Applicatives Methods (3B)

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The definition of Applicative

```
class (Functor f) => Applicative f where
    pure :: a -> f a
    (<*>) :: f (a -> b) -> f a -> f b
```

f (a -> b) :: a **function** wrapped in f

f a :: a **value** wrapped in f

The class has a two methods :

pure brings arbitrary values into the functor

(<*>) takes a **function** wrapped in a functor f
and a **value** wrapped in a functor f
and returns the result of the application
which is also wrapped in a functor f

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Maybe instance of Applicative

```
instance Applicative Maybe where
    pure                    = Just
    (Just f) <*> (Just x)   = Just (f x)
    _        <*> _          = Nothing
```

pure wraps the value with **Just**;

(<*>) applies

the function wrapped in **Just**

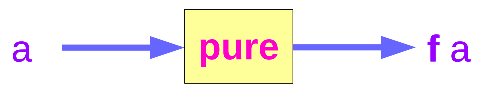
to the value wrapped in **Just** if both exist,

and results in **Nothing** otherwise.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

An Instance of the Applicative Typeclass

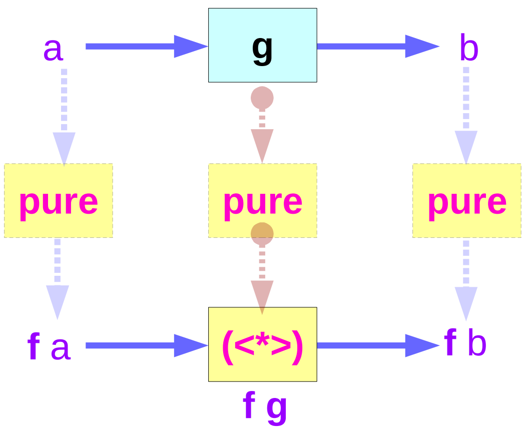
```
class (Functor f) => Applicative f where
    pure :: a -> f a
    (<*>) :: f (a -> b) -> f a -> f b
```



(Functor f) => Applicative f

f : Functor, Applicative

```
instance Applicative Maybe where
    pure = Just
    Nothing <*> _ = Nothing
    (Just f) <*> something = fmap f something
```



(Functor f) => Applicative f

f : function in a context

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Left associative <*>, fmap, and <$>

pure g <*> x <*> y <*> z

fmap g x <*> y <*> z

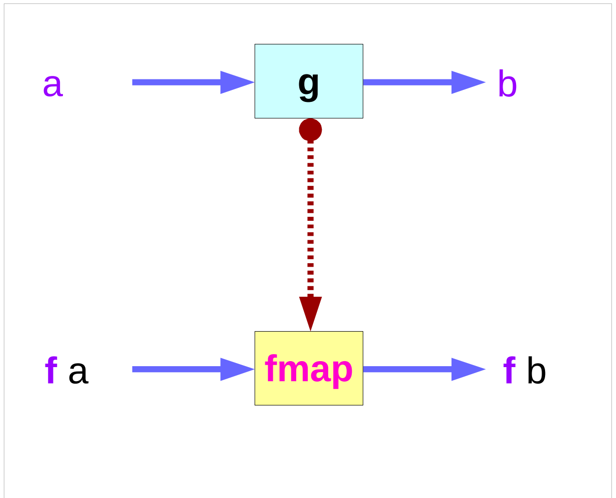
g <$> x <*> y <*> z

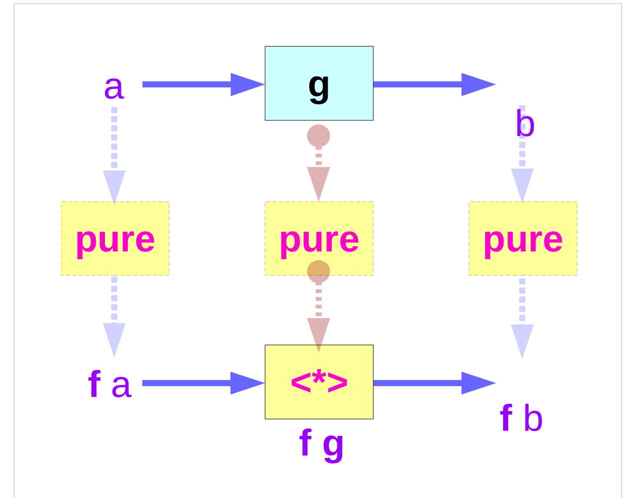
g :: f (a -> b -> c -> d)
x :: f a
y :: f b
z :: f c

infix operator <$>

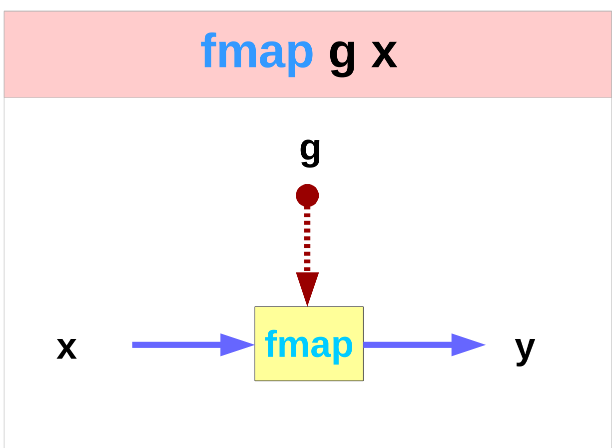
http://learnyouahaskell.com/functors-applicative-functors-and-monoids

fmap g x = (pure g) <*> x

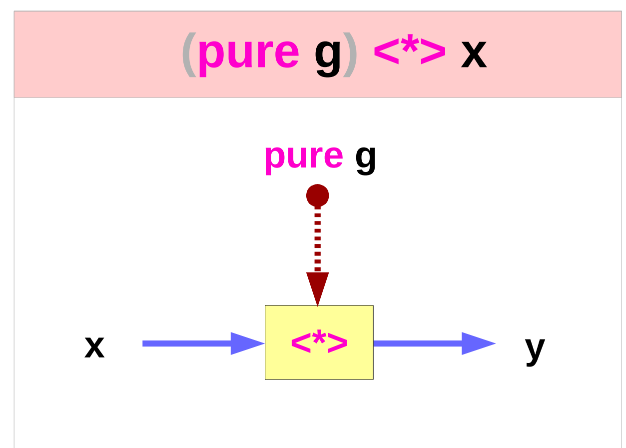




pure = f



x :: f a
y :: f b



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Left associative <*> examples

```
ghci> pure (+) <*> Just 3 <*> Just 5
Just 8
```

```
pure (+)  <*>  Just 3  <*>  Just 5
Just (+3)  <*>  Just 5
Just 8
```

```
ghci> pure (+) <*> Just 3 <*> Nothing
Nothing
```

```
ghci> pure (+) <*> Nothing <*> Just 5
Nothing
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Infix Operators <*> vs <$> - a type view

h <*> x <*> y

h :: **f** (a -> b -> c)
x :: **f** a
y :: **f** b

h :: **f** (a -> b -> c)
x :: **f** a
h <*> x :: **f** (b -> c)

h :: **f** (a -> b -> c)
x :: **f** a
h <*> x :: **f** (b -> c)
y :: **f** b
h <*> x <*> y :: **f** c

g <$> x <*> y

g :: (a -> b -> c)
x :: **f** a
y :: **f** b

g :: (a -> b -> c)
x :: **f** a
g <$> x :: **f** (b -> c)

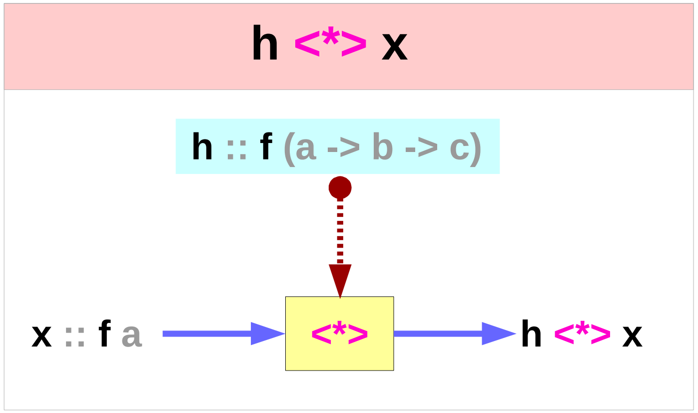
g :: (a -> b -> c)
x :: **f** a
g <$> x :: **f** (b -> c)
y :: **f** b
g <$> x <*> y :: **f** c

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Infix Operators <*> vs <$> - a curried function view

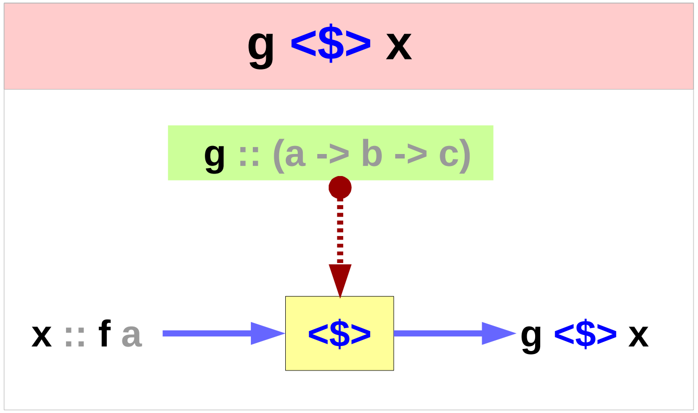
h <*> x <*> y

h :: **f** (a -> b -> c)
x :: **f** a
y :: **f** b



g <$> x <*> y

g :: (a -> b -> c)
x :: **f** a
y :: **f** b



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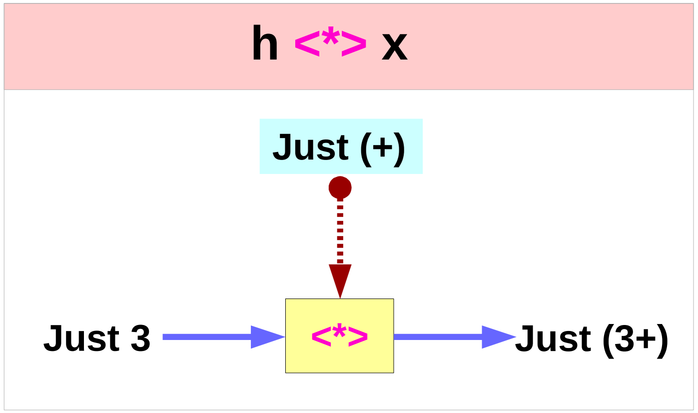
Infix Operators <*> vs <$> examples

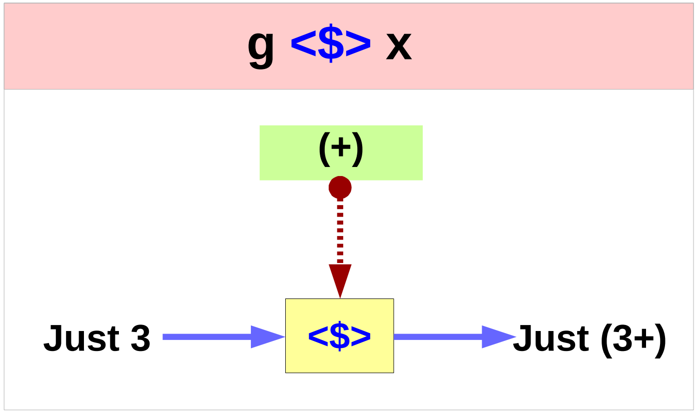
h <*> x <*> y

Just (+) <*> Just 3 <*> Just 2
Just (+3) <*> Just 2
Just 5



g <$> x <*> y

(+) <$> Just 3 <*> Just 2
Just (+3) <*> Just 2
Just 5



http://learnyouahaskell.com/functors-applicative-functors-and-monoids

the minimal complete definition

```
class (Functor f) => Applicative f where
    pure :: a -> f a
    (<*>) :: f (a -> b) -> f a -> f b
```

the minimal complete definition

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
g <$> x = fmap g x
```

Not in the minimal complete definition

g :: a -> b, x :: f a

```
instance Applicative Maybe where
    pure = Just
    Nothing <*> _ = Nothing
    (Just g) <*> something = fmap g something
```

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

The Applicative Typeclass

Applicative is a superclass of **Monad**.

every **Monad** is also a **Functor** and an **Applicative**

fmap, **pure**, **(<*>)** can all be used with **monad**s.

a **Monad** instance

requires **Functor** and **Applicative** instances.

defines the types and roles of **return** and **(>>)**

fmap :	defined in **Functors**
pure, (**<*>**) :	defined in **Applicatives**
return, (**>>**) :	defined in **Monads**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(<$>) vs ($)

(<$>) infix operator

(<$>) :: (Functor f) => (a -> b) -> f a -> f b

g <$> x = **fmap g x**

The $ operator is for avoiding parentheses

putStrLn (show (1 + 1))

putStrLn $ show (1 + 1)

putStrLn $ show $ 1 + 1 – right associative

($) calls the function which is its left-hand argument of $

on the value which is its right-hand argument of $

The Applicative Laws

The identity law:	pure id <*> v = v	id :: a -> a v :: f a
Homomorphism:	pure g <*> pure x = pure (g x)	g :: a -> b x :: a
Interchange:	u <*> pure y = pure ($ y) <*> u	u :: f (a -> b) y :: a
Composition:	u <*> (v <*> w) = pure (.) <*> u <*> v <*> w	w :: f a v :: f (a -> b) u :: f (b -> c)

Left associative   u <*> v <*> w = (u <*> v) <*> w

u :: f (c -> b -> a)
v :: f c
u <*> v :: f (b -> a)
w :: f b
u <*> v <*> w = f a

The Identity Law

The identity law	**pure id <*> v = v**	**id :: a -> a**	**v :: f a**

pure to inject values into the functor
in a *default*, *featureless* way,
so that the result is as close as possible
to the plain value.

applying the **pure id** morphism does nothing,
exactly like with the plain **id** function.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Homomorphism Law

The homomorphism law	**pure g <*> pure x = pure (g x)**	**g :: a -> b**	**x :: a**

applying a "**pure**" function to a "**pure**" value is the same as
applying the function to the value in the *ordinary way*
and then using **pure** on the result.
means **pure** preserves function application.

applying a non-effectful function **g**
to a non-effectful argument **x** in an effectful context **pure**
is the same as just **applying** the function **g** to the argument **x**
and then injecting the result **(f x)**
into the effectual context with **pure**.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Interchange Law

The interchange law **u <*> pure y = pure ($ y) <*> u** **u :: f (a -> b)** **y :: a**

($ **y**) is the *function* that supplies **y** as argument to another function – a higher order function

Function $ Argument
$ y
(**y**) as a single argument

applying a morphism **u** to a "**pure**" value **pure y** is the same as applying **pure** ($ **y**) to the morphism **u**

Just (+3) <*> Just 2
Just ($ 2) <*> Just (+3)

when evaluating the application of an effectful function (**u)** to a pure argument (**pure y**), the order doesn't matter – commutative.

u :: f (a -> b) **y :: a**
u <*> pure y :: f b **pure y :: f a**
pure ($ y) <*> u :: f b **pure ($ y) :: f (a)**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Composition Law

The composition law	**pure (.) <*> u <*> v <*> w = u <*> (v <*> w)**	w :: f a v :: f (a -> b) u :: f (b -> c)

pure (.) composes morphisms similarly
to how **(.)** composes functions:

applying the composed mourphism
pure (.) <*> u <*> v *to* **w**
gives the same result (**u <*> (v <*> w)**)
as applying **u** *to* the result **(v <*> w)**
of applying **v** to **w**

it is expressing a sort of associativity property of **(<*>)**.

w :: f a -- value
v :: f (a -> b) -- func1
u : f (b -> c) -- func2

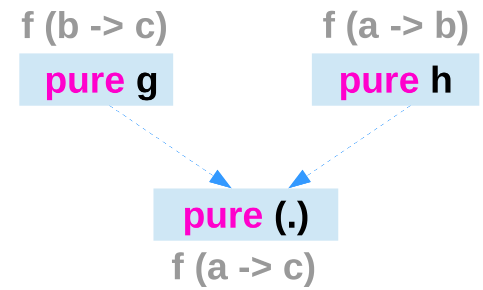
v <*> w :: f b
u <*> (v <*> w) :: f c

pure (.) <*> u <*> v :: f (a -> c)
pure (.) <*> u <*> v <*> w :: f c

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

The Composition Law and Left Associativity

The composition law pure (.) <*> u <*> v <*> w = u <*> (v <*> w) w :: f a v :: f (a -> b) u :: f (b -> c)



pure (.) <*> pure g <*> pure h <*> pure x (g . h) x

((pure (.) <*> pure g) <*> pure h) <*> pure x ⬍

= pure g <*> (pure h <*> pure x) g (h x)

u = pure g :: f (b -> c) g :: (b -> c)
v = pure h :: f (a -> b) h :: (a -> b)
w = pure x :: f a x :: a

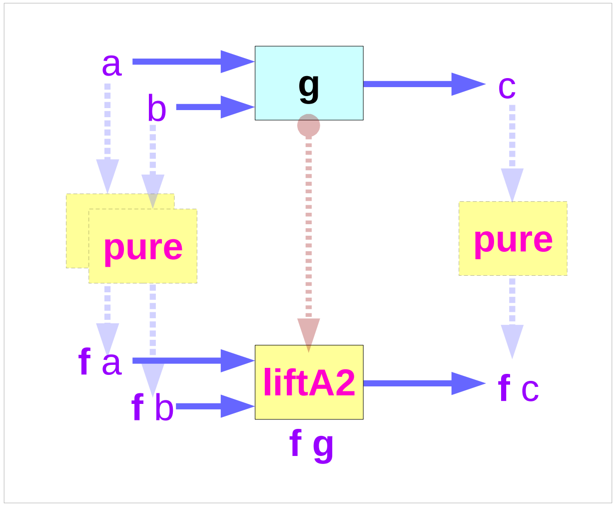
Left associative u <*> v <*> w = (u <*> v) <*> w u :: f (c -> b -> a) v :: f c w :: f b

u :: f (c -> b -> a)
v :: f c
u <*> v :: f (b -> a)
w :: f b
u <*> v <*> w = f a

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

liftA2

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

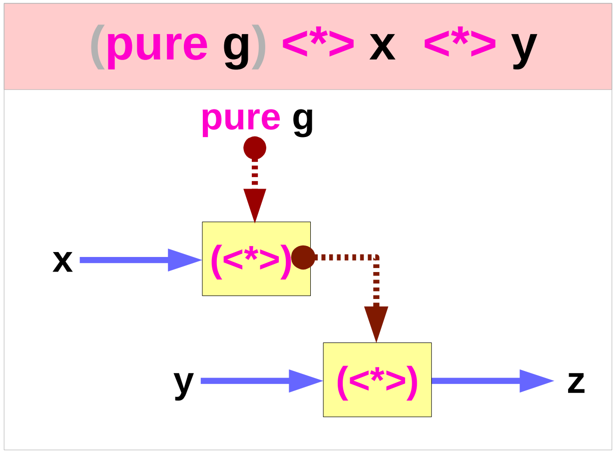


lift a binary function (**a->b->c**) to actions.

Some functors support an implementation of **liftA2** that is more efficient than the default one.

liftA2 may have an efficient implementation whereas **fmap** is an expensive operation,

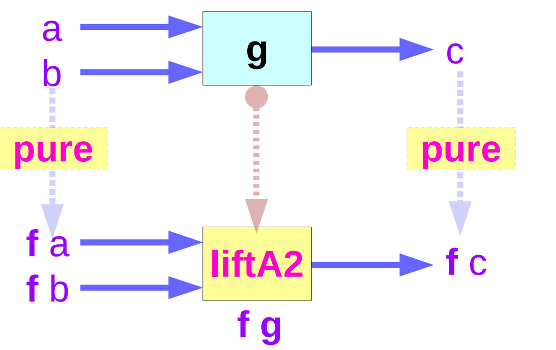
sometimes better to use **liftA2** than to use **fmap** over the structure and then use **<*>**.



http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

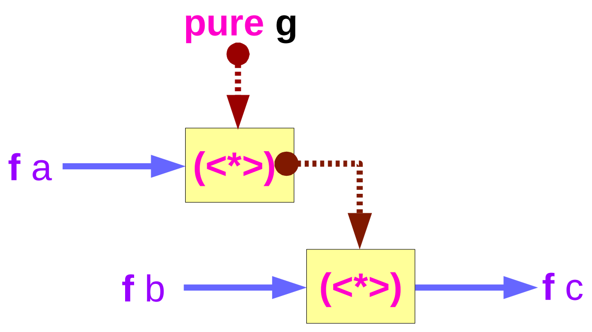
liftA2, <*>, fmap, <$>

liftA2 g x y

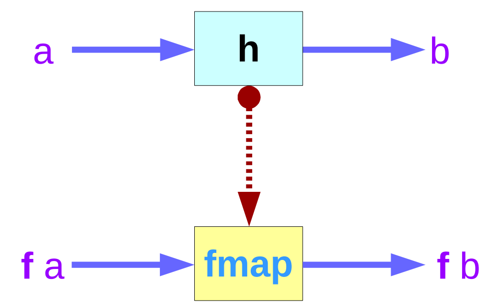


g :: a -> b -> c

(pure g) <*> x <*> y

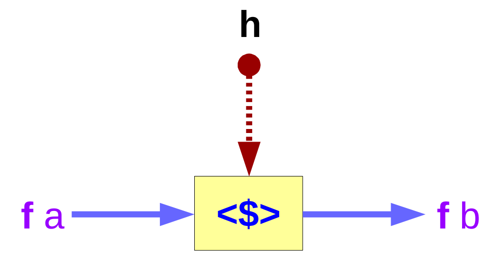


fmap h x

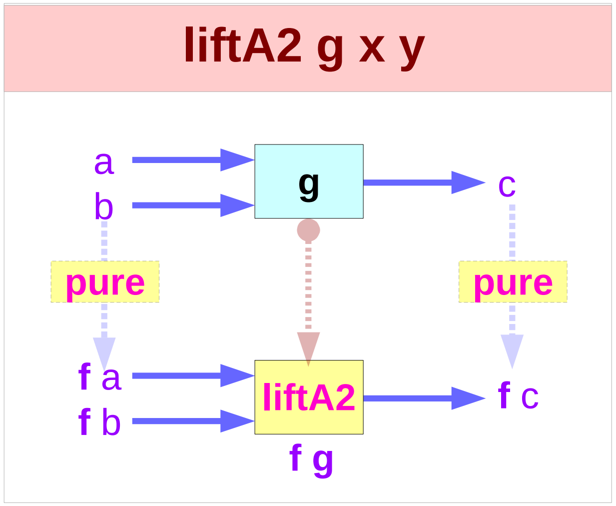


h :: a -> b

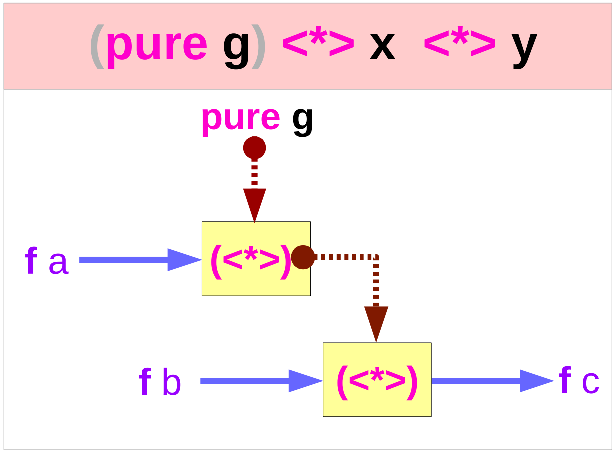
h <$> x

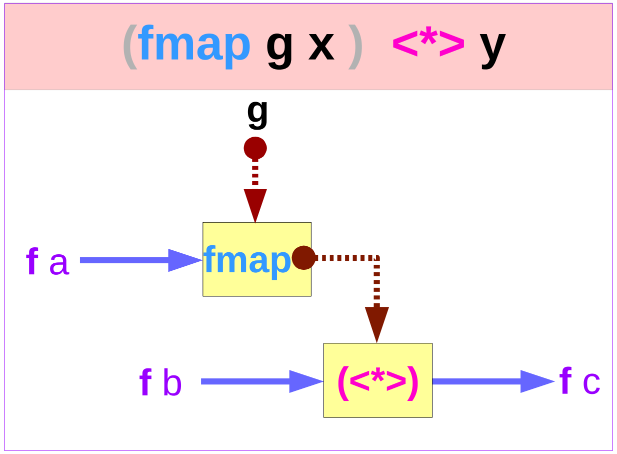


pure g <*> x <*> y = (fmap g x) <*> y

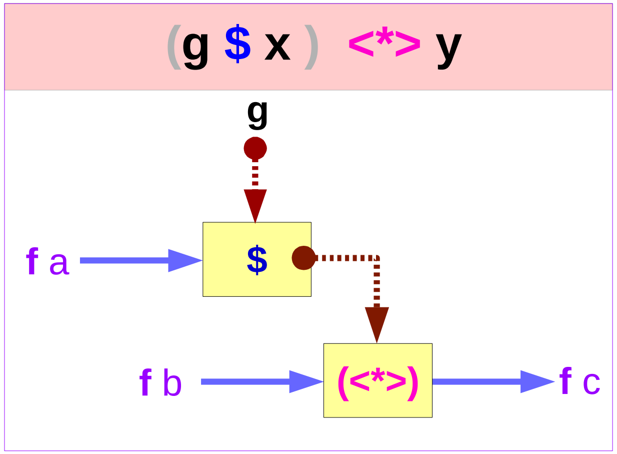


g :: a -> b -> c





g :: a -> b -> c



liftA2

liftA2 g x y

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

g :: a -> b -> c

x :: f a

y :: f b

liftA2 g x y :: f c

pure g <*> x <*> y

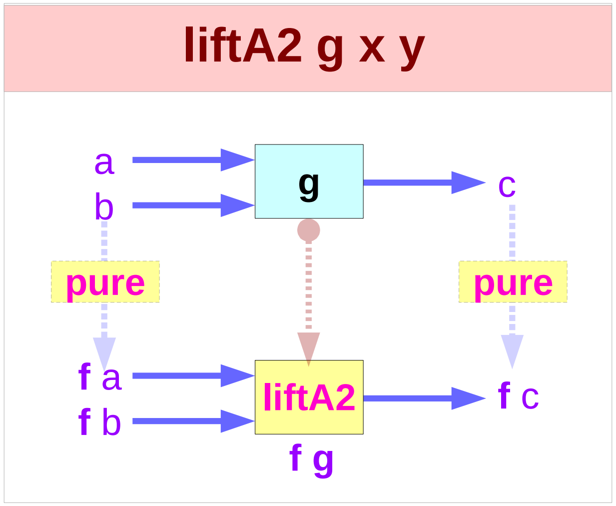
g :: a -> b -> c

x :: f a

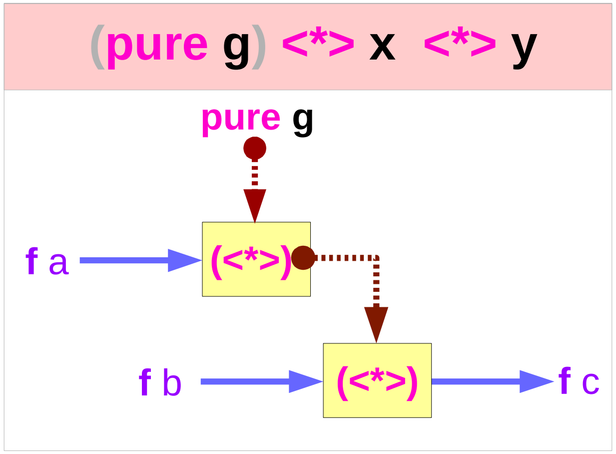
y :: f b

z :: f c

pure g <*> x <*> y :: f c



g :: a -> b -> c



https://wiki.haskell.org/Applicative_functor

liftA2

```
(a -> b -> c) -> (f a -> f b -> f c)

fmap :: (a -> b) -> (f a -> f b)

fmap2 :: Functor f => (a -> b -> c) -> (f a -> f b -> f c)
fmap2 h fa fb = undefined

h  :: a -> b -> c
fa :: f a
fb :: f b

h         :: a -> (b -> c)
fmap h    :: f a -> f (b -> c)
fmap h fa :: f (b -> c)
```

http://www.openhaskell.com/lectures/applicative.html

liftA2

```
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

pure  :: a             -> f a
fmap  :: (a -> b)      -> f a -> f b
fmap2 :: (a -> b -> c) -> f a -> f b -> f c

liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
liftA2 h fa fb = (h `fmap` fa) <*> fb

(<$>) :: Functor f => (a -> b) -> f a -> f b
(<$>) = fmap

liftA2 h fa fb = h <$> fa <*> fb

liftA3 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d
liftA3 h fa fb fc = ((h <$> fa) <*> fb) <*> fc
```

http://www.openhaskell.com/lectures/applicative.html

liftA2

liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c

liftA2 (+) (Just 5) (Just 6) = Just 11

<*> :: Applicative f => f (a -> b) -> f a -> f b

(Just (+5)) <*> (Just 6) = Just 11

let v1 = IO (Just (+5))

let v2 = IO (Just 6)

liftA2 (<*>) v1 v2 = IO (Just 11)

https://blog.ssanj.net/posts/2014-08-10-boosting-liftA2.html

<*> or liftA2 implementations

liftA2 :: (a -> b -> c) -> f a -> f b -> f c

A minimal complete definition :

either one of the two

1) pure and **<*>**

2) pure and **liftA2**

If it defines both, then they must behave the same

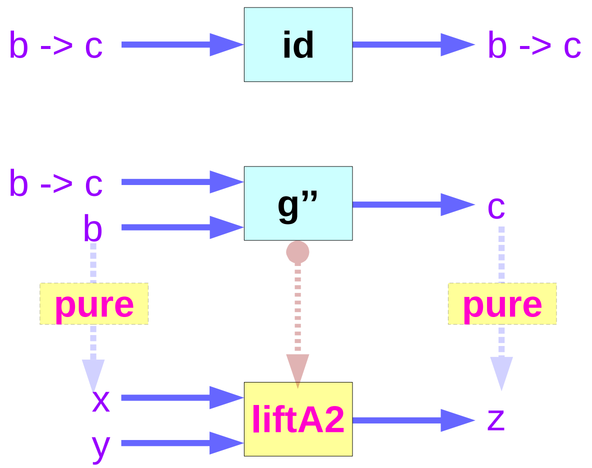
as their default definitions:

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

(<*>) = liftA2 id

liftA2 id x y = id <$> x <*> y = x <*> y

liftA2 id x y = x <*> y

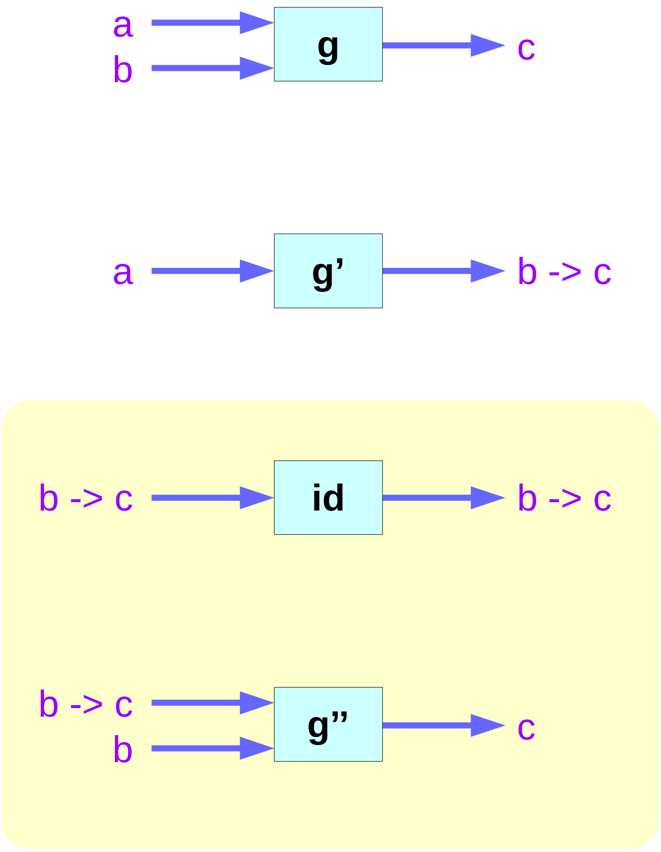


liftA2 g x y = g <$> x <*> y	g :: a -> b -> c	x :: f a	y :: f b
liftA2 g'' x y = g'' <$> x <*> y	g'' :: (b -> c) -> (b -> c)	x :: f (b -> c)	y :: f b
liftA2 id x y = id <$> x <*> y = x <*> y	id :: (b -> c) -> (b -> c)	x :: f (b -> c)	y :: f b

(<*>) = liftA2 id

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

g” :: (b->c) -> (b->c)



g :: a -> b -> c

view the function

as having one input only

g’ :: a -> (b -> c)

consider the case

when **a** is (**b -> c**)

id :: (b -> c) -> (b -> c)

Then **g”** is the same as **id**

g” :: (b -> c) -> (b -> c)

liftA2

Actually, using the **liftA** commands

we can pull results of applicative functors

into a scope where we can talk

exclusively about functor results **c**

and not about effects. **f c**

Note that functor results can also be functions. c

This scope is simply a function,

which contains the code that we used in the non-functorial setting.

```
liftA3
  (\x g h -> let y = g x in h y y)
  fx fg fh
```

The order of effects is entirely determined by the order of arguments to liftA3

.

http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2

liftA2

Consider the non-functorial expression:

```
x :: x
g :: x -> y
h :: y -> y -> z

let y = g x
in  h y y
```

Very simple. Now we like to generalize this to

```
fx :: f x
fg :: f (x -> y)
fh :: f (y -> y -> z)
```

https://wiki.haskell.org/Applicative_functor

liftA2

However, we note that

let fy = fg <*> fx

in fh <*> fy <*> fy

runs the effect of fy

twice. E.g. if fy

writes something to the terminal then fh <*> fy <*> fy

writes twice. This could be intended, but how can we achieve, that the effect is run only once and the result is used twice? Actually, using the liftA

commands we can pull results of applicative functors into a scope where we can talk exclusively about functor results and not about effects. Note that functor results can also be functions. This scope is simply a function, which contains the code that we used in the non-functorial setting.

liftA3

(\x g h -> let y = g x in h y y)

fx fg fh

The order of effects is entirely determined by the order of arguments to liftA3

.

https://wiki.haskell.org/Applicative_functor

liftA2(<*>)

10
down vote
accepted

The wiki article says that **liftA2 (<*>)**
can be used to compose applicative functors.
It's easy to see how to use it from its type:

```
o :: (Applicative f, Applicative f1) =>
    f (f1 (a -> b)) -> f (f1 a) -> f (f1 b)
o = liftA2 (<*>)
```

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

liftA2(<*>) Examples

So to if **f** is **Maybe** and **f1** is **[ ]** we get:

[(+1), (+6)] **[1, 6]**

> Just [(+1),(+6)] \`o\` Just [1, 6]
Just [2,7,7,12]

(+1) [1, 6]
(+6) [1, 6]

The other way around is:

> [Just (+1),Just (+6)] \`o\` [Just 1, Just 6]
[Just 2,Just 7,Just 7,Just 12]

[Just (+1),Just (+6)] **[Just 1, Just 6]**

Just (+1) [Just 1, Just 6]
Just (+6) [Just 1, Just 6]

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

LiftA2 (:)

your ex function is equivalent to **liftA2 (:)**:

```
test1 = liftA2 (:) "abc" ["pqr", "xyz"]
```

To use **(:)** with deeper applicative stack
you need multiple applications of **liftA2**:

```
*Main> (liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])
Just ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]
```

However it only works when both operands are equally deep.
So besides double liftA2 you should use pure to fix the level:

```
*Main> (liftA2 . liftA2) (:) (pure "abc") (Just ["pqr", "xyz"])
Just ["apqr","axyz","bpqr","bxyz","cpqr","cxyz"]
```

https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers

<$> related operators

Functor map <$>

(<$>) :: Functor f => (a -> b) -> f a -> f b

(<$) :: Functor f => a -> f b -> f a

($>) :: Functor f => f a -> b -> f b

The **<$>** operator is just a synonym
for the **fmap** function from the Functor typeclass.

This function generalizes the **map** function for lists
to many other data types, such as **Maybe**, **IO**, and **Map**.

https://haskell-lang.org/tutorial/operators

<$> examples

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
  putStrLn "Enter your year of birth"
  year <- read <$> getLine
  let age :: Int
      age = 2020 - year
  putStrLn $ "Age in 2020: " <> show age
```

https://haskell-lang.org/tutorial/operators

<$**, **$> operators

In addition, there are two additional operators provided

which replace a value inside a Functor

instead of applying a function.

This can be both more convenient in some cases,

as well as for some Functors be more efficient.

value <$ functor = const value <$> functor

functor $> value = const value <$> functor

x <$ y = y $> x

x $> y = y <$ x

https://haskell-lang.org/tutorial/operators

<*> related operators

Applicative function application <*>

(<*>) :: Applicative f => f (a -> b) -> f a -> f b

(*>) :: Applicative f => f a -> f b -> f b

(<*) :: Applicative f => f a -> f b -> f a

Commonly seen with **<$>**, **<*>** is an operator

that applies a wrapped function to a wrapped value.

It is part of the Applicative typeclass,

and is very often seen in code like the following:

foo <$> bar <*> baz

https://haskell-lang.org/tutorial/operators

<*> examples

For cases when you're dealing with a Monad, this is equivalent to:

```
do x <- bar
   y <- baz
   return (foo x y)
```

Other common examples including parsers and serialization libraries.

Here's an example you might see using the aeson package:

```
data Person = Person { name :: Text, age :: Int } deriving Show

-- We expect a JSON object, so we fail at any non-Object value.
instance FromJSON Person where
    parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
    parseJSON _ = empty
```

https://haskell-lang.org/tutorial/operators

*> operator

To go along with this, we have two helper operators that are less frequently used:

***>** ignores the value from the first argument. It can be defined as:

```
a1 *> a2 = (id <$ a1) <*> a2
```

Or in do-notation:

```
a1 *> a2 = do
    _ <- a1
    a2
```

For Monads, this is completely equivalent to **>>**.

https://haskell-lang.org/tutorial/operators

<* operator

<* is the same thing in reverse: perform the first action then the second,

but only take the value from the first action.

Again, definitions in terms of **<*>** and do-notation:

```
(<*) = liftA2 const
```

```
a1 <* a2 = do
    res <- a1
    _ <- a2
    return res
```

https://haskell-lang.org/tutorial/operators

(*> v.s. >>) and (pure v.s. return)

```
(*>) ::     Applicative f  =>  f a ->  f b ->  f b
(>>) ::     Monad m        =>  m a ->  m b ->  m b

pure ::     Applicative f  =>  a ->    f a
return ::   Monad m        =>  a ->    m a
```

the constraint changes from **Applicative** to **Monad**.

(*>) in **Applicative**	**(>>)** in **Monad**
pure in **Applicative**	**return** in **Monad**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf