\[ b_n(x) = \frac{1}{\phi_n^2(x)} \frac{(x-x_0)(x-x_2)}{(x_i-x_0)(x_i-x_2)} \]

\[ \phi_n \] is set of poly. of order \( n \) at node numbers

\[ \phi_n(x_0) = 1, \quad \phi_n(x_2) = \phi_n(x_3) = 0 \]

Use (1) p. 8.3 and (2) p. 8.8 to find expr. for \( a_0, a_1, a_2 \) in terms of \( (x_i, f(x_i)), \ i = 0,1,2 \).

Use (1) p. 8.3 to derive the simple Simpson’s rule (2) p. 7.6.
(1)-(2) p. 7-3: Gen. Newton (T.N)

Cases w/ (n+1) pts x₀, x₁, ..., xₙ

fₙ₊₁(x) = pₙ₊₁(x) ∈ $P_{n+1}$ = set of poly. of order ≤ n.

λₙ₊₁(x) ∈ $P_{n+1}$

HVI: $f(x) = \frac{x^n - 1}{x}$ on [0,1]

$x₀ = 0$, $xₙ = b = 1$

Consider: n = 1, 2, 4, 8, 16

Const. $fₙ(x) = \sum \lambdaₙ(x) f(x)$

Plot f, fₙ, n = 1, 2, 4, 8, 16

Comp. $Iₙ = \int fₙ(x) \, dx$, n = 1, 2, 4, 8

and compare $\int f \, dx$.

For n = 4, plot $f$, $f₁, f₂, f₄$.

Why not $f₃, f₅, f₆$?