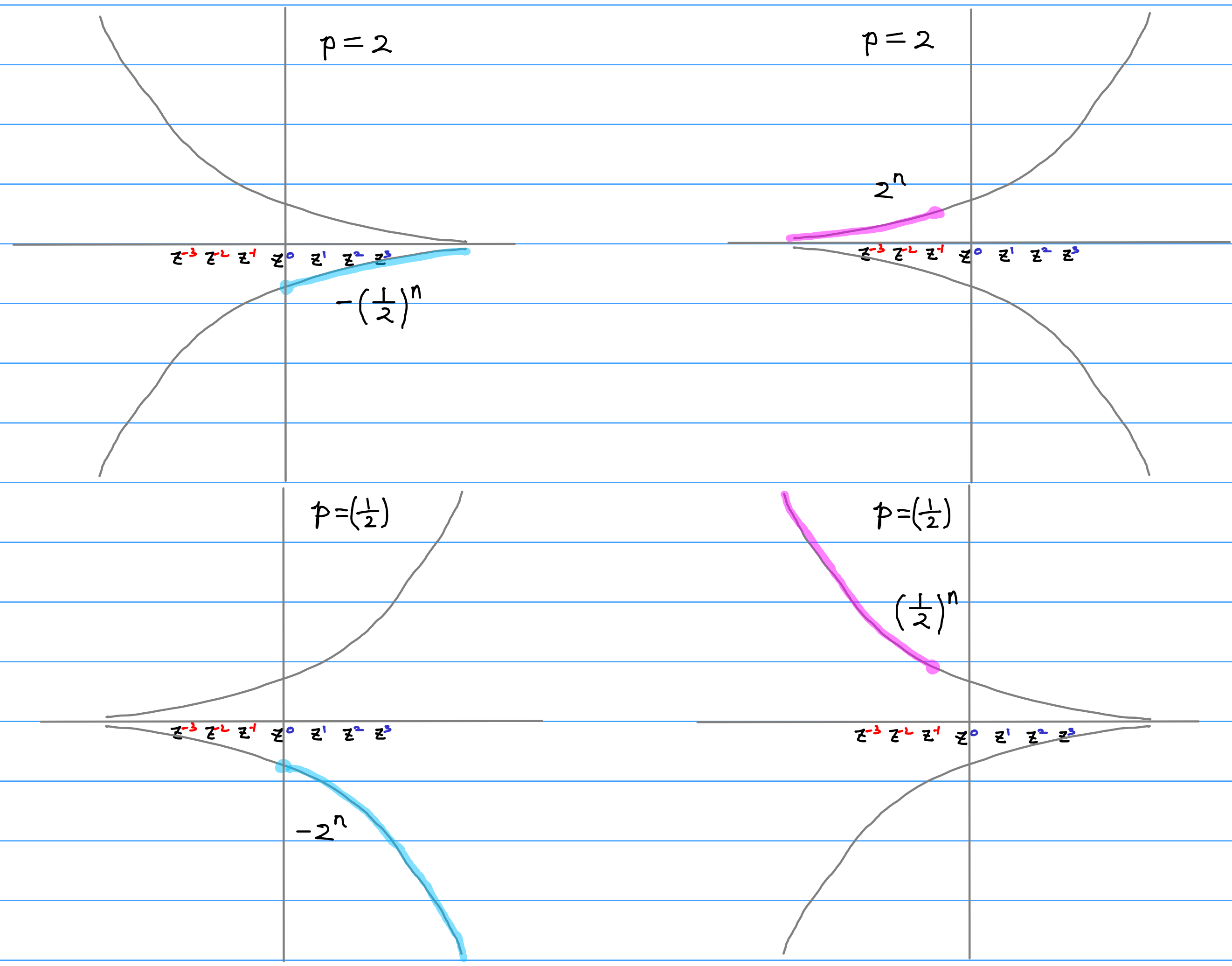
anti-causal f(z)	
$(p)^{-n-1}$	$\dfrac{z^{-1}}{1-pz^{-1}}$

$n < 0$ $|z| > p$

$$p^{0}z^{-1}+p^{1}z^{-2}+p^{2}z^{-3}+\cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^{n} \qquad n < 0$$

causal n=0,+1,+2,+3,.... $-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

anti-causal n=-1,-2,-3,.... $(p^{0}, p^{1}, p^{2}, \cdots)$



anti-causal $g(z)$ $(|z|>p^{-1})$

$f(z^{-1}) \longleftrightarrow -b_n$ $(n<1)$

causal $g(z)$ $(|z|<p^{-1})$

$g(z) \longleftrightarrow b_n$ $(n\geq 1)$

$|z|>p^{-1}$ n < 1

n < 1 $$-(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^{n}$$

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \qquad \boxed{\text{anti-causal } f(z) \quad -(p^{-1})^{-n+1}}$$

$\cdot\frac{p}{p}\frac{z}{z}$ $\cdot(-1)$

n ≥ 1 $$p^{0}z^{1}+p^{1}z^{2}+p^{2}z^{3}+\cdots = \sum_{n=1}^{\infty}(p)^{n-1}z^{n}$$

$$\boxed{\frac{z}{1-pz}} \qquad \boxed{(p)^{n-1} \quad \text{causal } f(z)}$$

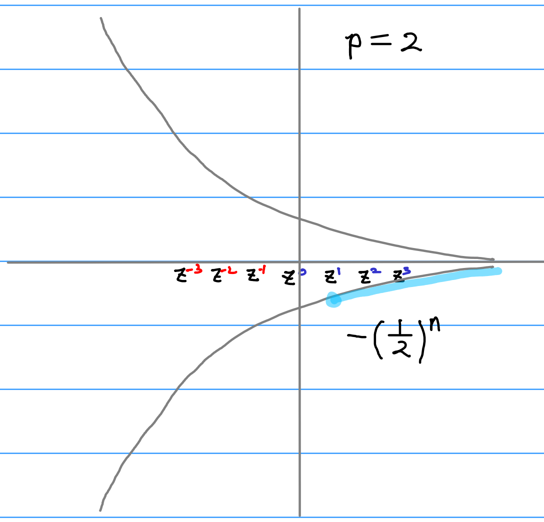
$|z|<p^{-1}$ n ≥ 1

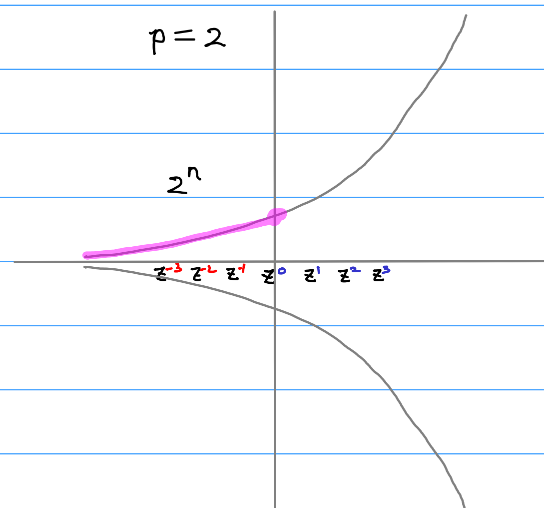
causal n=+1,+2,+3,....

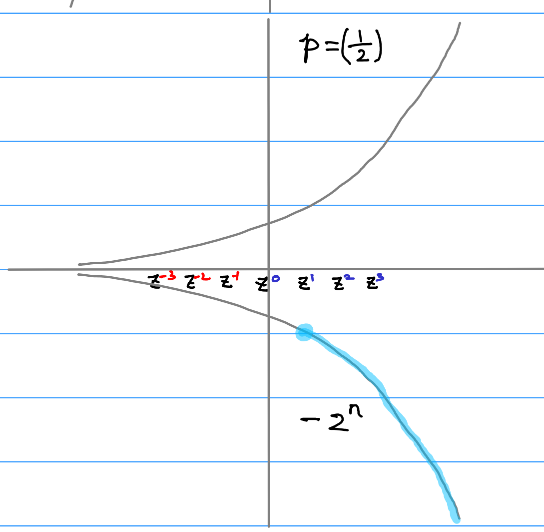
$-(p^{0}, p^{1}, p^{2}, \cdots)$

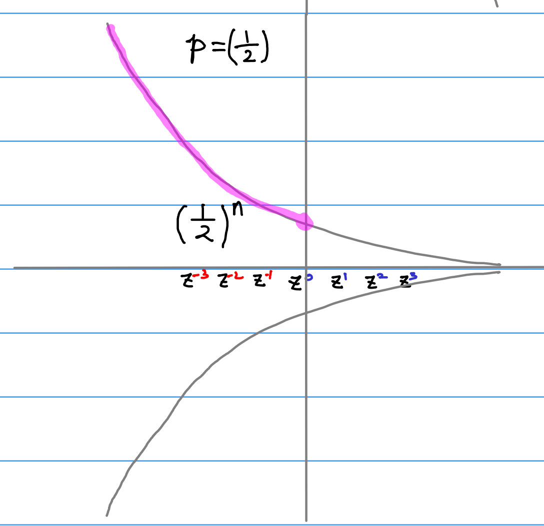
anti-causal n=0,-1,-2,-3,....

$(p^{-1}, p^{-2}, p^{-3}, \cdots)$



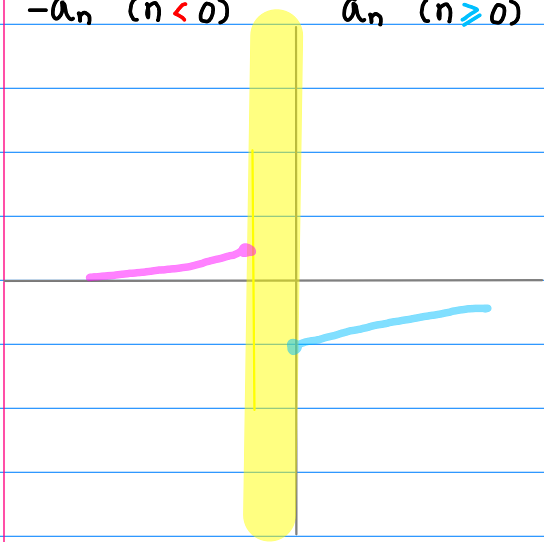






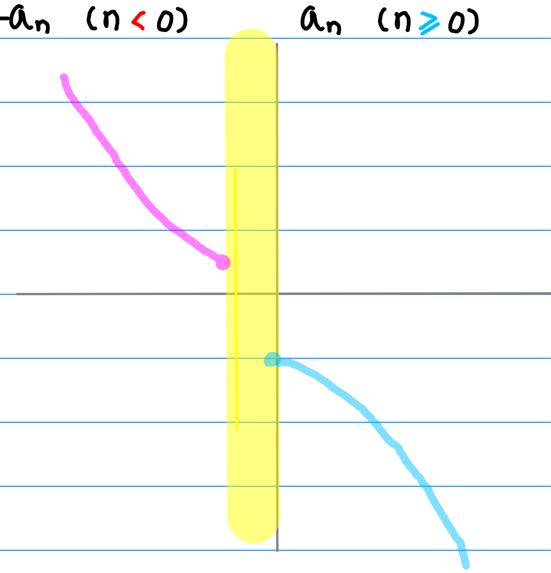
$f(z)$ $(|z|>p)$ $f(z)$ $(|z|<p)$

$-a_n$ $(n<0)$ a_n $(n\geq 0)$



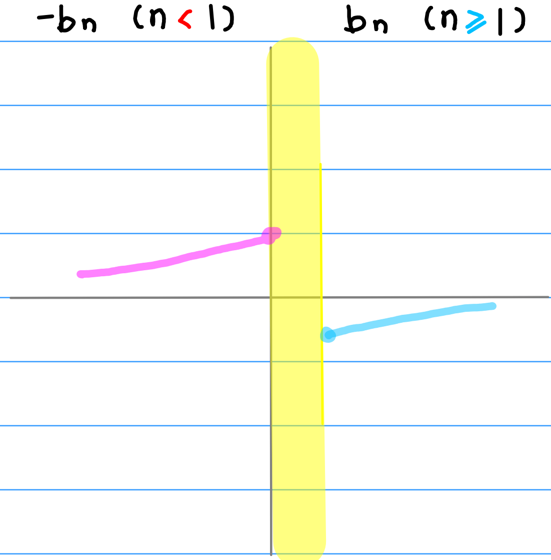
$f(z)$ $(|z|>p)$ $f(z)$ $(|z|<p)$

$-a_n$ $(n<0)$ a_n $(n\geq 0)$



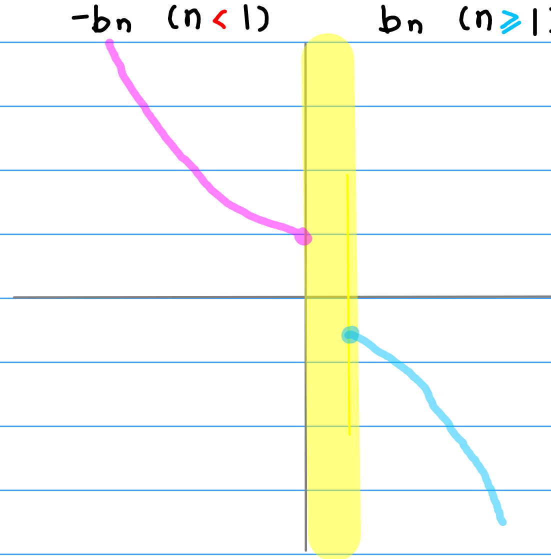
$g(z)$ $(|z|>p^{-1})$ $g(z)$ $(|z|<p^{-1})$

$-b_n$ $(n<1)$ b_n $(n\geq 1)$



$g(z)$ $(|z|>p^{-1})$ $g(z)$ $(|z|<p^{-1})$

$-b_n$ $(n<1)$ b_n $(n\geq 1)$



causal $f(z)$

$f(z) \leftrightarrow a_n \quad (n \geq 0)$

anti-causal $f(z)$

$f(z^{-1}) \leftrightarrow a_{-n} \quad (n < 1)$

$n \geq 0$ $\quad |z| < p$

$\boxed{\text{causal } f(z) \quad -(p^{-1})^{n+1}}$ $\boxed{-\frac{p^{-1}}{1-p^{-1}z}}$

$$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{n} \quad n \geq 0$$

$|z| > p^{-1}$ $\quad n < 1$

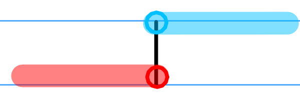
$n < 1$

$$-(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^{n}$$

$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\boxed{\text{anti-causal } f(z) \quad -(p^{-1})^{-n+1}}$

$n \geq 0$

$\boxed{\text{causal } f(z) \quad -p^{-n-1}}$



$-n$

$n < 1$

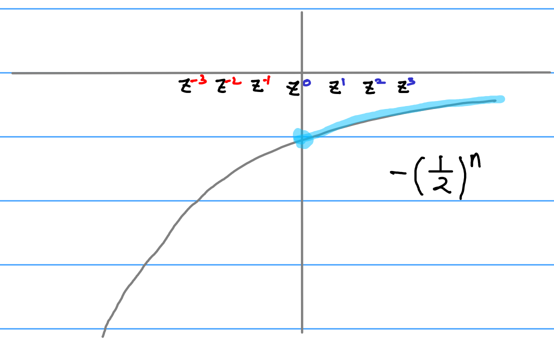
$\boxed{\text{anti-causal } f(z) \quad -p^{+n-1}}$

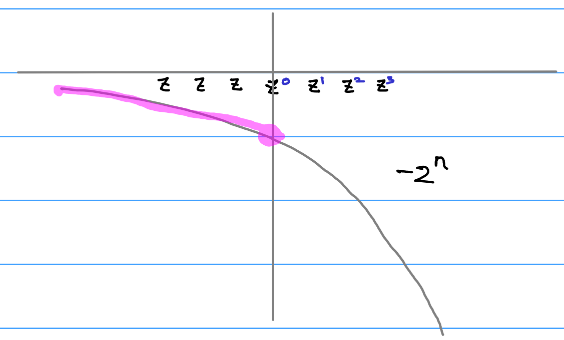
causal $\quad$ n=0,+1,+2,+3,....

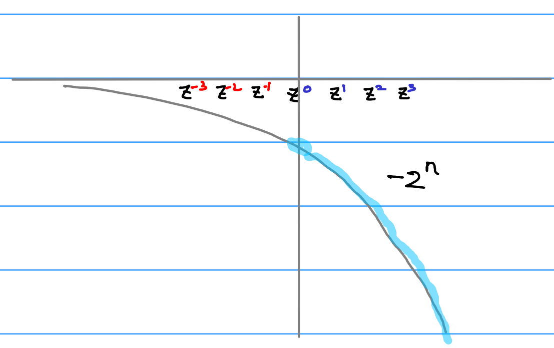
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

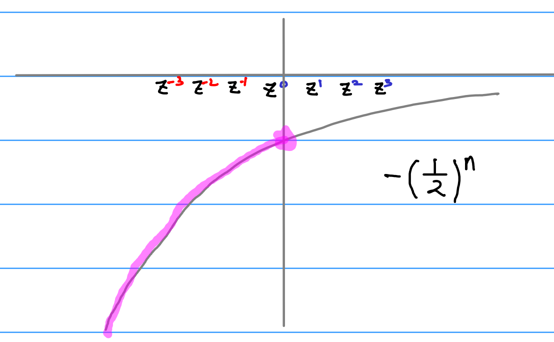
anti-causal $\quad$ n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$









anti-causal $g(z)$ causal $g(z)$

$g(z^{-1}) \leftrightarrow b_{-n}\ (n<0)$ $g(z) \leftrightarrow b_n\ (n \geq 1)$

$n<0$ $|z|>p$

$\boxed{(p)^{-n-1} \text{ anti-causal } f(z)}$ $\boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$

$$p^0 z^{-1} + p^1 z^{-2} + p^2 z^{-3} + \cdots = \sum_{n=-1}^{-\infty} (p)^{-n-1} z^n \qquad n<0$$

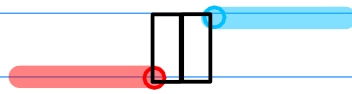
$|z|<p^{-1}$ $n \geq 1$

$$n \geq 1 \qquad p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=1}^{\infty} (p)^{n-1} z^n$$

$\boxed{\dfrac{z}{1-pz}}$ $\boxed{(p)^{n-1} \text{ causal } f(z)}$

$n<0$ $n \geq 1$

$\boxed{p^{-n-1} \text{ anti-causal } f(z)}$

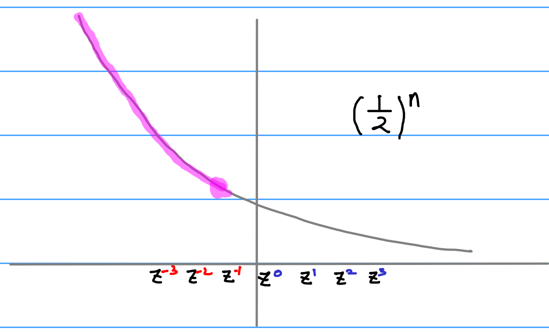


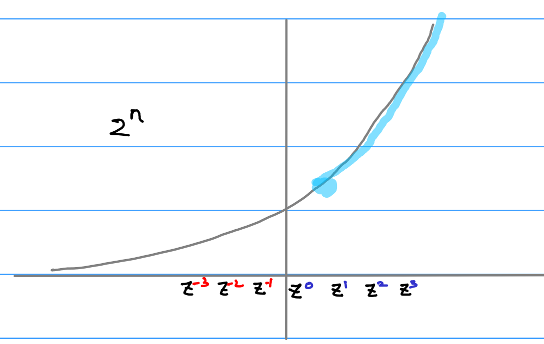
$-n$

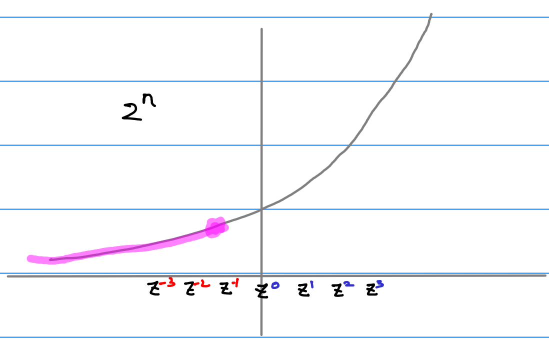
$\boxed{p^{n-1} \text{ causal } f(z)}$

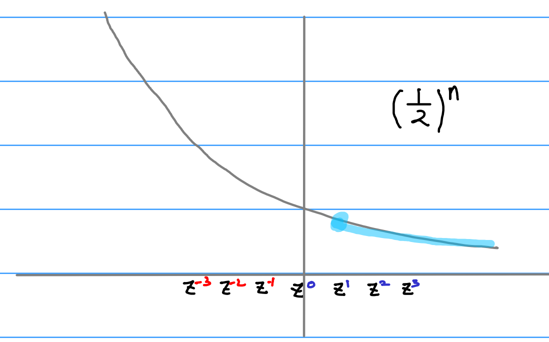
anti-causal $n=-1,-2,-3,\ldots.$ causal $n=+1,+2,+3,\ldots.$

$(p^{-2}, p^{-3}, p^{-4}, \cdots)$ $(p^{-2}, p^{-3}, p^{-4}, \cdots)$









$f(z^{-1})$ $(|z|>p^{-1})$ $f(z)$ $(|z|<p)$

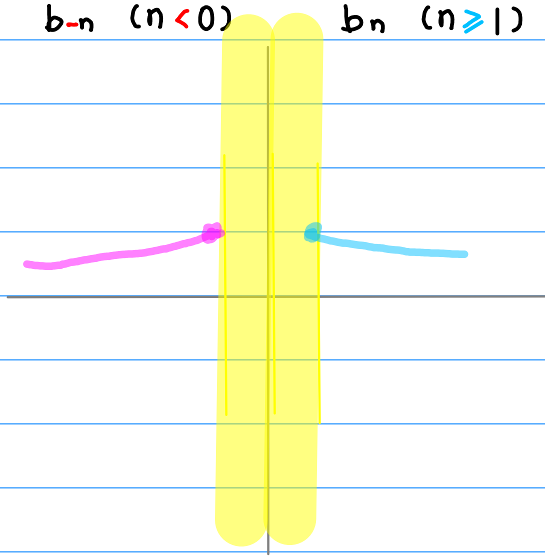
a_{-n} $(n<1)$ a_n $(n\geq 0)$

$f(z^{-1})$ $(|z|>p^{-1})$ $f(z)$ $(|z|<p)$

a_{-n} $(n<1)$ a_n $(n\geq 0)$

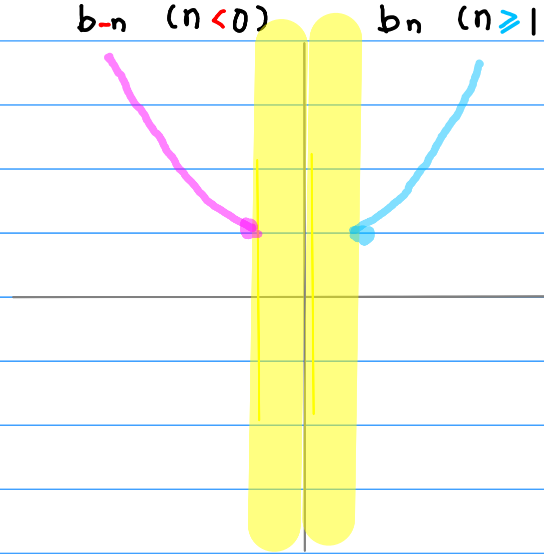
$g(z^{-1})$ $(|z|>p)$ $g(z)$ $(|z|<p^{-1})$

b_{-n} $(n<0)$ b_n $(n\geq 1)$



$g(z^{-1})$ $(|z|>p)$ $g(z)$ $(|z|<p^{-1})$

b_{-n} $(n<0)$ b_n $(n\geq 1)$



x_n		a_{-n}
y_n		b_{-n}
causal		anti-causal
$n \geq 0$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n < 1$
$n \geq 1$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n < 0$
anti-causal		causal
$n < 1$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n \geq 0$
$n < 0$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n \geq 1$

getting causal sequence

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift left, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[l, shift left] & = f(z) \leftrightarrow \boxed{?} \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[u, shift left] & &
\end{tikzcd}
$$

$$\| $$

$$X(z) \leftrightarrow \boxed{?}$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, shift left, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, shift left, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[l, shift left] & = Y(z) \leftrightarrow \boxed{?} \\
\boxed{\dfrac{z}{1-pz}} \arrow[u, shift left] & &
\end{tikzcd}
$$

$$\| $$

$$g(z) \leftrightarrow \boxed{?}$$

getting causal sequence w/o memorizing

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}} = f(z) \leftrightarrow \boxed{-(p^{-1})^{n+1}}$$

$$\boxed{\frac{z}{1-pz}} = g(z) \leftrightarrow \boxed{(p)^{n-1}}$$

Left shift

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} = X(z) \leftrightarrow \boxed{(p)^{n-1}}$$

Left shift

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = Y(z) \leftrightarrow \boxed{-(p^{-1})^{n+1}}$$

Getting anti-causal sequence

$$\boxed{\frac{1}{z-p}} \xleftrightarrow{\frac{p^{-1}}{p^{-1}}} \boxed{-\frac{p^{-1}}{1-p^{-1}z}} = f(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$Y(z^{-1}) \longleftrightarrow \boxed{?}$$

$$\boxed{\frac{1}{z-p}} \xleftrightarrow{\frac{z^{-1}}{z^{-1}}} \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} = g(z^{-1}) \longleftrightarrow \boxed{?}$$

$$X(z) \longleftrightarrow (p)^{n-1}$$

$$\boxed{\frac{1}{z^{-1}-p}} \xleftrightarrow{\frac{p^{-1}}{p^{-1}}} \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1}) \longleftrightarrow \boxed{?}$$

$$Y(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$\boxed{\frac{1}{z^{-1}-p}} \xleftrightarrow{\frac{z^{-1}}{z^{-1}}} \boxed{\frac{z}{1-pz}}$$

$$\boxed{\frac{z}{1-pz}} = g(z) \longleftrightarrow (p)^{n-1}$$

$$X(z^{-1}) \longleftrightarrow \boxed{?}$$

① $z \leftarrow z^{-1}$ ② $a_n \leftarrow a_{-n}$

$$\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[dd, dash, "z^{-1}"] \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[dd, dash, "z^{-1}"'] & \\
& \\
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \\
\boxed{\dfrac{z}{1-pz}} &
\end{tikzcd}$$

$$-\frac{p^{-1}}{1-p^{-1}z} = f(z) \leftrightarrow -(p^{-1})^{n+1}$$

$$Y(z^{-1}) \leftrightarrow \boxed{-(p^{-1})^{-n+1}}$$

$$\frac{z^{-1}}{1-pz^{-1}} = g(z^{-1}) \leftrightarrow \boxed{(p)^{-n-1}}$$

$$X(z) \leftrightarrow (p)^{n-1}$$

$$-\frac{p^{-1}}{1-p^{-1}z^{-1}} = f(z^{-1}) \leftrightarrow \boxed{-(p^{-1})^{-n+1}}$$

$$Y(z) \leftrightarrow -(p^{-1})^{n+1}$$

$$\frac{z}{1-pz} = g(z) \leftrightarrow (p)^{n-1}$$

$$X(z^{-1}) \leftrightarrow \boxed{(p)^{-n-1}}$$

Getting anti-causal sequence w/o memorizing

$$f(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \longrightarrow \boxed{-\frac{p^{-1}}{1-p^{-1}z}} = f(z)$$

$$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$$

$$a_{-n} = \boxed{-(p^{-1})^{-n+1}} \longleftarrow \boxed{-(p^{-1})^{n+1}} = a_n$$

$$g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}} \longrightarrow \boxed{\frac{z}{1-pz}} = g(z)$$

$$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$$

$$b_{-n} = \boxed{(p)^{-n-1}} \longleftarrow \boxed{(p)^{n-1}} = b_n$$

$$X(z^{-1}) = \boxed{\frac{z}{1-pz}} \longrightarrow \boxed{\frac{z^{-1}}{1-pz^{-1}}} = X(z)$$

$$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$$

$$x_{-n} = \boxed{(p)^{-n-1}} \longleftarrow \boxed{(p)^{n-1}} = x_n$$

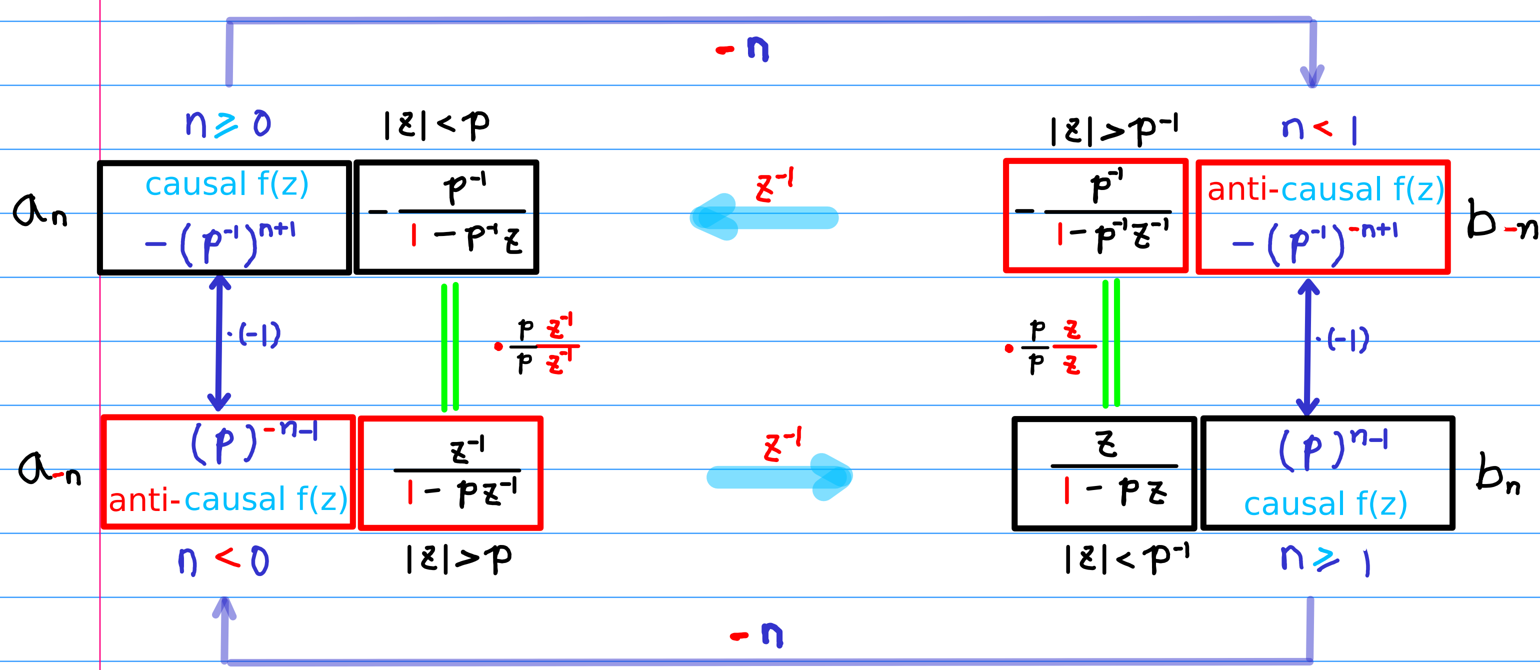
$$Y(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \longrightarrow \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = Y(z)$$

$$\updownarrow \qquad\qquad\qquad\qquad \updownarrow$$

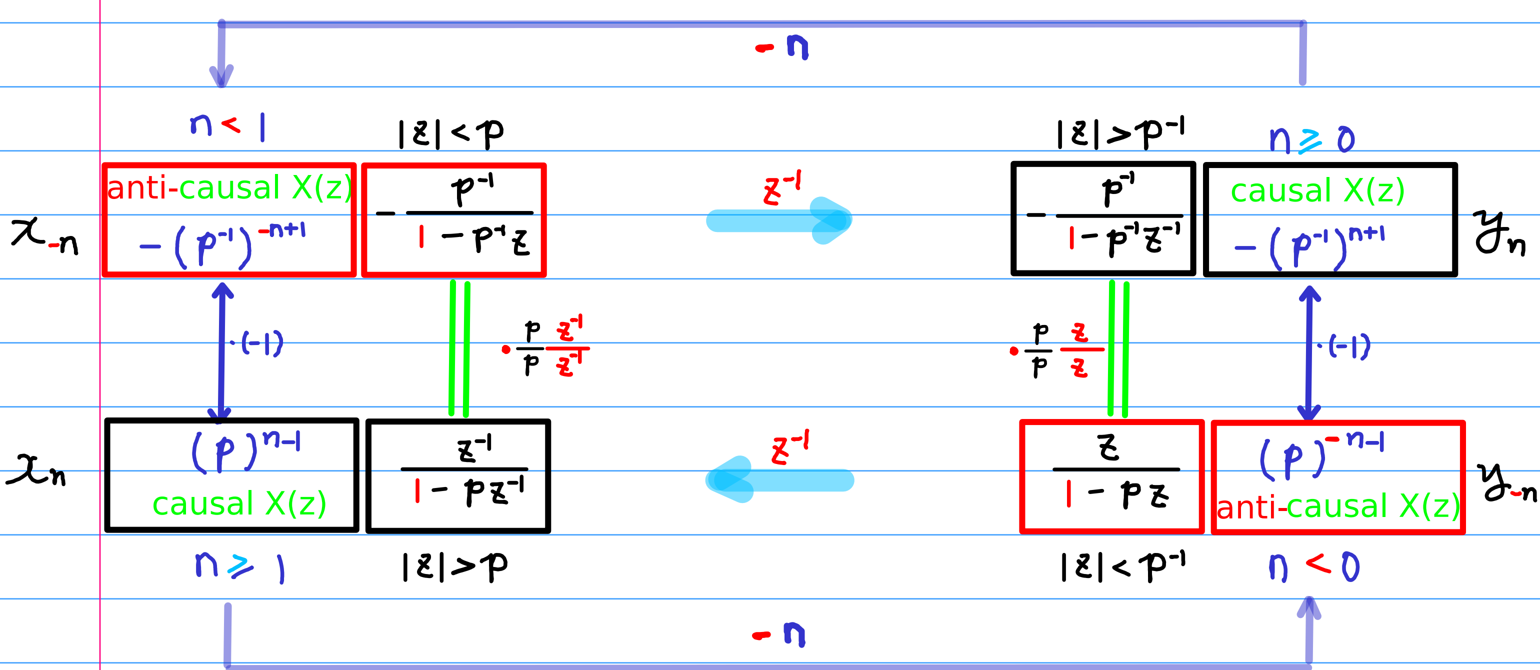
$$y_{-n} = \boxed{-(p^{-1})^{-n+1}} \longleftarrow \boxed{-(p^{-1})^{n+1}} = y_n$$

Getting anti-causal sequence

Laurent Series



Z-Transform



① $z \to z^{-1}$ to get causal $f(z)$, $|z| < a$

② $f(z) \leftrightarrow a_n$ $|z| < a$, $n \geq 0, 1$

③ $n \to -n$ to get anti-causal $n < 0, 1$

① $\dfrac{-1}{(z-1)(z-2)}$

② $\dfrac{-0.5z^2}{(z-1)(z-0.5)}$

$-\dfrac{1}{1-z}+\dfrac{0.5}{1-0.5z}$

$f(z)$	$\lvert z\rvert<1$	causal
$X(z)$	$\lvert z\rvert<1$	anti-causal

$+\dfrac{z}{1-z}-\dfrac{z}{1-2z}$

$f(z)$	$\lvert z\rvert<0.5$	causal
$X(z)$	$\lvert z\rvert<0.5$	anti-causal

$+\dfrac{z^{-1}}{1-z^{-1}}-\dfrac{z^{-1}}{1-2z^{-1}}$

$f(z)$	$\lvert z\rvert>1$	anti-causal
$X(z)$	$\lvert z\rvert>1$	causal

$-\dfrac{1}{1-z^{-1}}+\dfrac{1}{1-0.5z^{-1}}$

$f(z)$	$\lvert z\rvert>2$	anti-causal
$X(z)$	$\lvert z\rvert>2$	causal

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$f(z) \quad |z|<1$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$f(z) \quad |z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$g(z) \quad |z|<0.5$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$X(z) \quad |z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$V(z) \quad |z|>1$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$X(z) \quad |z|>2$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$X(z) \quad |z|<1$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$V(z) \quad |z|>2$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$X(z) \quad |z|<0.5$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$V(z) \quad |z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$W(z) \quad |z|>1$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$f(z) \quad |z|>1$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$g(z) \quad |z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$h(z) \quad |z|<0.5$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$f(z) \quad |z|>2$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$g(z) \quad |z|<1$

Ⓐ $f(z)$ Ⓑ $X(z)$

① $\frac{3}{2}\dfrac{-1}{(z-0.5)(z-2)} = \left(\dfrac{1}{z-0.5} - \dfrac{1}{z-2}\right)$

①-Ⓐ $|z|<0.5$ $f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $(n \geq 0)$

$f(z)$ $|z|>2$ $f(z) = \dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $\boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $(n<0)$

①-Ⓑ $|z|<0.5$ $X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n<1)$

$X(z)$ $|z|>2$ $X(z) = \dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $\boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq 1)$

② $\frac{3}{2}\dfrac{-z^2}{(z-2)(z-0.5)} = \left(\dfrac{0.5z}{(z-0.5)} - \dfrac{2z}{(z-2)}\right)$

②-Ⓐ $|z|<0.5$ $f(z) = -\dfrac{z}{1-2z} + \dfrac{z}{1-0.5z}$ $\boxed{-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}}$ $(n \geq 1)$

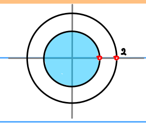
$f(z)$ $|z|>2$ $f(z) = \dfrac{0.5}{1-0.5z^{-1}} - \dfrac{2}{1-2z^{-1}}$ $\boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n-1}}$ $(n<1)$

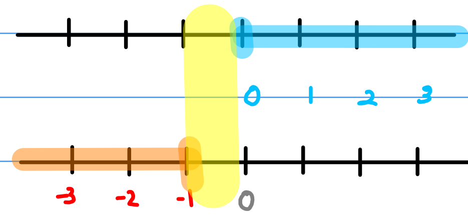
②-Ⓑ $|z|<0.5$ $X(z) = -\dfrac{z}{1-2z} + \dfrac{z}{1-0.5z}$ $\boxed{-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}}$ $(n<0)$

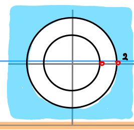
$X(z)$ $|z|>2$ $X(z) = \dfrac{0.5}{1-0.5z^{-1}} - \dfrac{2}{1-2z^{-1}}$ $\boxed{+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}}$ $(n \geq 0)$

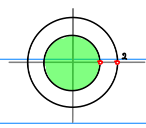
Ⓐ $f(z)$ Ⓑ $X(z)$ time domain view

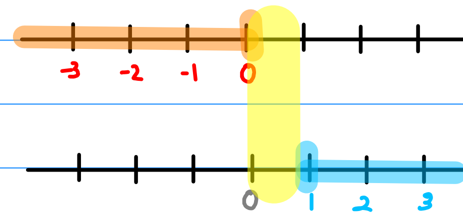
① $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$

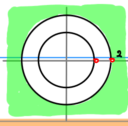
①-Ⓐ $|z| < 1$  $-\dfrac{1}{1-z} + \dfrac{0.5}{1-0.5z}$ $(n \geq 0)$



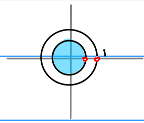
$f(z)$ $|z| > 2$  $(n < 0)$

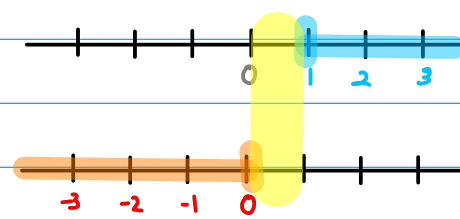
①-Ⓑ $|z| < 1$  $(n < 1)$

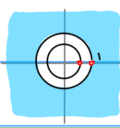


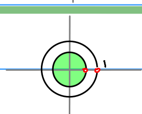
$X(z)$ $|z| > 2$  $\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}}$ $(n \geq 1)$

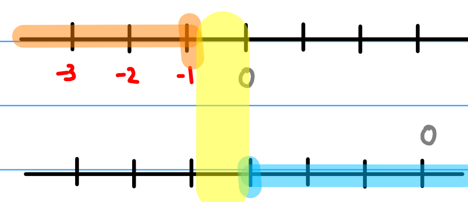
② $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

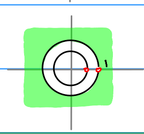
②-Ⓐ $|z| < 0.5$  $+\dfrac{z}{1-z} - \dfrac{z}{1-2z}$ $(n \geq 1)$



$f(z)$ $|z| > 1$  $(n < 1)$

②-Ⓑ $|z| < 0.5$  $(n < 0)$



$X(z)$ $|z| > 1$  $-\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}}$ $(n \geq 0)$

$z^{-1}X(z)$ Shifted Sequence

$$X(z) = \underset{|z|>1}{\frac{1}{1-z^{-1}}} - \underset{|z|>2}{\frac{1}{1-2z^{-1}}} \quad (|z|>2)$$

$$a_n = 1^n + 2^n \quad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots\right) + \left(2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots\right) \quad n \qquad n = 0, 1, 2, \cdots$$

$\cdot z^{-1}$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \quad (n \geq 1)$$

disjoint domains $\quad D_1 : |z| < p \quad D_2 : |z| > p^{-1} \quad 0 < p < 1$

$D_1 : |z| < p^{-1} \quad D_2 : |z| > p \quad p > 1$

disjoint domains $\quad N_1 : n \geq 0 \quad N_2 : n < 0$

$N_1 : n \geq 1 \quad N_2 : n < 1$

causal $\quad f(z) \quad D_1 : |z| < p \quad 0 < p < 1$

$D_1 : |z| < p^{-1} \quad p > 1$

$\dfrac{1}{1 - p^{-1} z} \quad N_1 : n \geq 0$

$\dfrac{z}{1 - p^{-1} z} \quad N_1 : n \geq 1$

causal $\quad X(z) \quad D_2 : |z| > p^{-1} \quad 0 < p < 1$

$D_2 : |z| > p \quad p > 1$

$\dfrac{1}{1 - p z^{-1}} \quad N_1 : n \geq 0$

$\dfrac{z^{-1}}{1 - p z^{-1}} \quad N_1 : n \geq 1$

$z\,f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \quad n = 0, 1, 2, \cdots$$

$\cdot z$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z\,f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$\bullet z$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \qquad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

z^{-1}

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \qquad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$\bullet\, z^{-1}$

$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2})$ n $n = 0, 1, 2, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$ $n-1$ $n = 1, 2, 3, \cdots$

$$z^{-1} X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots)$ n $n = 1, 2, 3, \cdots$

$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

z^{-1}

$(1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots) + (2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots)$ $-n$ $n = -1, -2, -3, \cdots$

$$z X(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$a_{-(n+1)}$

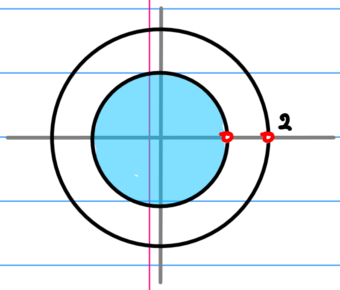
Causal $\quad f(z) \quad X(z)$

$|z|<1 \quad |z|>2$

① − Ⓐ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$|z|<1 \quad |z|<2$



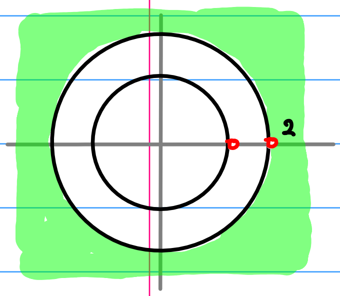
$$f(z) = (-1)\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad (|z|<1)$$

$$a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

① − Ⓑ

$$\frac{-1}{(z-1)(z-2)} = \left(\frac{1}{z-1} - \frac{1}{z-2}\right)$$

$|z|>1 \quad |z|>2$



$$X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$1^{n} - 2^{n} \quad (n \geq 0)$$

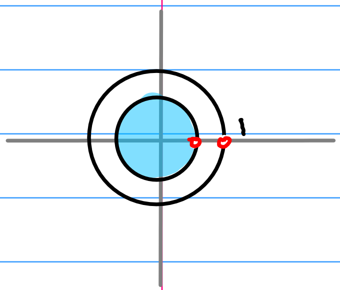
$$a_n = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \quad |z| > 1$

②-Ⓐ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|<1 \qquad |z|<0.5$



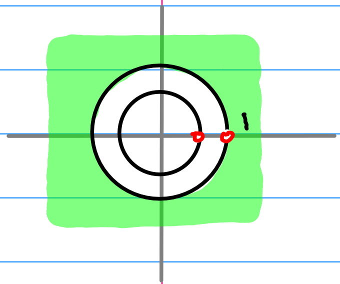
$f(z) = (+1)\dfrac{z}{1-z} - \dfrac{z}{1-2z} \quad (|z|<0.5)$

$1^n - 2^n \quad (n \geq 0)$

$a_n = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$

②-Ⓑ $\dfrac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\dfrac{z}{(z-1)} + \dfrac{0.5z}{(z-0.5)}\right)$

$|z|>1 \qquad |z|>0.5$



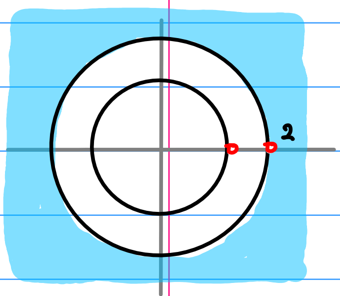
$X(z) = -\dfrac{1}{1-z^{-1}} + \dfrac{0.5}{1-0.5z^{-1}} \quad (|z|>1)$

$a_n = -1^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$

Anti-causal $f(z)$ $X(z)$

$|z| > 2$ $|z| < 1$

① - Ⓐ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$



$|z|>1$ $|z|>2$

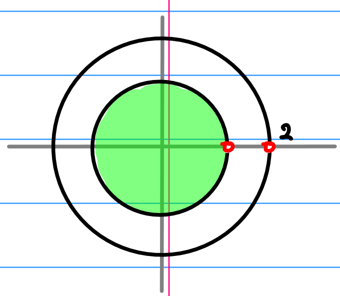
$$f(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad (|z|>2)$$

$$1^{n} - 2^{n} \quad (n \geq 0)$$

$$1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$a_n = 1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

① - Ⓑ $\dfrac{-1}{(z-1)(z-2)} = \left(\dfrac{1}{z-1} - \dfrac{1}{z-2}\right)$



$|z|<1$ $|z|<2$

$$X(z) = -\frac{1}{1-z} + \frac{0.5}{1-0.5z} \quad (|z|<1)$$

$$-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

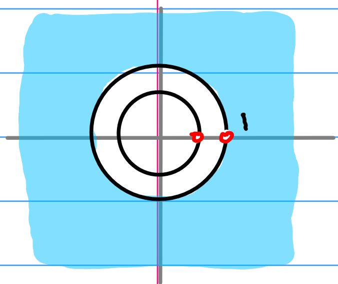
$$a_n = -1^{n-1} + 2^{n-1} \quad (n<1)$$

Anti-causal $f(z)$ $X(z)$

$|z|>1$ $|z|<0.5$

② - Ⓐ $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$|z|>1$ $|z|>0.5$



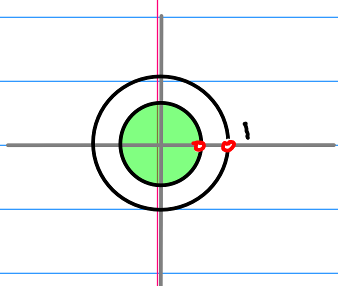
$$f(z) = (-1)\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad (|z|>1)$$

$$-1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$$a_n = -1^{n-1} + 2^{n-1} \quad (n<1)$$

② - Ⓑ $$\frac{-0.5z^2}{(z-1)(z-0.5)} = \left(-\frac{z}{(z-1)} + \frac{0.5z}{(z-0.5)}\right)$$

$|z|<1$ $|z|<0.5$



$$X(z) = \frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<1)$$

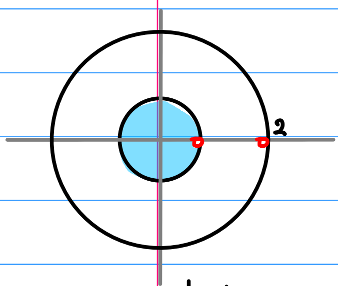
$$+1^{n} - 2^{n} \quad (n \geq 0)$$

$$+1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$$a_n = 1^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n<0)$$

①-Ⓐ $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$ $|z|<0.5$ causal $|z|>2$ anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

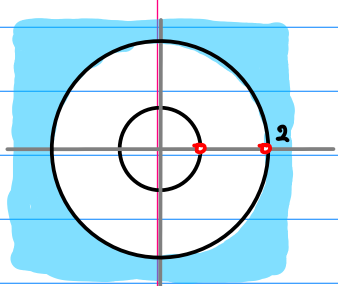


$|z|<0.5$

$$\begin{aligned} f(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^n \end{aligned}$$

$(n \geq 0)$ $a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z|>2$

$$\begin{aligned} f(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \quad \neq \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1}z^{-n} \\ &= \sum_{n=-1}^{-\infty}(2)^{n+1}z^n - \sum_{n=-1}^{-\infty}\left(\frac{1}{2}\right)^{n+1}z^n \end{aligned}$$

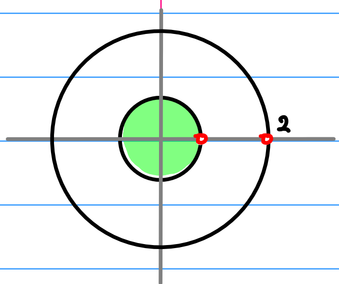
$(n<0)$ $a_n = 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$

① - Ⓑ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$

$|z| < 0.5$ anticausal

$|z| > 2$ causal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



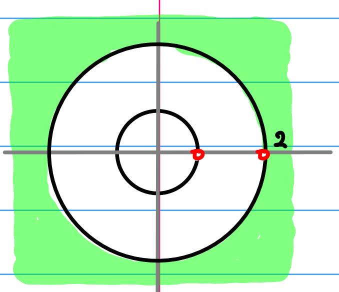
$|z|<0.5$

$$\boxed{\begin{aligned} X(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1}z^{-n} + \sum_{n=0}^{-\infty}(2)^{n-1}z^{-n}$$

$(n \le 0)$ $a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



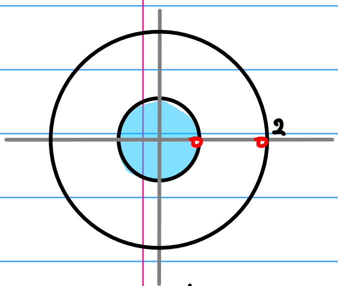
$|z|>2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1}z^{-n} \end{aligned}}$$ $\neq$

$(n > 0)$ $a_n = \left(\frac{1}{2}\right)^{n-1} - (2)^{n-1}$

② - Ⓐ $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

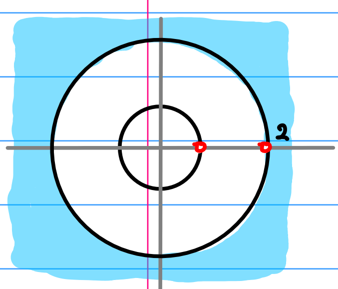


$|z| < 0.5$

$$f(z) = \boxed{\begin{aligned} &-\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n \end{aligned}} \qquad \neq$$

$(n > 0)$ $\quad a_n = -2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

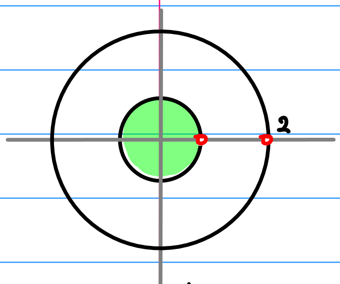
$$f(z) = \boxed{\begin{aligned} &\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)} \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}\left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$$= \sum_{n=0}^{-\infty}(2)^{n-1}z^n - \sum_{n=0}^{-\infty}\left(\tfrac{1}{2}\right)^{n-1}z^n$$

$(n \leq 0)$ $\quad a_n = 2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$

② - ⓑ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



$|z| < 0.5$

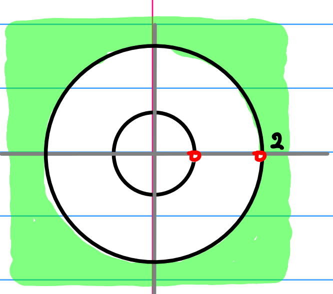
$$\boxed{\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^n \end{aligned}}$$

$\neq$

$$= -\sum_{n=-1}^{-\infty}\left(\frac{1}{2}\right)^{n+1}z^{-n} + \sum_{n=-1}^{-\infty}(2)^{n+1}z^{-n}$$

$(n < 0)$ $a_n = -\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\frac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$(n \geq 0)$ $a_n = \left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$