

Spectrum Representation (2B)

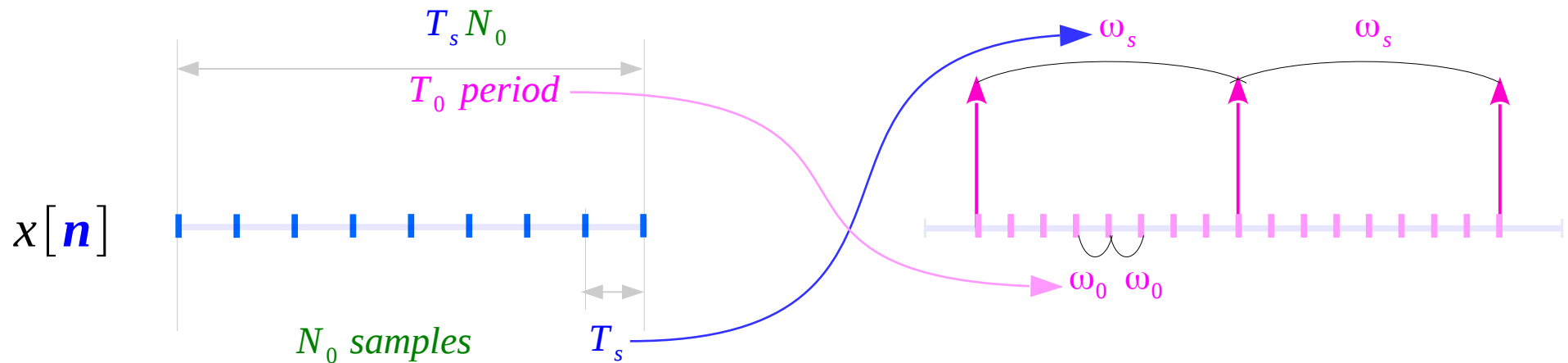
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ω_s and ω_0

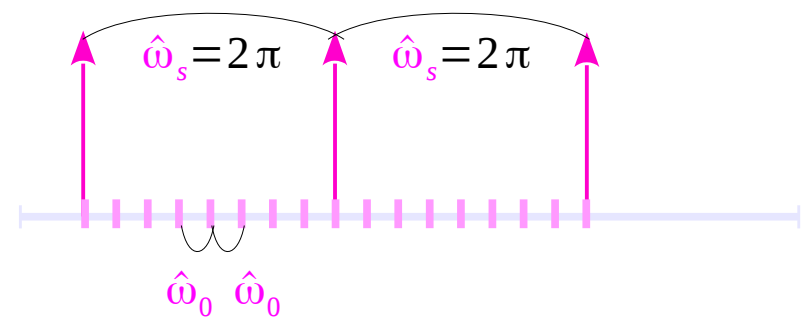


$$\omega_0 = \frac{2\pi}{T_0}$$

$$\omega_s = \frac{2\pi}{T_s}$$

$$\hat{\omega}_0 = \frac{2\pi}{N_0}$$

$$\hat{\omega}_s = \frac{2\pi}{1}$$



Fourier Transform Types

Continuous Time Fourier Series

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{k=-\infty}^{+\infty} C_k e^{+jk\omega_0 t}$$

Discrete Time Fourier Series

$$y[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\hat{\omega}_0 n} \quad \longleftrightarrow \quad x[n] = \sum_{k=0}^{N-1} y[k] e^{+jk\hat{\omega}_0 n}$$

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

Discrete Time Fourier Transform

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega} n} \quad \longleftrightarrow \quad x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(j\hat{\omega}) e^{+j\hat{\omega} n} d\hat{\omega}$$

Computation at $k\omega_0$

CTFS

Periodic $x(t)$

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \quad k\omega_0$$

$$@ \quad k\omega_0 = k \left(\frac{2\pi}{T} \right) \text{ rad/sec}$$

CTFT

Aperiodic $x(t)$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \omega \leftarrow k\omega_0$$

$$@ \quad k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{NT_s} \right) \text{ rad/sec}$$

DTFS

Periodic $x[n]$

$$y[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\hat{\omega}_0 n} \quad k\hat{\omega}_0$$

$$@ \quad k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{NT_s} \right) \text{ rad/sec}$$

DTFT

Aperiodic $x[n]$

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega} n} \quad \hat{\omega} \leftarrow k\hat{\omega}_0$$

$$@ \quad k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{NT_s} \right) \text{ rad/sec}$$

Computations using $X[k]$

CTFS

Periodic $x(t)$

$$C_k \approx \frac{1}{N} X[k]$$

$$k \omega_0$$

$$@ k \omega_0 = k \left(\frac{2\pi}{T} \right) \text{ rad/sec}$$

CTFT

Aperiodic $x(t)$

$$X(jk\omega_0) \approx T_s X[k]$$

$$\omega \leftarrow k \omega_0$$

$$@ k \omega_0 = k \hat{\omega}_0 f_s = k \left(\frac{2\pi}{N T_s} \right) \text{ rad/sec}$$

DTFS

Periodic $x[n]$

$$y[k] = \frac{1}{N} X[k]$$

$$k \hat{\omega}_0$$

$$@ k \omega_0 = k \left(\frac{2\pi}{T} \right) \text{ rad/sec}$$

DTFT

Aperiodic $x[n]$

$$X(jk\hat{\omega}_0) \approx X[k]$$

$$\hat{\omega} \leftarrow k \hat{\omega}_0$$

$$@ k \omega_0 = k \hat{\omega}_0 f_s = k \left(\frac{2\pi}{N T_s} \right) \text{ rad/sec}$$

Computations using DFT

CTFS

Periodic $x(t)$

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt$$

$$C_k \approx \frac{1}{N} \text{DFT}(x[n]) \quad k\omega_0$$

$$@ \quad k\omega_0 = k \left(\frac{2\pi}{T} \right) \text{ rad/sec}$$

CTFT

Aperiodic $x(t)$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$X(jk\omega_0) \approx T_s \text{DFT}(x[n]) \quad \omega \leftarrow k\omega_0$$

$$@ \quad k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{N T_s} \right) \text{ rad/sec}$$

DTFS

Periodic $x[n]$

$$y[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\hat{\omega}_0 n}$$

$$y[k] = \frac{1}{N} \text{DFT}(x[n]) \quad k\hat{\omega}_0$$

$$@ \quad k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{N T_s} \right) \text{ rad/sec}$$

DTFT

Aperiodic $x[n]$

$$X(j\hat{\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\hat{\omega} n}$$

$$X(jk\hat{\omega}_0) \approx \text{DFT}(x[n]) \quad \hat{\omega} \leftarrow k\hat{\omega}_0$$

$$@ \quad k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{N T_s} \right) \text{ rad/sec}$$

FFT Amplitude and Power Spectrum

Two-Sided Amplitude Spectrum

$$A_k = \frac{1}{N} |X[k]| \quad (V/Hz^{-1/2})$$
$$= \frac{1}{N} \sqrt{\Re^2\{X[k]\} + \Im^2\{X[k]\}}$$

$$k = 0, 1, 2, \dots, N/2, N/2+1, \dots, N-1$$

Two-Sided Power Spectrum

$$P_k = \frac{1}{N^2} |X[k]|^2 \quad (V^2/Hz^{-1})$$
$$= \frac{1}{N^2} (\Re^2\{X[k]\} + \Im^2\{X[k]\})$$

$$k = 0, 1, 2, \dots, N/2, N/2+1, \dots, N-1$$

One-Sided Amplitude Spectrum

$$\bar{A}_0 = \frac{1}{N} |X[0]| \quad k=0$$

$$\bar{A}_k = \frac{2}{N} |X[k]| \quad k=1, 2, \dots, N/2$$

One-Sided Power Spectrum

$$\bar{P}_0 = \frac{1}{N^2} |X[0]|^2 \quad k=0$$

$$\bar{P}_k = \frac{2}{N^2} |X[k]|^2 \quad k=1, 2, \dots, N/2$$

Frequency Bin

$$f = k \frac{f_s}{N}$$

Frequency Bin

$$f = k \frac{f_s}{N}$$

FFT Amplitude and Phase Spectrum

Two-Sided Amplitude Spectrum

$$\begin{aligned} A_k &= \frac{1}{N} |X[k]| \\ &= \frac{1}{N} \sqrt{\Re^2\{X[k]\} + \Im^2\{X[k]\}} \end{aligned}$$

$$k = 0, 1, 2, \dots, N/2, N/2+1, \dots, N-1$$

Two-Sided Phase Spectrum

$$\phi_k = \tan^{-1} \left(\frac{\Im\{X[k]\}}{\Re\{X[k]\}} \right)$$

$$k = 0, 1, 2, \dots, N/2, N/2+1, \dots, N-1$$

Frequency Bin

$$f = k \frac{f_s}{N}$$

CTFS and Power Spectrum

Two-Sided Power Spectrum

$$\frac{1}{N^2} |X[k]|^2 = |C_k|^2 = \frac{1}{4} (a_k^2 + b_k^2) = \frac{1}{4} |g_k|^2$$

Single-Sided Power Spectrum

$$\frac{2}{N^2} |X[k]|^2 = 2 |C_k|^2 = \frac{1}{2} |g_k|^2 = |g_{k,rms}|^2$$

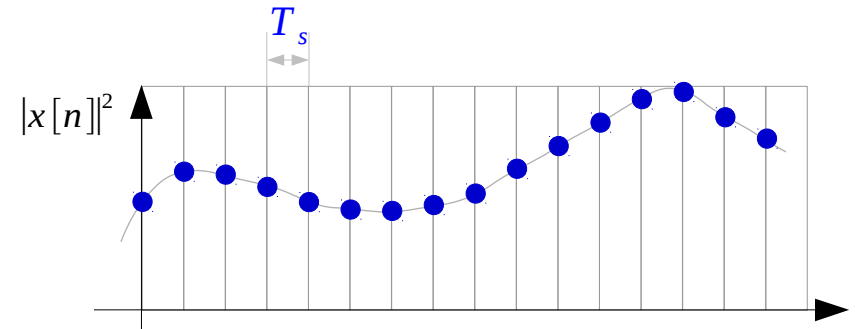
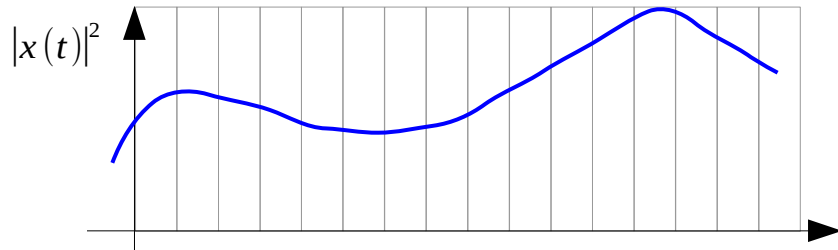
$$C_k = \frac{1}{2} g_{+k} e^{+j\phi_k} \quad (k > 0)$$
$$C_k = \frac{1}{2} g_{-k} e^{-j\phi_k} \quad (k < 0)$$

$$x(t) = g_0 + \sum_{k=1}^{\infty} g_k \cos(k\omega_0 t + \phi_k)$$

g_k each sinusoid's amplitude

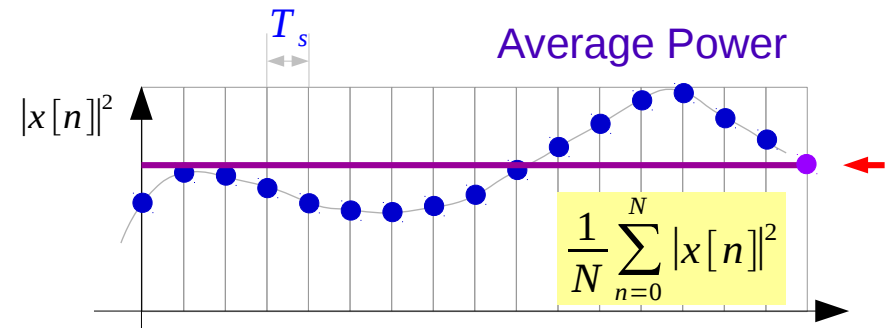
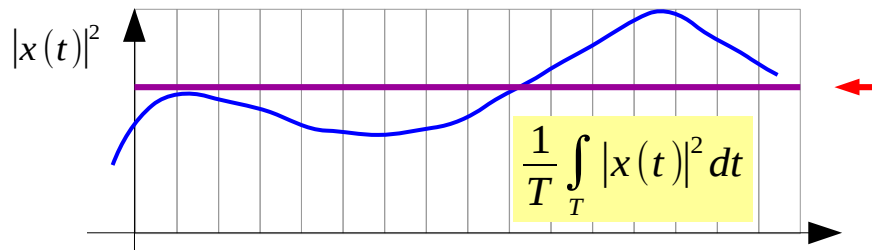
$g_{k,rms}$ each sinusoid's amplitude rms value

Average Power and Total Energy



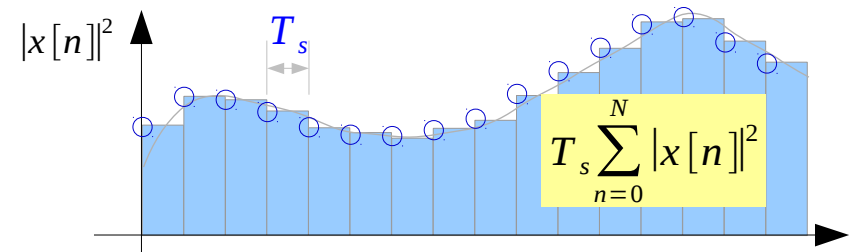
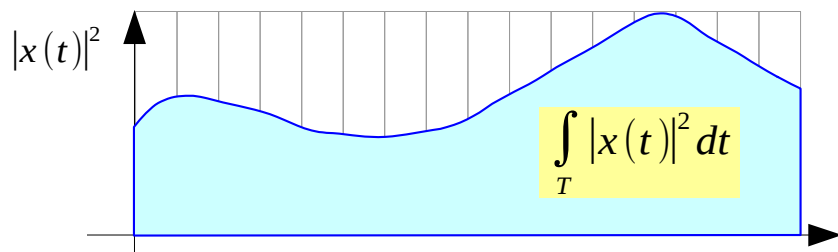
Average Power

Average Power

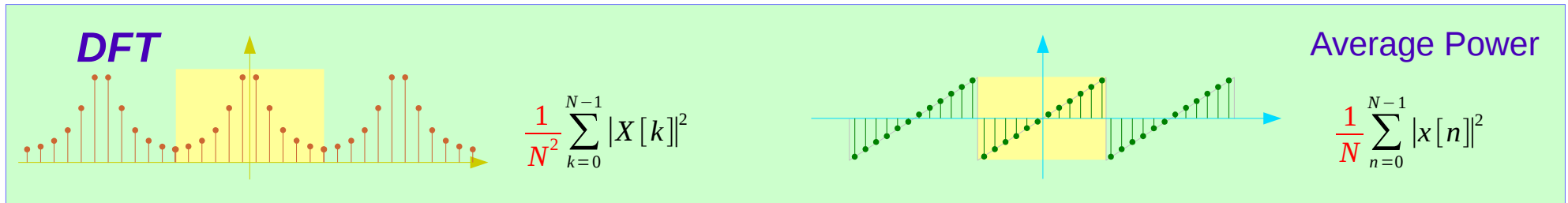


Total Energy

Total Energy



Periodogram



Average Power

$$\frac{1}{N} \sum_{k=0}^{N-1} P_{xx}[k] = \frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2$$



$$\frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

Average Power



Periodogram

$$P_{xx}[k] = \frac{1}{N} |X[k]|^2 = \left| \frac{X[k]}{\sqrt{N}} \right|^2$$



$$\frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 = \sum_{n=0}^{N-1} |x[n]|^2$$



Total Energy

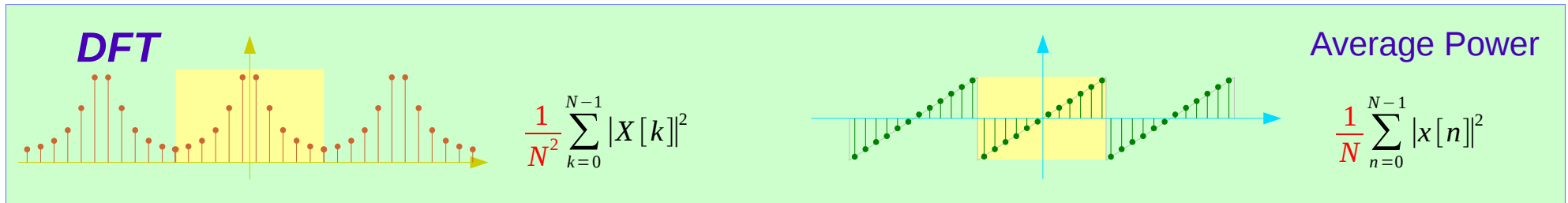
$$T_s \sum_{k=0}^{N-1} P_{xx}[k] = \frac{T_s}{N} \sum_{k=0}^{N-1} |X[k]|^2$$



$$\frac{T_s}{N} \sum_{k=0}^{N-1} |X[k]|^2 = T_s \sum_{n=0}^{N-1} |x[n]|^2$$

Total Energy

Periodogram



Average Power

$$\frac{1}{N} \sum_{k=0}^{N-1} P_{xx}[k]$$

=

$$\frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

Average Power

freq-domain samples

time-domain samples

Total Energy

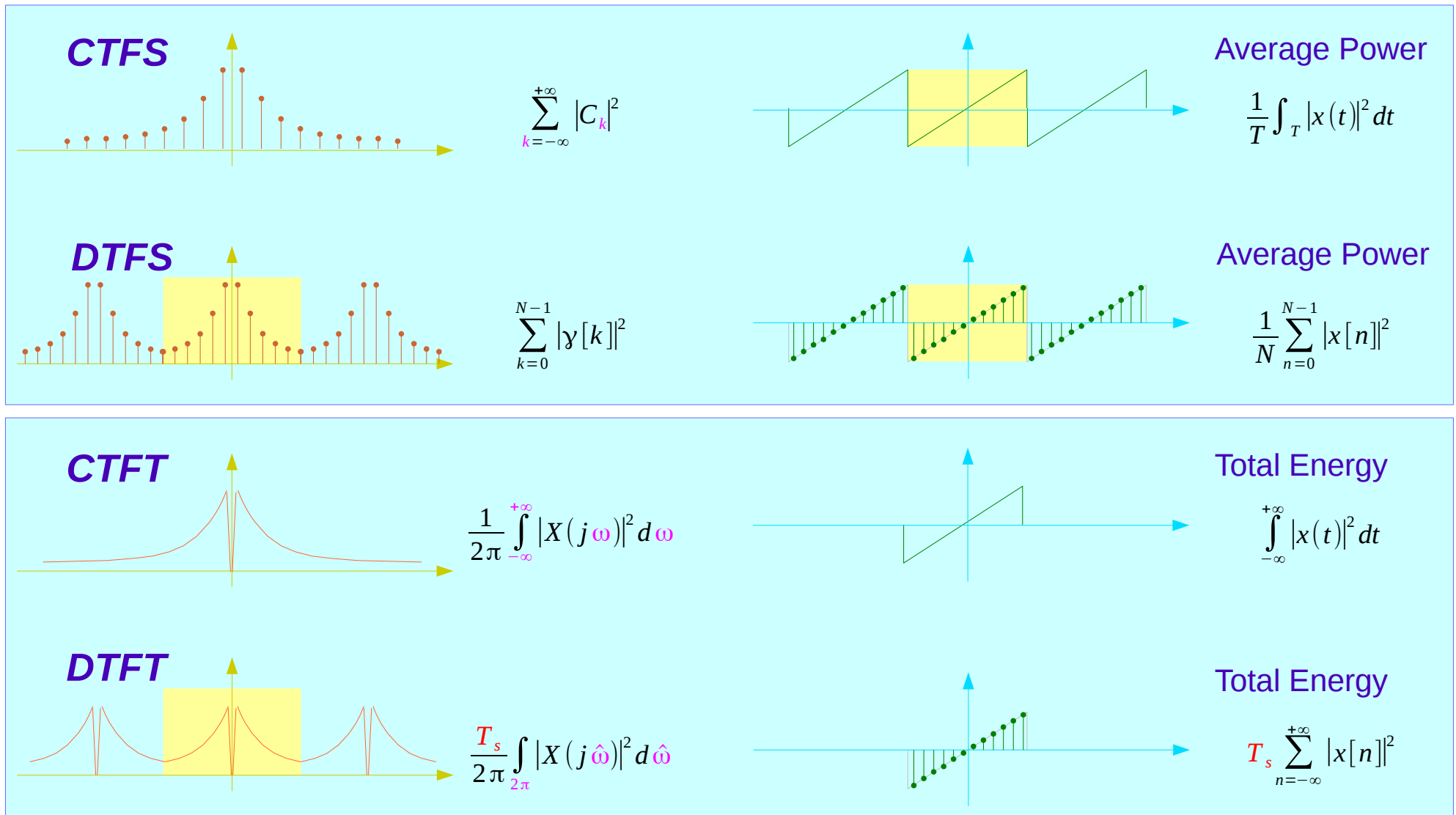
$$T_s \sum_{k=0}^{N-1} P_{xx}[k]$$

=

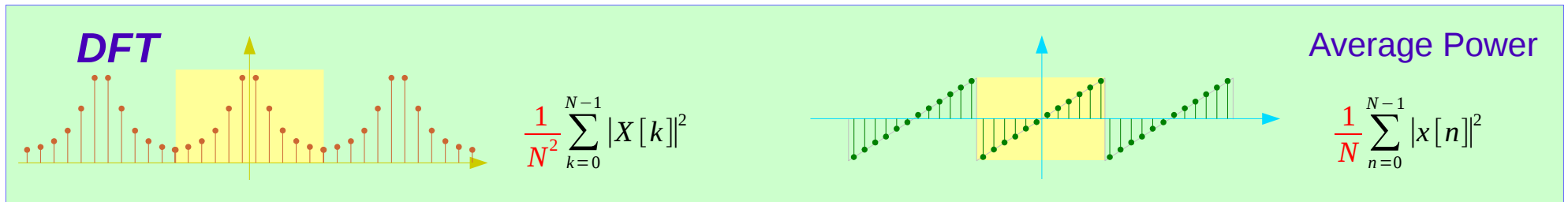
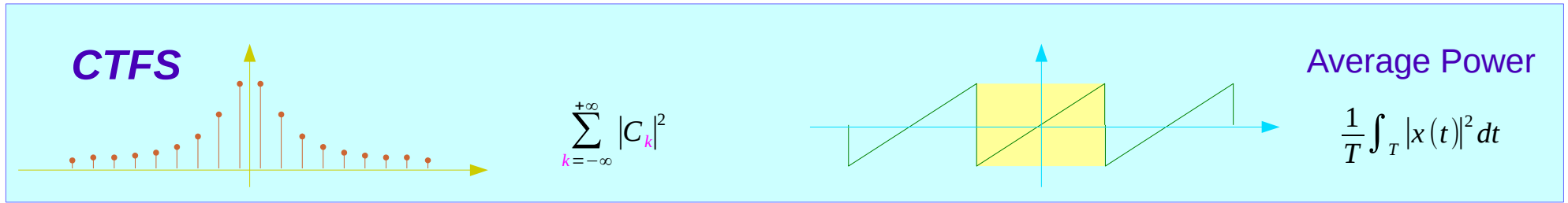
$$T_s \sum_{n=0}^{N-1} |x[n]|^2$$

Total Energy

Parseval's Theorem



Approximate CTFS Parseval's Theorem



$$c_k \approx \frac{X[k]}{N}$$

$$\sum_{k=0}^{N-1} |c_k|^2$$



$$\sum_{k=0}^{N-1} \left| \frac{X[k]}{N} \right|^2 = \frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2$$

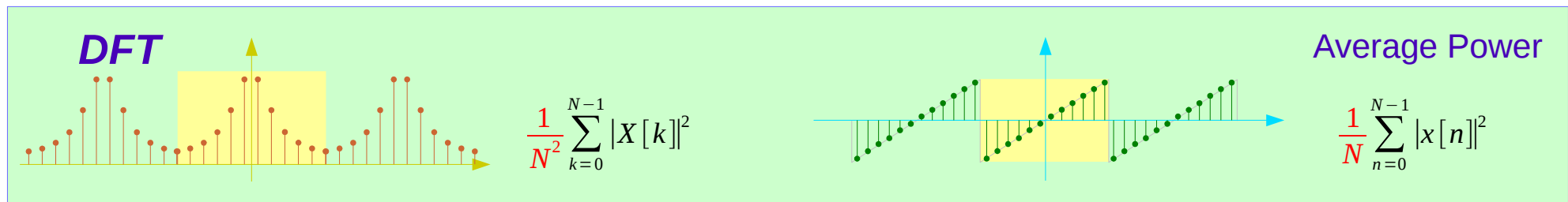
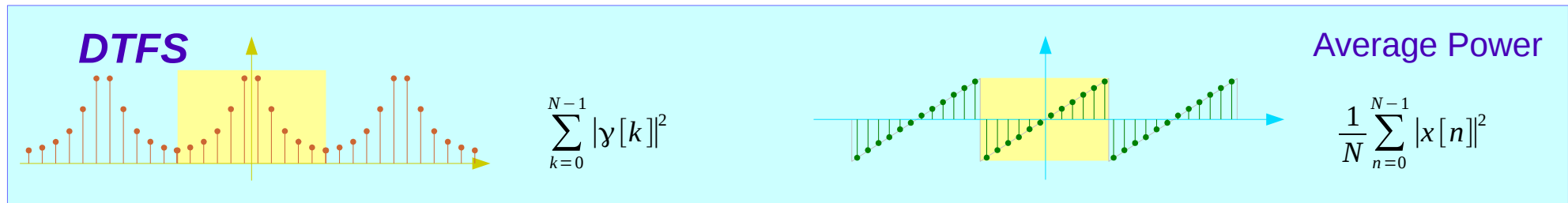


$$\frac{1}{T} \int_T |x(t)|^2 dt$$

$$\frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \quad \text{Average Power}$$

$$\frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 = \sum_{n=0}^{N-1} |x[n]|^2$$

DTFS Parseval's Theorem



$$y[k] = \frac{X[k]}{N}$$

$$\sum_{k=0}^{N-1} |y[k]|^2$$



$$\sum_{k=0}^{N-1} \left| \frac{X[k]}{N} \right|^2 = \frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2$$

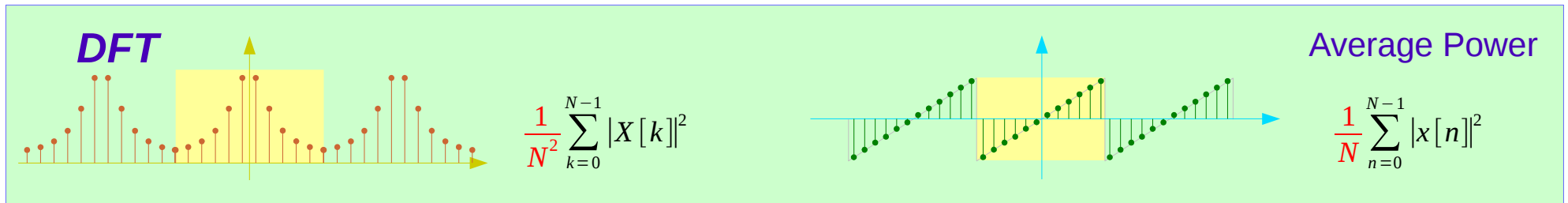
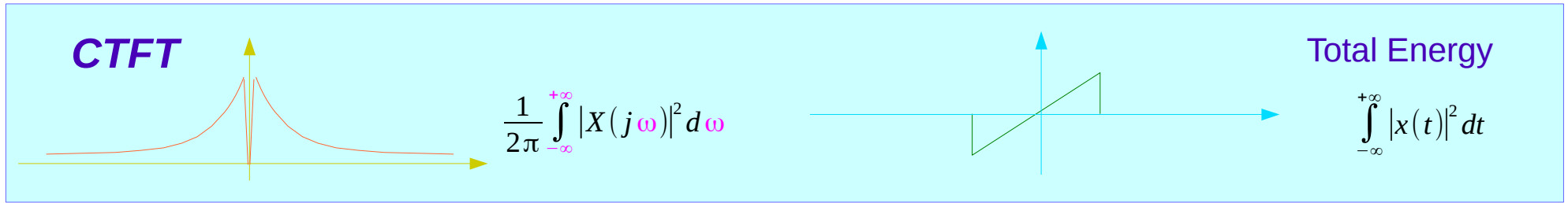


$$\frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

$$\frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2 = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \quad \text{Average Power}$$

$$\frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 = \sum_{n=0}^{N-1} |x[n]|^2$$

Approximate CTFT Parseval's Theorem



$$X(jk\omega_0) \approx T_s X[k]$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$



$$\sum_{k=0}^{N-1} \left| \frac{X[k]}{N} \right|^2 = \frac{T_s}{N^2} \sum_{k=0}^{N-1} |X[k]|^2$$



$$\int_{-\infty}^{+\infty} |x(t)|^2 dt$$

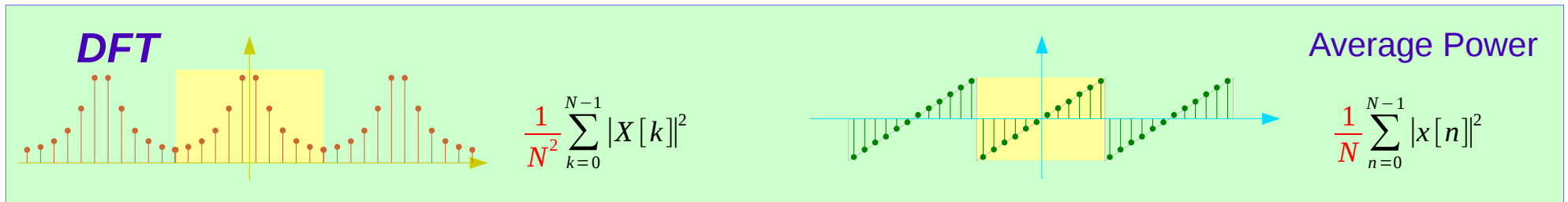
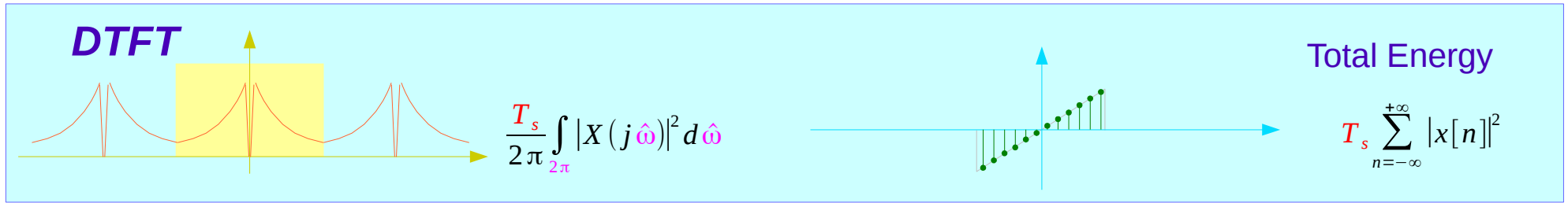
$$X(jk\omega_0) \approx T_s X[k]$$

$$\frac{T_s}{N} \sum_{k=0}^{N-1} |X[k]|^2 = T_s \sum_{n=0}^{N-1} |x[n]|^2$$

Average Power

$$\frac{1}{N} \sum_{k=0}^{N-1} |X[k]|^2 = \sum_{n=0}^{N-1} |x[n]|^2$$

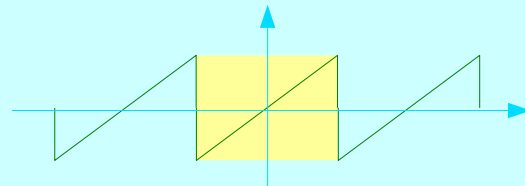
Parseval's Theorem



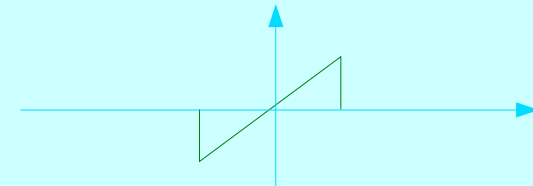
Periodic and Aperiodic Signals

Periodic Signals	Aperiodic Signals
Average Power	Total Energy

Continuous Time

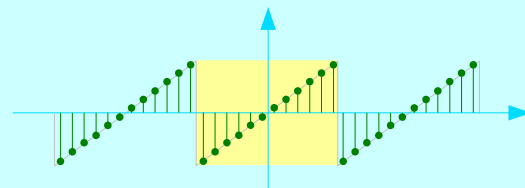


Average Power $\frac{1}{T} \int_T |x(t)|^2 dt$

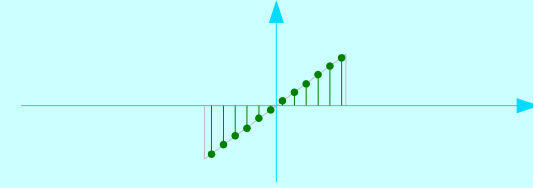


Total Energy $\int_{-\infty}^{+\infty} |x(t)|^2 dt$

Discrete Time



Average Power $\frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$



Total Energy $T_s \sum_{n=-\infty}^{+\infty} |x[n]|^2$

Periodic and Aperiodic Signals

Periodic Signals	Aperiodic Signals
Average Power	Total Energy

Continuous
Time

CTFS

$$C_k \approx \frac{1}{N} X[k]$$

$$\frac{1}{T} \int_T |x(t)|^2 dt$$

$$\sum_{k=-\infty}^{+\infty} |C_k|^2$$

CTFT

$$X(jk\hat{\omega}_0) \approx T_s X[k]$$

$$\int_T |x(t)|^2 dt$$

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

Discrete
Time

DTFS

$$y[k] = \frac{1}{N} X[k]$$

$$\frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2$$

$$\sum_{k=0}^{N-1} |y[k]|^2$$

DTFT

$$X(jk\hat{\omega}_0) \approx X[k]$$

$$T_s \sum_{n=0}^{N-1} |x[n]|^2$$

$$\frac{T_s}{2\pi} \int_{2\pi} |X(j\hat{\omega})|^2 d\hat{\omega}$$

Periodic and Aperiodic Signals

Periodic Signals	Aperiodic Signals
Average Power	Total Energy

Continuous
Time

CTFS

$$\frac{1}{T} \int_T |x(t)|^2 dt \cdot T$$



CTFT

$$\int_T |x(t)|^2 dt$$

Discrete
Time

DTFS

$$\frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 \cdot T$$

$$T = N \cdot T_s$$



DTFT

$$T_s \sum_{n=0}^{N-1} |x[n]|^2$$

FS Coefficients of Periodic and Aperiodic Signals

Periodic Signals

Aperiodic Signals

Frequency Spacing

$$\omega_0 = \frac{2\pi}{NT_s}$$

$$\omega_0 = \frac{2\pi}{NT_s} \quad \left(\hat{\omega}_0 = \frac{2\pi}{N} \right)$$

Two Sided F.S. Coefficient

$$\frac{1}{N} X(k) = y_k$$

$$\frac{T_s}{N} X(k) = T_s y_k$$

One Sided F.S. Coefficient

$$\frac{1}{N} X(k) \quad k=0, \frac{N}{2}$$

$$\frac{T_s}{N} X(k) \quad k=0, \frac{N}{2}$$

$$\frac{2}{N} X(k) \quad k=1, \dots, \frac{N}{2}-1$$

$$\frac{2T_s}{N} X(k) \quad k=1, \dots, \frac{N}{2}-1$$

Frequency Bin

$$k\omega_0 = k \left(\frac{2\pi}{NT_s} \right)$$

$$k\omega_0 = k\hat{\omega}_0 f_s = k \left(\frac{2\pi}{NT_s} \right)$$

Average Power – CTFS

Total Energy – CTFT

FS Coefficients of Periodic and Aperiodic Signals

Periodic Signals

Aperiodic Signals

Frequency Spacing

$$\Delta f = \frac{1}{N \Delta t}$$

$$\Delta f = \frac{1}{N \Delta t}$$

Two Sided F.S. Coefficient

$$\frac{1}{N} X(k)$$

$$\frac{\Delta t}{N} X(k)$$

One Sided F.S. Coefficient

$$\frac{1}{N} X(k) \quad k=0, \frac{N}{2}$$

$$\frac{\Delta t}{N} X(k) \quad k=0, \frac{N}{2}$$

$$\frac{2}{N} X(k) \quad k=1, \dots, \frac{N}{2}-1$$

$$\frac{2\Delta t}{N} X(k) \quad k=1, \dots, \frac{N}{2}-1$$

Frequency Bin

$$k \Delta f$$

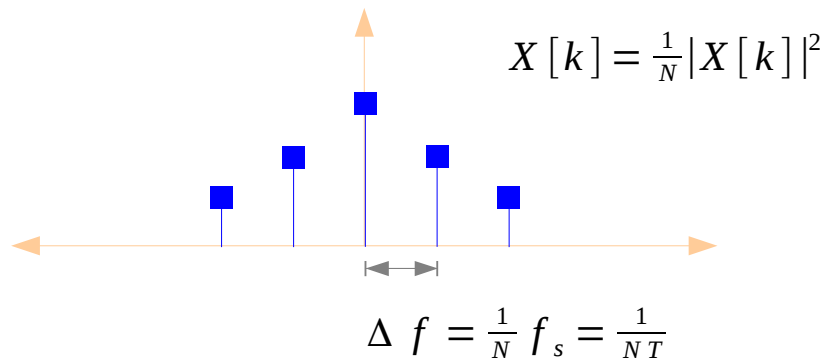
$$k \Delta f$$

Average Power

Total Energy

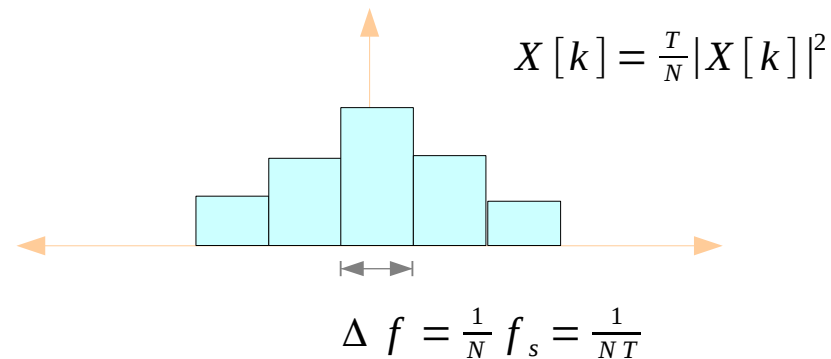
Power Spectrum and Power Spectral Density

Power Spectrum



Power Spectral Density

Hz



$$\frac{1}{N} \sum_{n=0}^{N-1} x^2[n] = \frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2$$

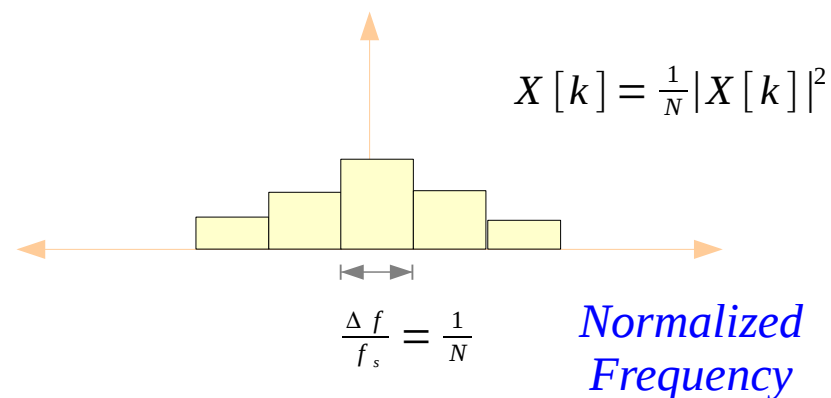
$$\Rightarrow \sum_{k=0}^{N-1} S[k] \Delta f$$

$$= \frac{1}{NT} \sum_{k=0}^{N-1} S[k] = \frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2$$

$$S[k] = \frac{T}{N} |X[k]|^2$$

Power Spectral Density

Hz · sec



Periodogram → Power Spectral Density

FS Coefficients of Random Signals

Random Signals

Frequency Spacing

$$\Delta f = \frac{1}{N \Delta t}$$

Two Sided
Power Spectral Density

$$P = \sum_{k=0}^{N-1} S(k) \Delta f$$

One Sided
Power Spectral Density

$$P = \sum_{k=0}^{N/2} S_1(k) \Delta f$$

$$S_1(k) = 2S(k) \quad k=0, \frac{N}{2}$$

$$S_1(k) = S(k) \quad k=1, \dots, \frac{N}{2}-1$$

$$\frac{1}{N \Delta t} \sum x^2 \Delta t$$

$$\sum S \Delta f = \frac{1}{N \Delta t} \sum S$$

$$S(k) = \frac{\Delta t}{N} |X(k)|^2$$

Frequency Bin

$$k \Delta f$$

Power Spectrum using FFT

Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j(2\pi/N)kn}$$

$$X = \text{fft}(x)$$

$$x = \text{ifft}(X)$$

$$x(t) = \sum_{k=-\infty}^{+\infty} C_k e^{+jk\omega_0 t} \quad \text{CTFS}$$

$$x_{FS}(t) = \sum_{k=-M}^{+M} y_k e^{+jk\omega_0 t} \quad \text{DTFS}$$

$$C_k \approx y_k = \frac{X[k]}{N} \quad \text{Approximated Fourier Coefficients}$$

$$|C_k|^2 \approx \frac{|X[k]|^2}{N^2} \quad \text{Approximated Power Spectrum}$$

Approximated
Fourier Series Coefficients

$$fc = \text{fft}(x)/N = X/N$$

$$x = \text{ifft}(fc)*N$$

Periodogram using FFT

$$C_k \approx y_k = \frac{X[k]}{N} \quad \text{Approximated Fourier Coefficients}$$

$$|C_k|^2 \approx \frac{|X[k]|^2}{N^2} \quad \text{Approximated Power Spectrum}$$

$$\frac{1}{N} \sum_{n=0}^{N-1} x^2[n] = \frac{1}{N^2} \sum_{k=0}^{N-1} |X[k]|^2 \quad \text{Average Power}$$

$$\rightarrow \left(\frac{\sum_{k=0}^{N-1} \frac{|X[k]|^2}{N}}{N} \right)^2 \quad \text{RMS of sq root Periodogram}$$

$$\frac{|X[k]|^2}{N} \quad k=0,1,\dots,N-1 \quad \text{Approximated Periodogram}$$

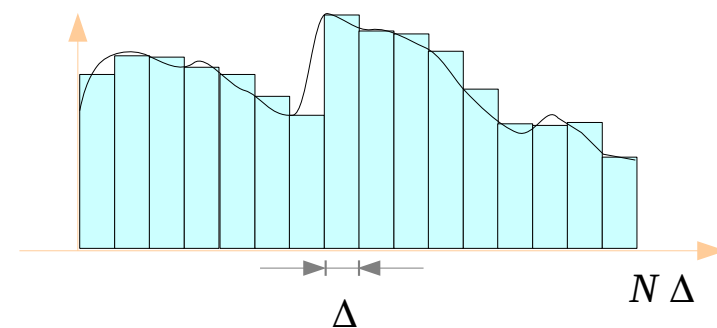
$$\frac{|X[k]|}{\sqrt{N}} \quad k=0,1,\dots,N-1 \quad \text{Square root Periodogram}$$

RMS in continuous time



$$\frac{1}{T} \int_0^T g^2(t) dt$$

RMS in discrete time



$$\frac{1}{N \Delta} \sum_{k=0}^{N-1} |g[k]|^2 \Delta = \frac{1}{N} \sum_{k=0}^{N-1} |g[k]|^2$$

From CTFS to CTFT

Continuous Time Fourier Series

$$C_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt \quad \longleftrightarrow \quad x(t) = \sum_{n=0}^{\infty} C_k e^{+jk\omega_0 t}$$

$$C_k = \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jk\omega_0 t} dt$$

$$x_{T_0}(t) = \sum_{n=0}^{\infty} C_k e^{+jk\omega_0 t} \cdot \frac{2\pi}{2\pi} \cdot \frac{T_0}{T_0}$$

$$C_k T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jk\omega_0 t} dt$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{k=0}^{\infty} C_k T_0 e^{+jk\omega_0 t} \cdot \frac{2\pi}{T_0}$$

$$T_0 \rightarrow \infty \quad \omega_0 = \frac{2\pi}{T_0} \rightarrow d\omega \quad C_k T_0 \rightarrow X(j\omega) \quad x_{T_0} \rightarrow x(t)$$

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \quad \longleftrightarrow \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

CTFS and CTFT

Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j(2\pi/N)kn}$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{+j(2\pi/N)kn}$$

$$C_k \approx \gamma_k = \frac{X[k]}{N} \quad \text{Approximated Fourier Coefficients}$$

$$x[n] = \sum_{k=-M}^{+M} \gamma_k e^{+j\left(\frac{2\pi}{N}\right)nk}$$

$$n = 0, 1, 2, \dots, N-1,$$

$$\gamma_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi}{N}\right)nk}$$

$$k = -M, \dots, 0, \dots, +M$$

Continuous Time Fourier Series

$$C_k T_0 = \int_{-T_0/2}^{+T_0/2} x_{T_0}(t) e^{-jk\omega_0 t} dt$$

$$x_{T_0}(t) = \frac{1}{2\pi} \sum_{k=0}^{\infty} C_k T_0 e^{+jk\omega_0 t} \cdot \frac{2\pi}{T_0}$$

Continuous Time Fourier Transform

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{+j\omega t} d\omega$$

$$T_0 \rightarrow \infty, \omega_0 \rightarrow 0 \quad (\omega_0 \rightarrow d\omega)$$

Signals without discontinuity
Signals with discontinuity

Sampling frequency is not an integer
multiple of the FFT length

Leakage

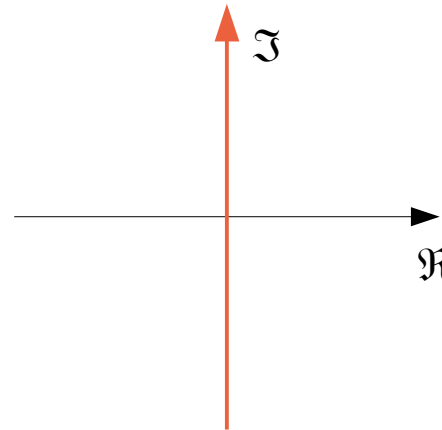
$$\left[0, \frac{f_s}{2}\right]$$

Fourier Transform

$f(t)$ A continuous sum of weighted exponential functions :

$$-\infty < \omega < +\infty$$

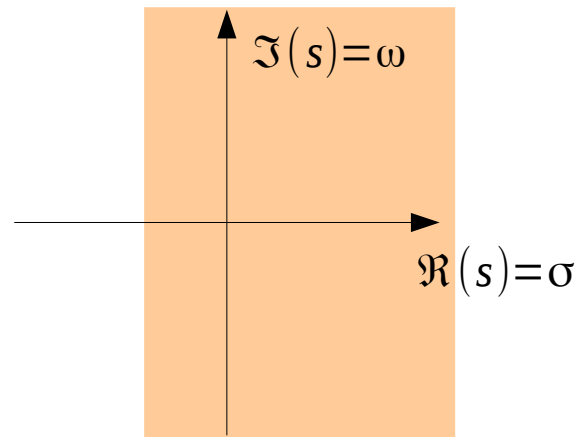
Not so useful in transient analysis



Laplace Transform

$f(t) e^{-st} = f(t) e^{-(\sigma + j\omega)t}$

Linear Time Domain
Analysis
Initial Condition



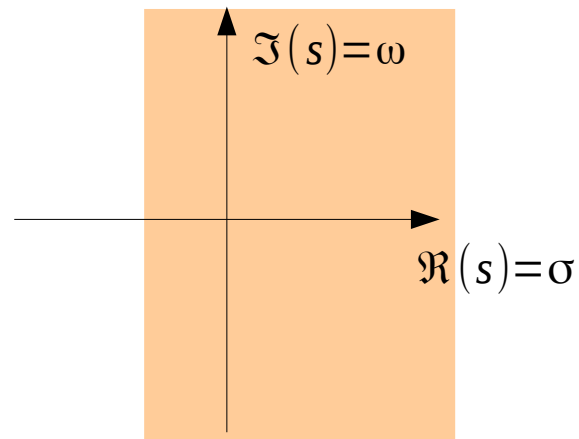
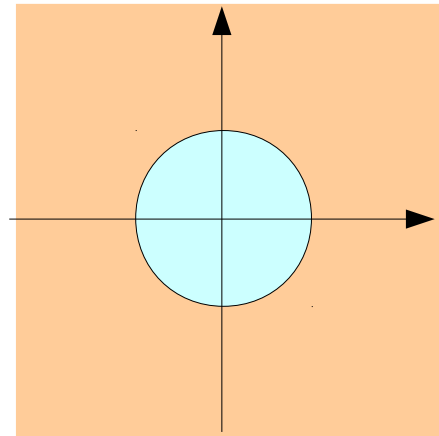
z Transform

$$f[n] z^{-n}$$

Discrete Time System

Difference Equation

$$z = e^{sT} = e^{\sigma T} e^{j\omega T}$$



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann