Applicative Sequencing (3C)

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This document was produced by using LibreOffice.

Based on

http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass

http://learnyouahaskell.com/functors-applicative-functors-and-monoids

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

<$> related operators

Functor map <$>

```
(<$>)  :: Functor f =>  (a -> b) -> f a  -> f b
(<$)   :: Functor f =>  a        -> f b  -> f a
($>)   :: Functor f => f a       -> b    -> f b
```

replace b in f b with a … f a

replace a in f a with b … f b

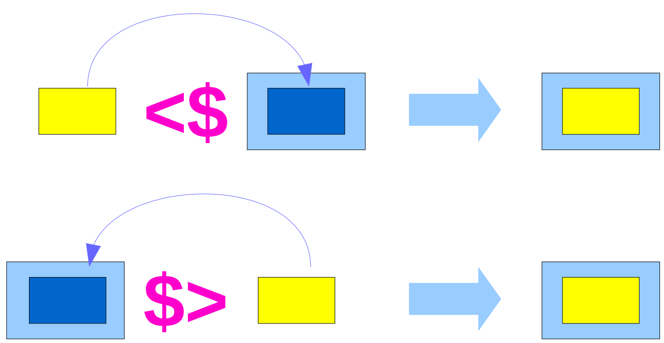
The <$> operator is just a synonym

for the fmap function in the Functor typeclass.

fmap generalizes map for **lists**

to other data types : **Maybe**, **IO**, **Map**.

Replacing the core



https://haskell-lang.org/tutorial/operators

<$ / <$> / $> operators

there are two additional operators provided

which replace a **value** inside a Functor

instead of applying a function.

This can be both more convenient in some cases,

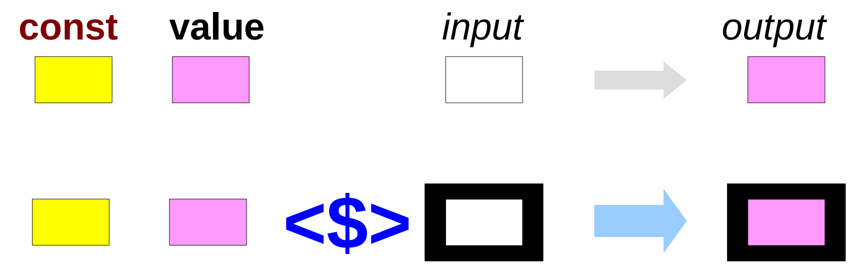
as well as for some Functors be more efficient.

value <$ functor = const value <$> functor

functor $> value = const value <$> functor

x <$ y = y $> x **y :: functor**

x $> y = y <$ x **x :: functor**



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<$ / <$> / $> operators examples

```
import Data.Functor

Prelude> Just 1 $> 2
Just 2
Prelude> Just 2 $> 1
Just 1

Prelude>  1 <$ Just 3
Just 1
Prelude>  3 <$ Just 1
Just 3

Prelude>  1 <$ Just 3
Just 1
Prelude>  3 <$ Just 1
Just 3
```

```
import Data.Functor

Prelude> (+1) <$> Just 2
Just 3
Prelude> (+1) <$> Just 3
Just 4

Prelude> (+1) <$> Nothing
Nothing

Prelude> const 2 <$> Just 111
Just 2
```

https://www.schoolofhaskell.com/school/to-infinity-and-beyond/pick-of-the-week/Simple%20examples

<$> examples

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
    putStrLn "Enter your year of birth"
    year <- read <$> getLine
    let age :: Int
        age = 2020 - year
    putStrLn $ "Age in 2020: " <> show age
```

```
getLine :: IO String
```

Input: read "12"::Double

Output: 12.0

```
-- this infix synonym for mappend is found in Data.Monoid
x <> y = mappend x y
infixr 6 <>
```

https://haskell-lang.org/tutorial/operators

<*> related operators

Applicative function application <*>

```
(<*>)  :: Applicative f => f (a -> b) -> f a -> f b
(*>)   :: Applicative f => f a        -> f b -> f b
(<*)   :: Applicative f => f a        -> f b -> f a
```

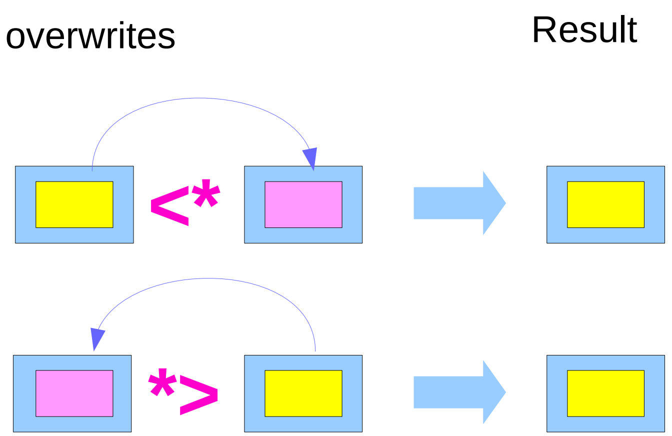
<*> is an operator that applies
a wrapped function to a wrapped value.

<*> is a part of the Applicative typeclass,

<*>is very often used as follows

foo <$> bar <*> baz

faa <*> bar <*> baz



https://haskell-lang.org/tutorial/operators

*> operator

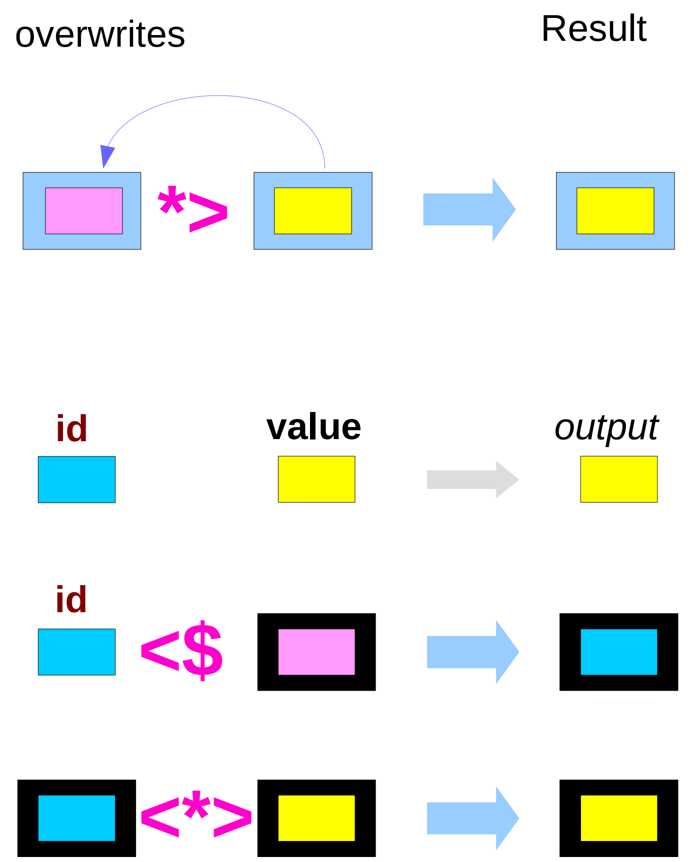
two helper operators

*> ignores the value from the first argument.

*> is completely equivalent to >> in Monad

a1 *> a2 = (id <$ a1) <*> a2

a1 *> a2 = do
_ <- a1
a2



(id <$ a1)

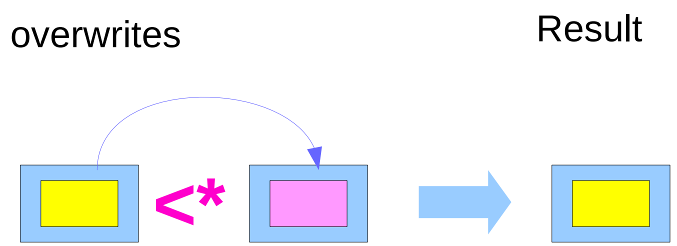
(id <$ a1) <*> a2

https://haskell-lang.org/tutorial/operators

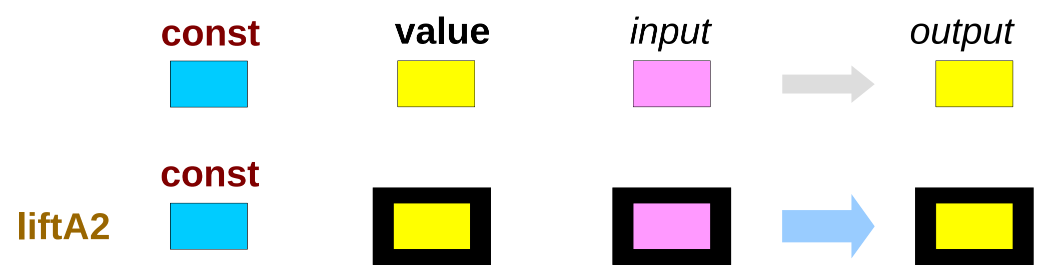
<* operator

<* is the same thing in reverse: perform the first action then the second, but only take the value from the first action.

(<*) = liftA2 const




```
a1 <* a2 = do
   res <- a1
     _ <- a2
   return res
```



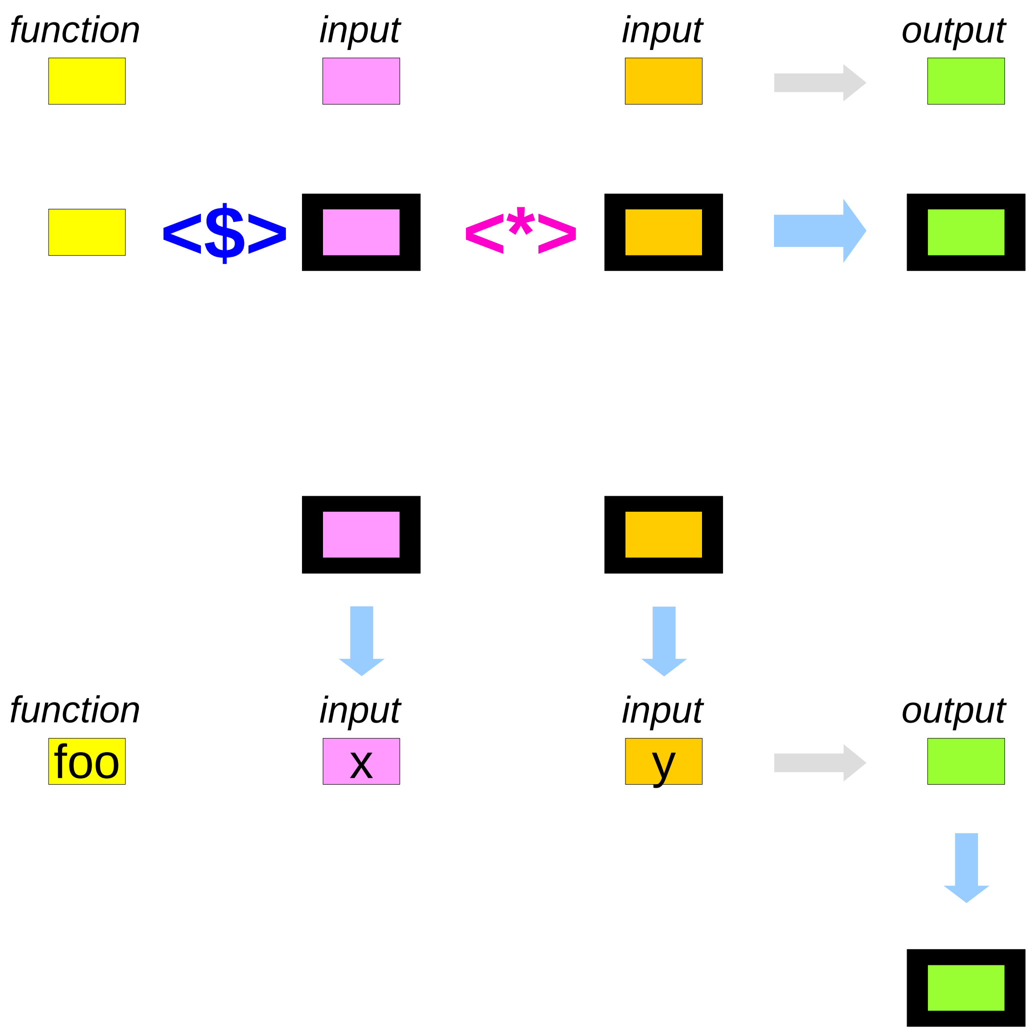
https://haskell-lang.org/tutorial/operators

<*> examples

foo <$> bar <*> baz

With a Monad, this is equivalent to:

```
do x <- bar
   y <- baz
   return (foo x y)
```



https://haskell-lang.org/tutorial/operators

<*> examples

examples including parsers and serialization libraries.

using the **aeson** package: (handling **JSON** data)

```
data Person = Person { name :: Text, age :: Int } deriving Show

-- We expect a JSON object, so we fail at any non-Object value.
instance FromJSON Person where
  parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
  parseJSON _ = empty
```

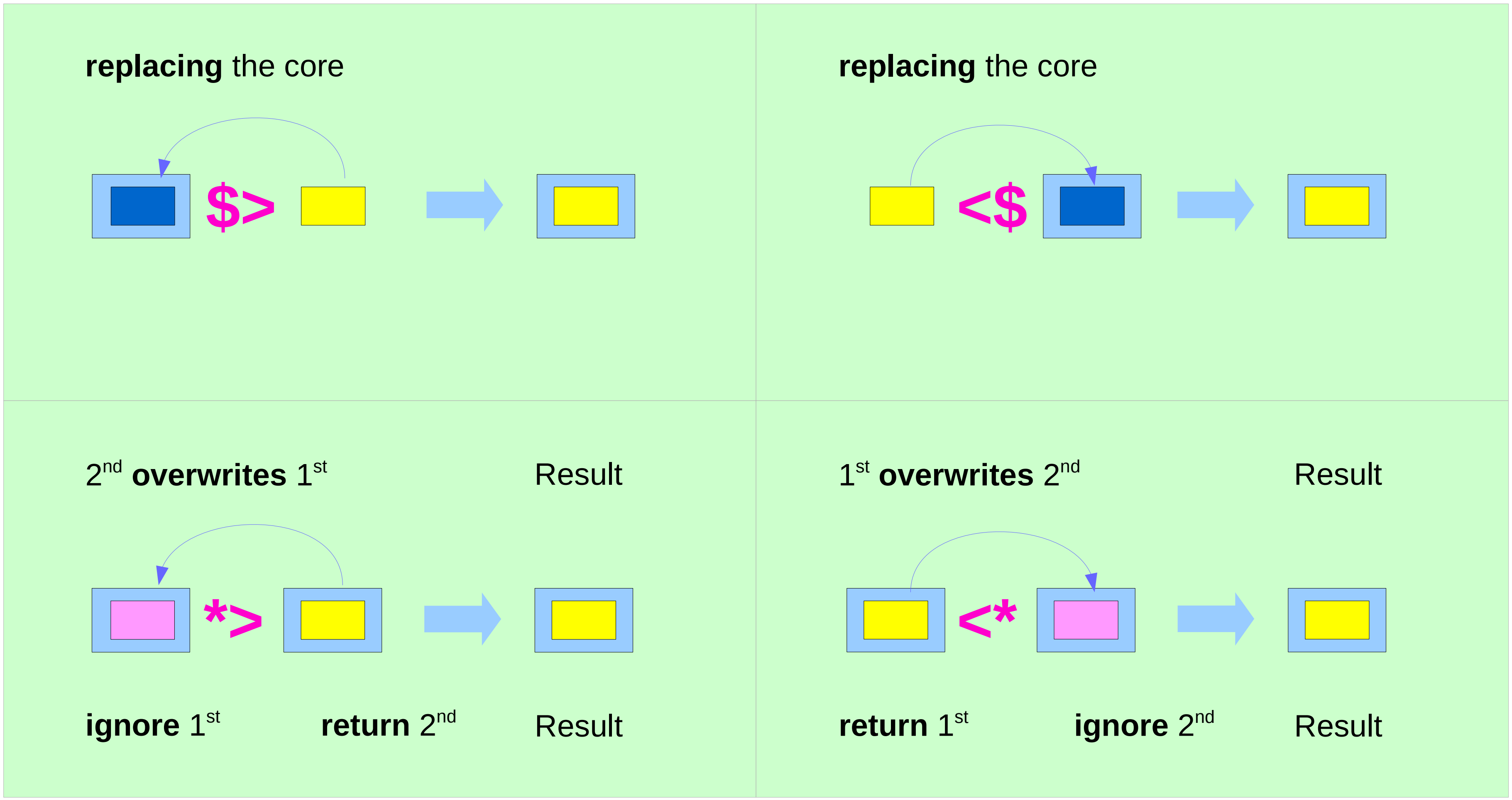
: append-head operator (cons)

. function composition operators

. name qualifier

https://haskell-lang.org/tutorial/operators

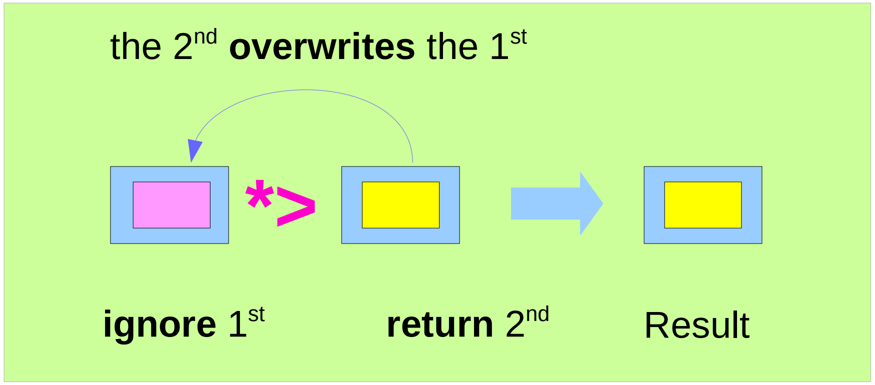
($> v.s. <$) and (*> v.s. <*)



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

(*> v.s. >>) and (pure v.s. return)

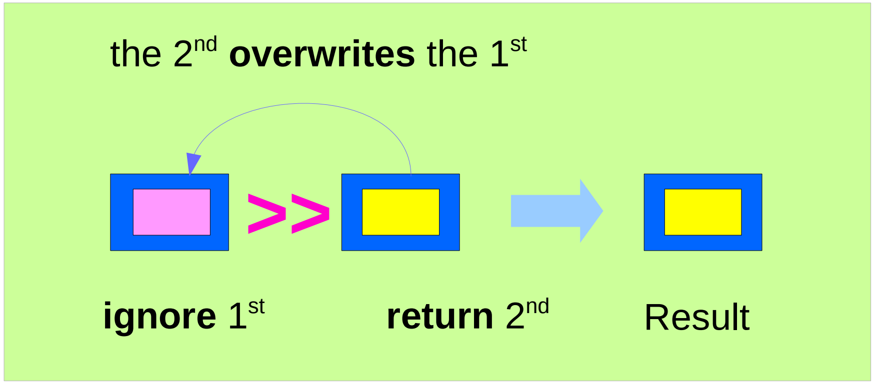
```
(*>) ::     Applicative f => f a -> f b -> f b
(>>) ::     Monad m       => m a -> m b -> m b

pure ::     Applicative f => a ->   f a
return ::   Monad m       => a ->   m a
```



the constraint changes from **Applicative** to **Monad**.

(*>) in Applicative ⟷ (>>) in Monad

pure in Applicative ⟷ return in Monad



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Sequencing of Effects

commutative monads in Haskell,

the concept involved is the same, only specialised to Monad.

Commutativity (or the lack thereof) affects

other functions which are derived from **(<*>)** as well.

(*>) is a clear example:

(*>) :: Applicative f => f a -> f b -> f b

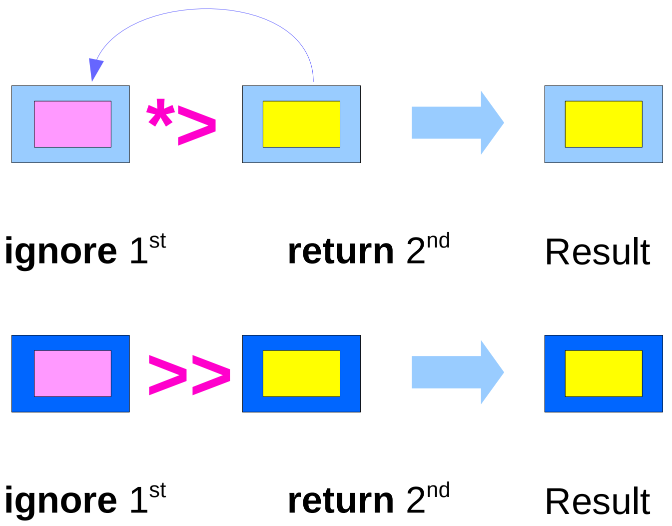
(*>) combines **effects** while preserving

only the **values** of its second argument.

For **monads**, it is equivalent to **(>>)**.

Here is a demonstration of it using Maybe,

which is commutative:



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Sequencing of Effects

Prelude> **[(2*),(3*)] <*> [4,5]**

1) [8,10,12,15] -- correct answer

2) [8,12,10,15]

The difference is that for the first (and correct) answer

the result is obtained

by taking the skeleton of the first list

and replacing each element (2*), (3*)

by all possible combinations [(2*) <*> 4, (2*) <*> 5, (3*) <*> 4, (3*) <*> 5]

with elements of the second list,

while for the other possibility

the starting point is the second list.

[(2*) <*> 4, (3*) <*> 4, (2*) <*> 5, (3*) <*> 5]

sequencing effects

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Non-commutative Functors

by **effects** we mean the functorial **context**,
as opposed to the **values** within the functor

some examples:

the skeleton of a list,
actions performed in the real world in IO,
the existence of a value in Maybe

The existence of two legal implementations of (<*>) for lists
only differ in the **sequencing** of **effects**
[] is a **non-commutative** applicative functor.

Prelude> **[(2*),(3*)] <*> [4,5]**

1) [8,10,12,15]

2) [8,12,10,15]

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Commutative Functors

a commutative applicative functor is
one for which the following holds:

```
liftA2 f u v = liftA2 (flip f) v u -- Commutativity
```

Or, equivalently,

```
f <$> u <*> v = flip f <$> v <*> u
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Commutative Monads (1)

```
do                                do
  a <- actA                         b <- actB
  b <- actB                         a <- actA
  return (a + b)                    return (a + b)
```

commutative if the order of **side effects** is not important.

there are many monads that commute (e.g. Maybe, Random).
If the monad is commutative, then the operations captured within it
can be computed in parallel.

No good syntax for monads that commute
still an open research problem

https://stackoverflow.com/questions/5897845/relax-ordering-constraints-in-monadic-computation

Commutative Monads (2)

Commutative monads are monads
for which the order of actions makes no difference
(they commute), that is when following code:

```
do                      do
  a <- actA               b <- actB
  b <- actB               a <- actA
  m a b                   m a b
```

commutative if the order of **side effects** is not important.

Examples of commutative include:

Reader monad
Maybe monad

https://wiki.haskell.org/Monad#Commutative_monads
https://stackoverflow.com/questions/6089997/how-do-i-find-out-whether-a-monad-is-commutative

Left-to-right sequencing

The convention in Haskell is to always implement (<*>)

and other applicative operators using **left-to-right sequencing**.

Even though this convention helps reducing confusion,

it also means appearances sometimes are misleading.

For instance, the **(<*)** function is not flip **(*>)**,

as it sequences effects from left to right just like **(*>)**:

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

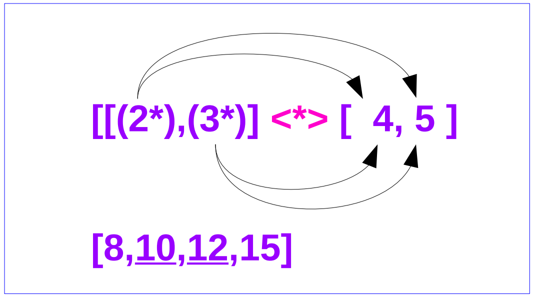
<**> operators

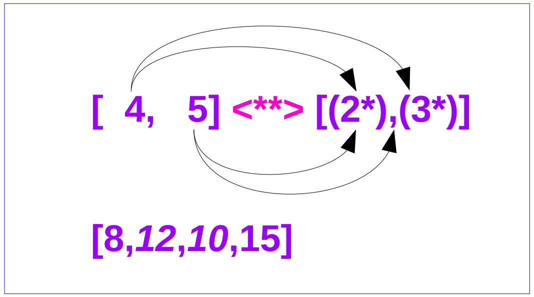
```
(<**>) :: Applicative f =>  f a         -> f (a -> b)  -> f b
(<*>)  :: Applicative f =>  f (a -> b)  -> f a         -> f b
```

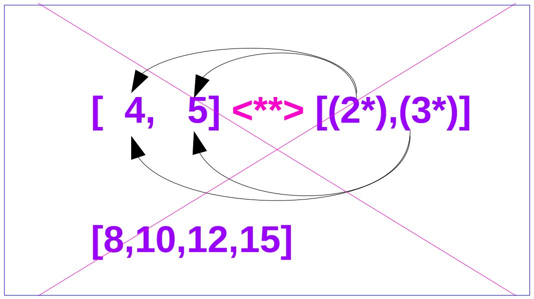
from **Control.Applicative**

not flip **(<*>)**

a way of inverting the sequencing







https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Sequencing examples (1)

Prelude> **[(2*),(3*)] <*> [4,5]**

[8,10,12,15]

Prelude> **[4,5] <**> [(2*),(3*)]**

[8,12,10,15]

Prelude> **Just 2 *> Just 3**

Just 3

Prelude> **Just 3 *> Just 2**

Just 2

Prelude> **Just 2 *> Nothing**

Nothing

Prelude> **Nothing *> Just 2**

Nothing

[(2*)] <*> [4,5], [(3*)] <*> [4,5]

[4] <> [(2*),(3*)], [5] <**> [(2*),(3*)]**

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Sequencing examples (2)

```
Prelude> (print "foo" *> pure 2) *> (print "bar" *> pure 3)
"foo"
"bar"
3
Prelude> (print "bar" *> pure 3) *> (print "foo" *> pure 2)
"bar"
"foo"
2
```

```
Prelude> (print "foo" *> pure 2) <* (print "bar" *> pure 3)
"foo"
"bar"
2
```

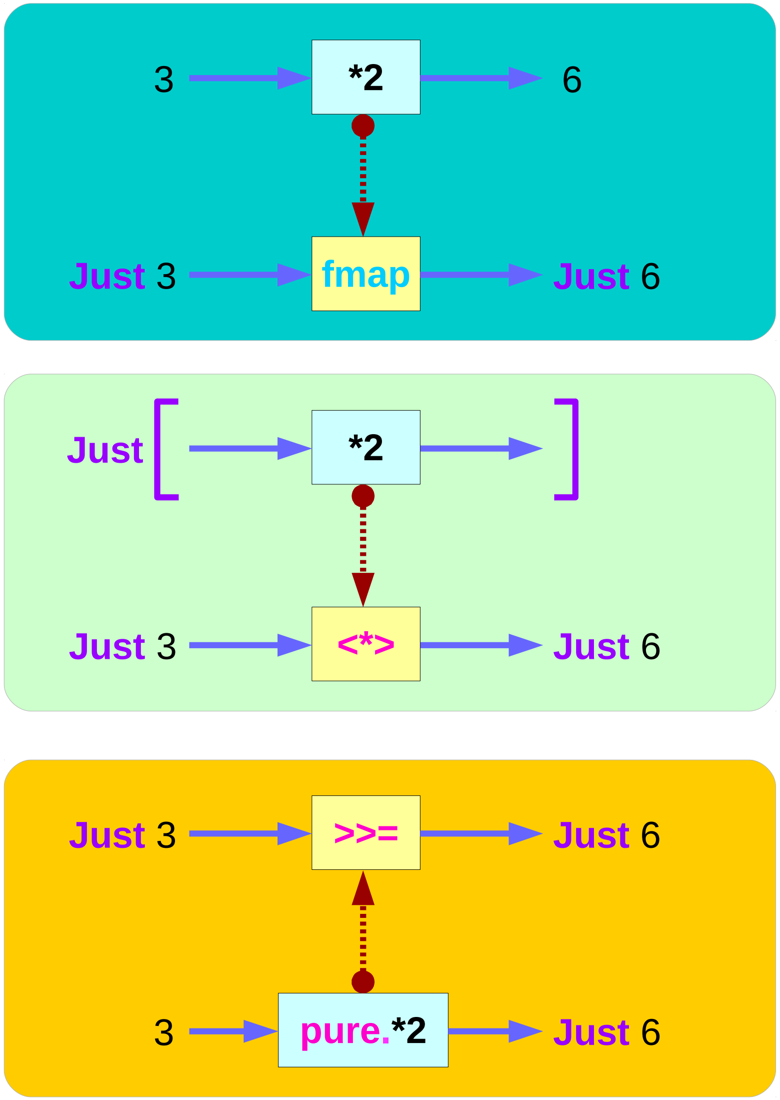
https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Functors, Applicative, and Monad

fmap :: Functor f => (a -> b) -> f a -> f b

(<*>) :: Applicative f => f (a -> b) -> f a -> f b

(>>=) :: Monad m => m a -> (a -> m b) -> m b



https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Functors, Applicative, and Monad Examples

```
fmap  :: Functor     f => (a -> b)   -> f a            -> f b
```

```
Prelude> fmap (*2) (Just 3)
Just 6
```

```
(<*>) :: Applicative f => f (a -> b) -> f a            -> f b
```

```
Prelude> (Just (*2)) <*>  (Just 3)
Just 6
```

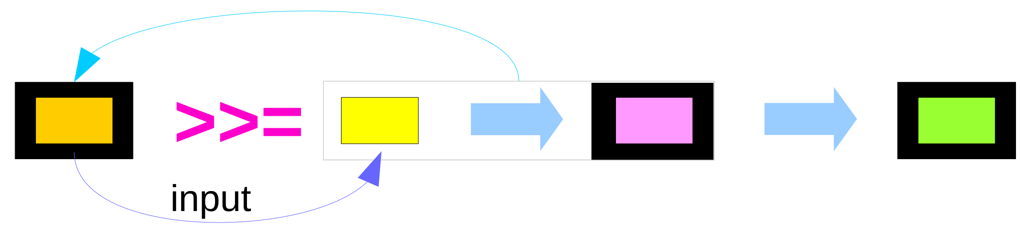
```
(>>=) :: Monad       m => m a        -> (a -> m b)     -> m b
```

```
Prelude> (Just 3) >>= (pure . (*2))
Just 6
Prelude> (Just 3) >>= (return . (*2))
Just 6
```

(=<<) : the flipped version of (>>=)

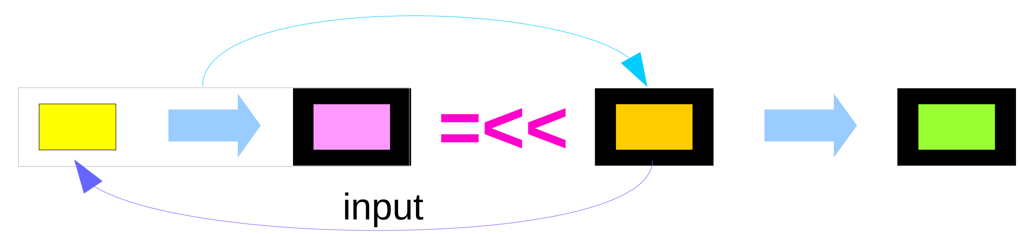
```
(>>=) ::   Monad   m   => m a   -> (a -> m b)   -> m b
```

maps **a -> m b** function at the right
over monadic **ma** functors at the left




```
(=<<) ::   Monad   m   => (a -> m b) -> m a   -> m b
```

maps **a -> m b** function at the left
over monadic **ma** functors at the right



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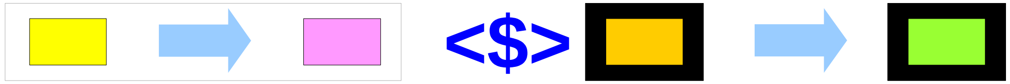
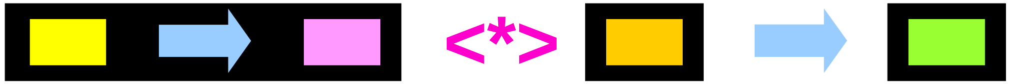
<$>, <*>, >>=, and =<< examples

```
Prelude> (*2) <$> (Just 3)
Just 6
```

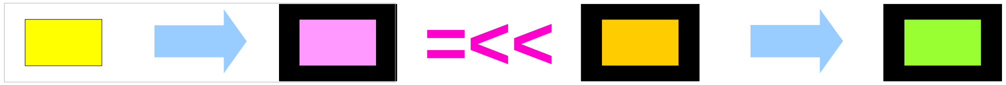



```
Prelude> (Just (*2)) <*>  (Just 3)
Just 6
```




```
Prelude> (Just 3) >>= (pure . (*2))
Just 6
```


```
Prelude> (pure . (*2)) =<< (Just 3)
Just 6
```



Comparing the three characteristic methods

replace **fmap** by its infix synonym, **(<$>)**

replace **(>>=)** by its flipped version, **(=<<)**

```
fmap  :: Functor      f  =>   (a -> b)  -> f a           -> f b
(<*>) :: Applicative  f  => f (a -> b)  -> f a           -> f b
(>>=) :: Monad        m  => m a         -> (a -> m b)    -> m b

(<$>) :: Functor      t  =>   (a ->   b)     -> (t a -> t b)
(<*>) :: Applicative  t  => t (a ->   b)     -> (t a -> t b)
(=<<) :: Monad        t  =>   (a -> t b)     -> (t a -> t b)
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

All mapping functions over Functors

fmap, **(<*>)** and **(=<<)** are all mapping functions over **Functors**.

The differences between them are in what is being mapped (functions) over in each case:

(<$>) ::	**Functor t**	**=>**	**(a -> b)**	**->**	**(t a -> t b)**
(<*>) ::	**Applicative t**	**=>**	**t (a -> b)**	**->**	**(t a -> t b)**
(=<<) ::	**Monad t**	**=>**	**(a -> t b)**	**->**	**(t a -> t b)**

fmap maps **(a -> b)** arbitrary functions over functors.

(<*>) maps **t (a -> b)** morphisms over (applicative) functors.

(=<<) maps **a -> t b** functions over (monadic) functors.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

From **fmap** to **(<*>)** and then to **(>>=)**

The differences of **Functor**, **Applicative** and **Monad** follow from what these three mapping functions allow you to do.

As you move from **fmap** to **(<*>)** and then to **(>>=)**, you gain in power, versatility and control, at the cost of guarantees about the results.

We will now slide along this scale.
While doing so, we will use the contrasting terms

> **values** to refer to plain values within a functor
> **contexts** to whatever surrounds them, respectively.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

No changing the context

The type of **fmap** ensures that it is impossible

to use it to **change the context**,

no matter which function it is given.

In **(a -> b) -> t a -> t b**, the **(a -> b)** function

has nothing to do with the **t** context of the **t** a functorial value,

and so applying it cannot affect the context.

For that reason, if you do **fmap** **f xs** on some list **xs**

the number of elements of the list will never change.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Changing the context

fmap cannot change the **context**

the **(a -> b)** function has no relation with the **t context**

the application of this function does not affect the **context t**

for example, the number of elements of the list will never change

Prelude> **fmap (2*) [2,5,6]**	*a list with 3 elements*
[4,10,12]	*a list with 3 elements*

That could be a safety guarantee or an unfortunate restriction

depending on your purpose

(<*>) is clearly able to change the **context**:

Prelude> **[(2*),(3*)] <*> [2,5,6]**	*two lists each with 3 elements*
[4,10,12,6,15,18]	*a list with 6 elements*

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Carrying a context

The **t (a -> b)** morphism carries a **context** of its own,
which is combined (applied) with the **context** of the **t** a functorial **value**.

(<*>), however, is subject to a more subtle restriction

while **t (a -> b)** morphisms carry **context**,
within them there are plain **(a -> b)**,
which are still unable to modify the **context**.

this means the changes to the **context** **(<*>)** performs
are fully determined by the **context** of its arguments,
and the **values** have no influence over the resulting context.

t (a->b) or **t b**
(a->b) or **a**

Prelude> **[(2*),(3*)]** **<*>** **[2,5,6]** *two lists each with 3 elements*
[4,10,12,6,15,18] *a list with 6 elements*

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Carrying a context examples

```
Prelude> (print "foo" *> pure (2*)) <*> (print "bar" *> pure 3)        (pure (2*)) <*> (pure 3)
"foo"
"bar"
6
Prelude> (print "foo" *> pure 2) *> (print "bar" *> pure 3)            (pure 2) *> (pure 3)
"foo"
"bar"
3
Prelude> (print "foo" *> pure undefined) *> (print "bar" *> pure 3)
"foo"                                                                   (pure undefined) *> (pure 3)
"bar"
3
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Creating a context

Prelude> **[(2*),(3*)] <*> [2,5,6]** *two lists each with 3 elements*

[4,10,12,6,15,18] *a list with 6 elements*

with **list (<*>)** you know that the length of the resulting list
will be the product of the lengths of the original lists,

with **IO (<*>)** you know that all real world effect
will happen as long as the evaluation terminates, and so forth.

with **Monad**, however, it is very different
(>>=) takes a (**a -> t b)** function, and so it is able
to create **context** from **values**
which means a lot of flexibility:

a -> t b *creaing* ***context t***

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Creating a context examples

```
Prelude> [1,2,5] >>= \x -> replicate x x          [ replicate 1 1, replicate 2 2, replicate 5,5 ]
[1,2,2,5,5,5,5,5]

Prelude> [0,0,0] >>= \x -> replicate x x          [ replicate 0 0, replicate 0 0, replicate 0,0 ]
[]

Prelude> return 3 >>= \x -> print $ if x < 10 then "Too small" else "OK"
"Too small"

Prelude> return 42 >>= \x -> print $ if x < 10 then "Too small" else "OK"
"OK"
```

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Flexibility

the extra **flexibility**

the less **guarantees** about

- whether your functions are able to unexpectedly erase parts of a data structure for pathological inputs
- whether the control flow in your application remains intelligible

performance implications

- the complex **data dependencies** of monadic codes might prevent **refactoring** and **optimizations**.

it is a good idea to only use as much power as needed for the task at hand.

If you do need the extra capabilities, use **Monad**
however, it is often worth it to check
whether **Applicative** or **Functor** are sufficient.

https://en.wikibooks.org/wiki/Haskell/Applicative_functors

Monadic binding / composition operators

```
(>>=)  :: Monad m => m a          -> (a -> m b)  -> m b
(=<<)  :: Monad m => (a -> m b)   -> m a         -> m b
(>>)   :: Monad m => m a          -> m b         -> m b
(>=>)  :: Monad m => (a -> m b)   -> (b -> m c)  -> (a -> m c)
(<=<)  :: Monad m => (b -> m c)   -> (a -> m b)  -> (a -> m c)
```

https://haskell-lang.org/tutorial/operators

Monadic binding operators (1)

```
(>>=)  :: Monad m => m a          -> (a -> m b)  -> m b
(=<<)  :: Monad m => (a -> m b) -> m a           -> m b
(>>)   :: Monad m => m a          -> m b         -> m b
```

monadic binding operators

The two most basic are **>>=** and **>>**

>>=, **>>**, **=<<** can be expressed in **do-notation**

>> is just a synonym for ***>** from the **Applicative** class

=<< is just **>>=** with the arguments reversed

https://haskell-lang.org/tutorial/operators

Monadic binding operators (2)

```
(>>=)  :: Monad m => m a          -> (a -> m b)  -> m b
(=<<)  :: Monad m => (a -> m b) -> m a          -> m b
(>>)   :: Monad m => m a          -> m b          -> m b
```

```
m1 >>= func = do                                      m1 :: m a
    x <- m1           -- extract the value            x  :: a
    func x                                            func :: a -> m b
```

```
m1 >> m2 = do                                         m1 :: m a
    _ <- m1           -- side effect only, ignore the value    m2 :: m b
    m2
```

```
func =<< m1 = do                                      m1 :: m a
    x <- m1           -- extract the value            x  :: a
    func x                                            func :: a -> m b
```

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Monadic composition operators (1)

```
(>=>)  :: Monad m => (a -> m b)  -> (b -> m c)  -> (a -> m c)
(<=<)  :: Monad m => (b -> m c)  -> (a -> m b)  -> (a -> m c)
```

composition operators for <u>two</u> **monadic functions**

- **>=>=**, **<=<** can be expressed in **do-notation**
- **>=>** **pipes** the result from the left side to the right side
- **<=<** **pipes** the result from the right side to the left side

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Monadic composition operators (2)

```
(>=>)  :: Monad m => (a -> m b) -> (b -> m c)  -> (a -> m c)
(<=<)  :: Monad m => (b -> m c) -> (a -> m b)  -> (a -> m c)
```

`f >=> g = \x -> do`
`    y <- f x`
`    g y`
	f :: a -> m b, x :: a, f x :: m b
	g :: b -> m c, y :: b, g y :: m c

`g <=< f = \x -> do`
`    y <- f x`
`    g y`
	f :: a -> m b, x :: a, f x :: m b
	g :: b -> m c, y :: b, g y :: m c

```
f >=> g = g <=< f
g >=> f = f <=< g
```

First f

Then g

https://haskell-lang.org/tutorial/operators

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf