

# Monad P3 : Primitive Types (1B)

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# Based on

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Haskell in 5 steps

[https://wiki.haskell.org/Haskell\\_in\\_5\\_steps](https://wiki.haskell.org/Haskell_in_5_steps)

# Lifting (1)

Typical data type with a parameter

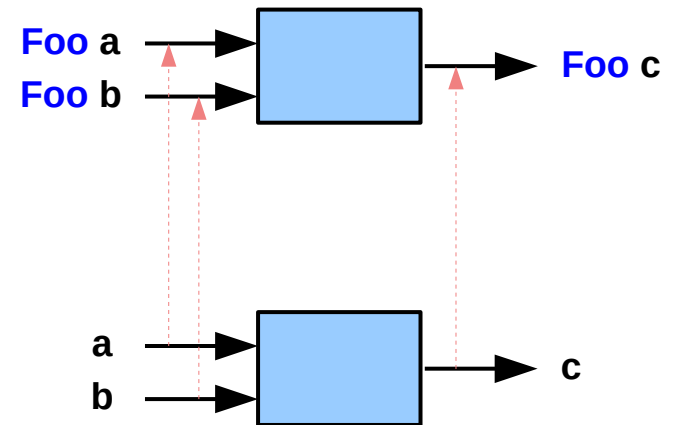
```
data Foo a = Foo { ...stuff here ...}
```

Suppose that a lot of uses of Foo take numeric types (Int, Double etc) and you keep having to write code that *unwraps* these numbers, adds or multiplies them, and then *wraps* them back up.

You can short-circuit this by writing the *unwrap-and-wrap* code once.

This function is traditionally called a "lift" because it looks like this:

```
liftFoo2 :: (a -> b -> c) -> Foo a -> Foo b -> Foo c
```



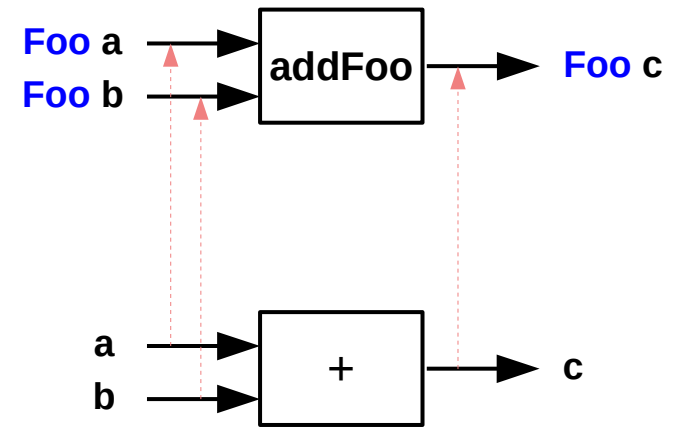
<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

# Lifting (2)

`liftFoo2 :: (a -> b -> c) -> Foo a -> Foo b -> Foo c`

in other words you have a function  
which takes a two-argument function  
(such as the (+) operator) and  
turns it into the equivalent function for **Foos**.

`addFoo = liftFoo2 (+)`



<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

# Bottom

Haskell allows you to use a special value called **undefined**.

This is sometimes also referred to as **bottom**,  $\perp$ , or  $\_ \_$

## Member of all types

```
Prelude> i = undefined
Prelude> i + 1
-- error!
Prelude> l = [1,2,3,4,undefined]
Prelude> l !! 3
4
Prelude> l !! 4
-- error!
```

## Laziness

```
Prelude> head [1, undefined]
1
Prelude> head [undefined, 1]
-- error!
```

## As a return value

```
Prelude> stupid = sum [1..]
Prelude> stupid
-- infinite loop
```

## As an argument to a function

```
Prelude> weird x = 3

Prelude> weird . sum $ [1..]
3

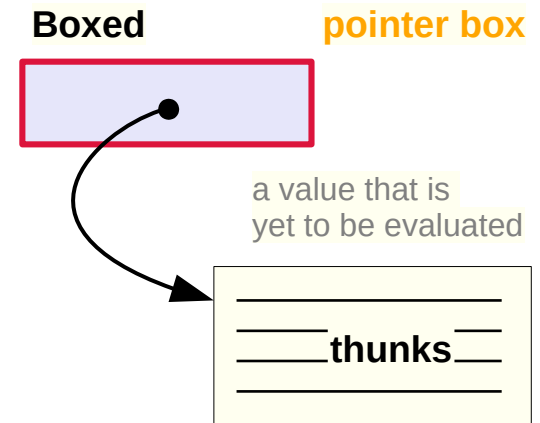
Prelude> weird undefined
3
```

<https://andre.tips/wmh/brief-note-undefined/>

# Box

In most implementations of **lazy evaluation**, **values** are represented at runtime as **pointers** to either their **value**, or **code** for computing their value.

This extra level of **indirection**, together with any extra tags needed by the **runtime**, is known as a **box**.



<https://en.wikibooks.org/wiki/Haskell/Libraries/Arrays>

# Boxed representation

the expressiveness of **non-strict arrays** comes at a price, especially if the array elements are simple numbers (**values**).

Instead of direct storing those numeric elements, **non-strict** arrays require a **boxed representation**

the elements are **pointers to heap objects** containing the numeric values.

This **additional indirection** requires extra **memory** and drastically reduces the **efficiency** of array access, especially in **tight loops**.

<https://www.tweag.io/posts/2017-09-27-array-package.html>



# Boxed vs Unboxed Kinds

```
> :k Int
Int :: *
> :k Int#
Int# :: #
```

**Int#** has a different **kind** than normal Haskell datatypes: **#**.

<https://haskell-lang.org/tutorial/primitive-haskell>

# Boxed vs Unboxed Types

values of **boxed** type are represented

by a **pointer** to a **heap object**

The representation of a Haskell **Int** is

a two-word **heap** object

An **unboxed** type is represented

by the **value** itself,

no **pointers** or **heap allocation** are involved.

**unboxed** types correspond to the “**raw machine**” types in C

**Int#**            (long int)

**Double#**        (double)

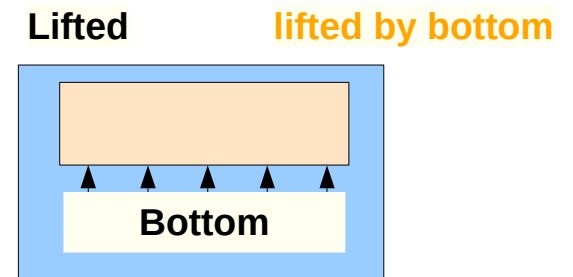
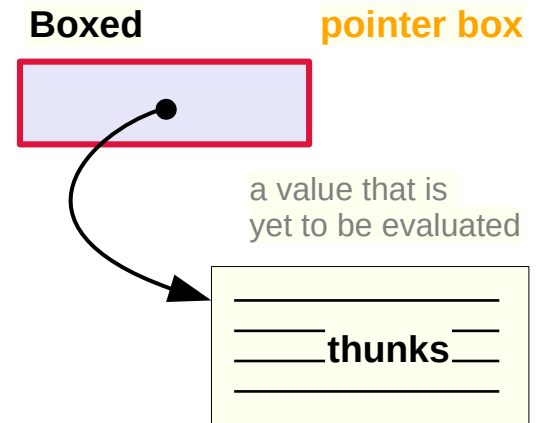
**Addr#**           (void \*)

Most **types** in GHC are **boxed**,

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Classifying types – Summary

|                  |   |
|------------------|---|
| <b>Boxed</b>     | a <b>pointer</b> to a <b>heap</b> object. |
| <b>Unboxed</b>   | no <b>pointer</b>                         |
| <b>Lifted</b>    | <b>bottom</b> as an element.              |
| <b>Unlifted</b>  | no <b>extra values</b> .                  |
| <b>Algebraic</b> | <u>one or more constructors</u> ,         |
| <b>Primitive</b> | a <b>built-in type</b>                    |



Undefined  
Infinite loop  
Exception

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

# Classifying types – Boxed, Unboxed

## Boxed types

a **value** is represented by a **pointer** to a **heap** object.

## Unboxed types

a type is **unboxed** iff its representation is other than a **pointer**.

<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

# Classifying types – Lifted, Unlifted

## Lifted types

A type is **lifted** iff it has **bottom** as an element.

A **value** of a **lifted type** can be **bottom**.

it can be **undefined**, or perhaps a computation that **never finishes**, or one that **throws** an **exception**.

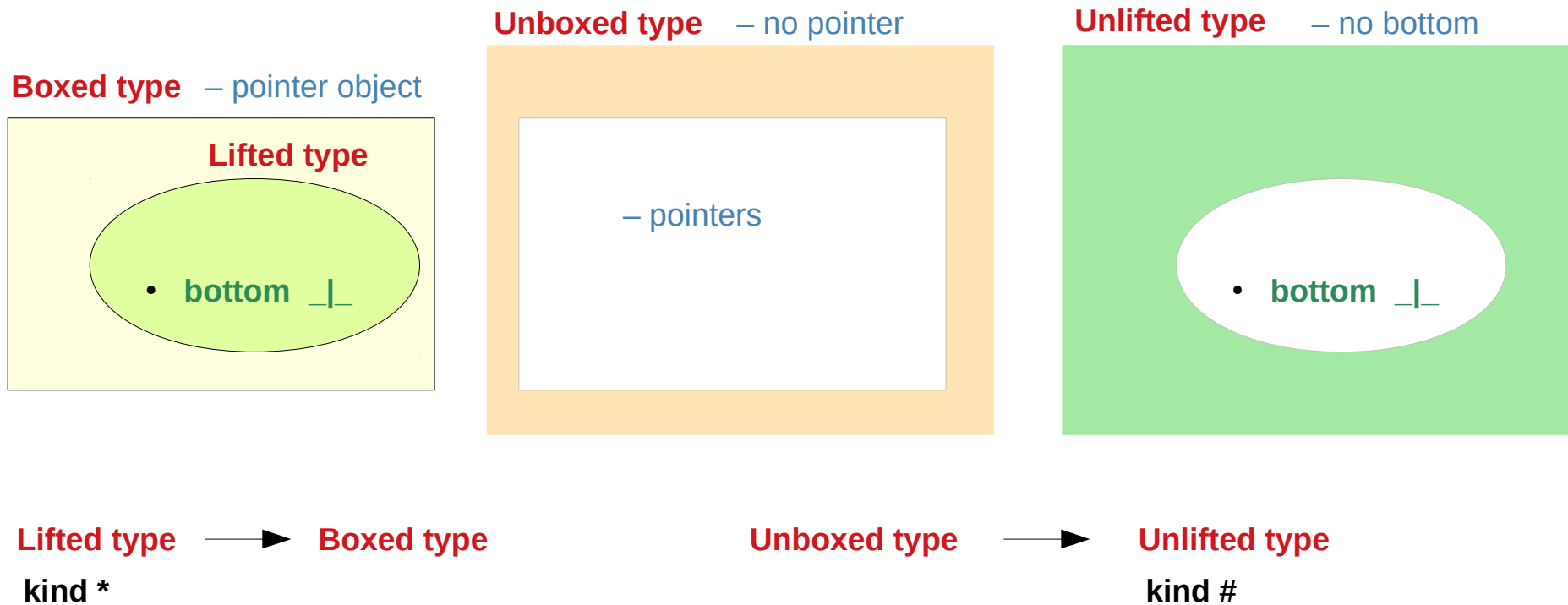
## Unlifted types

do not have these potentially troublesome **extra values**.

<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

# (Un)Lifted and (Un)Boxed types



<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

# Applications of Unboxed types

## Unboxed types

cannot have **thunks**

since **thunks** are **pointers** to data

telling you how to produce the value

cannot exploit **laziness**

really just hold **values**.

they can be **faster**.

<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

# Applications of lifted types

**Closures** always have **lifted** types: i.e.  
any **let-bound** identifier in **Core** must have a **lifted** type.

Operationally, a lifted object is one that can be entered.

Only **lifted** types may be unified with a **type variable**.

**Polymorphism** does not play with **unlifted types**.  
**parametric type** must be **lifted**.

Something like `id 0 :: Int#` does not work.

<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>



# Applications of unlifted types

Unlifted types do not have **bottom** as a value

This can be useful in a **purely "semantic" level**  
(if you don't want those extra values) and

it can also facilitate more **efficient implementations**  
by reducing **costly indirections**.

A GHC optimization called the **worker-wrapper transformation**  
exploits this

<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

# Classifying types – Algebraic

## **Algebraic**

a data type with one or more **constructors**,  
whether declared with **data** or **newtype**.

An **algebraic** type is one that can be deconstructed  
with a **case** expression.

**Algebraic** is NOT the same as **lifted**  
because **unboxed** (and thus **unlifted**) **tuples** count as "**algebraic**".

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

# Classifying types – Primitive

## Primitive

a type is **primitive** iff it is a **built-in type** that can't be expressed in Haskell.

Currently, all **primitive** types are **unlifted**, but that's not necessarily the case.

(E.g. **Int** could be **primitive**.)

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

# Type classification examples

|                   | Primitive | Boxed | Lifted | Algebraic |
|-------------------|-----------|-------|--------|-----------|
| <b>Int#</b>       | Yes       | No    | No     | No        |
| <b>Array#</b>     | Yes       | Yes   | No     | No        |
| <b>(# a, b #)</b> | Yes       | No    | No     | Yes       |
| <b>( a, b )</b>   | No        | Yes   | Yes    | Yes       |
| <b>[a]</b>        | No        | Yes   | Yes    | Yes       |

Some **primitive types** are **unboxed**, such as **Int#**, whereas some are **boxed** but **unlifted** (such as **Array#**). The only **primitive types** that we classify as **algebraic** are the **unboxed tuples**.

|                   |                |                 |                       |
|-------------------|----------------|-----------------|-----------------------|
| <b>Array#</b>     | <b>Boxed</b>   | <b>Unlifted</b> | pointer, no bottom    |
| <b>ByteArray#</b> | <b>Unboxed</b> | <b>Unlifted</b> | no pointer, no bottom |

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

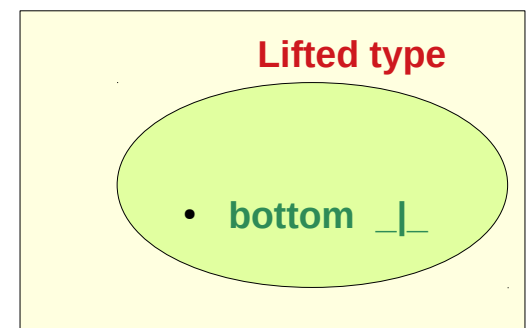
Primitive types  $\approx$

Unboxed types  $\approx$

Unlifted types

Lifted type  $\longrightarrow$  Boxed type  
 Unlifted type  $\longleftarrow$  Unboxed type

**Boxed type** – pointer object



# Int# not normal data type

The **Int# constructor** is actually just a normal **data constructor** in Haskell with a **#**

**Int#** is not a normal **data type**

In **GHC.Prim**, it's implementation is:

**data Int#**

- like everything else in **GHC.Prim** is really a **lie**.
- is provided by the **implementation**,
- is in fact a normal **long int** from C

**Int#**

Normal **data constructor**

Not normal **data type**

<https://haskell-lang.org/tutorial/primitive-haskell>

# Magic hash

By convention, all **unlifted types** end with a **#**,  
called the **magic hash**,  
enabled by the **MagicHash extension**.  
examples include **Char#** and **Int#**.

to distinguish **unboxed operations** – functions with **#**

**(+#) :: Int# -> Int# -> Int#**

**(+#) = let x = x in x**

You can even have

**unboxed tuples (# a, b #)**

**unboxed sums (# a | b #)**

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

<https://haskell-lang.org/tutorial/primitive-haskell>

# Primitive Operations

The **primitive operations** (**PrimOps**) on **primitive types**

e.g., **(+#)** is **addition** on **Int#s**

the **machine-addition**

– usually one instruction.

the **standard + operator** and **Int data type**

are actually themselves defined in normal Haskell code,

which provides many benefits:

**standard type class** support, **laziness**, etc.

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# PrimOps

**primops**, short for **primitive operations**,  
are core pieces of **functionality** provided by **GHC** itself.

They are the *magical, elegant boundary*  
between "things we do in Haskell itself"  
and "things which our implementation provides."

<https://haskell-lang.org/tutorial/primitive-haskell>



# Functions in primitive operations

Look at the implementation of other functions in **GHC.Prim**;  
they're *all* defined as **let x = x in x**.

```
and# :: Word# -> Word# -> Word#
```

```
and# = let x = x in x
```

When GHC reaches a call to one of these **primops**,  
it automatically replaces it with the **real implementation**,  
- an **assembly code**, an **LLVM code**, or something else

**dummy implementation** to give **Haddock** documentation

**GHC.Prim** is processed by **Haddock** more or less  
like any other module; but is effectively ignored by GHC itself.

```
let x = x in x
```

<https://haskell-lang.org/tutorial/primitive-haskell>

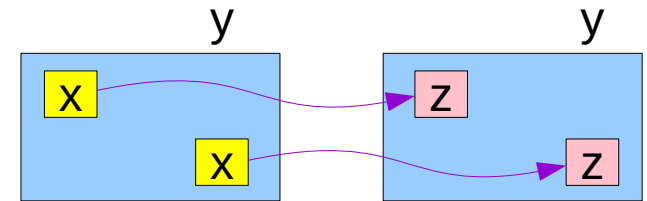
# let $x = z$ in $y$

**let  $x = z$  in  $y$**

change the variable  $x$  to the expression  $z$

wherever  $x$  occurs in the expression  $y$

Considered as the **reduction rule** for the **application** of the lambda abstraction  $\lambda x \rightarrow y$  to the **term**  $z$



<https://haskell-lang.org/tutorial/primitive-haskell>

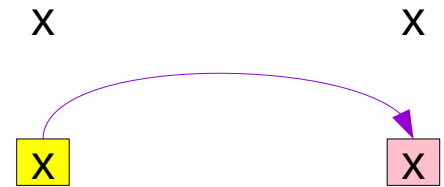
# let x = x in x

**let x = x in x**

these data declarations/functions are  
to provide access to the **raw compiler internals**.

**GHC.Prim** exists to export these primitives,  
it doesn't actually implement them or anything  
(eg its code isn't actually useful).  
All of that is done in the **compiler**.

It's meant for code that needs to be extremely optimized.



**let x = x in x**

<https://haskell-lang.org/tutorial/primitive-haskell>

# Primitive Types

## **Primitive (unlifted, unboxed) types**

cannot be defined in Haskell, and thus  
are built into the language and compiler.

**Primitive types** are always **unlifted**; that is,  
**bottom** cannot be a **value** of a **primitive type**

We use the convention  
that **primitive types**, **values**, and **operations**  
have a **# suffix**.

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Primitive Values

**Primitive values** are often represented by a simple bit-pattern, such as **Int#**, **Float#**, **Double#**.

But **Array#** is not necessarily the case: a **primitive value** might be represented by a **pointer** to a **heap-allocated object**. .... (Boxed)

Examples include **Array#**, the type of **primitive arrays**.

|                   | Primitive | Boxed | Lifted | Algebraic |
|-------------------|-----------|-------|--------|-----------|
| <b>Int#</b>       | Yes       | No    | No     | No        |
| <b>Array#</b>     | Yes       | Yes   | No     | No        |
| <b>(# a, b #)</b> | Yes       | No    | No     | Yes       |

Primitive types ≈

Unboxed types ≈

Unlifted types

**Int#** → **Primitive**  
**Array#** → **Boxed Arrays**  
**(# a, b #)** → **Unboxed Tuples**

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Primitive types are faster

```
--boxed.hs
fac :: Int -> Int
fac 0 = 1
fac n = n * fac (n - 1)

main = print (fac 10)

--unboxed.hs
import GHC.Exts
fac :: Int# -> Int#
fac 0# = 1#
fac n  = n *# fac (n -# 1#)

main = print (I# (fac 10#))
```

```
$ ghc boxed.hs
$ ghc -XMagicHash unboxed.hs
$ time ./boxed
$ time ./unboxed
```

The language extension `-XMagicHash` allows `"#"` as a **postfix modifier** to identifiers.

in `GHC.Exts`

**data Int**

A fixed-precision integer type with at least the range  $[-2^{29} .. 2^{29}-1]$ .  
The exact range by using `minBound` and `maxBound`

Constructors

**I# Int#**                      **I#(500#)**

**500# :: Int#**  
**I#(500#) :: Int**

can't compute `fac(500)` ... overflow

<https://moserei.de/2012/04/03/haskell-boxed-vs-unboxed.html>

# Restrictions on Primitive Types (1)

cannot pass a **primitive value** to a **polymorphic function** or  
cannot store a **primitive value** in a **polymorphic data type**.  
cannot use a **primitive value** in a **list type**.

**lists** of **primitive integers** are not possible : **[Int#]**

**polymorphic arguments** and **constructor fields**  
are assumed to be **pointers**:

Nevertheless, A **numerically-intensive** program  
using **unboxed types** can go a lot faster  
than its “standard” counterpart

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Restrictions on Primitive Types (2)

**polymorphic arguments** and **constructor fields**

are assumed to be **pointers**:

If an **unboxed** integer is used in such fields  
the **garbage collector** would attempt  
to follow an **unboxed** integer, *dereference*  
leading to unpredictable **space leaks**.

a **seq** operation on the **polymorphic component** may attempt  
to **dereference** the pointer, with disastrous results.

Even worse, the **unboxed value** might be larger than a **pointer**  
(Double# for instance).

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)



# Primitive Arrays

A **primitive array** is **heap-allocated** because it is too big a value to fit in a **register**, and would be too expensive to copy around;

in a sense, it is accidental that it is represented by a **pointer**.

If a **primitive value** is represented by a **pointer** ... *Array#*  
then the pointer really does point to that value

- no unevaluated thunks, no indirections...
- nothing can be at the other end of the **pointer** but the **primitive value**.

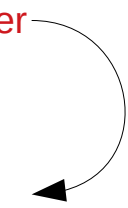
**Array#**

**Primitive**

**Boxed** ..... using a **pointer**

**Unlifted** ... no bottom

**primitive values**  
Int#, Float#, Double#



[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Primitive Arrays – `Array# obj` and `ByteArray#` (1)

A **primitive array** is **heap-allocated**

type `Array# obj`

**primitive arrays** of (**boxed**) Haskell objects `obj`

**Primitive**

**Boxed** .... use a pointer

**Unlifted** ... no bottom

type `ByteArray#`

**primitive arrays** of **bytes** ... similar to C arrays

**Primitive**

**Unboxed** .... no pointer

**Unlifted** ..... no bottom

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Primitive Arrays – Array# obj and ByteArray# (2)

A **primitive array** is **heap-allocated**

type **Array# obj**

**primitive arrays** of (**boxed**) Haskell objects **obj**

type **ByteArray#**

**primitive arrays** of **bytes** (no pointer)

|                   | Primitive | Boxed | Lifted | Algebraic |
|-------------------|-----------|-------|--------|-----------|
| <b>Int#</b>       | Yes       | No    | No     | No        |
| <b>ByteArray#</b> | Yes       | No    | No     | No        |
| <b>Array#</b>     | Yes       | Yes   | No     | No        |

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Primitive Arrays – `Array# obj` and `ByteArray#` (3)

A **primitive array** is **heap-allocated**

`type Array# obj` .... **boxed** Haskell objects `obj`  
`type ByteArray#` .... **unboxed bytes** (no pointer)

| <code>Array# obj</code>          | <code>ByteArray#</code>         |
|----------------------------------|---------------------------------|
| <b>Primitive</b>                 | <b>Primitive</b>                |
| <b>Boxed</b> ..... use a pointer | <b>Unboxed</b> ..... no pointer |
| <b>Unlifted</b> ... no bottom    | <b>Unlifted</b> ... no bottom   |

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# Boxed Arrays

GHC **heap** contains two kinds of objects  
some are just byte sequences,  
other contains pointers to other objects (so called "boxes").

These segregation allows to find chains of references  
when performing **garbage collection** and **update** these pointers  
when memory used by heap is compacted and  
objects are moved to new places.

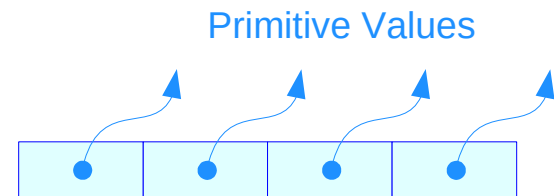
**Internal (raw)** GHC's type **Array#** represents  
a sequence of object **pointers (boxes)**.

The **Array#** type is used inside **Array** type  
which represents **boxed immutable arrays**.

Therefore,

|                       |                   |
|-----------------------|-------------------|
| <b>Unboxed Arrays</b> | <b>ByteArray#</b> |
| <b>Boxed Arrays</b>   | <b>Array#</b>     |

**Array#** : sequence of pointers (boxes)



<https://en.wikibooks.org/wiki/Haskell/Libraries/Arrays>

# Unboxed Arrays

**Unboxed** arrays (defined in `Data.Array.Unboxed`)

are more like `arrays in C`

they contain just the **plain values**

without the extra level of **indirection**, ... no pointer (box)

for example, an array of 1024 values of type `Int32`  
will use only 4 kb of memory.

- **indexing** of such arrays can be significantly **faster**.
- only of plain values having a **fixed size**
- must be **evaluated** when the array is evaluated

**Unboxed arrays** are represented by the `ByteArray#` type

<https://en.wikibooks.org/wiki/Haskell/Libraries/Arrays>.

<https://wiki.haskell.org/Arrays>

# Array#

**Array#** is more primitive than a Haskell array

- an **Array#** is indexed only by **Int#**s, starting at zero.
- **unboxed** but is a **heap allocated** object
- **unboxed** but is represented by
  - a **pointer** to the array itself
  - not to a **thunk** or to **bottom**
- the components of an **Array#** are themselves are **boxed**

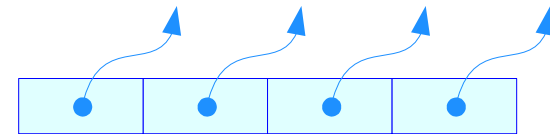
the Haskell Array interface is implemented using Array#

The type **Array# obj** is

the type of **primitive, unpointed arrays** of values of type **obj**.

**Array#** : sequence of pointers (**boxes**)

Points to value itself  
No thunks no bottom  
**Unboxed elements**



**Boxed arrays**  
sequence of pointers

[https://downloads.haskell.org/~ghc/5.04.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/5.04.1/docs/html/users_guide/primitives.html)

# ByteArray#

**Unboxed arrays** are represented by the **ByteArray#** type.

It's just a plain memory area in the heap, like the C's array.

**ByteArray#** is **unboxed** but **unlifted**

There are two primitive **operations**

that creates a **ByteArray#** of specified size

**newByteArray**

**newPinnedByteArray**

<https://en.wikibooks.org/wiki/Haskell/Libraries/Arrays>



# ByteArray# – normal heap

One **primitive operation** allocates memory in **normal heap** and so this **byte array** can be moved each time when garbage collection occurs.

This prevents converting of **ByteArray#** to **plain memory pointer** that can be used in C procedures

although it's still possible to pass current **ByteArray# pointer** to "unsafe foreign" procedure if it don't try to store this **pointer** somewhere

`newByteArray`

<https://en.wikibooks.org/wiki/Haskell/Libraries/Arrays>

# ByteArray# – pinned heap area

The second **primitive operation** `allocates ByteArray#` of specified size in **pinned heap** area, which contains objects with fixed place.

Such byte array will never be moved by garbage collection so its address can be used as plain Ptr and shared with **C world**.

`newPinnedByteArray`

<https://en.wikibooks.org/wiki/Haskell/Libraries/Arrays>

# Unboxed tuple – multiple return value

**(# e\_1, ..., e\_n #)**

**e\_1 .. e\_n** are **expressions** of any type (**primitive** or **non-primitive**).

**Unboxed tuples** are used for

**functions** that need to *return multiple values*,

but they avoid the **heap allocation** of

fully-fledged **tuples** (**boxed real tuple**)

when an **unboxed tuple** is returned,

the **components** are put *directly*

*into registers or on the stack*;

[https://downloads.haskell.org/~ghc/7.0.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/7.0.1/docs/html/users_guide/primitives.html)

# Unboxed tuple – no heap allocation

the **unboxed tuple** itself

does not have a **composite representation**.

no tuples within tuples representation

Many of the **primitive operations**

return **unboxed tuples**.

In particular, the **IO** and **ST monads**

use **unboxed tuples**

to avoid unnecessary allocation

during sequences of operations.

[https://downloads.haskell.org/~ghc/7.0.1/docs/html/users\\_guide/primitives.html](https://downloads.haskell.org/~ghc/7.0.1/docs/html/users_guide/primitives.html)

# Unboxed tuple examples

```
newtype IO a = IO (State# RealWorld -> (# State# RealWorld, a #))
```

The first primitive is the **unboxed tuple**, seen in code as **(# x, y #)**.

1. **State# RealWorld**
2. **a**

a **multiple value return** syntax

But not actual **real tuples** and  
can't be put in **variables** as such.

**Boxed real tuple** incurs **heap allocation**  
whenever an **IO action** is performed,

<http://blog.ezyang.com/2011/05/unraveling-the-mystery-of-the-io-monad/>

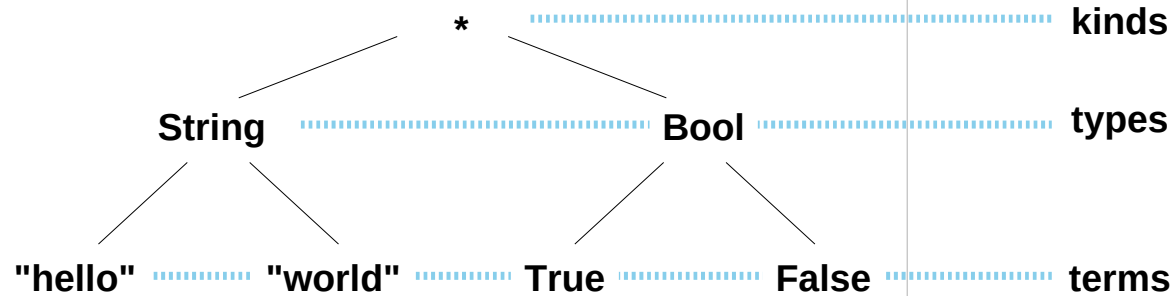
# Types and Kinds

Just like **values / terms** can be classified into **types**,  
**types** can be classified into **kinds**.

The **values** "hello" and "world" are of **type** **String**.

The **values** **True** and **False** are of **type** **Bool**.

Similarly, the **types** **String** and **Bool** are  
of **kind** **\***, pronounced "star".



<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# \* kind

**\***, pronounced "**star**", is

the **kind** of **all data types**  
seen as **nullary type constructors**, and  
also called **proper types** in this context.

this normally includes **function types**

all lifted inhabited type

**Inhabitable  
Lifted type**

bottom

**kind \***

**Inhabitable  
Unlifted type**

no bottom

**kind #**

[https://en.wikipedia.org/wiki/Kind\\_\(type\\_theory\)](https://en.wikipedia.org/wiki/Kind_(type_theory))

# :type and :kind

**:t** or **:type** to check the **type** of a term

**:k** or **:kind** to check the **kind** of a type.

**λ> :t True**

```
True :: Bool
```

```
Term  :: Type  
(value)
```

**λ> :k Bool**

```
Bool :: *
```

```
Type  ;; Kind
```

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>



# Kind encode type representation

Kinds are like types for types

lifted inhabitable types have the **kind \***

```
'c' :: Char :: *
```

```
Just 1 :: Maybe Int :: *
```

Type constructors, on the other kind, contain the **arrow symbol**

```
Maybe :: * -> *
```

```
Either :: * -> * -> *
```

Unlifted types are of the **# kind**

```
'c'# :: Char # :: #
```

**Inhabitable  
Lifted type**

bottom

**kind \***

**Inhabitable  
Unlifted type**

no bottom

**kind #**

Haskell High Performance Programming,, Samuli Tomason, 2016

# Inhabited types

In standard Haskell, all **inhabited types**  
(types that have **at least 1 value**) are of **kind \***

**Int**

**Int -> String**

**[Int]**

**Maybe Int**

**Either Int Int**

each of these types has **at least one term**  
therefore all these types are of **kind \***

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# Uninhabited types

**Maybe** and **Either** are **uninhabited**.

But they are **type constructors**

There is **no term** of type **Maybe**, not even the **infinite loop!**

```
λ> x = undefined :: Maybe
```

```
<interactive>:9:18: error
```

- Expecting **one more argument** to 'Maybe'

```
λ> f x = f x :: Maybe
```

```
<interactive>:10:14: error:
```

- Expecting **one more argument** to 'Maybe'

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# Terms

Just as **expressions** denote values,  
**type expressions** are **syntactic terms**  
that denote **type values** (or just **types**).

Examples of **type expressions** include

the **atomic types**

**Integer** (infinite-precision integers), **Char** (characters),

**Integer->Integer** (functions mapping Integer to Integer),

the **structured types**

**[Integer]** (homogeneous lists of integers) and

**(Char,Integer)** (character, integer pairs).

<https://www.haskell.org/tutorial/goodies.html>

# Type Constructors with type arguments

A **type constructor** takes one or more **type arguments**, and produces a **data type** when enough arguments are supplied, i.e. it supports **partial application** thanks to **currying**.

This is how Haskell achieves **parametric types**.

For instance, the **type []** is a **type constructor** - it takes a **single argument** to specify the type of the elements of the list.

Hence, **[Int]**, **[Float]** and even **[[Int]]** are valid applications of the **[] type constructor**.

[https://en.wikipedia.org/wiki/Kind\\_\(type\\_theory\)](https://en.wikipedia.org/wiki/Kind_(type_theory))

# Type Constructors and data constructors

a **nullary / unary type constructor** (or simply a **type**).

has zero / one argument

**data Bool = True | False**

a nullary type constructor ... **Bool**

two nullary data constructors ... **True** and **False**

**data Tree a = Tip | Node a (Tree a) (Tree a)**

a unary type constructor ... **Tree**

a nullary data constructors ... **Tip**

a unary data constructors ... **Node**

<https://wiki.haskell.org/Constructor>

# Data constructors - first class values

## Data constructors

are **first class values** in Haskell and actually have a **type**.

For instance, the type of the **Left constructor** of the **Either data type** is:

```
data Either a b = Left a | Right b
Left :: a -> Either a b
```

As **first class values**, they may be

- passed to functions,
- held in a list,
- data elements of other algebraic data types, and so forth.

<https://wiki.haskell.org/Constructor>

# Data constructors – not types

**Data constructors** are not **types**  
they denote **values**.

**Node a** (~~**Node a**~~) (~~**Node a**~~)

It is illegal because the **type** is **Tree**, not **Node**.

```
data Tree a = Tip | Node a (Tree a) (Tree a)
```

<https://wiki.haskell.org/Constructor>



# Type constructors and Kinds

$* \rightarrow *$  is the **kind** of a **unary type constructor**,  
e.g. of a **list type constructor**.

$* \rightarrow * \rightarrow *$  is the **kind** of a **binary type constructor** (via currying),  
e.g. of a **pair type constructor**, and also  
that of a **function type constructor**  
(not to be confused with the result of its **application**,  
which itself is a function type, thus of kind  $*$ )

$( * \rightarrow * ) \rightarrow *$  is the **kind** of a **higher-order type operator**  
from **unary type constructors** to **proper types**.

[https://en.wikipedia.org/wiki/Kind\\_\(type\\_theory\)](https://en.wikipedia.org/wiki/Kind_(type_theory))

# Kind examples (1)

Haskell's kind system has just two rules:

\* pronounced "**type**" is the **kind** of all *lifted* data types.

$k1 \rightarrow k2$  is the kind of a **unary type constructor**,  
which takes a type of kind  $k1$  and  
produces a type of kind  $k2$

[https://en.wikipedia.org/wiki/Kind\\_\(type\\_theory\)](https://en.wikipedia.org/wiki/Kind_(type_theory))

# Kind examples (2)

$[]$  is a **type** of **kind**  $* \rightarrow *$  .

Because **Int** has **kind**  $*$  ,  
applying **type constructor**  $[]$  to it  
results in **[Int]**, of **kind**  $*$  .

The **2-tuple constructor**  $( , )$  has kind  $* \rightarrow * \rightarrow *$  ,

the **3-tuple constructor**  $( , , )$  has kind  $* \rightarrow * \rightarrow * \rightarrow *$  and so on.

[https://en.wikipedia.org/wiki/Kind\\_\(type\\_theory\)](https://en.wikipedia.org/wiki/Kind_(type_theory))

# Inhabited types with kind \*

An **inhabited type**

a type which has values.

a so called **proper types** in Haskell)

For instance, ignoring type classes

**4** is a **value** of type **Int**,

**[1, 2, 3]** is a **value** of type **[Int]** (list of Ints).

all **inhabited lifted types** are of **kind \***

**Int** and **[Int]** have **kind \***

any function type has **kind \***

for instance **Int -> Bool** or even **Int -> Int -> Bool**.

[https://en.wikipedia.org/wiki/Kind\\_\(type\\_theory\)](https://en.wikipedia.org/wiki/Kind_(type_theory))

# Inhabited types with kind #

all inhabited *lifted* types are of **kind \***

\* is the **kind** of all inhabited **boxed** (or **lifted**) types.

However, in GHC's version of Haskell,  
there are also some **inhabited types**  
that are not of **kind \***  
**unboxed / unlifted / primitive types**  
are of **kind #**

Q these are defined in the **GHC.Prim** module  
from the ghc-prim package.

**Inhabitable  
Lifted type**

bottom

**kind \***

**Inhabitable  
Unlifted type**

no bottom

**kind #**

**Primitive** ≈

**Unboxed** ≈

**Unlifted types**

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# \* kind and # kind

So `ByteArray#`, the type of raw blocks of memory, is ~~boxed~~ because it is represented as a ~~pointer~~, but **unlifted** because **bottom** is not an element. unboxed

```
> undefined :: ByteArray#
```

```
Error: Kind incompatibility when matching types:
```

```
  a0 :: *
```

```
  ByteArray# :: #
```

Therefore it appears that the old User's Guide definition is more accurate than the GHC Commentary one:

**\*** is the **kind of lifted types**.

(And, conversely, **#** is the **kind of unlifted types**.)

**Unboxed Arrays**

**ByteArray#**

**Boxed Arrays**

**Array#**

<https://stackoverflow.com/questions/27095011/what-exactly-is-the-kind-in-haskell>

# Kind and runtime representation

Each **unlifted type** has a **kind**  
that describes its **runtime representation**.

*Is this a **pointer** to something in the **heap**?*

*Is it a signed/unsigned **word-sized value**?*

The compiler then uses that **type's kind**  
to decide which **machine code** it needs to produce -  
this is called "**kind-directed compilation**".

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# Runtime representation of values

GHC maintains a property that  
the **kind** of all **inhabited types** tells us  
the **runtime representation of values of that type**.  
(as distinct from **type constructors** or **type-level data**)

## Inhabited types – instance

**kind** tells the runtime representation  
of values of that type

<http://hackage.haskell.org/package/base-4.12.0.0/docs/GHC-Exts.html#:MutVar-35->



# Unified types and kinds

Starting with GHC8, **types** and **kinds** have been unified.

a single indexed type of types

```
data TYPE a                :: RuntimeRep -> *  
  
data TYPE (a :: RuntimeRep) :: RuntimeRep -> Type
```

a single type  
indexed by a

a kind

\* -> \*

Haskell High Performance Programming,, Samuli Tomason, 2016

# The **kind \*** - the synonym **Type**

Recently, the **kind \*** is often referred to as **Type**  
(do not confuse with **TYPE r**).

these are **synonyms** for now,  
and the plan is to gradually phase out **\*** in favour of **Type**.

```
data TYPE a                :: RuntimeRep -> *  
data TYPE (a :: RuntimeRep) :: RuntimeRep -> Type
```

here, all inhabited types are of **kind \***  
not just inhabited lifted types

Old usage

```
Kind *  
    for all lifted inhabitable types  
Kind #  
    for all unlifted inhabitable types
```

Recent usage

```
Kind * or Type  
    for all inhabitable types  
    either lifted or unlifted
```

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# Kind **TYPE r**

**TYPE IntRep** has the **kind** of **unlifted integers**,  
**TYPE FloatRep** has the **kind** of **unlifted floats**, etc.  
**TYPE LiftedRep** has the **kind** for **all lifted types** -  
in fact, the **\* kind** is nothing more than a synonym for **TYPE LiftedRep**

**TYPE r** enables us to abstract  
not only over all **unlifted** types,  
but also over **lifted** ones.

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# Kind **TYPE** LiftedRep

```
True :: Bool           :t True   to check the type of a term
Bool :: *              :k Bool   to check the kind of a type.
```

```
data TYPE (a :: RuntimeRep) :: RuntimeRep -> Type
```

```
type Type = TYPE LiftedRep
```

The **kind** of **types** with **lifted values**. For example

```
Int :: Type
```

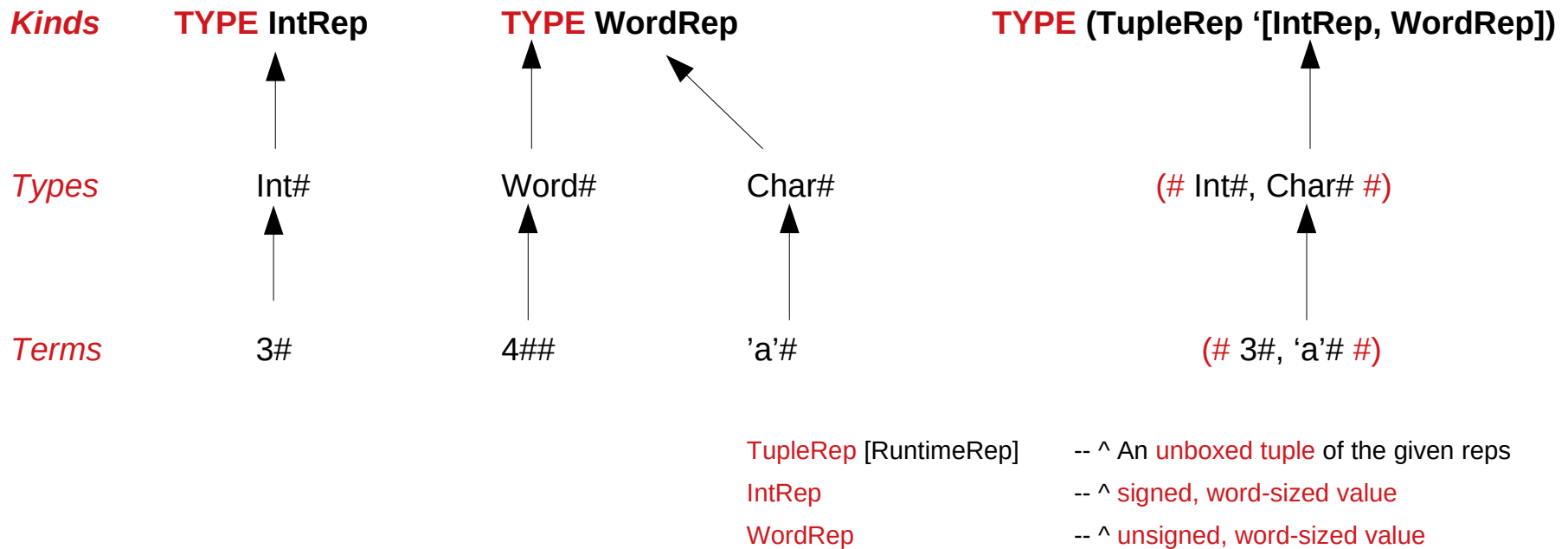
```
Int :: TYPE LiftedRep
```

*type :: kind*  
*term :: type*

<https://hackage.haskell.org/package/ghc-prim-0.6.1/docs/GHC-Types.html#v:LiftedRep>

# Inhabited types with kind **TYPE r**

Here are some examples:



<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# RuntimeRep – TYPE

This datatype encodes **the choice of runtime value**.

Note that **TYPE** is parameterised by **RuntimeRep**;

```
data TYPE a                :: RuntimeRep -> *
data TYPE (a :: RuntimeRep) :: RuntimeRep -> Type
type Type = TYPE LiftedRep
```

this is precisely what we mean by the fact  
that a **type's kind** encodes the **runtime representation**.

A **type synonym** is a new name for an existing type.

```
type MyChar = Char
```

<http://hackage.haskell.org/package/base-4.12.0.0/docs/GHC-Exts.html#:MutVar-35->

# The data type `Type`

The single data type `Type` is used to represent

- **types** (possibly of **higher kind**);  
e.g. `[Int]`, `Maybe`
- **kinds** (which classify **types** and **coercions**);  
e.g. `(* -> *)`, `T ::= [Int]`.
- **sorts** (which classify types);  
e.g. `TY`, `CO`

<https://gitlab.haskell.org/ghc/ghc/-/wikis/commentary/compiler/type-type>

# Kind Rationale

Haskell has a very powerful and expressive **static type system**.

The Haskell kind system has been extended to overcome an unsatisfactorily inexpressiveness in **programming at the type level**,

<https://gitlab.haskell.org/ghc/ghc/-/wikis/kind-system>

Note: As of June 2013, this page is rather out of date.

This page is currently a WIP ..



# Tools for programming in type level

- |                                   |                            |
|-----------------------------------|----------------------------|
| • <b>Data constructors</b>        | <b>Type constructors</b>   |
| • <b>Type signatures</b>          | <b>Kind signatures</b>     |
| • <b>High Order Functions</b>     | <b>Higher Kinded Types</b> |
| • <b>Other kinds except *</b>     |                            |
| • <b>Unboxed / Unlifted types</b> |                            |
| • <b>Constraints</b>              |                            |
| • <b>Datatype Promotion</b>       |                            |
| • <b>GHC.TypeList</b>             |                            |
| • <b>Kind polymorphism</b>        |                            |
| • <b>Levity polymorphism</b>      |                            |

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# RuntimeRep – constructors

|  |   |
|--|---|
| <b>VecRep</b> <b>VecCount</b> <b>VecElem</b> | a SIMD vector type                      |
| <b>TupleRep</b> [ <b>RuntimeRep</b> ]        | An unboxed tuple of the given reps      |
| <b>SumRep</b> [ <b>RuntimeRep</b> ]          | An unboxed sum of the given reps        |
| <b>LiftedRep</b>                             | lifted; represented by a pointer        |
| <b>UnliftedRep</b>                           | unlifted; represented by a pointer      |
| <b>IntRep</b>                                | signed, word-sized value                |
| <b>WordRep</b>                               | unsigned, word-sized value              |
| <b>Int64Rep</b>                              | signed, 64-bit value (on 32-bit only)   |
| <b>Word64Rep</b>                             | unsigned, 64-bit value (on 32-bit only) |
| <b>AddrRep</b>                               | A pointer, but not to a Haskell value   |
| <b>FloatRep</b>                              | a 32-bit floating point number          |
| <b>DoubleRep</b>                             | a 64-bit floating point number          |

<http://hackage.haskell.org/package/base-4.12.0.0/docs/GHC-Exts.html#:MutVar-35->

# Kind **TYPE** **r**

the kind **TYPE** **r**

this kind is *parameterised* over **r :: RuntimeRep**,

**RuntimeRep**

describes a type's **runtime representation**

can be one of the following:

```
data RuntimeRep = VecRep VecCount VecElem      -- a SIMD vector type
  | TupleRep [RuntimeRep]  -- An unboxed tuple of the given reps
  | SumRep [RuntimeRep]   -- An unboxed sum of the given reps
  | LiftedRep             -- lifted; represented by a pointer
  | UnliftedRep         -- unlifted; represented by a pointer
  | IntRep               -- signed, word-sized value
  | WordRep             -- unsigned, word-sized value
  | Int64Rep            -- signed, 64-bit value (on 32-bit only)
  | Word64Rep          -- unsigned, 64-bit value (on 32-bit only)
  | AddrRep            -- A pointer, but not to a Haskell value
  | FloatRep           -- a 32-bit floating point number
  | DoubleRep         -- a 64-bit floating point number
```

<https://diogocastro.com/blog/2018/10/17/haskells-kind-system-a-primer/>

# RuntimeRep – boxed values

```
data Array# (a :: Type)    :: Type -> TYPE UnliftedRep
data ByteArray#           :: TYPE UnliftedRep
data Char#                :: TYPE WordRep
data Double#              :: TYPE DoubleRep
data Float#               :: TYPE FloatRep
data Int#                  :: TYPE IntRep
data Int32#                :: TYPE IntRep
data Int64#                :: TYPE Int64Rep

data TYPE (a :: RuntimeRep) :: RuntimeRep -> Type
```

<http://hackage.haskell.org/package/base-4.12.0.0/docs/GHC-Exts.html#:MutVar-35->

# Closure (1)

A **closure**, the opposite of a **combinator**, is a function that makes use of **free variables** in its **definition**. It '**closes**' around some portion of its environment. for example

```
f x = (λy -> x + y)
```

**f** returns a **closure**, because the variable **x**, which is **bound outside** of the **lambda** abstraction is **used inside** its **definition**.

An interesting side note: the context in which **x** was bound shouldn't even exist anymore, and wouldn't, had the lambda abstraction not closed around x.

<https://stackoverflow.com/questions/39985296/what-are-lifted-and-unlifted-product-types-in-haskell>

# Closure (2)

```
mkAdder :: Int -> (Int -> Int)
```

```
mkAdder y = \x -> x + y
```

**mkAdder** takes an **Int** as an **argument**, and returns a function (**Int -> Int**) as a result.

the returned function  $\lambda x \rightarrow x + y$  has a **free variable** (**y**) which refers to its **environment**.

calling **mkAdder** with a particular **argument** (say, **3**), returns a **closure**, containing the function  $\lambda x \rightarrow x + y$  together with the environment (**y = 3**).

<https://mail.haskell.org/pipermail/beginners/2009-July/002067.html>

# Closure (3)

**mkAdder** is really just (+), written in a funny way!  
So this isn't a contrived example;  
closures are quite fundamental in Haskell.

<https://mail.haskell.org/pipermail/beginners/2009-July/002067.html>

# Combinator (1)

There are two distinct meanings of the word "combinator"

The first is a narrow, technical meaning, namely:

A function or definition with no free variables.

A "function with **no free variables**" is a **pure lambda-expression** that refers only to its **arguments**, like

**$\lambda a \rightarrow a$**

**$\lambda a \rightarrow \lambda b \rightarrow a$**

**$\lambda f \rightarrow \lambda a \rightarrow \lambda b \rightarrow f\ b\ a$**

and so on. The study of such things is called combinatory logic.

<https://wiki.haskell.org/Combinator>



# Combinator (1)

The second meaning of "combinator" is a more informal sense referring to the combinator pattern, a style of organizing libraries centered around the idea of combining things.

This is the meaning of "combinator" which is more frequently encountered in the Haskell community.

Usually there is some type  $T$ , some functions for constructing "primitive" values of type  $T$ , and some "combinators" which can combine values of type  $T$  in various ways to build up more complex values of type  $T$ .

<https://wiki.haskell.org/Combinator>

# Let binding

A **let binding** is very similar to a **where binding**.

A **where binding** is a syntactic construct that binds variables at the end of a function and the whole function (or a whole pattern-matching subpart) can see these variables, including all the guards

A **let binding** binds variables anywhere and is an expression itself, but its scope is tied to where the let expression appears.

So if it's defined within a guard, its scope is **local** and it will not be available for another guard.

But it can also take **global** scope over all pattern-matching clauses of a function definition if it is defined at that level.

<https://chercher.tech/haskell/let-bindings>

# Case expression

A case expression must have at least one alternative  
and each alternative must have at least one body.  
Each body must have the same type,  
and the type of the whole expression is that type.

```
aaa x = case x of
```

```
  1 -> "A"
```

```
  2 -> "B"
```

```
  3 -> "C"
```

Input: aaa 3

Output: "C"

[http://zvon.org/other/haskell/Outputsyntax/caseQexpressions\\_reference.html](http://zvon.org/other/haskell/Outputsyntax/caseQexpressions_reference.html)

# Polymorphism

A **value** is **polymorphic** if there is more than one type it can have. Polymorphism is widespread in Haskell and is a key feature of its type system.

**Parametric polymorphism** refers to when the **type** of a **value** contains one or more (unconstrained) **type variables**, so that the **value** may adopt any type that results from substituting those variables with **concrete types**.

**Ad-hoc polymorphism** refers to when a **value** is able to adopt any one of several types because it, or a value it uses, has been given a **separate definition** for each of those types.

<https://wiki.haskell.org/Polymorphism>

# Parametric Polymorphism

the function **id** :: **a -> a**

- contains an **unconstrained type variable** **a**

the empty list **[]** :: **[a]** belongs to every list type

the **polymorphic function** **map** :: **(a -> b) -> [a] -> [b]**

may operate on any function type.

if a **type variable** appears multiple times,

it must take the same type everywhere it appears,

the **result type** of **id** must be the same as the **argument type**,

the **input** and **output types** of the **function** given to **map**

must match up with the **list types**.

**id** :: **a -> a**

**Char -> Char**

**Integer -> Integer**

**(Bool -> Maybe Bool) ->**

**(Bool -> Maybe Bool)**

<https://wiki.haskell.org/Polymorphism>

# Ad-hoc Polymorphism

For example, the **+** **operator** essentially does something entirely different when applied to **floating-point values** as compared to when applied to **integer values**

Most languages support at least some ad-hoc polymorphism,

if a **type** can be compared for equality then an **instance declaration** of the **Eq** class is given if the behaviour of the **== operator** on the given **type** is specified, all sorts of functions defined using that operator can be accessed checking if a value of the type is present in a list, or looking up a corresponding value in a list of pairs.

<https://wiki.haskell.org/Polymorphism>

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## References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>