Temporal Characteristics of Random Processes

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

1 The concepts of the random process

Ranadom Variable Definition

Definition

a real random variable

a real **function** over a sample space $S = \{s_1, s_2, s_3, ..., s_n\}$

a real random variable : capital letter X

a particular value : a lowercase letter x

a sample space $S = \{s_1, s_2, s_3, ..., s_n\}$

an element of S : s

Ranadom Variable Example

Example

$$X(s_1) = x_1 \qquad s_1 \longrightarrow x_1$$
$$X(s_2) = x_2 \qquad s_2 \longrightarrow x_2$$
$$\cdots \qquad \cdots$$
$$X(s_n) = x_n \qquad s_n \longrightarrow x_n$$

$S = \{s_1, s_2, s_3, \ldots, s_n\}$ a sample space

X a random variable

Random variables with time (1)

N Gaussian random variables

Definition

a function of both outcome s and time t
assign a time function to every outcome s

$$x(t, s)$$

the family of such time functions is called a random process

$$X(t, s)$$

the short-form notation $x(t)$ to present a specific waveform of a random process

$$X(t)$$

Random variables with time(2)

N Gaussian random variables

Definition

$X(t,s)$ represents a family or ensemble of time functions
$x(t,s)$a sample function, an ensemble member, a realization of the process
a random process $X(t,s)$ also represents a single time function whtn t is a variable and s is fixed at an outcome

Ranadom Process Example

Example

$$X(t, s_1) = x_1(t) \qquad s_1 \longrightarrow x_1(t)$$
$$X(t, s_2) = x_2(t) \qquad s_2 \longrightarrow x_2(t)$$
$$\cdots \qquad \cdots$$
$$X(t, s_n) = x_n(t) \qquad s_n \longrightarrow x_n(t)$$

$S = \{s_1, s_2, s_3, \ldots, s_n\}$ a sample space

$X(t)$ a random process

Random variables with time
N Gaussian random variables

Definition

$$X_i = X(t_i, s) = X(t_i) \qquad \textit{random variable}$$

$$X(t, s) = X(t) \qquad \textit{random process}$$

Classification of Random Processes (1)

N Gaussian random variables

- a continuous alphabet continuous time random process
- a discrete alphabet continuous time random process
- a continuous alphabet discrete time random process
- a discrete alphabet discrete time random process

An alphabet
N Gaussian random variables

Definition

the alphabet of $X(t)$ is the set of its possible values
classify random processes according to
the values of t for which the process is defined
the alphabet of the random variable $X = X(t)$ at time t

Classification of Random Processes(2)

N Gaussian random variables

- a continuous alphabet continuous time random process
  - $X(t)$has a continuous alphabet and t has continuous values
- a discrete alphabet continuous time random process
  - $X(t)$has a discrete alphabet and t has continuous values
- a continuous alphabet discrete time random process
  - $X(t)$has a continuous alphabet and t has discrete values
- a discrete alphabet discrete time random process
  - $X(t)$has a discrete alphabet and t has discrete values

Deterministic and Non-deterministic Processes
N Gaussian random variables

a sample function
if future values of any sample function cannot be predicted exactly from observed past values, the process is called non-determinisitic
A process is deterministic if future values of any sample function can be predicted from past values

Deterministic Random Process Example

N Gaussian random variables

$$X(t) = A\cos(\omega_0 + \Theta)$$

A, Θ, or ω_0(or all) can be random variables. Any one sample function corresponds to the above equation with particular values of these random variables. Therefore the knowledge of the sample function prior to any time instance fully allows the prediction of the sample function's future values because all the necessary information is known