Maximum Signal-to-Noise Ratio

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Definition

$$\left(\frac{\widehat{S}_o}{N_o}\right) = \frac{|x_o(t_o)|^2}{E\left[N_o^2(t)\right]}$$

$$\widehat{S}_o = |x_o(t_o)|^2$$

$$N_o = E\left[N_o^2(t)\right]$$

Definition

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) H(\omega) e^{j\omega t} d\omega$$

$$N_o = E\left[N_a^2(t)\right] = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{NN}(\omega) |H(\omega)|^2 d\omega$$

$$\left(\frac{S_o}{N_o}\right) = \frac{|\frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) H(\omega) e^{j\omega t_0} d\omega|^2}{\frac{1}{2\pi} \int_{-\infty}^{+\infty} S_{NN}(\omega) |H(\omega)|^2 d\omega}$$

Definition

$$\int_{-\infty}^{+\infty} A(\omega)B(\omega)d\omega \le \int_{-\infty}^{+\infty} |A(\omega)|^2 d\omega \int_{-\infty}^{+\infty} |B(\omega)|^2 d\omega$$

$$B(\omega) = CA^*(\omega)$$

$$A(\omega) = \sqrt{S_{NN}(\omega)}H(\omega)$$

$$A(\omega) = \frac{X(\omega)e^{j\omega t_0}}{2\pi\sqrt{S_{NN}(\omega)}}$$

Definition

$$\left| \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) H(\omega) e^{j\omega t} d\omega \right|^2 \leq$$

$$\int_{-\infty}^{+\infty} S_{NN}(\omega) |H(\omega)|^2 d\omega \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{|X(\omega)|^2}{S_{NN}(\omega)} d\omega$$

$$\left(\frac{\overline{S}_o}{N_o} \right) = \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \frac{|X(\omega)|^2}{S_{NN}(\omega)} d\omega$$

$$H_{opt}(\omega) = \frac{1}{2\pi C} \frac{X^*(\omega)}{S_{NN}(\omega)} e^{-j\omega t_o}$$

Definition

$$H_{opt}(\omega) = KX^*(\omega)e^{-j\omega t_0}$$

$$h_{opt}(t) = Kx^*(t_0 - t)$$

$$h_{opt}(t) = Kx(t_0 - t)$$