A Sudoku Solver – Pruning (3A)

- Richard Bird Implementation

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice.

Based on

Thinking Functionally with Haskell, R. Bird

https://wiki.haskell.org/Sudoku

http://cdsoft.fr/haskell/sudoku.html

https://gist.github.com/wvandyk/3638996

http://www.cse.chalmers.se/edu/year/2015/course/TDA555/lab3.html

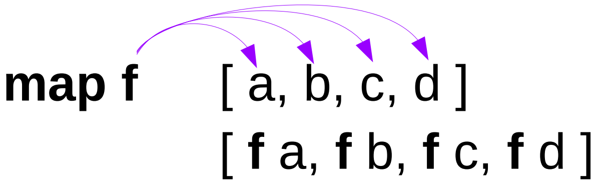
concat, map, filter

```
concat :: [[a]] -> [a]
concat []           = []
concat (xs:xss)  = xs ++ concat xss
```

concat [[a, b], [c], [d, e, f]]

[a, b, c, d, e, f]

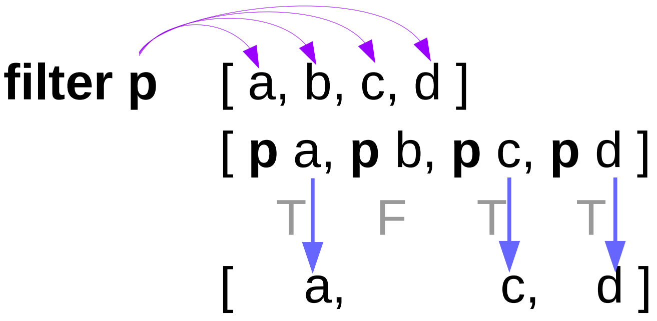
```
map :: (a -> b) -> [a] -> [b]
map f []           = []
map f (xs:xss)    = f xs : map f xss
```



map f [a, b, c, d]

[f a, f b, f c, f d]

```
filter :: (a ->bool) -> [a] -> [a]
filter p []           = []
filter p (xs:xss)  = if p xs then xs : filter p xss
                            else filter p xss
```



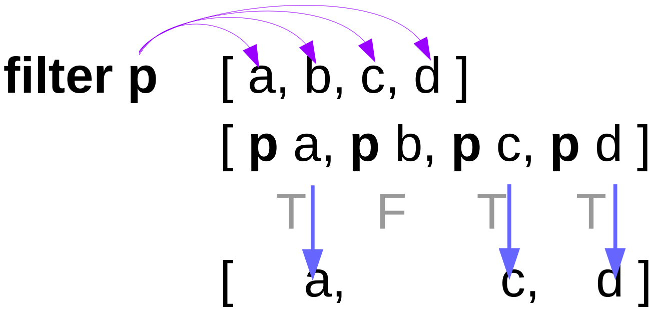
filter p [a, b, c, d]

[p a, p b, p c, p d]

[a, c, d]

Definitions of filter

```
filter :: (a ->bool) -> [a] -> [a]
filter p []            = []
filter p (xs:xss)      = if p xs then xs : filter p xss
                          else filter p xss
```



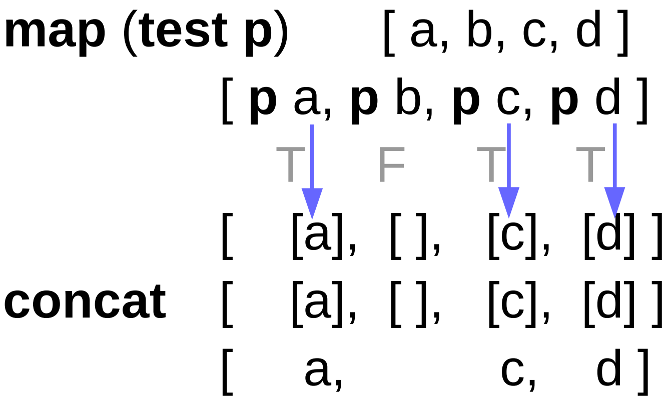
filter p [a, b, c, d]

[p a, p b, p c, p d]

T F T T

[a, c, d]

```
filter p = concat . map (test p)
test p x = if p x then [x] else [ ]
```



map (test p) [a, b, c, d]

[p a, p b, p c, p d]

T F T T

[[a], [ ], [c], [d]]

concat [[a], [ ], [c], [d]]

[a, c, d]

Definitions of filter

```
test p . f = map f . test (p . f)

test p x = if p x then [x] else [ ]
```

```
test p . f x
= test p (f x)
= if p (f x) then [f x] else [ ]
```

```
test (p . f) x = if (p . f) x then [x] else [ ]
= if p (f x) then [x] else [ ]
```

```
map f . test (p . f) x =
= if p (f x) then map f [x] else map f [ ]
= if p (f x) then [f x] else [ ]
```

concat, map, filter

map id = id

map (f . g) = **map** f . **map** g

map id [a, b, c, d]

[**id** a, **id** b, **id** c, **id** d]

[a, b, c, d]

map(**f**.**g**) [a, b, c, d]

[**f**.**g** a, **f**.**g** b, **f**.**g** c, **f**.**g** d]

[**f** (**g** a), **f** (**g** b), **f** (**g** c), **f** (**g** d)]

map g [a, b, c, d]

[**g** a, **g** b, **g** c, **g** d]

map f . map g [a, b, c, d]

map f [**g** a, **g** b, **g** c, **g** d]

[**f** (**g** a), **f** (**g** b), **f** (**g** c), **f** (**g** d)]

concat, map, filter

f . head = head . map f

f . head [a, b, c, d] = **f** a
head . map f [a, b, c, d] =
head [**f** a, **f** b, **f** c, **f** d] = **f** a

map f . tail = tail . map f

map f . tail [a , b, c, d] =
map f [b, c, d] = [**f** b, **f** c, **f** d]
tail . **map f** [a, b, c, d] =
tail [**f** a, **f** b, **f** c, **f** d] = [**f** b, **f** c, **f** d]

map f . concat =
concat . map (map f)

map f . **concat** [[a], [b], [c], [d]] =
map f . [a , b, c, d] = [**f** a, **f** b, **f** c, **f** d]
concat . **map (map f)** [[a], [b], [c], [d]] =
concat . [**map f** [a], **map f** [b], **map f**[c], **map f**[d]] =
concat [[**f** a], [**f** b], [**f** c], [**f** d]] =[**f** a, **f** b, **f** c, **f** d]

concat . concat

concat . map concat = concat . concat

concat . map concat [[[a]], [[b]], [[c]], [[d]]]

remove inside [ ] first

concat . [concat [[a]], concat [[b]], concat [[c]], concat [[d]]]

concat [[a], [b], [c], [d]]

[a, b, c, d]

concat . concat [[[a]], [[b]], [[c]], [[d]]]

remove outside [ ] first

concat [[a], [b], [c], [d]]

[a, b, c, d]

map f . concat

concat . map (map f) = map f . concat

concat . map (map f)   [[a], [b], [c], [d]]

concat   [map f [a], map f [b], map f [c], map f [d]]

concat   [[f a], [f b], [f c], [f d]]

[f a, f b, f c, f d]

map f . concat [[a], [b], [c], [d]]

map f   [a, b, c, d]

[f a, f b, f c, f d]

Strict Function

f . head = head . map f

f (head [ ]) = head (map f [ ]) = head [ ] (undefined)

concat, map, filter

```
tail      :: [a] -> [a]
reverse   :: [a] -> [a]
```

```
map f . tail      =       tail  . map f
map f . reverse   =    reverse . map f
```

```
head      ::  [a]  ->  a
concat    :: [[a]] -> [a]
```

```
      f . head    =      head . map f
  map f . concat  =   concat  . map (map f)
concat . concat   =   concat  . map concat
```

concat, map, filter

```
filter p . map f = map f . filter (p . f)

filter p . map f
= concat . map (test p) . map f

= concat . map (test p . f)
= concat . map (map f . test (p . f))

= concat . map (map f) . map (test (p . f))

= map f . concat .  map (test (p . f))

= map f . filter (p . f)
```

```
filter p = concat . map (test p)
test p x = if p x then [x] else [ ]

map m . map n = map m . n

test p . f = map f . test (p . f)
map m . map n = map m . n
concat . map (map f) = map f . concat

filter p = concat . map (test p)
```

Single-Cell Expansion

```
(f . g) xs = f (g xs)

map (f . g) xs = map f (map g xs)

filter p . map f = map f . filter (p . f)

filter p . map f
= concat . map (test p) . map f
= concat . map (test p . f)
= concat . map (map f . test (p . f))
= concat . map (map f) . map (test (p . f))
= map f. concat . map (test (p . f))
= map f. filter (p . f)
```

Single-Cell Expansion

```
prune :: Matrix Choices -> Matrix Choices
prune = pruneBy boxs . pruneBy cols . pruneBy rows
    where pruneBy f = f . map pruneRow . f

pruneRow :: Row Choices -> Row Choices
pruneRow row = map (remove ones) row
   where ones = [d | [d] <- row]
```

Single-Cell Expansion

```
solve :: Grid -> [Grid]
solve = filter valid . expand. Choices

prune :: Matrix [Digit] -> Matrix [Digit]
filter valid . expand = filter valid . expand . prune

pruneRow :: Row [Digit] -> Row [Digit]
pruneRow row = map (remove fixed) row
      where fixed = [d | [d] <- row]

remove :: [Digit] -> [Digit] -> [Digit]
remove ds [x] = [x]
remove ds xs = filter (`notElem` ds) xs

notElem :: (Eq a) => a -> [a] -> Bool
notElem x xs = all (/= x) xs
```

Single-Cell Expansion

pruneRow [[6], [1,2], [3], [1,3,4], [5,6]]
[[6], [1,2], [3], [1,4], [5]]

PruneRow [[6], [3,6], [3], [1,3,4], [4]]
[[6], [], [3], [1], [4]]

filter nodups . cp = filter nodups . cp . PruneRow

Single-Cell Expansion

```
f . f = id  assumed

filter (p . f) = map f . filter p . map f
filter (p . f) . map f = map f . filter p

filter p . map f = map f . filter (p . f)

map f. filter p . map f
= map f . map f . filter (p . f)
= filter (p . f)

filter valid . expand
= filter (all nodups . boxs) .
   filter (all nodups . cols) .
   filter (all nodups . rows) . expand
```

Single-Cell Expansion

```
filter (all nodups  . boxs) . expand
= map boxs . filter (all nodups) . map boxs . expand
= map boxs . filter (all nodups) . cp . map cp . boxs
= map boxs . cp . map (filter nodups) .map cp . boxs
= map boxs .cp . map (filter nodups . cp) . boxs

boxs . boxs = id
map boxs . expand = expand . boxs
filter (all p) . cp = cp . map . (filter p)

filter nodups . cp = filter nodups . cp . prunerow

map boxs . cp . map (filter nodups . cp . prunerow) . boxs
```

Single-Cell Expansion

```
map boxs . cp . map (filter nodups . cp . pruneRow) . box =
map boxs .cp . map (filter nodups) . map (cp . pruneRow) . boxs =
map boxs . filter (all nodups) . cp . map (cp . pruneRow) . boxs =
map boxs . filter (all nodups) . cp . map cp . map pruneRow . boxs =
map boxs. filter (all nodups) . expand . map pruneRow . boxs =
filter (all nodups . boxs) . map boxs . expand . map pruneRow . boxs =
filter (all nodups . boxs) . expand . bosx . map pruneRow . boxs =
filter (all nodups . boxs) .expand . pruneBy boxs =

filter (all nodups . boxs) . expand =
filter (all nodups . boxs) . expand . pruneBy boxs

filter valid . expand = filter valid . expand . prune

prune = pruneBy boxs . pruneBy cols . pruneBy rows
```

Single-Cell Expansion

```
cp . map (filter p) = filter (all p) . cp
boxs . boxs = id
boxs . expand = expand . boxs
boxs . boxs = id
pruneBy f = f . pruneRow . f
```

Single-Cell Expansion

```
solve = filter valid . expand . prune . choices

many :: (eq a) => (a -> a) -> a -> a
many f x = if x == y then x else many f y
     where y = f x

solve = filter valid . expand . many prune . choices
```

Single-Cell Expansion

```
expand1   :: Matrix Choices -> [Matrix Choices]
expand1 rows =
  [rows1 ++ [row1 ++ [c]:row2] ++ rows2 | c <- cs]
  where
  (rows1,row:rows2) = break (any smallest) rows
  (row1,cs:row2)       = break smallest row
  smallest cs             = length cs == n
  n                            = minimum (counts rows)

 counts                       = filter (/=1) . map length . concat
```

Single-Cell Expansion

```
> solve2 :: Grid -> [Grid]
> solve2 =  search . choices

> search :: Matrix Choices -> [Grid]
> search cm
>  |not (safe pm)  = []
>  |complete pm    = [map (map head) pm]
>  |otherwise      = (concat . map search . expand1) pm
>  where pm = prune cm

> complete :: Matrix Choices -> Bool
> complete = all (all single)

> single [_] = True
> single _   = False
```

Single-Cell Expansion

```
> solve2 :: Grid -> [Grid]
> solve2 =  search . choices

> search :: Matrix Choices -> [Grid]
> search cm
>  |not (safe pm)  = []
>  |complete pm    = [map (map head) pm]
>  |otherwise      = (concat . map search . expand1) pm
>  where pm = prune cm

> complete :: Matrix Choices -> Bool
> complete = all (all single)

> single [_] = True
> single _   = False
```

Single-Cell Expansion

```
> safe :: Matrix Choices -> Bool
> safe cm = all ok (rows cm) &&
>           all ok (cols cm) &&
>           all ok (boxs cm)

> ok row = nodups [d | [d] <- row]
```

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf