Monad P3 : Types (1A)

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Based on

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

data

data - creates new **algebraic type** with **value constructors**

- Can have several **value constructors**
- **Value constructors** are lazy
- **Values** can have several fields
- Affects both compilation and runtime, have runtime overhead
- Created type is a distinct new type
- Can have its own **type class instances**
- When pattern matching against **value constructors**,
  WILL be evaluated at least to weak head normal form (**WHNF**) *
- Used to create *new data type*
  (example: Address { zip :: String, street :: String })

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

newtype

newtype - creates new "**decorating**" **type** with **value constructor**

- Can have only one **value constructor**
- **Value constructor** is strict
- **Value** can have only one field
- Affects only compilation, no runtime overhead
- Created type is a distinct new type
- Can have its own **type class instances**
- When pattern matching against **value constructor**,
  CAN be not evaluated at all *
- Used to create *higher level concep*t based on existing type
  with distinct set of supported operations or
  that is not interchangeable with original type
  (example: Meter, Cm, Feet is Double)

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

type

type - creates an **alternative name** (synonym) for a **type** (typedef in C)

- No value constructors
- No fields
- Affects only compilation, no runtime overhead
- No new type is created (only a new name for existing type)
- Can NOT have its own **type class instances**
- When pattern matching against **data constructor**,
  behaves the same as original type
- Used to create higher level concept based on existing type
  with the same set of supported operations
  (example: String is [Char])

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

data, newtype, type

data: zero or more **constructors**,
each can contain zero or more **values**.

newtype: similar to above
but exactly one **constructor**
and only one **value** in that constructor,
and has the exact **same runtime representation**
as the value that it stores.

type: **type synonym**, compiler more or less
forgets about it once it is expanded.

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

data (lazy), newtype (strict)

Both **newtype** and the single-constructor **data**
introduce a single **data constructor**,
but the **data constructor** introduced by **newtype** is **strict**
and the **data constructor** introduced by **data** is **lazy**.

```
data    D = D Int    -- lazy
newtype N = N Int    -- strict
```

Then **N undefined** is equivalent to **undefined**
and causes an **error** when **evaluated**.

But **D undefined** is not equivalent to **undefined**,
and it can be **evaluated** as long as you don't try to peek inside.

https://www.reddit.com/r/haskell/comments/6xri4d/whats_the_difference_between_newtype_type_and_data/

Data definition without data constructors (1)

a **data definition** without **data constructors**

cannot be instantiated

data B

a new **type constructor B**,

but no **data constructors** to produce **values** of **type B**

In fact, such a data type is declared in the Haskell base: **Void**

ghci> **import Data.Void**

ghci> **:i Void**

data Void -- Defined in ‘Data.Void’

https://stackoverflow.com/questions/45385621/data-declaration-with-no-data-constructor-can-it-be-instantiated-why-does-it-c

Data definition without data constructors (2)

Being able to have **uninhabited types** turns out
to be useful in some areas

passing an **uninhabited type** as a **type parameter**
to another **type constructor**

https://stackoverflow.com/questions/45385621/data-declaration-with-no-data-constructor-can-it-be-instantiated-why-does-it-c

Data definition with data constructors

data B = String

defines

a **type constructor B** and

a **data constructor String**,

both taking no arguments.

Note that the **String** you define is in the **value namespace**,

so is different from the usual **String type constructor**.

ghci> data B = String

ghci> x = String

ghci> :t x

x :: B

https://stackoverflow.com/questions/45385621/data-declaration-with-no-data-constructor-can-it-be-instantiated-why-does-it-c

Algebraic type

This is a type where we specify the **shape** of each of the **elements**.

Algebraic refers to the **property** that
an **Algebraic Data Type** is created by **algebraic operations**.

The **algebra** here is **sums** and **products**:

sum is **alternation (A | B**, meaning A or B but not both)
product is **combination** (**A B**, meaning A and B together)

http://wiki.haskell.org/Algebraic_data_type

Algebraic type

data Pair = P Int Double

a **pair** of numbers, an **Int** and a **Double** together.
The **tag P** is used (in **constructors** and **pattern matching**)
to combine the contained values into a single structure
that can be assigned to a variable.

data Pair = I Int | D Double

just one number, either an **Int** or else a **Double**.
the tags **I** and **D** are used (in **constructors** and **pattern matching**)
to distinguish between the two alternatives.

http://wiki.haskell.org/Algebraic_data_type

Algebraic type

Sums and **products** can be repeatedly combined
into an arbitrarily large structures.

Algebraic Data Type is not to be confused with ***Abstract* Data Type**,
which (ironically) is its opposite, in some sense.

The initialism **ADT** usually means ***Abstract* Data Type**,
but **GADT** usually means **Generalized Algebraic Data Type**.

http://wiki.haskell.org/Algebraic_data_type

Decorate type

types, functions and values

Type variables in Haskell are typically named starting at **a**, **b**, etc.
They are sometimes (but not often)
decorated with numbers like **a1** or **b3**.

http://wiki.haskell.org/Algebraic_data_type

Decorate type

Functions used as higher-order arguments
are typically named starting at **f**, **g**, etc.

They will sometimes be decorated with numbers like type variables
and will also be decorated with the **'** character like **g'**.

You would read this latter example as "Jee-prime"
and it is typically a function that is
in some way related to g used as a helper or the like.

Occasionally functions may be given names
that are not on this continuum as an aide memoir,
for example a function parameter used internally
as a predicate may be given the name p.

http://wiki.haskell.org/Algebraic_data_type

Decorate type

Arguments to functions, or variables used exclusively
inside short functions, are often given names starting at x, y, etc.,
again occasionally decorated by numbers.
Other single-letter variable names may be chosen
if they can act as a mnemonic for their role such
as using a variable named p for a value known to be prime.

Note that these are guidelines and not rules.
Any of them can and will be ignored,
modified and/or abused in any given piece of Haskell code.
(A quick look at the Standard Prelude
as provided in the Haskell 98 Report should
be convincing enough for this.)

http://wiki.haskell.org/Algebraic_data_type

Newtype

newtype is used when you want to wrap one type in another type and nothing more complicated than that. Because of this restriction, it can be represented the same as the original type in memory, meaning there is zero runtime penalty for using a newtype

newtype Dollars = Dollars Int

Here, newtype is being used to take the somewhat uninformative type Int and create a more descriptive type, Dollars. To make a value of Dollars, one might write Dollars 3.)

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

Sometimes you want to make a type that's almost the same as another type. For example imagine our program calls for a Dollar type, a Yen type, and a Euro type, which are all just wrappers around Double. And let's say also we had a Currency typeclass with a convertToDollars and convertFromDollars function. We'd like to add, subtract, and multiply our currency like we could regular numbers.

One way to make our types would be as follows:

```
data Dollar = Dollar Double  deriving (Read, Show)
data Euro = Euro Double      deriving (Read, Show)
data Yen = Yen Double        deriving (Read, Show)
```

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

Now, this has two issues. The first is that we can't add or subtract two Dollars. We'd have to make each of these an instance of the Num typeclass, like this.

```
data Dollar = Dollar Double  deriving (Read, Show)
data Euro = Euro Double      deriving (Read, Show)
data Yen = Yen Double        deriving (Read, Show)

-- This, but for every currency:
instance Num Dollar where
    (Dollar a) + (Dollar b) = Dollar (a + b)
    (Dollar a) - (Dollar b) = Dollar (a - b)
    (Dollar a) * (Dollar b) = Dollar (a * b)
    negate (Dollar a) = Dollar (-a)
    abs (Dollar a) = Dollar (abs a)
```

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

same for every currency! Wrapping one type (in our case, Double) in another (in our case, Dollar) is such a common need that we have special syntax for it, newtype.

```
newtype Dollar = Dollar Double  deriving (Read, Show)
newtype Euro = Euro Double      deriving (Read, Show)
newtype Yen = Yen Double        deriving (Read, Show)
```

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

The main difference between using newtype and data is that newtype only works with the very simple situation of wrapping one type in one other type. You can’t use sum types or have multiple types wrapped up in one. And there’s a special GHC feature that makes newtype much more useful by letting it automatically derive typeclasses for you, and you turn it on by putting {-# LANGUAGE GeneralizedNewtypeDeriving #-} at the top of your code. This is called a pragma to turn on a language extension. We’ll discuss these later, for now just know that putting that at the top of a file turns on an extra feature of GHC. Here’s how it looks in action:

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

```
{-# LANGUAGE GeneralizedNewtypeDeriving #-}

newtype Dollar = Dollar Double  deriving (Read, Show, Num)
newtype Euro = Euro Double      deriving (Read, Show, Num)
newtype Yen = Yen Double        deriving (Read, Show, Num)
```

With GeneralizedNewtypeDeriving turned on, we were able to add Num to our list of typeclasses we'd like to be automatically derived, which is very useful! We'd be able to run (Dollar 3) + (Dollar 4) to get Dollar 7.0.

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

There's one other difference between newtype and data. Specifically, whether the constructor is strict or lazy. Imagine the following:

```
data D = D Int
newtype N = N Int
```

Now, you may remember that Haskell tries to only evaluate things when really necessary, so if you write 1+2 it won't actually evaluate that until it needs to. Haskell also has a special value named undefined which you can pass to any function and causes your program to instantly crash when it's evaluated.

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Newtype

data is for making new, complicated types, like data Person = Bob | Cindy | Sue.

newtype is for “decorating” or making a copy of an existing type, like newtype Dollar = Dollar Double.

type is for renaming a type, like type Polygon = [Point], which just makes Dollar be equivalent to Double and is mostly only used for making certain code easier to read.

https://webcache.googleusercontent.com/search?q=cache:_5Dl-cKznPcJ:https://andre.tips/wmh/newtype/+&cd=12&hl=en&ct=clnk&gl=us

Type class instances

Type classes allow us

to declare which types are **instances** of which class, and

to provide **definitions** of the **overloaded operations**
associated with a **class**.

https://www.haskell.org/tutorial/classes.html

Type class instances

For example, let's define a **type class** containing an **equality operator**:

```
class Eq a where
  (==)              :: a -> a -> Bool
```

Eq is the **name** of the **class** being defined,

== is the single **operation** in the **class**.

a **type a** is an **instance** of the **class Eq**

if there is an (**overloaded**) **operation ==**,

of the appropriate **type**, defined on it.

(Note that == is only defined on pairs of objects of the same type.)

type

class **Eq** **a**

class name

class instance

https://www.haskell.org/tutorial/classes.html

Type class instances

Eq a expresses a **constraint** that

a **type a** must be an **instance** of the **class Eq**

Eq a

is not a **type expression**

expresses a **constraint** on a **type**

called a **context**

placed at the front of **type expressions**

https://www.haskell.org/tutorial/classes.html

Type class instances

For example, the effect of the above class declaration
is to assign the following type to ==:

(==) :: (Eq a) => a -> a -> Bool

for every **type a** that is an **instance** of the **class Eq**,
== has type **a->a->Bool**

elem :: (Eq a) => a -> [a] -> Bool

for every **type a** that is an **instance** of the **class Eq**,
elem has type **a->[a]->Bool**

https://www.haskell.org/tutorial/classes.html

Type class instances

An **instance declaration** specifies

which types are **instances** of the **class Eq**, and

the actual behavior of == on each of those **types**

instance Eq Integer where

x == y = x `integerEq` y

the **definition** of **==** is called a **method**.

integerEq happens to be the **primitive function**

in general, any valid expression for a function definition

type

instance Eq integer

class name

class instance

https://www.haskell.org/tutorial/classes.html

Type class instances

```
instance Eq Integer where
x == y      =  x `integerEq` y
```

the **type Integer** is an **instance** of the **class Eq**
the definition of the **method ==**

```
instance Eq Float where
x == y      =  x `floatEq` y
```

the **type Float** is an **instance** of the **class Eq**
the definition of the **method ==**

https://www.haskell.org/tutorial/classes.html

Type class instances

simply substituting **type class** for **class**, and **type** for **object**,
yields a valid summary of Haskell's **type class mechanism**:

"**Classes** capture common sets of operations.
A particular **object** may be an **instance** of a **class**,
and will have a **method** corresponding to each **operation**.
Classes may be arranged **hierarchically**,
forming notions of **superclasses** and **sub classes**,
and permitting **inheritance** of operations/methods.
A **default method** may also be associated with an operation."

Haskell	OOP
type class	class
type	object

https://www.haskell.org/tutorial/classes.html

Type class instances

In contrast to OOP, it should be clear that **types** are not **objects**,
and in particular there is no notion of an object's or
type's **internal mutable state**.

An advantage over some OOP languages is that
methods in Haskell are completely type-safe:
any attempt to apply a **method** to a **value**
whose **type** is not in the required **class**
will be detected at compile time instead of at runtime.

In other words, **methods** are not "looked up" at runtime
but are simply passed as **higher-order functions**.

https://www.haskell.org/tutorial/classes.html

Type class instances

parametric polymorphism is useful in defining families of types
by universally quantifying over all types.

Sometimes, however, it is necessary
to quantify over some smaller set of types,
eg. those types whose elements can be compared for equality.

https://www.haskell.org/tutorial/classes.html

Type class instances

type classes can be seen as providing a structured way
to quantify over a constrained set of types

Indeed, we can think of **parametric polymorphism**
as a kind of **overloading** too!

an **overloading** occurs implicitly over all types
a **type class** for a constrained set of types

https://www.haskell.org/tutorial/classes.html

Polymorphic Types

types that are universally quantified in some way over all types.
polymorphic type expressions essentially describe families of types.

For example, **(forall a) [a]** is the family of types
consisting of, for every **type a**, the **type of lists of a**.

Lists of integers (e.g. **[1,2,3]**), lists of characters (**['a','b','c']**),
even lists of lists of integers, etc., are all members of this family.

(Note, however, that [2,'b'] is not a valid example,
since there is *no single type* that contains both 2 and 'b'.)

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

Identifiers such as **a** above are called **type variables**,
and are uncapitalized to distinguish them
from specific types such as **Int**.

since Haskell has only universally quantified **types**,
there is no need to explicitly write out the symbol
for **universal quantification**,
and thus we simply write **[a]** in the example above.

In other words, all **type variables** are implicitly universally quantified

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

Lists are a commonly used data structure in functional languages,
and are a good vehicle for explaining the principles of polymorphism.

The list **[1,2,3]** in Haskell is actually shorthand for the list **1:(2:(3:[]))**,
where **[]** is the **empty list** and **:** is the **infix operator**
that adds its first argument to the front of its second argument (a list).

Since **:** is right associative, we can also write this list as **1:2:3:[]**.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

```
length              :: [a] -> Integer
length []           =  0
length (x:xs)       =  1 + length xs

length [1,2,3]    =>    3
length ['a','b','c']      =>    3
length [[1],[2],[3]]      =>    3
```

an example of a polymorphic function.

It can be applied to a list containing elements of any type,

for example **[Integer]**, **[Char]**, or **[[Integer]]**.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

```
length          :: [a] -> Integer
length []       =  0
length (x:xs)     =  1 + length xs
```

The left-hand sides of the equations contain
patterns such as [] and x:xs.

In a function application these patterns are
matched against actual parameters in a fairly intuitive way

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

```
length          :: [a] -> Integer
length []       =  0
length (x:xs)     =  1 + length xs
```

[] only matches the empty list,

x:xs will successfully match any list with at least one element,

binding x to the first element and xs to the rest of the list

If the match succeeds,

the right-hand side is evaluated

and returned as the result of the application.

If it fails, the next equation is tried,

and if all equations fail, an error results.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

Function head returns the first element of a list,
function tail returns all but the first.

```
head            :: [a] -> a
head (x:xs)       =  x

tail            :: [a] -> [a]
tail (x:xs)       =  xs
```

Unlike length, these functions are not defined
for all possible values of their argument.
A runtime error occurs when these functions
are applied to an empty list.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

With polymorphic types, we find that
some types are in a sense strictly more general than others
in the sense that the set of values they define is larger.

For example, the type **[a]** is more general than **[Char]**.
In other words, the latter type can be derived from the former
by a suitable substitution for **a**.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

With regard to this **generalization ordering**,
Haskell's type system possesses two important properties:

First, every well-typed expression is guaranteed
to have a **unique principal type** (explained below),

and second, the **principal type** can be inferred automatically.

In comparison to a monomorphically typed language such as C,
the reader will find that polymorphism improves expressiveness,
and **type inference** lessens the burden of types on the programmer.

https://www.haskell.org/tutorial/goodies.html

Polymorphic Types

An expression's or function's **principal type** is
the least general type that, intuitively,
"contains all instances of the expression".

For example, the principal type of head is **[a]->a**; **[b]->a**, **a->a**,
or even **a** are correct types, but too general,
whereas something like **[Integer]->Integer** is too specific.
The existence of unique **principal types** is the hallmark feature
of the **Hindley-Milner type system**,
which forms the basis of the type systems of Haskell,
ML, Miranda, ("Miranda" is a trademark of Research Software, Ltd.)
and several other (mostly functional) languages.

https://www.haskell.org/tutorial/goodies.html

Explicitly Quantifying Type Variables

to explicitly bring fresh **type variables** into **scope**.

Example: **Explicitly quantifying** the **type variables**

map :: forall a b. (a -> b) -> [a] -> [b]

for any combination of types **a** and **b**

choose **a** = **Int** and **b** = **String**

then it's valid to say that map has the type

(Int -> String) -> [Int] -> [String]

Here we are **instantiating** the general type of **map**

to a more specific type.

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Implicit **forall**

any introduction of a **lowercase type parameter**

implicitly begins with a **forall** keyword,

Example: Two equivalent type statements

id :: a -> a

id :: forall a . a -> a

We can apply additional **constraints**

on the quantified **type variables**

https://en.wikibooks.org/wiki/Haskell/Existentially_quantified_types

Existential Types

Normally when creating a new type
using **type**, **newtype**, **data**, etc.,
every **type variable** that appears on the right-hand side
must also appear on the left-hand side.

```
newtype ST s a = ST (State# s -> (# State# s, a #))
```

Existential types are a way of escaping

Existential types can be used for several different purposes.
But what they do is to **hide** a **type variable** on the right-hand side.

https://wiki.haskell.org/Existential_type

Type Variable Example – (1) error

Normally, any type variable appearing on the right must
also appear on the left:

data Worker x y = Worker {buffer :: b, input :: x, output :: y}

This is an **error**, since the **type** of the **buffer**
isn't specified on the right (it's a type variable rather than a type)
but also isn't specified on the left (there's no '**b**' in the left part).

In Haskell98, you would have to write

data Worker b x y = Worker {buffer :: b, input :: x, output :: y}

https://wiki.haskell.org/Existential_type

Type Variable Example – (2) explicit type signature

However, suppose that a **Worker** can use any type '**b**'
so long as it belongs to some particular class.
Then every **function** that uses a Worker will have a type like

foo :: (Buffer b) => Worker b Int Int

In particular, failing to write an **explicit type signature** **(Buffer b)**
will invoke the dreaded monomorphism restriction.

Using **existential types**, we can avoid this:

https://wiki.haskell.org/Existential_type

Type Variable Example – (3) existential type

```
data Worker x y = forall b. Buffer b =>
                  Worker {buffer :: b, input :: x, output :: y}

foo :: Worker Int Int
```

The **type** of the **buffer** (**Buffer**) now does not appear
in the **Worker** type at all.

https://wiki.haskell.org/Existential_type

Type Variable Example – (4) characteristics

```
data Worker x y = forall b. Buffer b =>
                  Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

- it is now impossible for a function
  to demand a **Worker** having a specific type of **buffer**.

- the **type** of **foo** can now be derived automatically
  without needing an explicit **type signature**.
  (No **monomorphism** restriction.)

https://wiki.haskell.org/Existential_type

Type Variable Example – (4) characteristics

```
data Worker x y = forall b. Buffer b =>
                  Worker {buffer :: b, input :: x, output :: y}
foo :: Worker Int Int
```

- since code now has no idea
  what **type** the buffer function returns,
  you are more limited in what you can do to it.

https://wiki.haskell.org/Existential_type

Hiding a type

In general, when you use a '**hidden**' type in this way,
you will usually want that **type** to belong to a **specific class**,
or you will want to **pass some functions** along
that can work on that type.

Otherwise you'll have some value belonging
to a **random unknown type**,
and you won't be able to do anything to it!

https://wiki.haskell.org/Existential_type

Conversion to less a specific type

Note: You can use **existential types**

to **convert** a **more specific type**

into a **less specific one**.

There is no way to perform the reverse conversion!

https://wiki.haskell.org/Existential_type

A heterogeneous list example

This illustrates **creating a heterogeneous list**,
all of whose members implement "**Show**",
and progressing through that list to show these items:

```
data Obj = forall a. (Show a) => Obj a

xs :: [Obj]
xs = [Obj 1, Obj "foo", Obj 'c']

doShow :: [Obj] -> String
doShow [] = ""
doShow ((Obj x):xs) = show x ++ doShow xs
```

With output: **doShow xs ==> "1\"foo\"'c'"**

https://wiki.haskell.org/Existential_type

Bottom

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Bottom

bottom in Haskell specifically called **undefined**.

This is only one form of it

though technically **bottom** is also

a non-terminating computation, such as length [1..]

bottom is used to represent an expression which is

- not computable
- runs forever
- never returns a value
- throws an exception
- etc.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom represents computations

The term **bottom** refers to

a **computation** which never completes successfully.

a **computation** that fails due to some kind of error,

a **computation** that just goes into an infinite loop

(without returning any data).

The mathematical symbol for bottom is '⊥'.

In plain ASCII, '_|_'.

https://wiki.haskell.org/Bottom

Bottom – a member of any type

Bottom is a **member** of any type,

even the trivial type **()** or

the equivalent simple type:

data Unary = Unary

https://wiki.haskell.org/Bottom

Bottom – definitions

Bottom can be expressed in Haskell thus:

```
bottom = bottom
bottom = error "Non-terminating computation!"
```

Indeed, the Prelude exports a function

```
undefined = error "Prelude.undefined"
```

Other implementations of Haskell, such as Gofer, defined bottom as:

```
undefined | False = undefined
```

The type of bottom is arbitrary, and defaults to the most general type:

```
undefined :: a
```

https://wiki.haskell.org/Bottom

Bottom – Usage

As **bottom** is an **inhabitant** of every **type** *a value of every type*

bottoms can be used wherever a value of that type would be.

This can be useful in a number of circumstances:

```
-- For leaving a todo in your program to come back to later:
foo = undefined

-- When dispatching to a type class instance:
print (sizeOf (undefined :: Int))

-- When using laziness:
print (head (1 : undefined))
```

https://wiki.haskell.org/Bottom

Bottom Rule

if **x** is computable,

then **strict f x** evaluates to **f x**,

but if **x** is not computable, *undefined*

then **strict f x** evaluates to "not computable". *undefined*

for example, **f x = 2 * x**.

consider **f (1 / 0)**

can't evaluate it because you can't evaluate (1 / 0)

(1 / 0) not computable

f (1 / 0) not computable

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

strict f x

Sometimes it is necessary
to control order of evaluation in a **lazy** functional program.

Use the computable function **strict**,
strict f x = if x ≠ ⊥ then f x else ⊥.

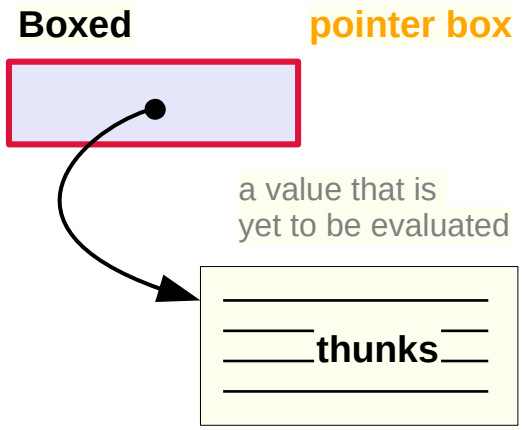
Operationally, **strict f x** is reduced by
first reducing **x** to **weak head normal form** (**WHNF**)
and then reducing the application **f x**.

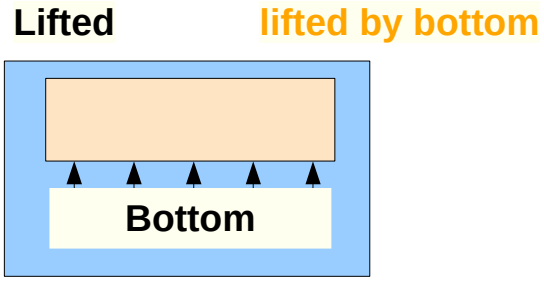
Alternatively, it is safe to reduce **x** and **f x** in parallel,
but not allow access to the result until **x** is in **WHNF**.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Classifying types – Summary

Boxed	a **pointer** to a **heap** object.
Unboxed	no **pointer**
Lifted	**bottom** as an element.
Unlifted	no **extra values**.
Algebraic	one or more **constructors**,
Primitive	a **built-in type**



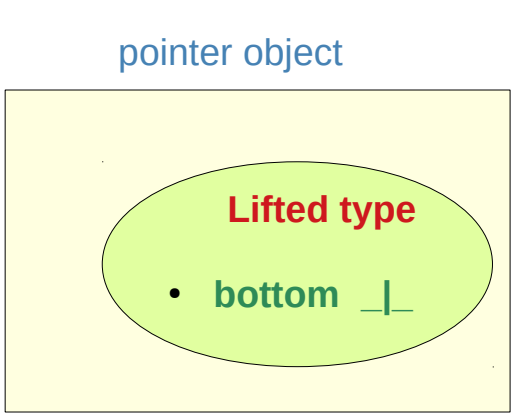


Undefined
Infinite loop
Exception

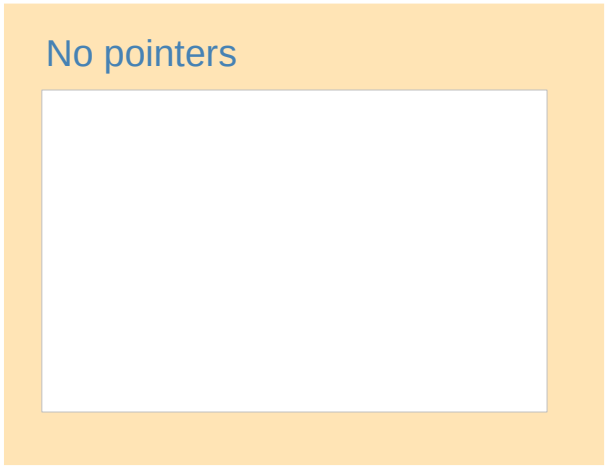
https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type

(Un)Lifted and (Un)Boxed types

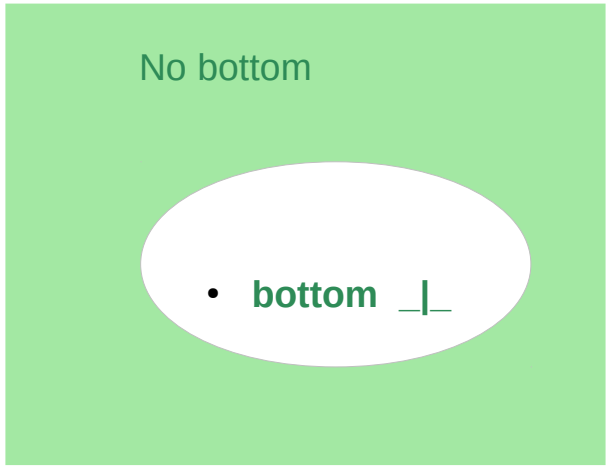
Boxed type



Unboxed type



Unlifted type



Lifted type → **Boxed type**

kind *

Unboxed type → **Unlifted type**

kind #

https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell

Bottom in a programming language

programming language :
bottom refers to a value that is less defined than any other.

It's common to assign the **bottom value** to every computation
that either produces an **error** or **fails** to **terminate**,

because trying to distinguish these conditions
which greatly weakens
the mathematics and
complicates program analysis.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom in an order theory

order theory (particularly **lattice theory**) :

The **bottom** element of a partially ordered set,

if one exists, is the one that precedes all others.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom in a lattice theory

Lattice theory

the logical **false** value

is the **bottom element** of a **lattice** of **truth values**,

and **true** is the **top element**

classical logic

these are the only two – **true** and **false**

but one can also consider logics

with infinitely many truthfulness values,

such as **intuitionism** and various forms of **constructivism**.

These take the notions in a rather different direction.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom in a standard Boolean logic

standard Boolean logic

the symbol ⊥ read **falsum** or **bottom**,

is simply a statement which is always false,

the equivalent of the false constant in programming languages.

The form is an inverted (upside-down) version of the symbol ⊤

(**verum** or **top**), which is the equivalent of true -

and there's mnemonic value in the fact that the symbol looks

like a capital letter T.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom – verum an falsum

The names **verum** and **falsum** are Latin for "**true**" and "**false**";
the names "**top**" and "**bottom**" come from
the use of the symbols in the **theory** of **ordered sets**,
where they were chosen based on
the location of the horizontal crossbar

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

Bottom – computability theory

computability theory, ⊥ is also
the value of an **uncomputable computation**,
so you can also think of it as the **undefined value**.

It doesn't matter why the computation is uncomputable -
whether because it has **undefined inputs**,
or **never terminates**, or whatever.

it defines **strict** as a **function**
that makes any computation (another function) **undefined**
whenever its inputs (arguments) are **undefined**.

https://stackoverflow.com/questions/26428828/what-does-%E2%8A%A5-mean-in-the-strictness-monad-from-p-wadlers-paper

WHNF (Weak Head Normal Form)

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Normal Form

An **expression** in **normal form**

is fully evaluated,

contains no un-evaluated thunks

no **sub-expression** could be evaluated any further

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Normal Form Examples

in normal form:

42

(2, "hello")

\x -> (x + 1)

not in normal form:

1 + 2	-- we could evaluate this to 3
(\x -> x + 1) 2	-- we could apply the function
"he" ++ "llo"	-- we could apply the (++)
(1 + 1, 2 + 2)	-- we could evaluate 1 + 1 and 2 + 2

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Head – outermost function application

The **head** in **WHNF** (Weak Head Normal Form)

does not refer to the **head** of a **list**,

but to the **outermost** **function application**.

thunks

generally refer to **unevaluated expressions**

HNF (Head normal form) is irrelevant for Haskell.

It differs from **WHNF** in that

the **bodies** of lambda expressions

are also evaluated *to some extent*.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

NF is WHNF

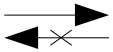
An **expression** in **WHNF** (weak head normal form)

has been evaluated to the outermost

data constructor or **lambda abstraction** (the **head**).

sub-expressions may or may not have been evaluated.

Normal form (NF)



Weak Head Normal Form (WHNF)

No unevaluated subexpressions

No unevaluated head expression

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Weak Head Normal Form Test

To determine whether an expression is in weak head normal form,
we only have to look at the **outermost part** of the expression.

If the **outermost** part of the expression
- is a **data constructor** or a **lambda**,
  - then it is in **weak head normal form**.
- is a **function application**,
  - then it is not in **weak head normal form**.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Evaluation Example

outermost application

from left to right;

lazy evaluation.

Example:

take 1 (1:2:3:[]) => { apply **take** }

1 : take (1-1) (2:3:[]) => { apply (-) }

1 : take 0 (2:3:[]) => { apply **take** }

1 : []

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Reduced Normal Form

evaluation stops when there are

no more function applications left to replace.

the result is in **normal form**

(or reduced normal form, **RNF**).

no unevaluated subexpressions

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Lazy Evaluation

No matter in which **order** you evaluate an expression,

you will always end up with the same **normal form**

(but only if the evaluation terminates).

There is a slightly different description for **lazy evaluation**.

Namely, it says that you should evaluate everything

to weak head normal form (**WHNF**) only.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

The head of the expression

There are precisely **three cases** for an expression to be in **WHNF**:

A **constructor**: constructor expression_1 expression_2 ...

A **built-in function** with too few arguments, like (+) 2 or sqrt

A **lambda-expression**: \x -> expression

In other words, the **head** of the **expression**

(i.e. the **outermost function application**)

cannot be evaluated any further,

but the function argument may contain

unevaluated expressions.

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

Weak Head Normal Form Test

in weak head normal form:

(1 + 1, 2 + 2) -- the outermost part is the data constructor (,)

\x -> 2 + 2 -- the outermost part is a lambda abstraction

'h' : ("e" ++ "llo") -- the outermost part is the data constructor (:)

As mentioned, all the normal form expressions listed above
are also in weak head normal form.

not in weak head normal form:

1 + 2 -- the outermost part here is an application of (+)

(\x -> x + 1) 2 -- the outermost part is an application of (\x -> x + 1)

"he" ++ "llo" -- the outermost part is an application of (++)

in normal form:
42
(2, "hello")
\x -> (x + 1)

https://stackoverflow.com/questions/6872898/what-is-weak-head-normal-form

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf

[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf