

# FPGA Carry Chain Adder (1A)

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# FPGA Carry Chain Cell



$$s_i = (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i$$

$$c_{i+1} = (a_i \cdot b_i) + (a_i \oplus b_i) c_i = \bar{p}_i \cdot g_i + p_i \cdot c_i = \bar{p}_i \cdot a_i + p_i \cdot c_i = \bar{p}_i \cdot b_i + p_i \cdot c_i$$

when  $\bar{p}_i = 1$ , then  $a_i = b_i$

when  $g_i = 1$ , then  $a_i = b_i = 1$

$p(i)$	0	1
0	0	1
1	1	0

$g(i)$	0	1
0	0	0
1	0	1

# FPGA Carry Chain Cell



Synthesis of Arithmetic Circuits: FPGA, ASIC and Embedded Systems, J-P Deschamps et al

# FPGA Carry Chain

FPGAs generally contain dedicated computation resources for generating fast adders

The Virtex family programmable arrays include logic gates (**XOR**) and **multiplexers** that along with the general purpose **lookup tables** allow one to build effective carry-chain adders

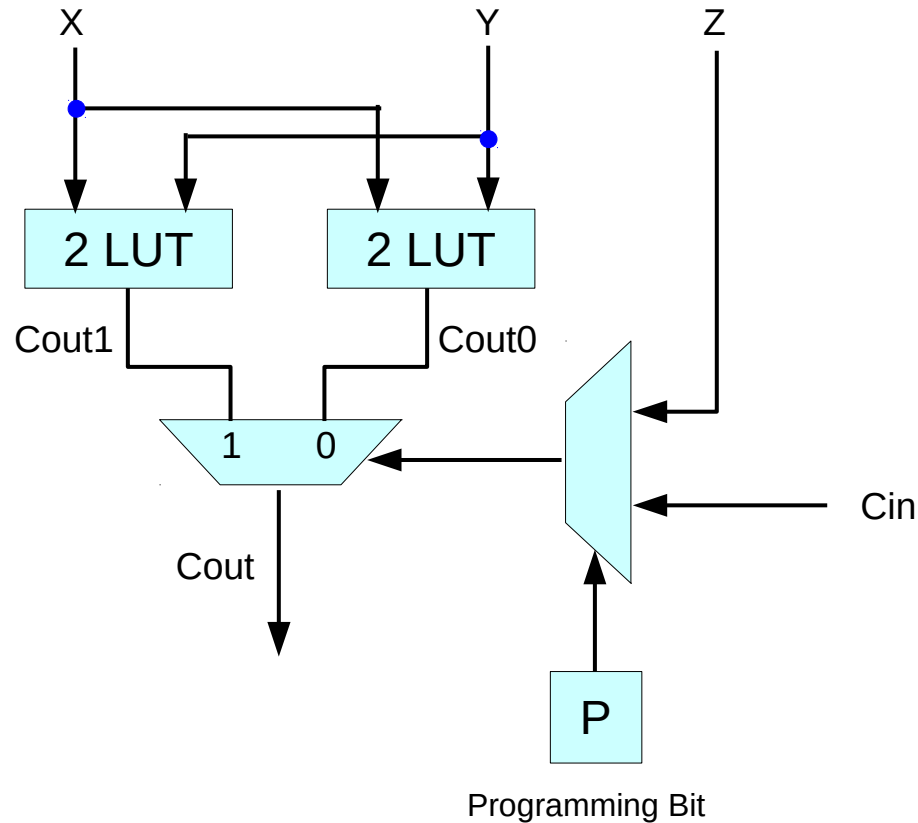
The carry chain is made up of multiplexers belonging to adjacent configurable blocks

the lookup table is used for implementing the exclusive or function

$$p(i) = x(i) \text{ xor } y(i)$$

[https://en.wikipedia.org/wiki/Carry-lookahead\\_adder](https://en.wikipedia.org/wiki/Carry-lookahead_adder)

# FPGA Carry Chain Cell



Cout1, Cout2 : functions of X, Y, Cin

Cout1 = X+Y when Cin=1

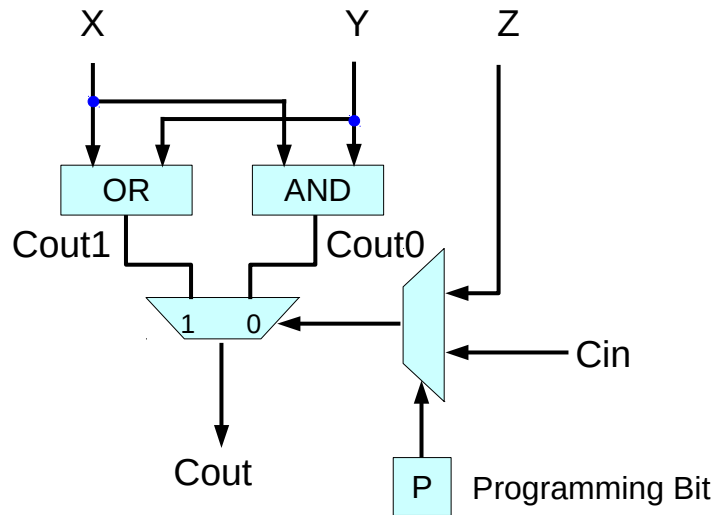
Cout0 = X Y when Cin=0

Cout = (X + Y) Cin + X Y  $\overline{\text{Cin}}$

Cout = P' Cin + G  $\overline{\text{Cin}}$  ... P' = relaxed P

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

# FPGA Carry Chain Cell



X	Y	Cin	$\overline{\text{Cin}}$	$\overline{X} \overline{Y}$
		Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X} Y$
1	0	1	0	$X \overline{Y}$
1	1	1	1	$X Y$

Cout : functions of X, Y, Cin

$$\text{Cout}(X, Y, 1) = \text{Cout1} = X + Y$$

$$\text{Cout}(X, Y, 0) = \text{Cout0} = X Y$$

$$\text{Cout1} = X + Y \text{ when Cin}=1$$

$$\text{Cout0} = X Y \text{ when Cin}=0$$

$$\text{Cout1} = P' \text{Cin} \dots P' = \text{relaxed } P$$

$$\text{Cout0} = G \overline{\text{Cin}}$$

If  $\overline{\text{Cin}}$ , then  $\text{Cout} = (\overline{X} Y + X \overline{Y} + X Y)$   
 If  $\text{Cin}$ , then  $\text{Cout} = X Y$

$$\text{Cin} (X + Y) + \overline{\text{Cin}} X Y$$

$$\text{Cin} (\overline{X} Y + X \overline{Y} + X Y) + \overline{\text{Cin}} X Y$$

$$\text{Cin} (\overline{X} Y + X \overline{Y}) + (\text{Cin} + \overline{\text{Cin}}) X Y$$

$$P \text{Cin} + G$$

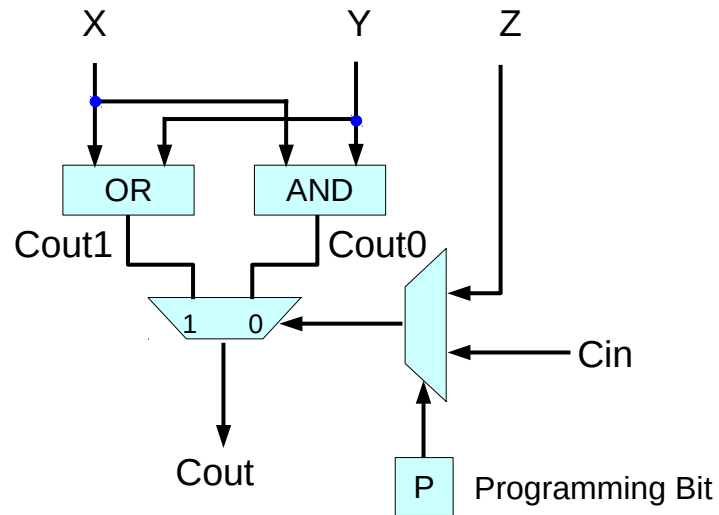
$$\text{Cin} (X + Y) + \overline{\text{Cin}} X Y$$

$$\text{Cin } P' + \overline{\text{Cin}} G$$

...  $P'$  : relaxed  $P$

High Performance Carry Chains for FPGAs, S. Hauck, M. M. Hosler, T. W. Fry

# FPGA Carry Chain Cell



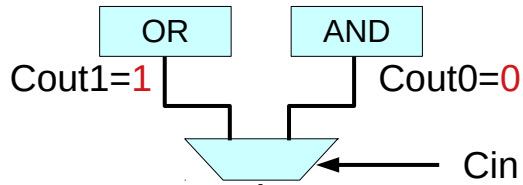
X	Y	Cin	$\overline{\text{Cin}}$	$\overline{X} \overline{Y}$
		Cout1	Cout0	
0	0	0	0	$\overline{X} \overline{Y}$
0	1	1	0	$\overline{X} Y$
1	0	1	0	$X \overline{Y}$
1	1	1	1	$X Y$

X	Y	Cin	Cout
0	0	0	Cout0
0	1	0	Cout0
1	0	0	Cout0
1	1	0	Cout0
0	0	1	Cout1
0	1	1	Cout1
1	0	1	Cout1
1	1	1	Cout1

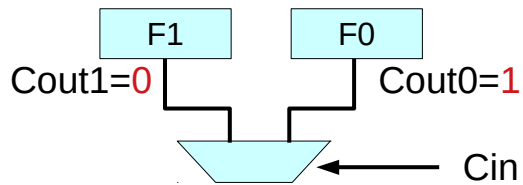
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# FPGA Carry Chain Cell



Cout1=1 when Cin=1  
 Cout0=0 when Cin=0  
 Cout = Cin



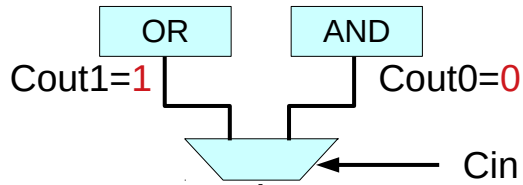
Cout1=0 when Cin=1  
 Cout0=1 when Cin=0  
 Cout =  $\overline{\text{Cin}}$

Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Propagate
1	0	$\overline{\text{Cin}}$	Inverse Propagate
1	1	1	Generate

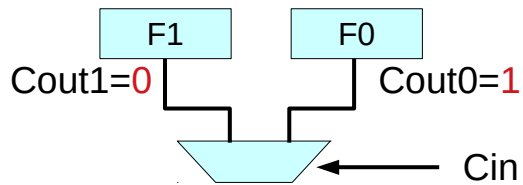
Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	$\overline{\text{Cin}}$	Propagate
1	1	1	Generate

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# FPGA Carry Chain Cell



Cout1=1 when Cin=1  
 Cout0=0 when Cin=0  
 Cout = Cin



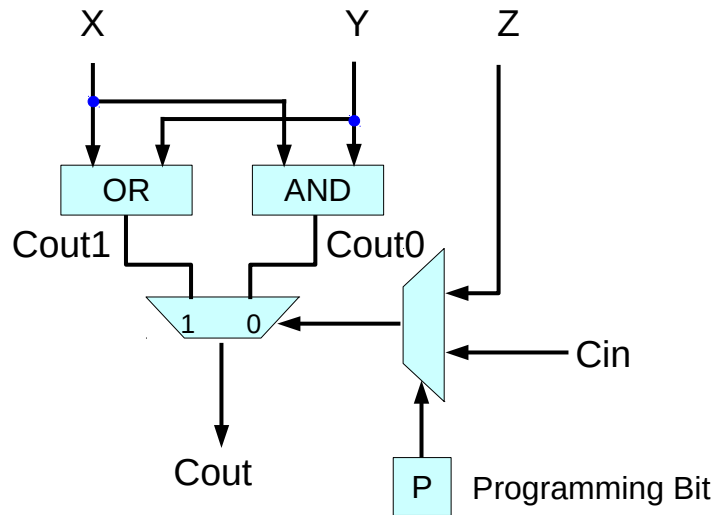
Cout1=0 when Cin=1  
 Cout0=1 when Cin=0  
 Cout =  $\overline{\text{Cin}}$

Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Propagate
1	0	$\overline{\text{Cin}}$	Inverse Propagate
1	1	1	Generate

X	Y	Cin	Cout		Cout1	Cout0
0	0	0	0	Cout0	0	0
0	1	0	0	Cout0	1	0
1	0	0	0	Cout0	1	0
1	1	0	1	Cout0	1	1
0	0	1	0	Cout1	0	0
0	1	1	1	Cout1	1	0
1	0	1	1	Cout1	1	0
1	1	1	1	Cout1	1	1

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# Carry Chain



Carry Out

X	Y	Cin	Cout
0	0	Cin	$\overline{\text{Cin}}$
0	1	Cin	$\overline{\text{Cin}}$
1	0	Cin	$\overline{\text{Cin}}$
1	1	Cin	Cin

X	Y	Cin	$\overline{\text{Cin}}$	$\overline{\text{X}} \overline{\text{Y}}$
		Cout1	Cout0	
0	0	0	0	$\overline{\text{X}} \overline{\text{Y}}$
0	1	1	0	$\overline{\text{X}} \text{Y}$
1	0	1	0	$\text{X} \overline{\text{Y}}$
1	1	1	1	$\text{X} \text{Y}$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Cout1=1 when Cin=1

Cout0=0 when Cin=0

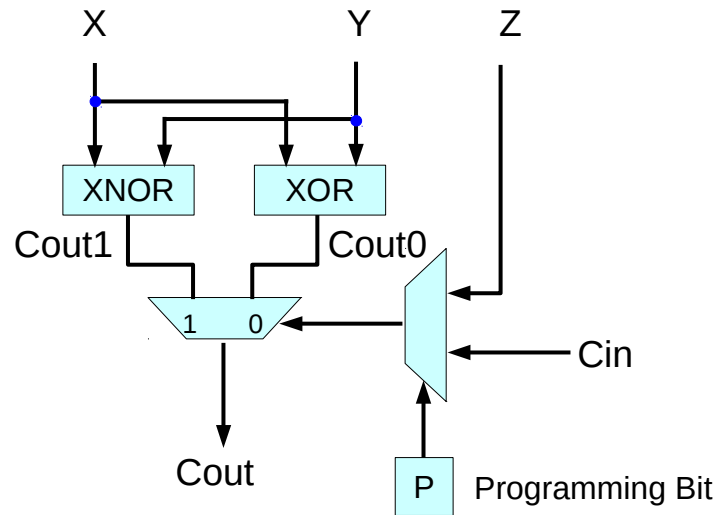
Cout = Cin propagate

Cout1=0 when Cin=1

Cout0=1 when Cin=0

Cout =  $\overline{\text{Cin}}$  inverse propagate

# Parity Checker



X	Y	Cin	$\overline{\text{Cin}}$	
		Cout1	Cout0	
0	0	1	0	$\overline{X} \overline{Y}$
0	1	0	1	$\overline{X} Y$
1	0	0	1	$X \overline{Y}$
1	1	1	0	$X Y$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

## Computing Parity

$X \oplus Y \oplus \text{Cin}$	
$0 \oplus 0 \oplus \text{Cin}$	$\overline{\text{Cin}}$
$0 \oplus 1 \oplus \text{Cin}$	$\overline{\text{Cin}}$
$1 \oplus 0 \oplus \text{Cin}$	$\overline{\text{Cin}}$
$1 \oplus 1 \oplus \text{Cin}$	Cin

Cout1=1 when Cin=1

Cout0=0 when Cin=0

Cout = Cin propagate

Cout1=0 when Cin=1

Cout0=1 when Cin=0

Cout =  $\overline{\text{Cin}}$  inverse propagate

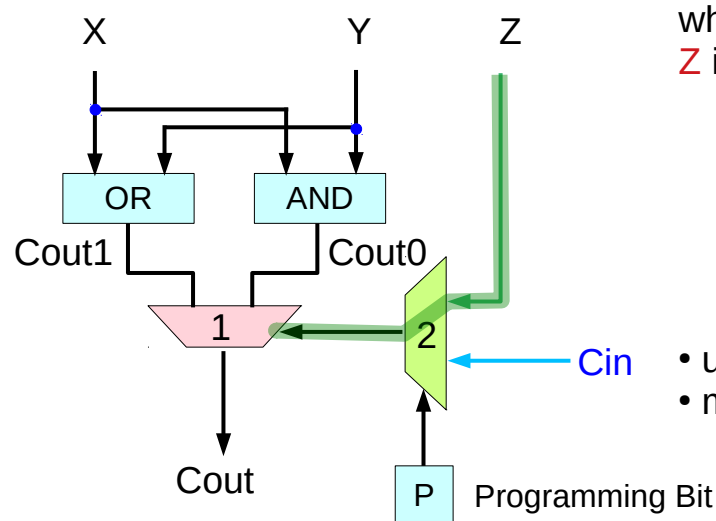
# Ripple Carry Chain



the **first** cell in the chain

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# FPGA Carry Chain Cell



when **Cin** is ignored,  
**Z** is routed to **mux1**

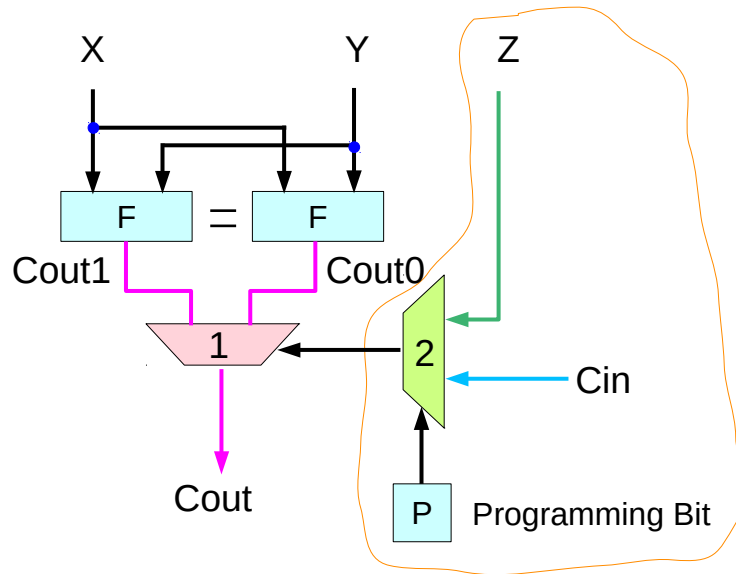
- used in **combined adder/subtractors**
- must be ignored, otherwise

the logic cells - resources to compute a function  
the exact location of logic cells depends on the user.  
a user can start or end a carry computation  
at any place in an fpga.

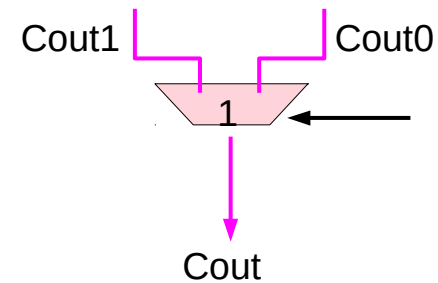
But in many carry computations,  
the first cell has only 2 inputs,  
and forcing the carry chain  
to wait for the arrival of an additional,  
unnecessary input **Z** will only needlessly  
slow down the circuit's computation.

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# FPGA Carry Chain Cell



when Cin is ignored,  
Z can also be ignored  
by having the same LUTs



the **first cell** in the chain

the same LUTs

the same output  
regardless of Z and Cin

Cout1 = Cout0 = Cout  
regardless of the select

# Ripple Carry Chain

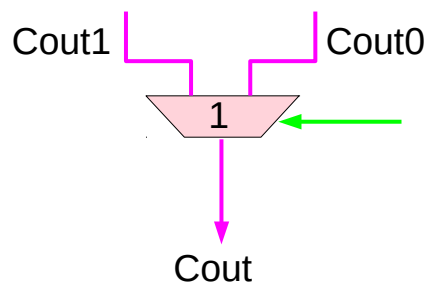
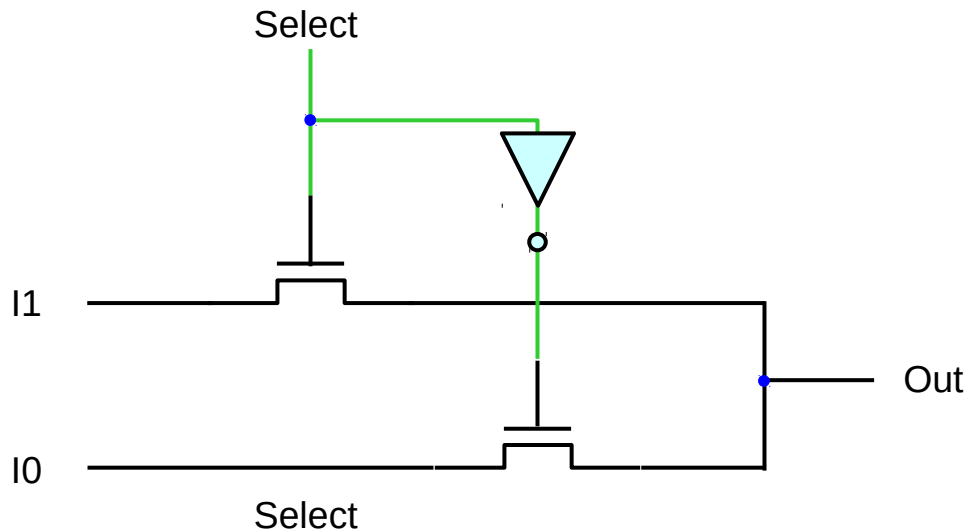


fig1b shows an implementation of a mux that does not obey this requirement

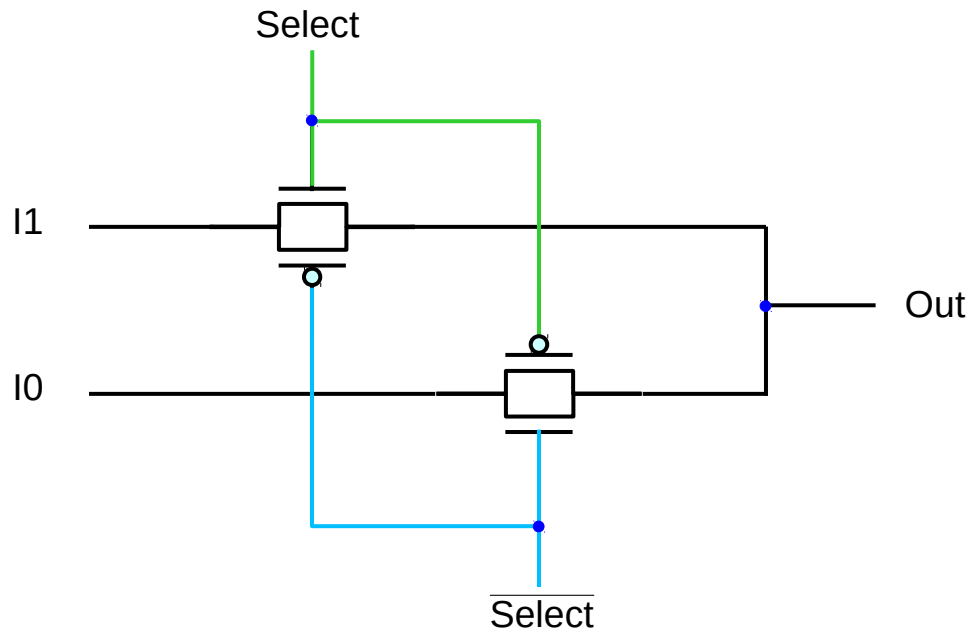
since the carry chain is part of an fpga, the input to this mux could be connected to some **unused logic** in another row which is generating **unknown values**.

if that unused logic had **multiple transitions** which caused the signal to change **quicker** than the gate could react, then it is possible that **the select signal** to this mux could be stuck midway between true and false (2.5V for 5V CMOS)

in this case, it will not be able to pass a true value from the input to the output and thus will not function properly for this application.



# Ripple Carry Chain



however a mux built with both n-transistor and p-transistor pass gates will operate properly for this case

assume this mux implementation will be used

tristate driver based muxes could be used, which restore signal drive and cut series RC chains

# Unit Gate Delay Model

All simple gate of two or three inputs that are directly implementable in **one logic level** in CMOS are considered to have a **delay of one**.

All other gate must be implemented by such gates, and have the delay of the underlying circuit.

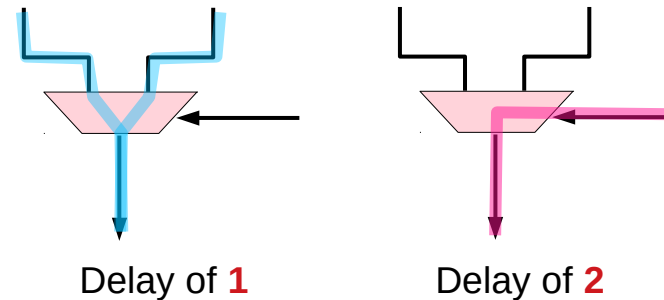
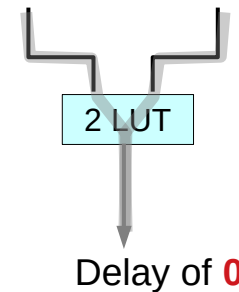
## Delay of one

- inverters and
- 2 to 3 input NAND
- 2 to 3 input NOR gates

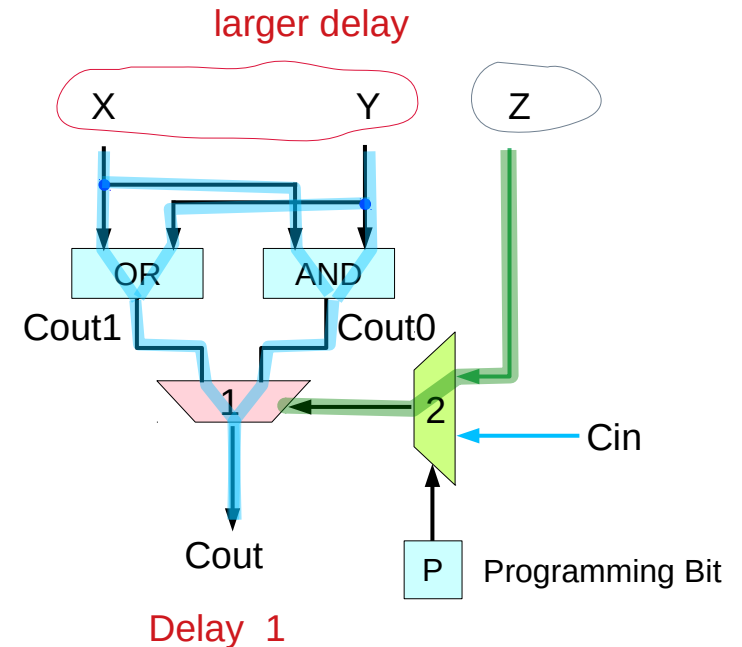
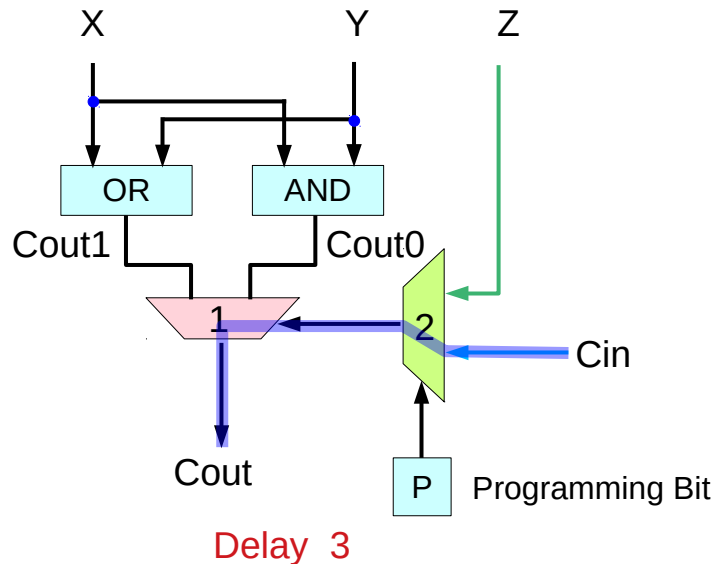
A **2:1 mux** has a **delay of one** from the I0 or I1 inputs to the output, But has a **delay of two** from the select input to the output due to the Inverter delay

## Delay of zero (constant delay)

- the delay of the 2-LUTs,
- any routing leading to them,



# FPGA Carry Chain Cell



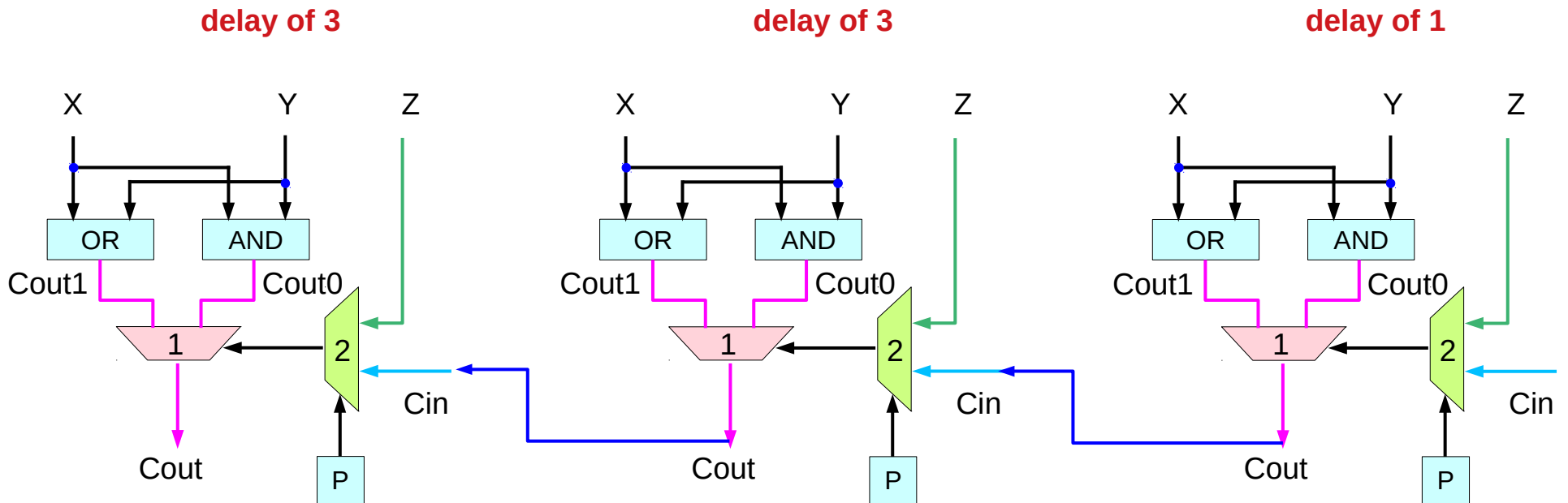
Significantly slower  
two muxes on the carry chain in each cell

Delay 1 for first cell  
Delay 3 for each additional cell in the carry chain  
delay 1 for mux2  
delays 2 for mux1

Overall  $3n-2$  for an n-cell carry chain

The critical path comes from the 2-LUTs and not from the input Z since the delay through the 2-LUTs will be larger than through mux 2 in the first cell

# FPGA Carry Chain Cell



**delay of  $3n-2$**  for an  $n$ -bit ripple carry chain

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# FPGA Carry Chain Cell

the linear delay growth of ripple carry adders

optimize a ripple carry chain structure for use in FPGAs

while this provides some performance gain  
over the basic ripple carry scheme  
found in many current FPGAs,

still much slower than what is done in custom logic

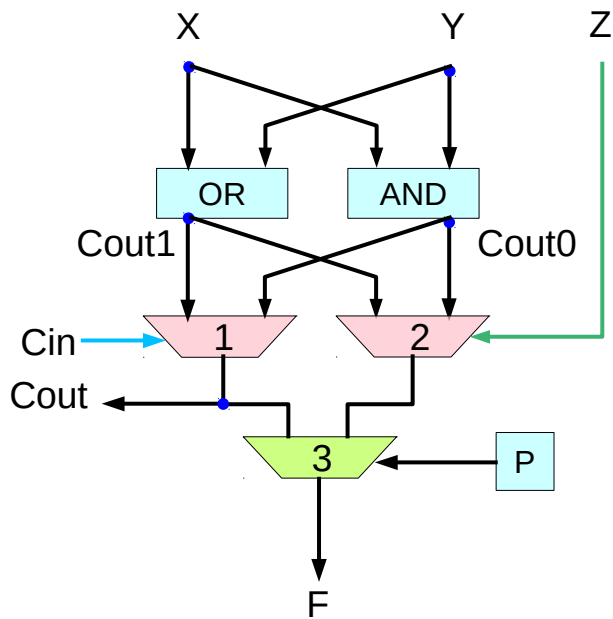
advanced adder techniques in custom logic  
can be integrated into reconfigurable logic

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# FPGA Carry Chain Cell

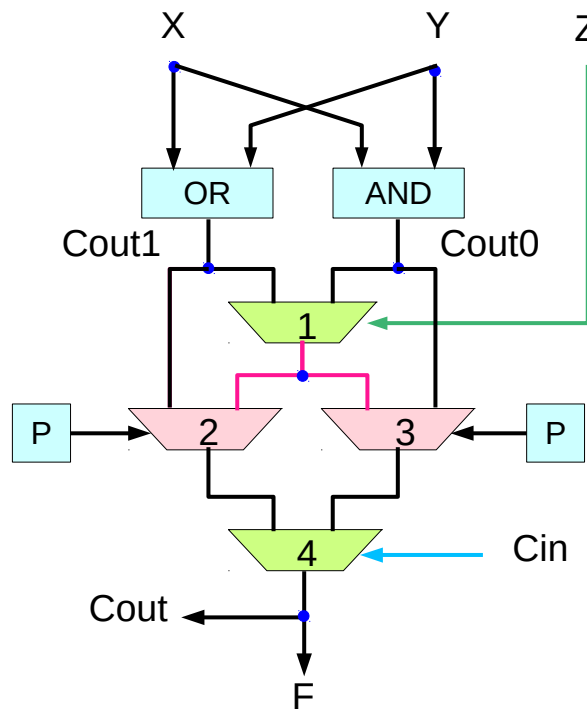
## Design A

$2n / 2n+2$



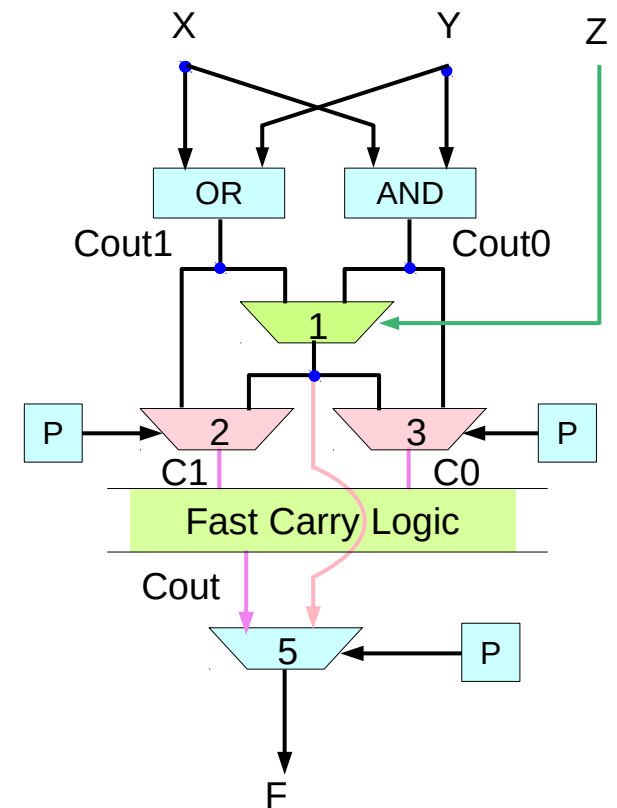
## Design B

$2n / 2n+1$



## Design C

$2n+2$



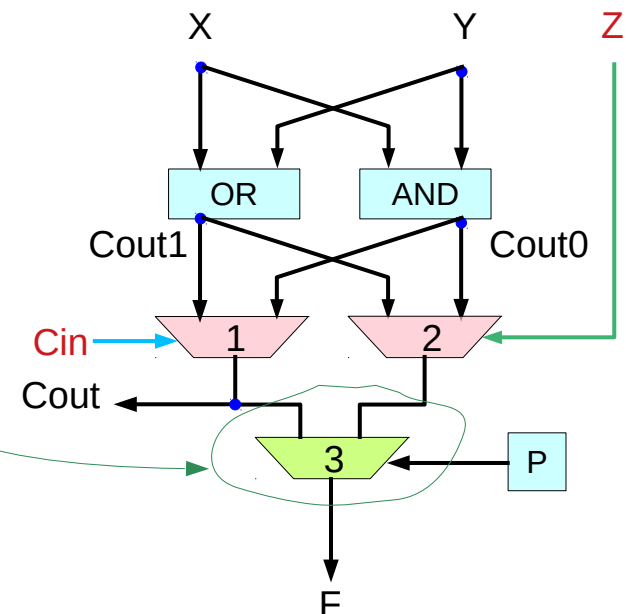
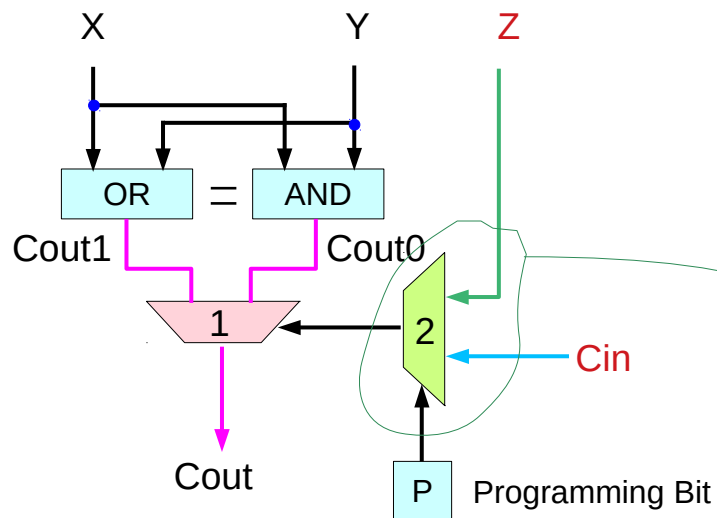
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# Design A (1)

to reduce the delay of the ripple carry chain

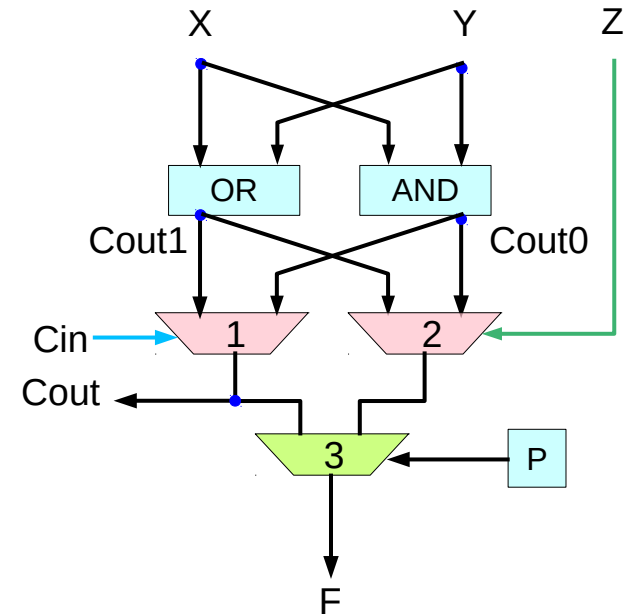
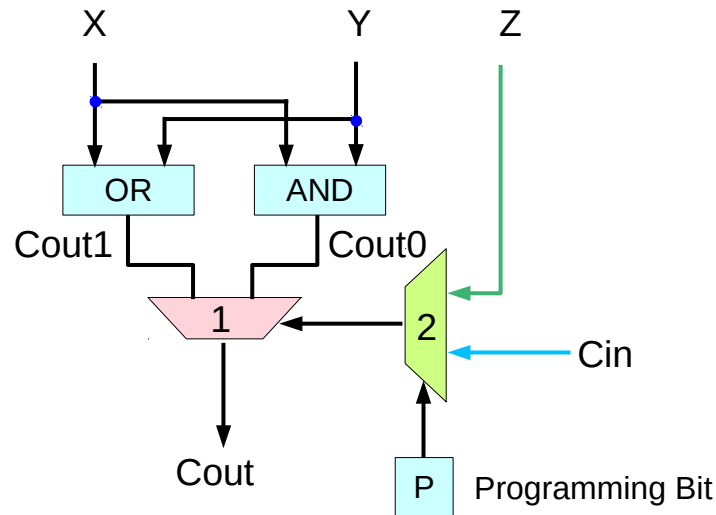
- remove **mux2** from the carry path.
- no need to choose between **Cin** and **Z** for the select line to the output **mux1**

- two separate muxes, **mux1** and **mux2**, controlled by **Cin** and **Z**, respectively.
- the circuit chooses between these outputs with **mux3**.



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# Design A (2)

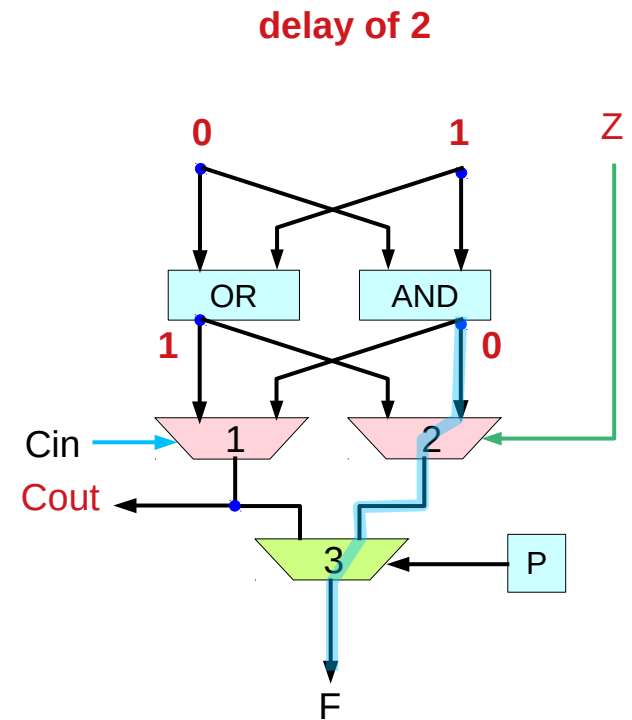
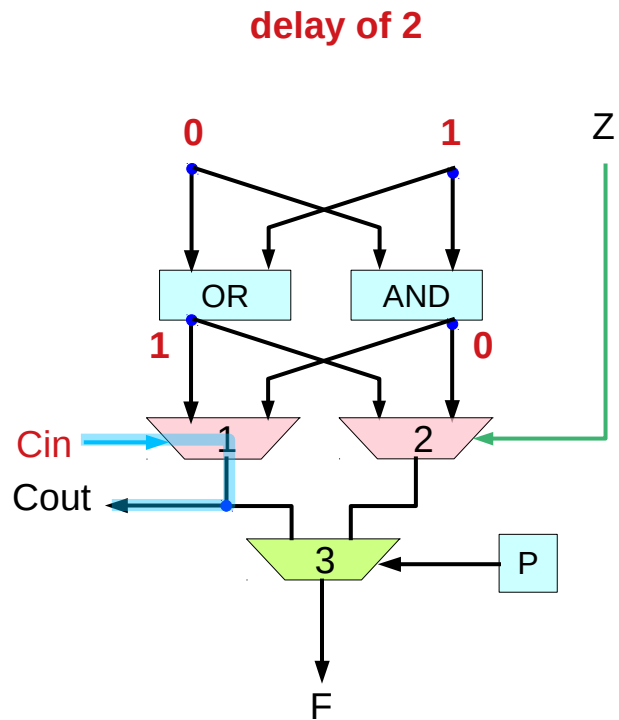


- not logically equivalent
- the **Z** input in the first cell cannot be used
  - **Z** is only attached to **mux2**
  - **mux2** does not lead to the carry cells
  - not connected to **Cout**

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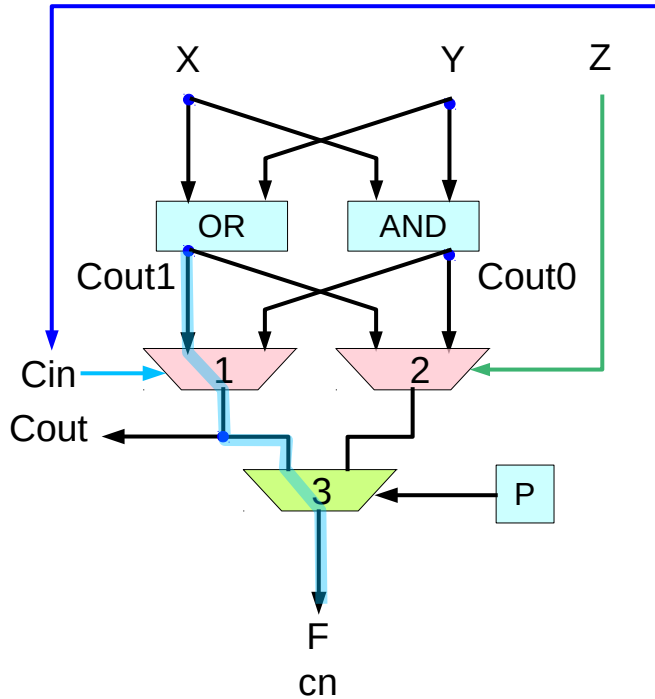


# Design A (3)



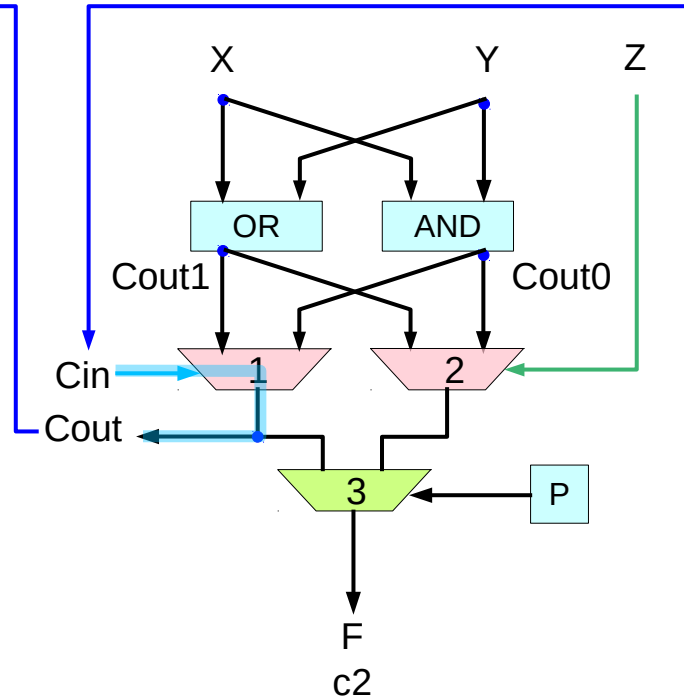
# Design A (4)

delay of 3



(2 for mux1, 1 for mux3)

delay of 2



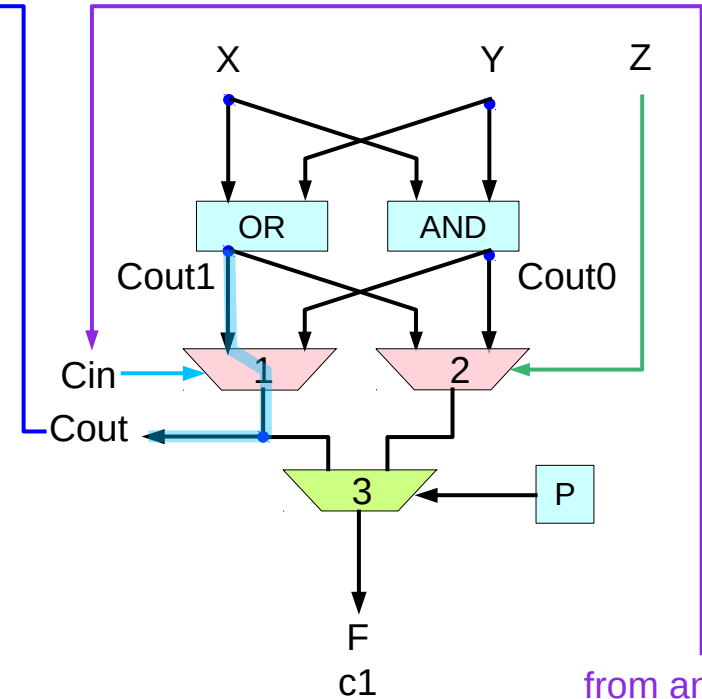
50% faster circuit than the original design

**delay of  $2n$**  for an  $n$ -bit ripple carry chain

**delay of  $2(n+1)$**

the first cell

delay of 1

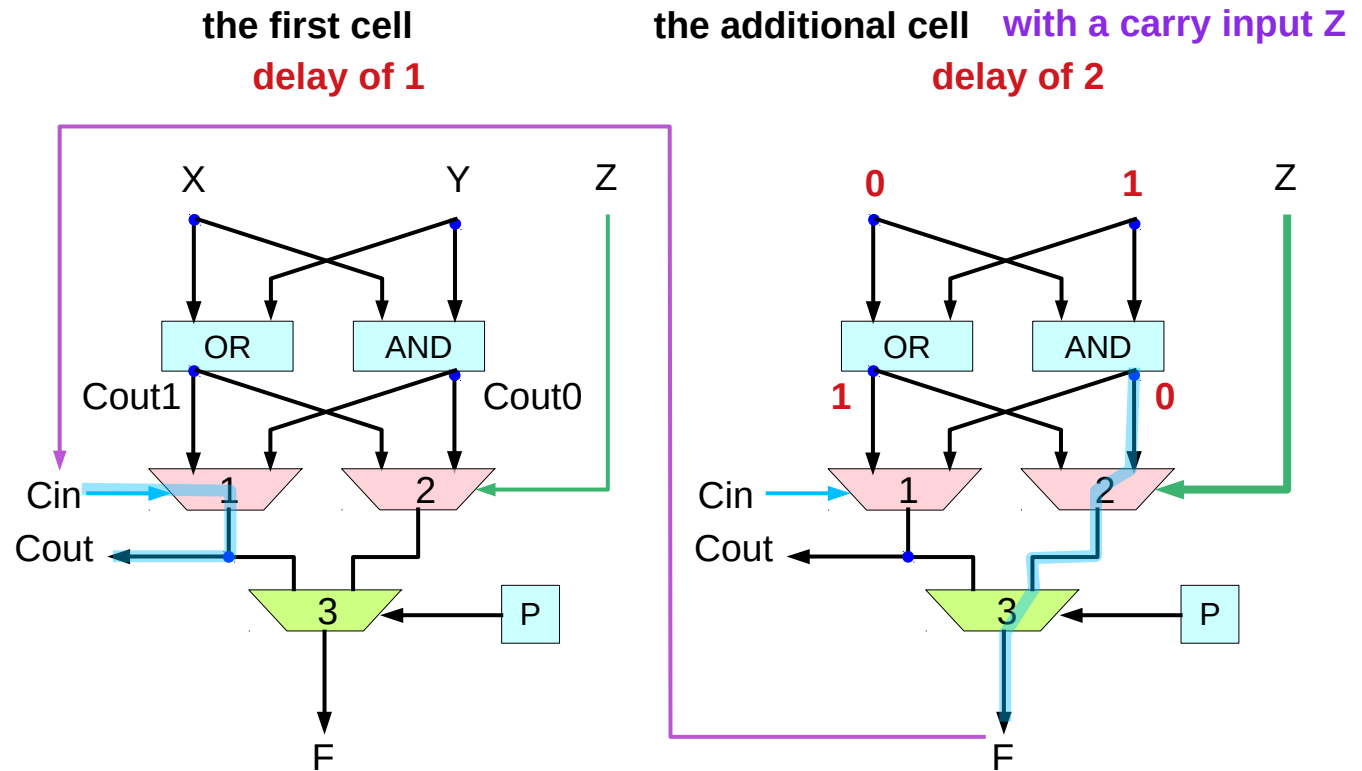


from an additional cell

without a carry input Z

with a carry input Z

# Design A (5)

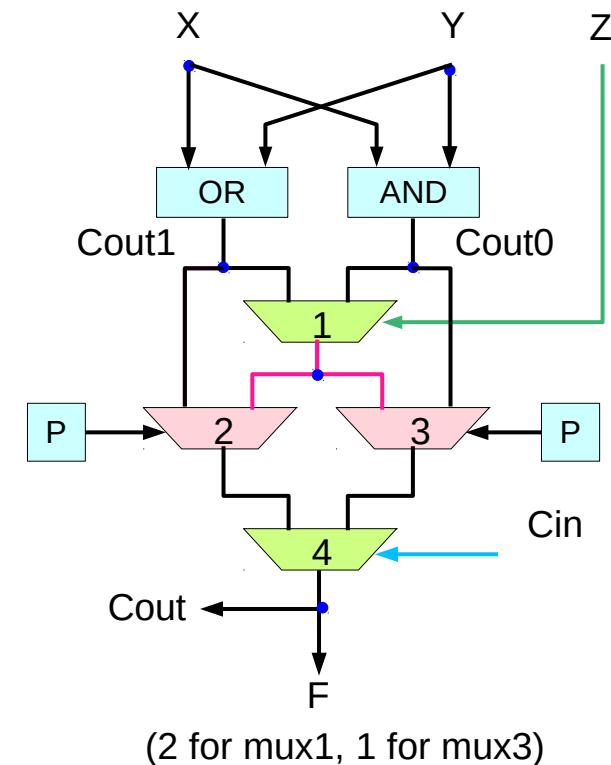


delay of  $2(n+1)$  for an  $n$ -bit ripple carry chain with a carry input

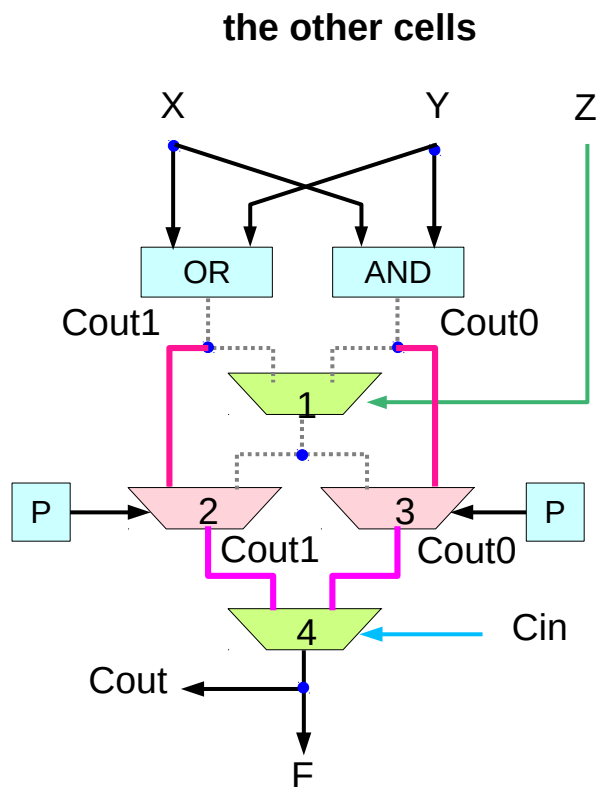
# Design B (1)

although this design is 1 gate delay slower than that of fig 2a, it provides the ability to have a **carry input** to the **first cell** in a **carry chain**, something that is important in many computations.

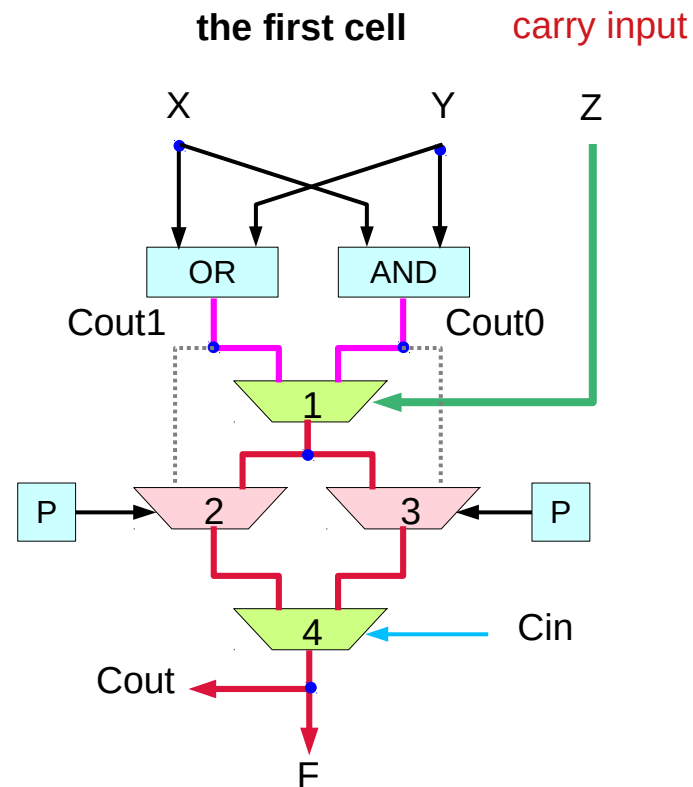
Also, for carry computations that do not need this feature, without a **carry input** the first cell in a **carry chain** built from fig 2b can be configured to bypass **mux1**, reducing the overall delay to  $2n$ , which is identical to that of fig2a.



# Design B (2)



for cells in the middle of a carry chain  
mux2 passes Cout1  
mux3 passes Cout0  
mux4 receives Cout1 and Cout0  
provides a standard ripple carry path.

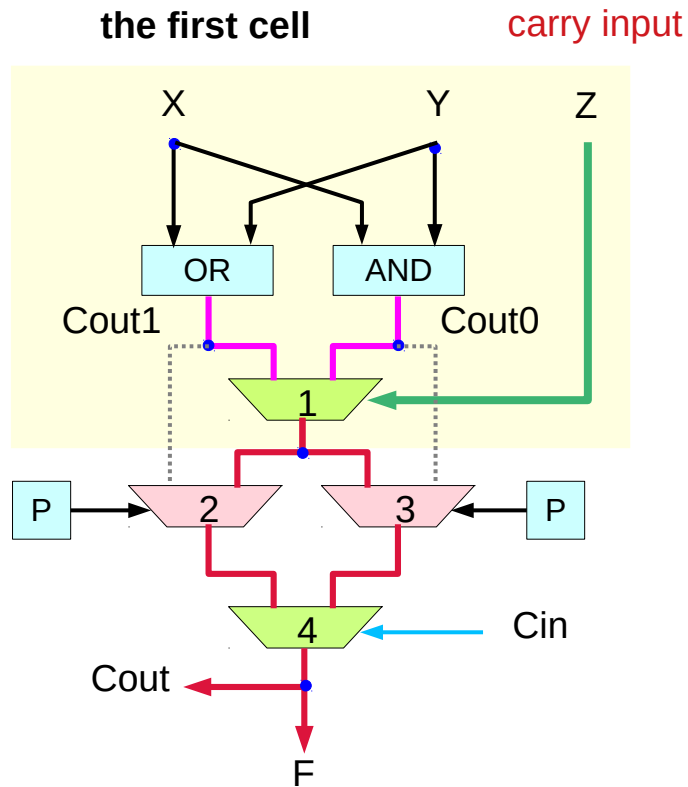


For the first cell in a carry chain  
with a carry input (provided by input Z),  
mux2 and mux3 both pass the value from mux1

the two main inputs to mux4 are identical  
the output of mux4 (Cout) will be the same  
as the output of mux1 (ignoring Cin)

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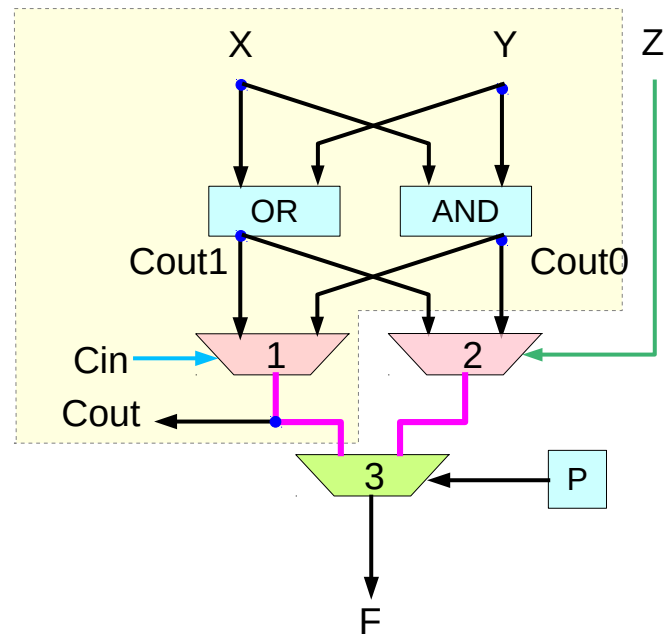
# Design B (3)



**mux1**'s main inputs are driven by two 2-LUTs (OR, AND) controlled by **X** and **Y**  
**mux1** forms a **3-LUT** with the other 2-LUTs

When **mux2** and **mux3** pass the value from **mux1** (**Cout1** and **Cout2** respectively)  
the circuit is configured to continue the carry chain

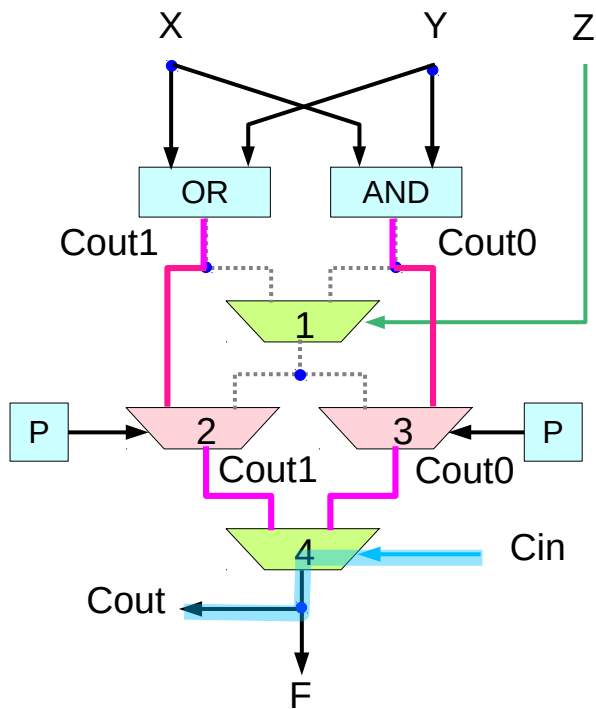
Functionally equivalent



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# Design B (4)

delay of 2 the other cells

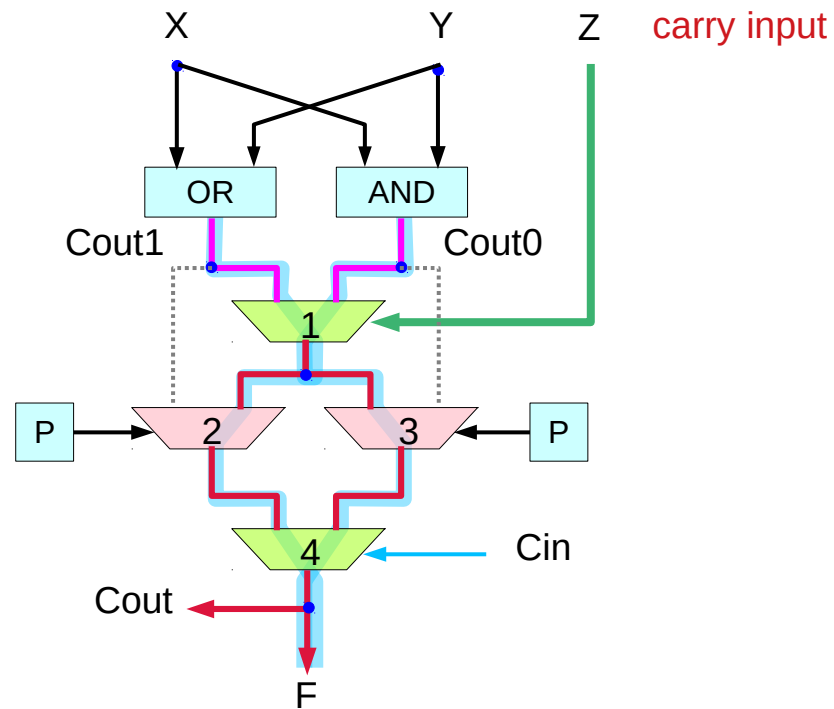


A delay of 2 in all other cells  
except the first cell in the carry chain

an total delay of  $2n+1$  for an n-bit carry chain  
 when a carry input to the first cell is enabled

1 gate delay slower than that of fig 2a,

delay of 3 the first cell with a carry input

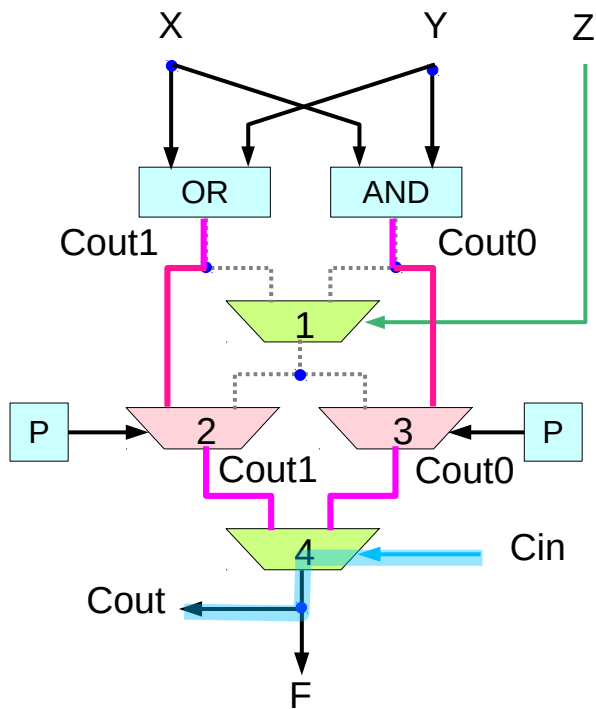


a delay of 3 in the first cell  
 1 in mux1, 1 in mux2, 1 in mux4

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# Design B (5)

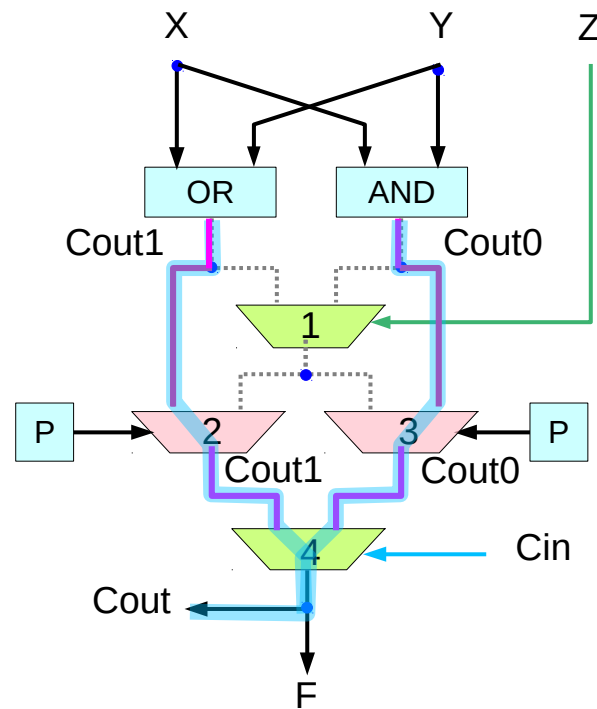
delay of 2 the other cells



A delay of 2 in all other cells  
except the first cell in the carry chain

an total delay of **2n** for an n-bit carry chain  
when a carry input to the first cell is **disabled**

delay of 2 the first cell without a carry input

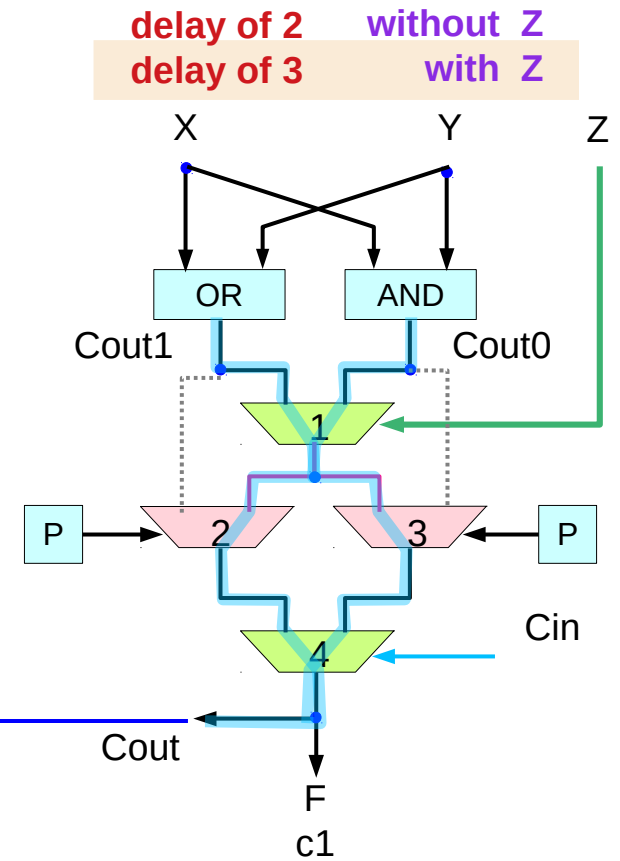
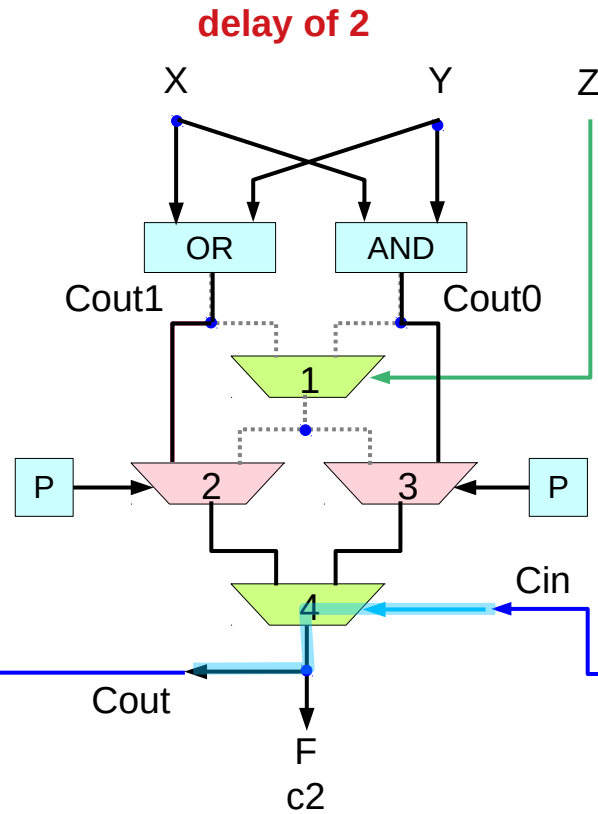
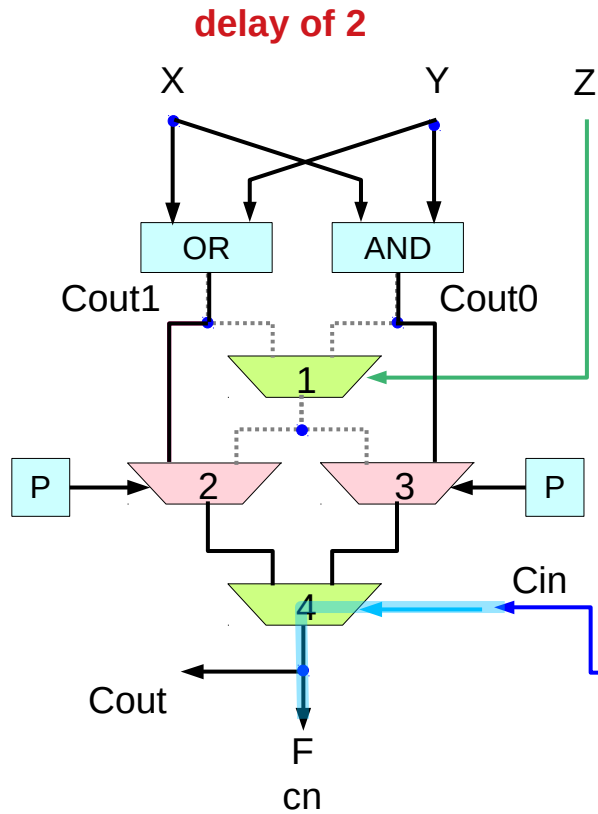


a delay of 2 in the first cell  
when a carry input is not used

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# Design B (6)



delay of  $2n$  for an  $n$ -bit ripple carry chain

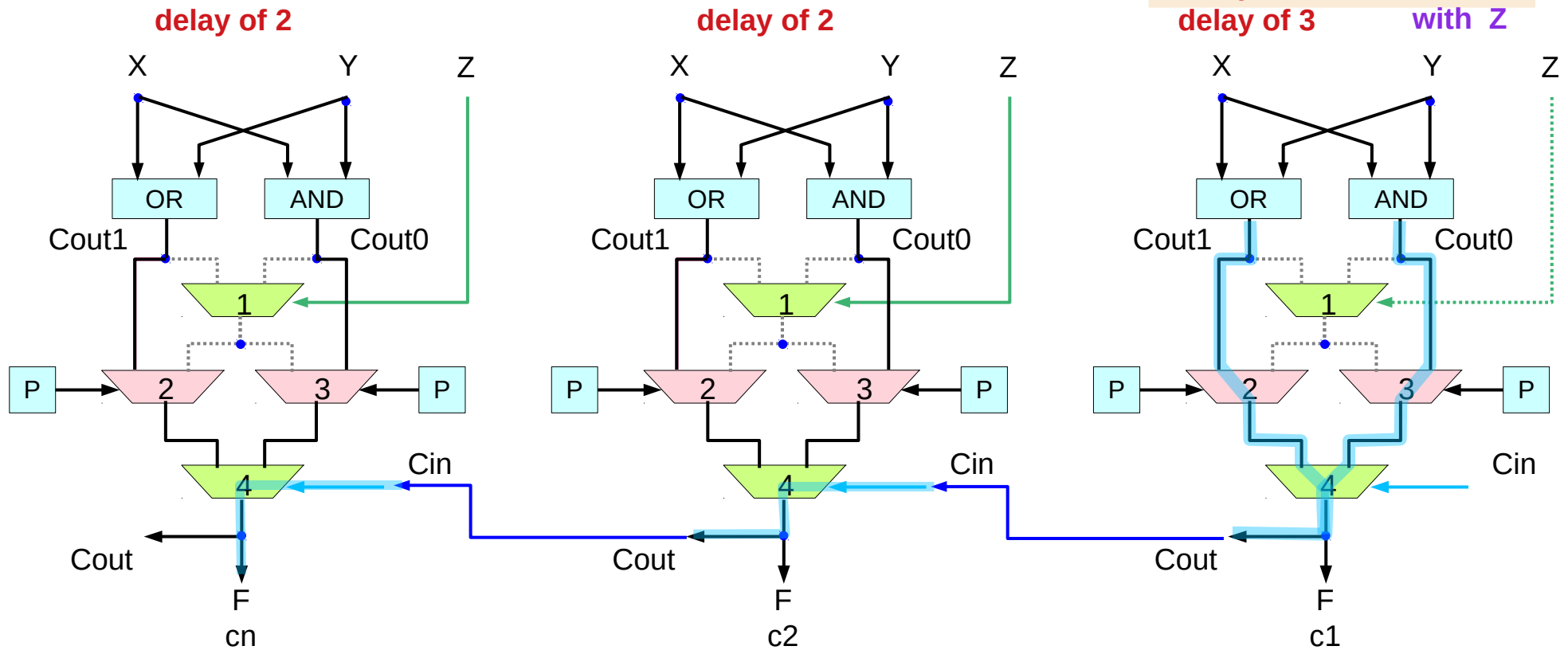
without a carry input Z

delay of  $2n+1$

with a carry input Z

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# Design B (7)



delay of  $2n$  for an  $n$ -bit ripple carry chain

without a carry input  $Z$

delay of  $2n+1$

with a carry input  $Z$

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# Design C (1)

the actual carry chain (mux4) in Design B has been replaced by

- an **abstract fast carry logic unit**
- **mux5** has been added

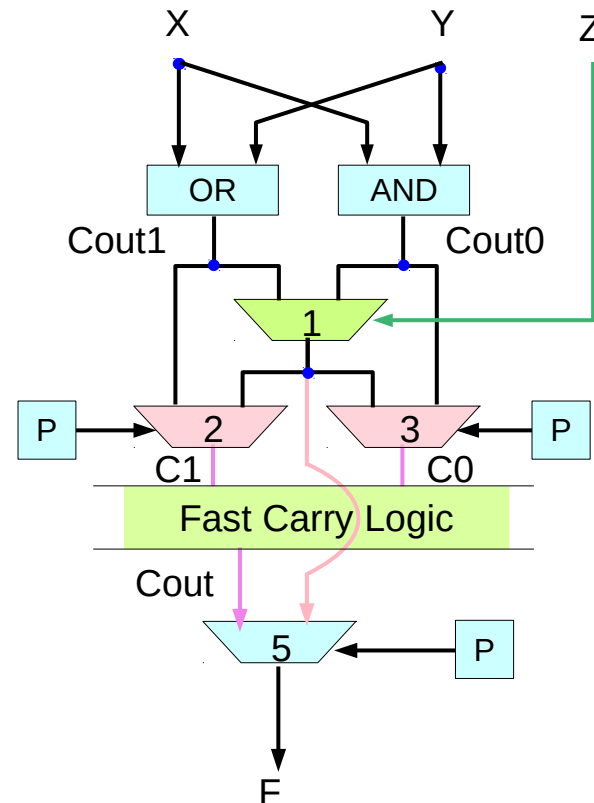
to the **abstract fast carry logic units**, various high performance carry chains can be applied

**mux5** is present because

- significant delay for **non-carry computations**
- much faster **carry propagation** for long carry chains

when used as a simple normal **3 LUT**, using inputs X, Y, and Z

**mux5** allows us to bypass the carry chain by selecting the output of **mux1**



# Design C (2)

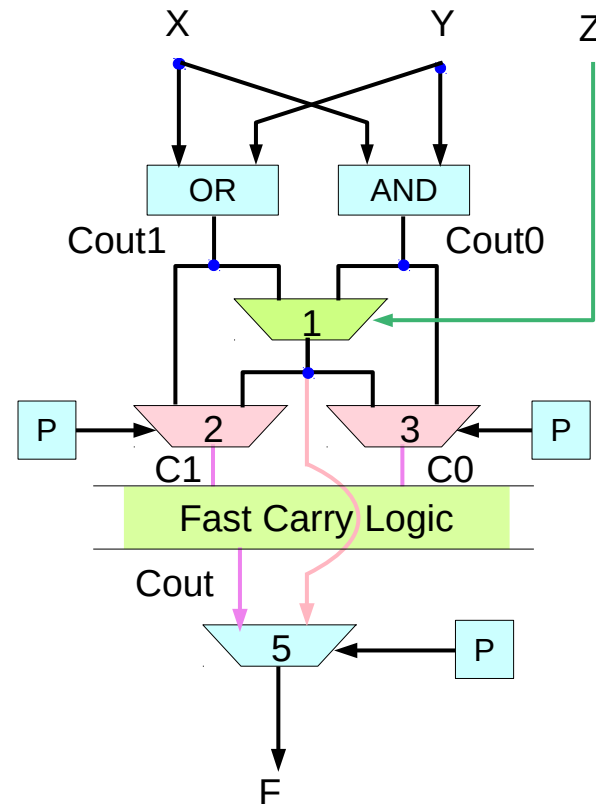
All the developed fast carry logic units in Design C that can compute the following value, can provide the functionality necessary to support the needs of FPGA carry chain computations

$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

where  $i$  is the position of the cell within the carry chain,

thus, the **fast carry logic unit** can contain any logic structure implementing this equation (including Brent-Kung, Variable Bit, and Ripple Carry).

Note that because of the needs and requirements of carry chains for FPGAs, new circuits are developed, by utilizing the standard adder structures, but which are more appropriate for FPGAs



# Design C (3)

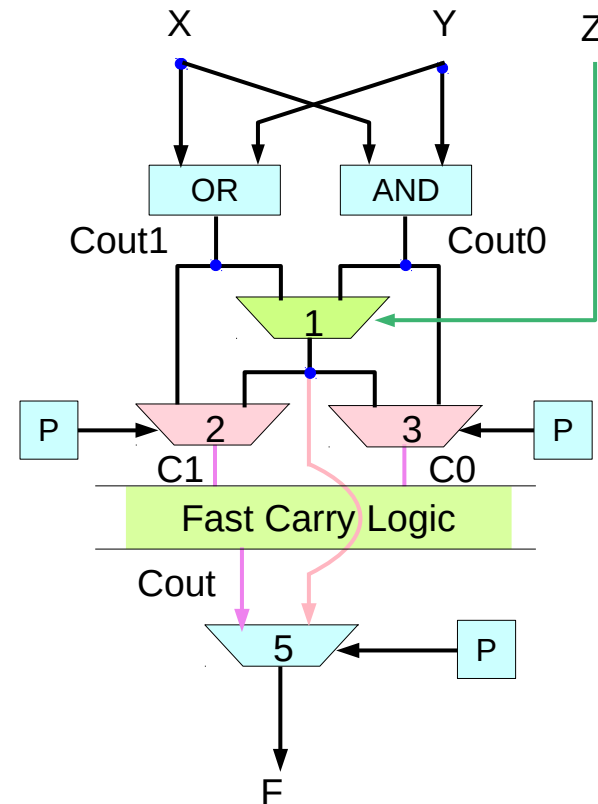
the main difference is to support all states

- Generate
- Propagate
- Kill
- Inverse Propagate

These 4 states are encoded on signals **C1** and **C0**

Also, while standard adders are concerned only with the maximum delay through an entire **n-bit adder** structure, the delay concerns for FPGAs are more complicated

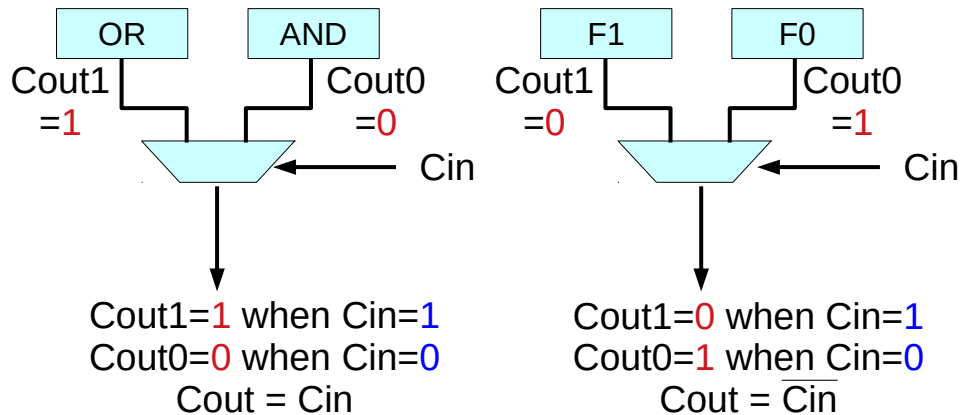
Specifically, when an **n-bit carry chain** is built into the architecture of an FPGA it does not represent an actual computation, but only the potential for a computation.



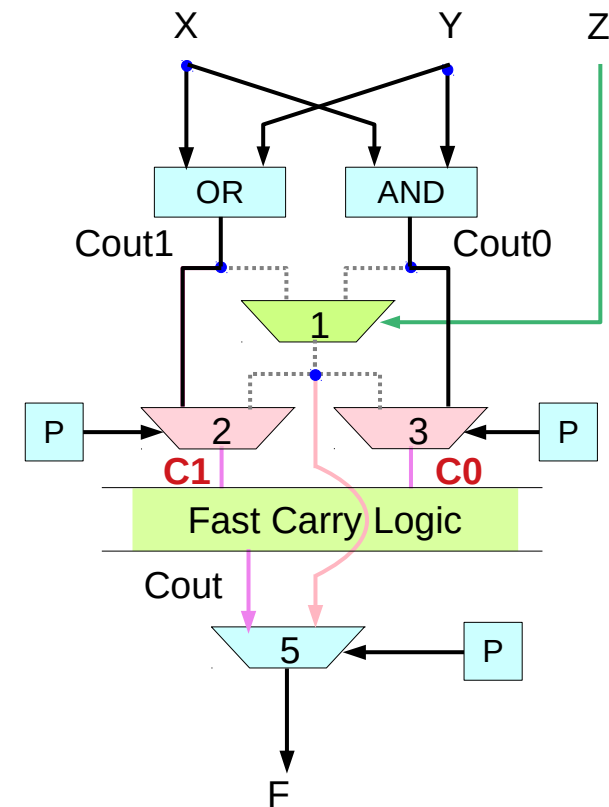
# Design C (4)

X	Y	Cin	Cout0	Cout1
0	0	0	0	0
0	1	1	0	1
1	0	1	0	1
1	1	1	1	1

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{Cin}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



C1	C0	Name
0	0	Kill
0	1	$\overline{Cin}$ Inverse Propagate
1	0	Cin Propagate
1	1	1 Generate



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# Design C (5)

X	Y	C1	C0	
0	0	0	0	$\bar{X}\bar{Y}$
0	1	1	0	$\bar{X}Y$
1	0	1	0	$X\bar{Y}$
1	1	1	1	$XY$

$$C1_i = X_i + Y_i$$

$$C0_i = X_i \cdot Y_i$$

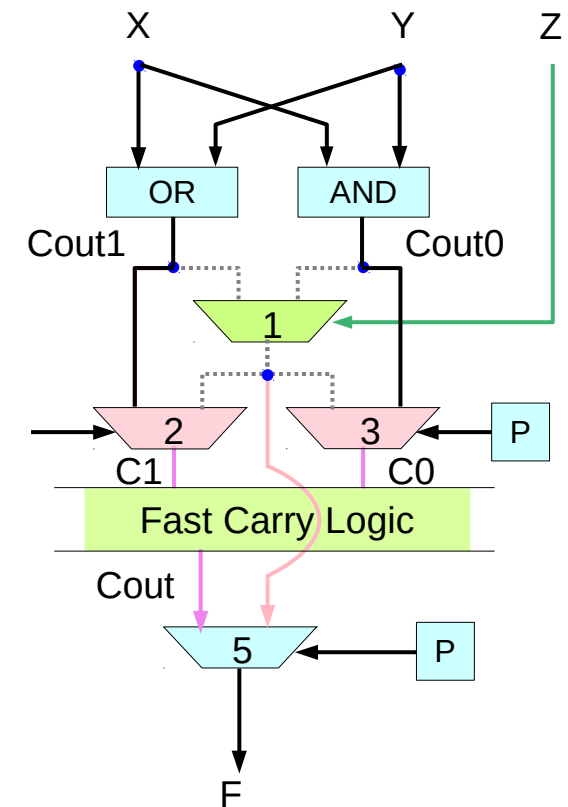
C1	C0		Name
0	0	0	Kill
0	1	$\overline{Cin}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$(Cout_{i-1} \cdot C1_i) = Cout_{i-1} \cdot (\bar{X}Y + X\bar{Y} + XY)$$

$$(\overline{Cout_{i-1}} \cdot C0_i) = \overline{Cout_{i-1}} \cdot XY$$

X	Y	Cout <sub>i</sub>	Cout <sub>i+1</sub>
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	1
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



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# Design C (6) - Complements of C0 and C1

$$C1 = \bar{X}Y + X\bar{Y} + XY$$

X	Y	C1
0	0	0
0	1	1
1	0	1
1	1	1

$$C0 = XY$$

X	Y	C0
0	0	0
0	1	0
1	0	0
1	1	1

$$C1 = \bar{X}Y + X\bar{Y} + XY$$

$$C0 = XY$$

$$\bar{C1} = \overline{(\bar{X}Y) + (X\bar{Y}) + (XY)} = \bar{X}\bar{Y}$$

$$\bar{C0} = \bar{X} + \bar{Y} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

$$\bar{C1} = \bar{X}\bar{Y}$$

X	Y	$\bar{C1}$
0	0	0
0	1	1
1	0	1
1	1	1

$$\bar{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

X	Y	$\bar{C0}$
0	0	0
0	1	1
1	0	1
1	1	1

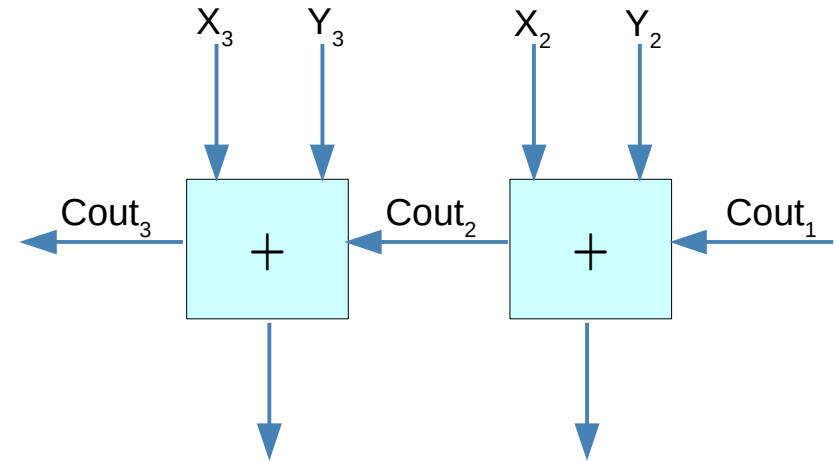
$$Cout_3 = (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \bar{C1}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \bar{C0}_2))$$

$$= (Cout_1 \cdot (C1_3 \cdot (\bar{X}_2Y_2 + X_2\bar{Y}_2 + X_2Y_2) + C0_3 \cdot \bar{X}_2\bar{Y}_2)) + (\bar{Cout}_1 \cdot (C1_3 \cdot X_2Y_2 + C0_3 \cdot (\bar{X}_2Y_2 + X_2\bar{Y}_2 + \bar{X}_2\bar{Y}_2)))$$



# Design C (7) - Cout<sub>3</sub> in terms of Cout<sub>1</sub>

X <sub>3</sub> Y <sub>3</sub>	X <sub>2</sub> Y <sub>2</sub>	Cout <sub>2</sub>	Cout <sub>3</sub>	Cout <sub>3</sub>
0 0	0 0	0	0	0
0 0	0 1	Cout <sub>1</sub>	0	0
0 0	1 0	Cout <sub>1</sub>	0	0
0 0	1 1	1	0	0
0 1	0 0	0	Cout <sub>3</sub>	0
0 1	0 1	Cout <sub>1</sub>	Cout <sub>3</sub>	Cout <sub>1</sub>
0 1	1 0	Cout <sub>1</sub>	Cout <sub>3</sub>	Cout <sub>1</sub>
0 1	1 1	1	Cout <sub>3</sub>	1
1 0	0 0	0	Cout <sub>3</sub>	0
1 0	0 1	Cout <sub>1</sub>	Cout <sub>3</sub>	Cout <sub>1</sub>
1 0	1 0	Cout <sub>1</sub>	Cout <sub>3</sub>	Cout <sub>1</sub>
1 0	1 1	1	Cout <sub>3</sub>	1
1 1	0 0	0	1	1
1 1	0 1	Cout <sub>1</sub>	1	1
1 1	1 0	Cout <sub>1</sub>	1	1
1 1	1 1	1	1	1



$$Cout_3 = (Cout_1 \cdot (C_{13} \cdot C_{12} + C_{03} \cdot \overline{C_{12}})) + (\overline{Cout_1} \cdot (C_{13} \cdot C_{02} + C_{03} \cdot \overline{C_{02}}))$$

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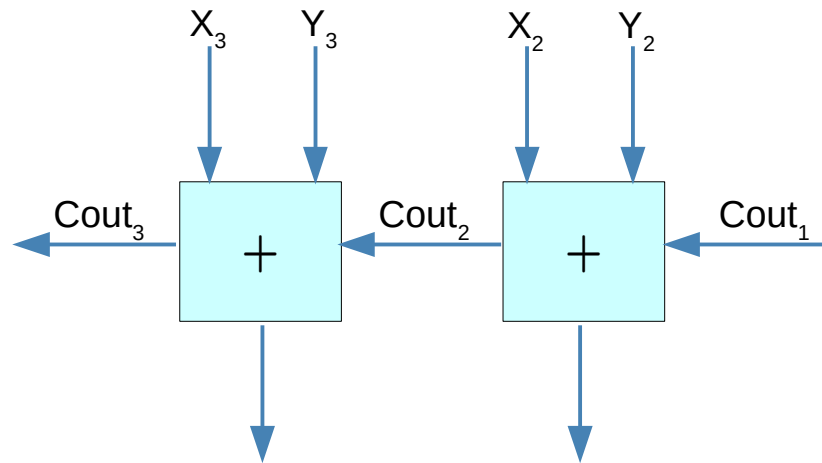
# Design C (8) - Cout<sub>3</sub> in terms of Cout<sub>1</sub>

X <sub>3</sub> Y <sub>3</sub>	X <sub>2</sub> Y <sub>2</sub>	C <sub>13</sub>	C <sub>03</sub>	C <sub>12</sub>	C <sub>02</sub>	Cout <sub>1</sub>		$\overline{\text{Cout}}_1$		Cout <sup>3</sup>
						C <sub>13</sub> C <sub>12</sub>	C <sub>03</sub> $\overline{\text{C}}_1$	C <sub>13</sub> C <sub>02</sub>	C <sub>03</sub> $\overline{\text{C}}_0$	
0 0	0 0	0	0	0	0	0	0	0	0	0
0 0	0 1	0	0	1	0	0	0	0	0	0
0 0	1 0	0	0	1	0	0	0	0	0	0
0 0	1 1	0	0	1	1	0	0	0	0	0
0 1	0 0	1	0	0	0	0	0	0	0	0
0 1	0 1	1	0	1	0	1	0	0	0	Cout <sub>1</sub>
0 1	1 0	1	0	1	0	1	0	0	0	Cout <sub>1</sub>
0 1	1 1	1	0	1	1	1	0	1	0	1
1 0	0 0	1	0	0	0	0	0	0	0	0
1 0	0 1	1	0	1	0	1	0	0	0	Cout <sub>1</sub>
1 0	1 0	1	0	1	0	1	0	0	0	Cout <sub>1</sub>
1 0	1 1	1	0	1	1	1	0	1	0	1
1 1	0 0	1	1	0	0	0	1	0	1	1
1 1	0 1	1	1	1	0	1	0	0	1	1
1 1	1 0	1	1	1	0	1	0	0	1	1
1 1	1 1	1	1	1	1	1	0	1	0	1

$$\text{Cout}_3 = (\text{Cout}_1 \cdot (C_{13} \cdot C_{12} + C_{03} \cdot \overline{C_{12}})) + (\overline{\text{Cout}}_1 \cdot (C_{13} \cdot C_{02} + C_{03} \cdot \overline{C_{02}}))$$

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# Design C (7) - Cout<sub>3</sub> in terms of Cout<sub>1</sub>



$$P = X \oplus Y$$

$$G = X \cdot Y$$

$$C1 = P + G$$

$$C0 = G$$

$$P = C1\overline{C0}$$

$$G = C0$$

$$(Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2})) + (\overline{Cout_1} \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}))$$

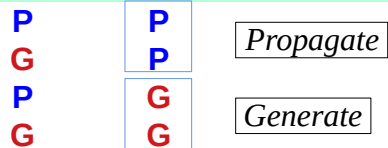
$$\begin{matrix} P & P & G & \overline{PG} \\ P & G & & \\ G & P & & \\ G & G & & \end{matrix}$$

$$\begin{matrix} P & G & G & \overline{PG} \\ G & G & G & P \end{matrix}$$

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# Generate and Kill conditions

$$C1_3 \cdot C1_2 \cdot Cout_1$$



$$C1_3 \cdot C0_2 \cdot \overline{Cout_1}$$



$$P = X \oplus Y$$

$$G = X \cdot Y$$

$$C0_3 \cdot \overline{C1_2} \cdot Cout_1$$



$$C0_3 \cdot \overline{C0_2} \cdot \overline{Cout_1}$$



$$C1 = P + G$$

$$C0 = G$$

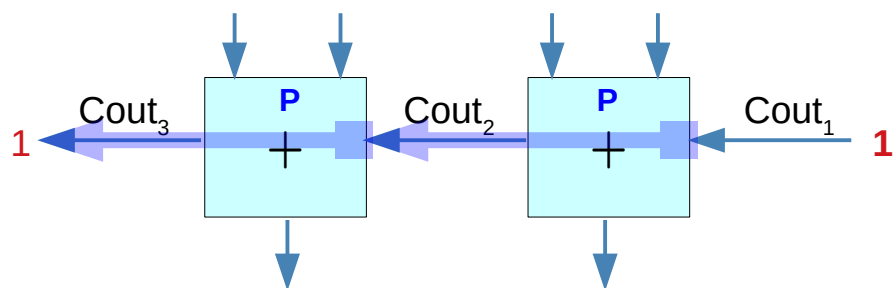
$$P = C1\overline{C0}$$

$$G = C0$$

$$(C1_3 C1_2 + C0_3 \overline{C1_2}) Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2}) \overline{Cout_1}$$

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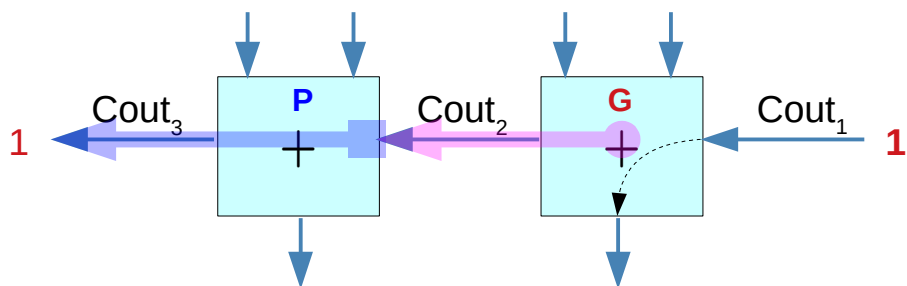
# Design C (9) - When Cout1 = 1



$$C1_3 \cdot C1_2 \cdot Cout_1$$

OR OR

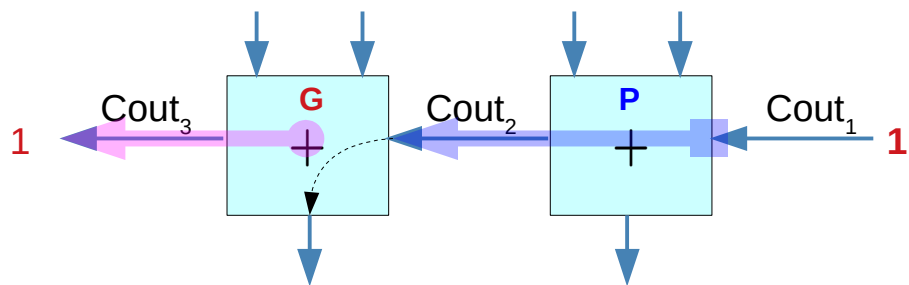
$$P \begin{matrix} \overline{X_3}Y_3 \\ X_3\overline{Y_3} \\ X_3Y_3 \end{matrix} \begin{matrix} \overline{X_2}Y_2 \\ X_2\overline{Y_2} \\ X_2Y_2 \end{matrix} P$$



$$C1_3 \cdot C1_2 \cdot Cout_1$$

OR OR

$$P \begin{matrix} \overline{X_3}Y_3 \\ X_3\overline{Y_3} \\ X_3Y_3 \end{matrix} \begin{matrix} \overline{X_2}Y_2 \\ X_2\overline{Y_2} \\ X_2Y_2 \end{matrix} G$$



$$C1_3 \cdot C1_2 \cdot Cout_1$$

OR OR

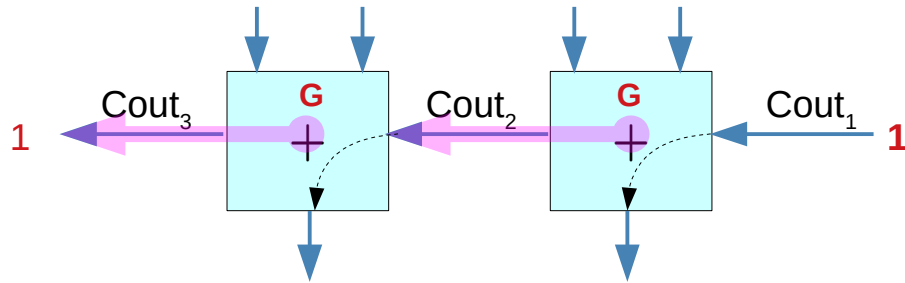
$$G \begin{matrix} \overline{X_3}Y_3 \\ X_3\overline{Y_3} \\ X_3Y_3 \end{matrix} \begin{matrix} \overline{X_2}Y_2 \\ X_2\overline{Y_2} \\ X_2Y_2 \end{matrix} P$$

$$P = X \oplus Y \quad C1 = P + G$$

$$G = X \cdot Y \quad C0 = G$$

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# Design C (10) - When Cout1 = 1



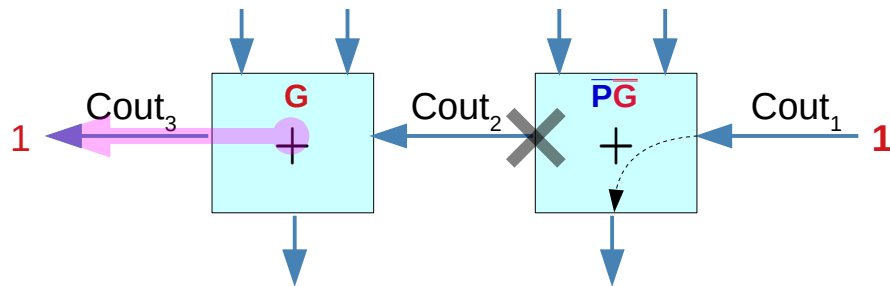
$$C1_3 \cdot C1_2 \cdot Cout_1$$

OR OR

$$\overline{X_3}Y_3 \quad \overline{X_2}Y_2$$

$$X_3\overline{Y_3} \quad X_2\overline{Y_2}$$

$$G \quad X_3Y_3 \quad X_2Y_2 \quad G$$



$$C0_3 \cdot \overline{C1_2} \cdot Cout_1$$

AND NOR

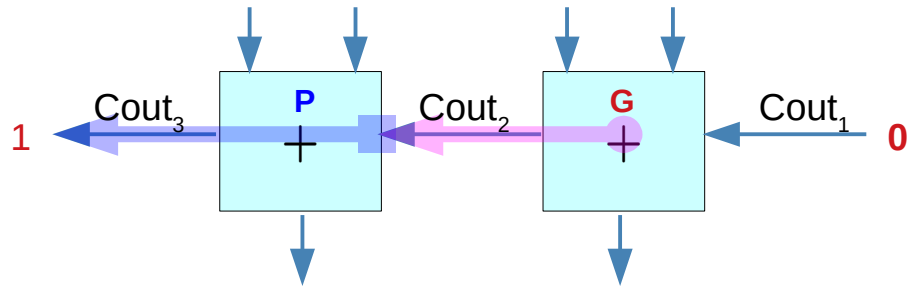
$$G \quad X_3Y_3 \quad \overline{X_2}Y_2 \quad \overline{PG}$$

$$P = X \oplus Y \quad C1 = P + G$$

$$G = X \cdot Y \quad C0 = G$$

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# Design C (11) - When Cout1 = 0



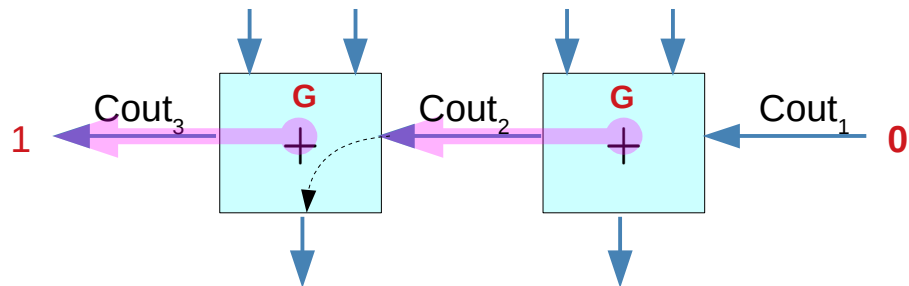
$$C1_3 \cdot C0_2 \cdot \overline{Cout_1}$$

OR AND

$$P \quad \overline{X_3}Y_3 \quad X_2Y_2 \quad G$$

$$X_3\overline{Y_3}$$

$$X_3Y_3$$



$$C1_3 \cdot C0_2 \cdot \overline{Cout_1}$$

OR AND

$$G \quad \overline{X_3}Y_3 \quad X_2Y_2 \quad G$$

$$X_3\overline{Y_3}$$

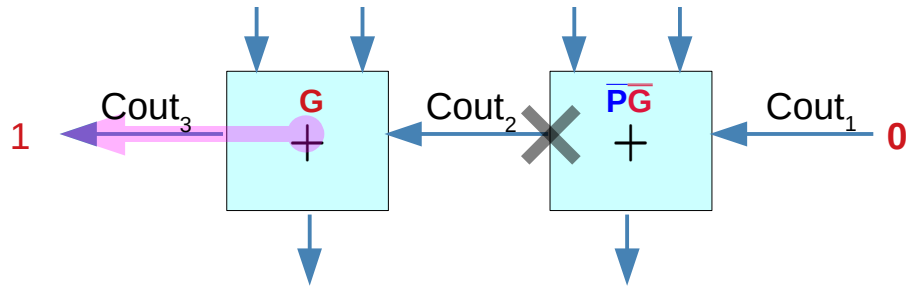
$$X_3Y_3$$

$$P = X \oplus Y \quad C1 = P + G$$

$$G = X \cdot Y \quad C0 = G$$

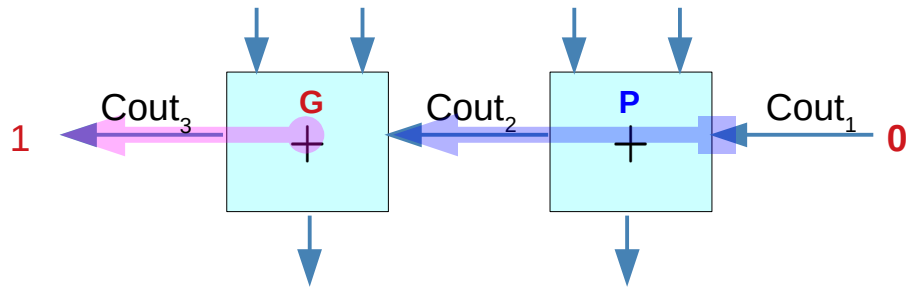
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# Design C (12) - When Cout1 = 0



$$C0_3 \cdot \overline{C0_2} \cdot \overline{Cout_1}$$

AND	NAND
$G \cdot X_3 Y_3$	$\overline{X_2 Y_2} \cdot \overline{PG}$
	$\frac{X_2 \overline{Y_2}}{X_2 Y_2}$



$$C0_3 \cdot \overline{C0_2} \cdot \overline{Cout_1}$$

AND	NAND
$G \cdot X_3 Y_3$	$\overline{X_2 Y_2} \cdot P$
	$\frac{X_2 \overline{Y_2}}{X_2 Y_2}$

$P = X \oplus Y$	$C1 = P + G$
$G = X \cdot Y$	$C0 = G$

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# FPGA Carry Chain Cell

$$C1 = \bar{X}Y + X\bar{Y} + XY \quad C0 = XY$$

$$\bar{C1} = \bar{X}\bar{Y}$$

$$\bar{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$$

**C1 and C0 are not mutually exclusive**

**C1 includes C0**

$$C1 \cdot C0 = C0$$

$$C1 + C0 = C1$$

$$\bar{C1} \cdot \bar{C0} = \bar{C1}$$

$$\bar{C1} + \bar{C0} = \bar{C0}$$

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