

Tapped Delay

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Based on

Introduction to Signal Processing

S. J. Ofranidis

The necessities in DSP C Programming

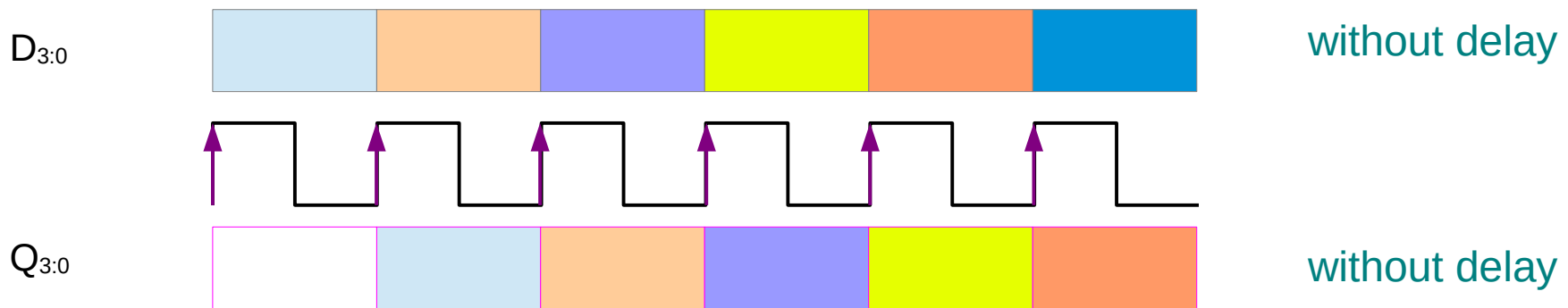
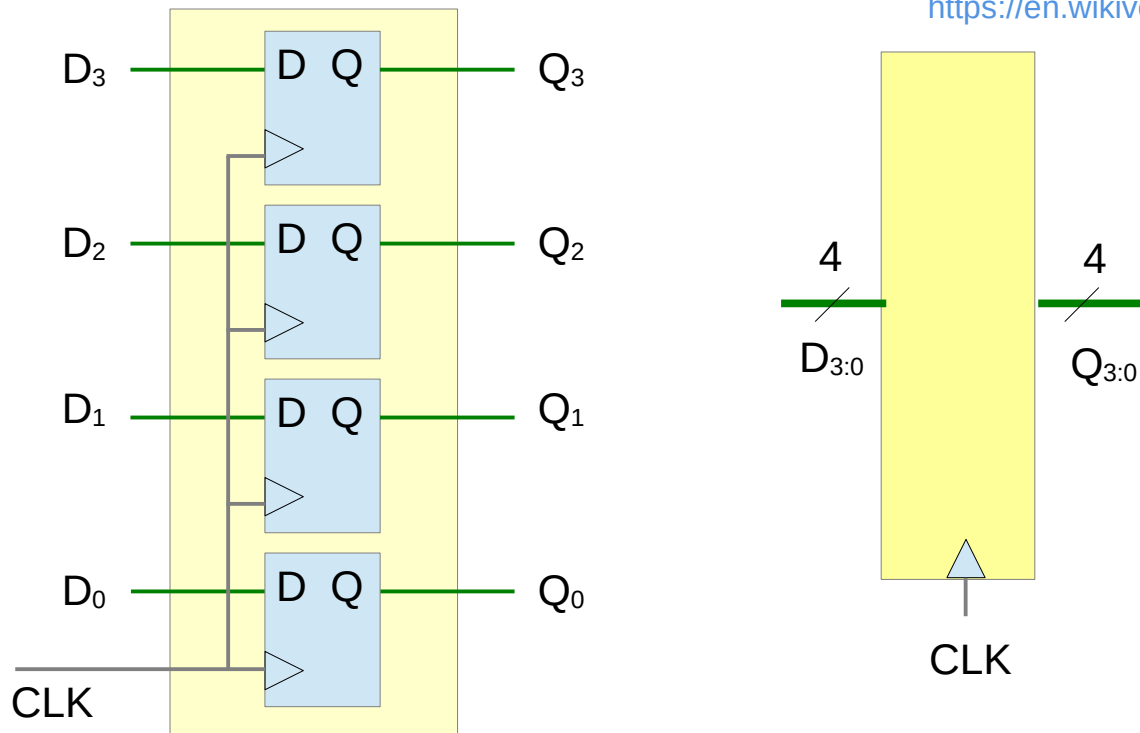
FIR Filter (A.pdf) 20191114

D Flip Flop

Considering the widely used
Edge triggered
D-type Flip Flops

Register

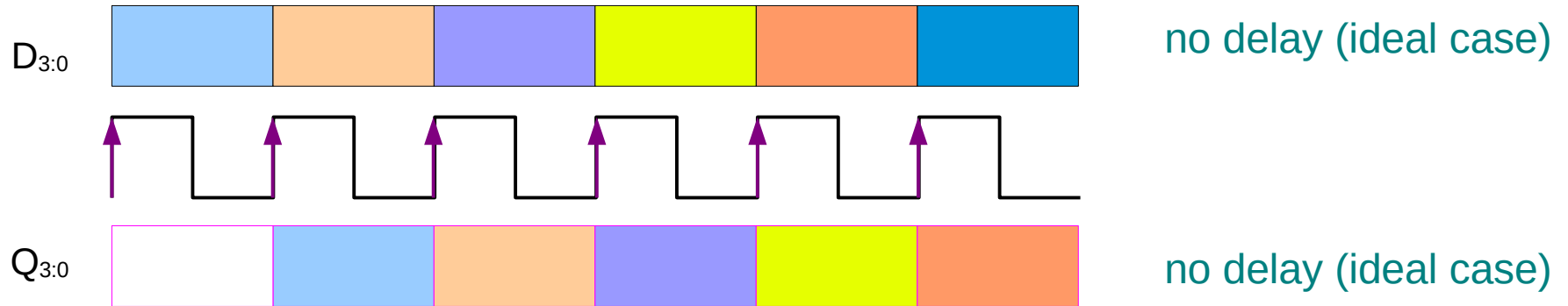
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



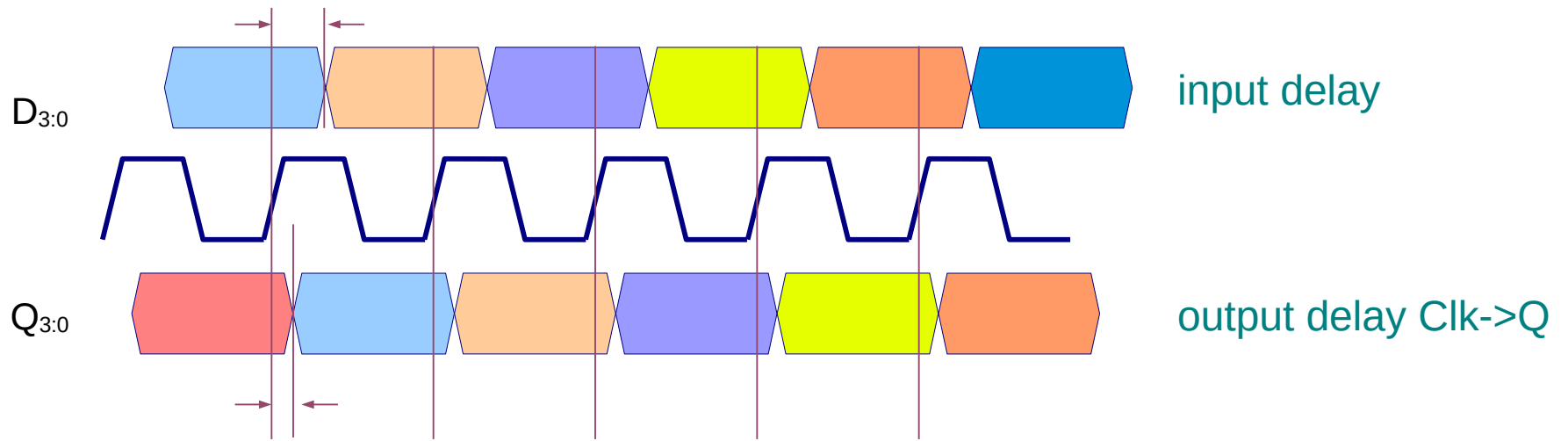
Types of Timing Diagrams

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

a timing diagram without delays

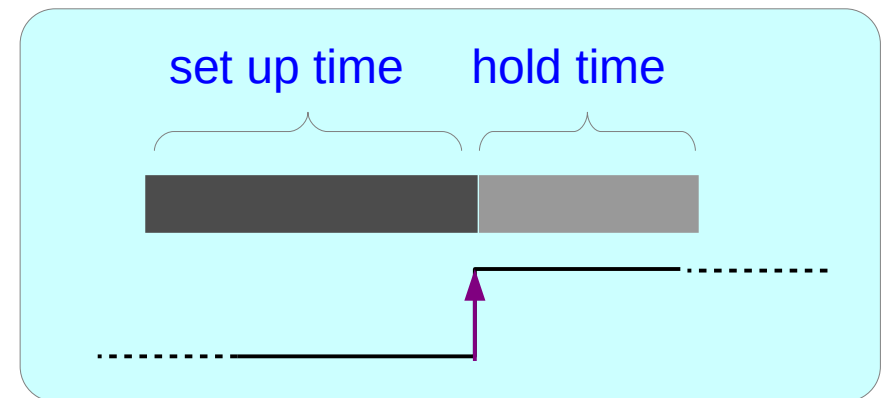
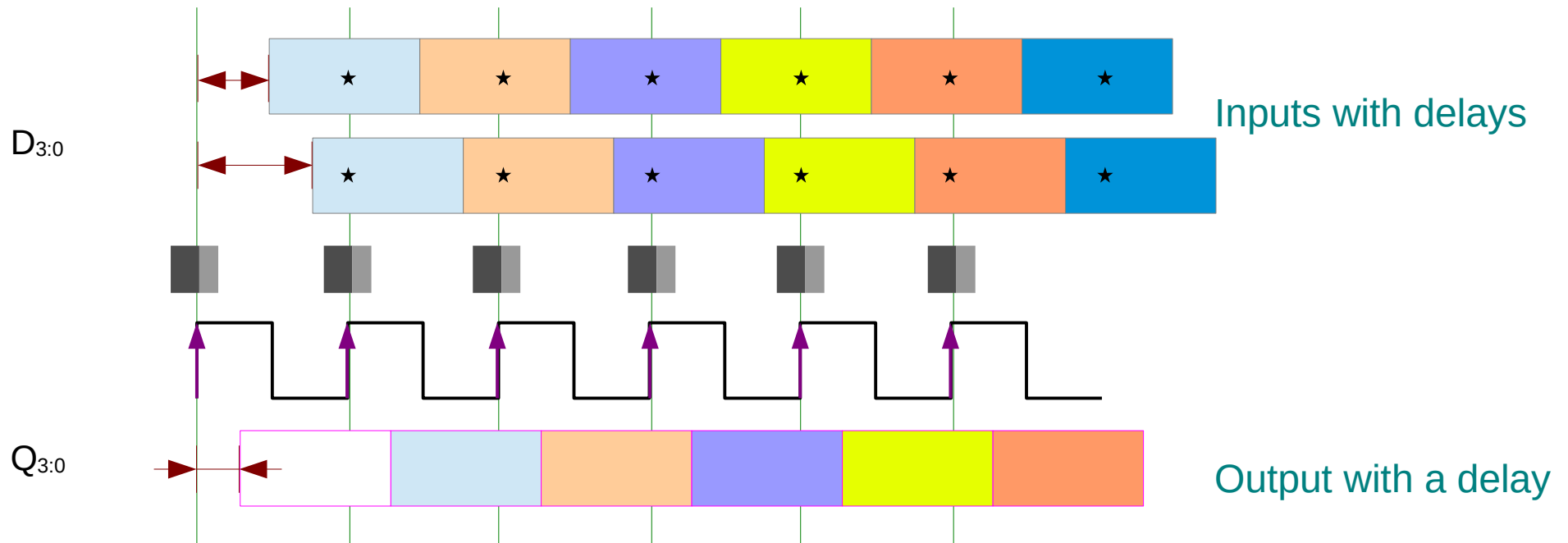


a timing diagram with delays



Setup & Hold Time (1)

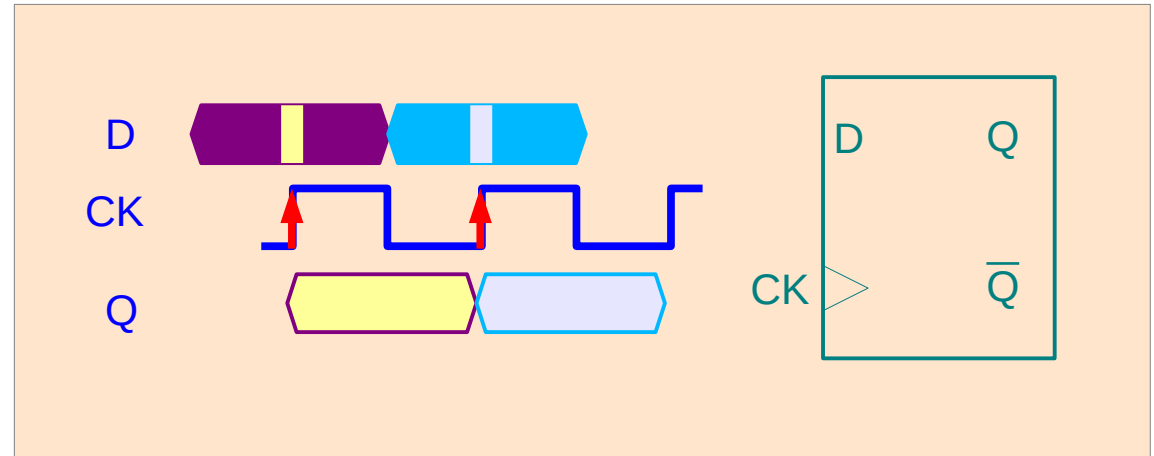
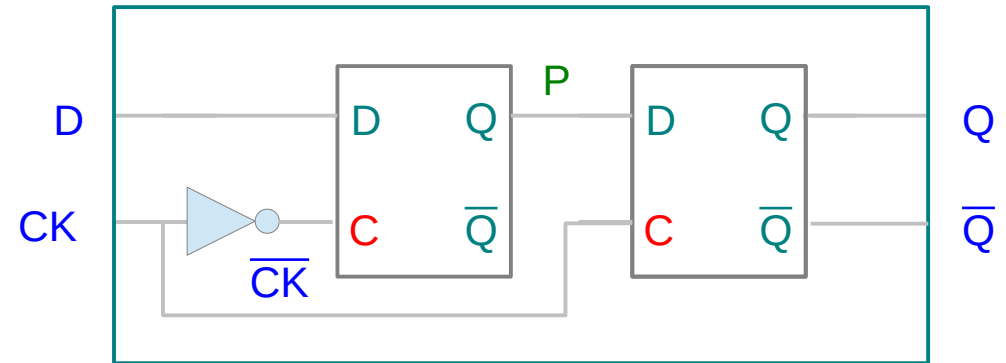
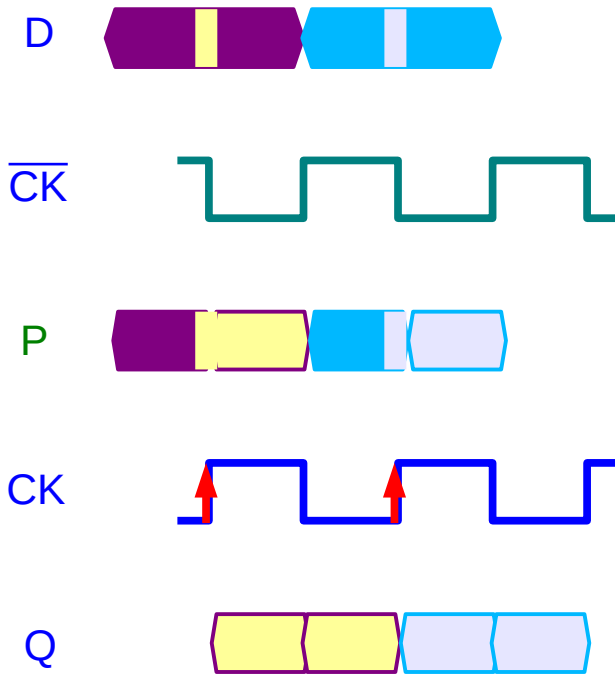
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



Master-Slave D FlipFlop – Rising Edge

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

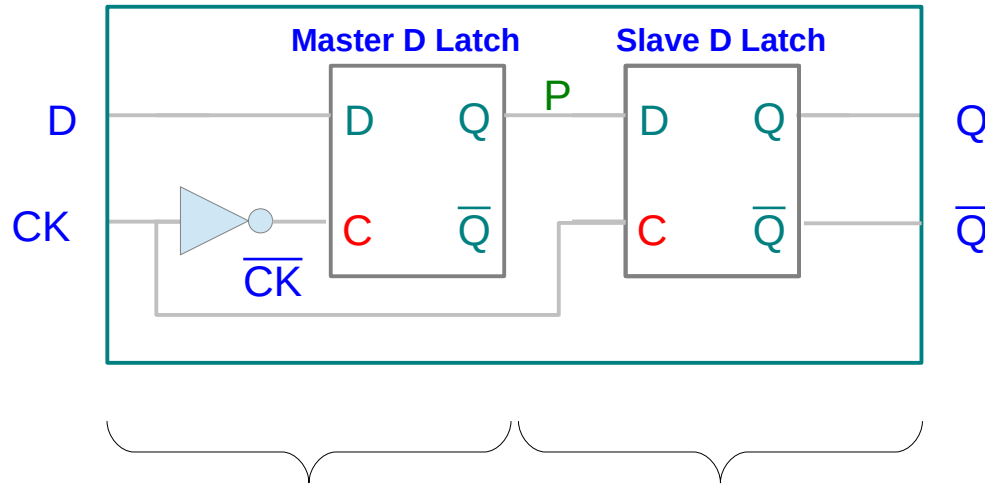
Master D Latch



Slave D Latch

Master-Slave D FlipFlop – Rising Edge

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

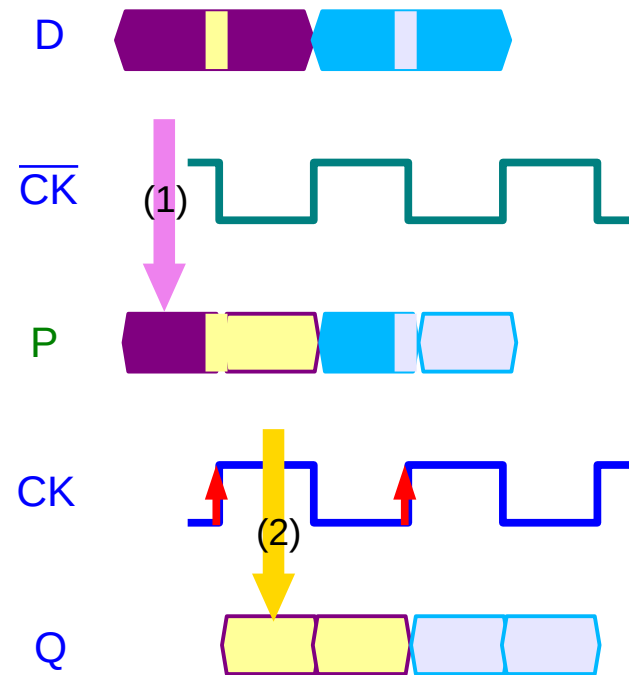


(1) the current input D gets stored in the master latch

(2) the current content P is clocked out to the output Q

Using **inverted clocks enable** (1) and (2) to be executed sequentially

Master D Latch

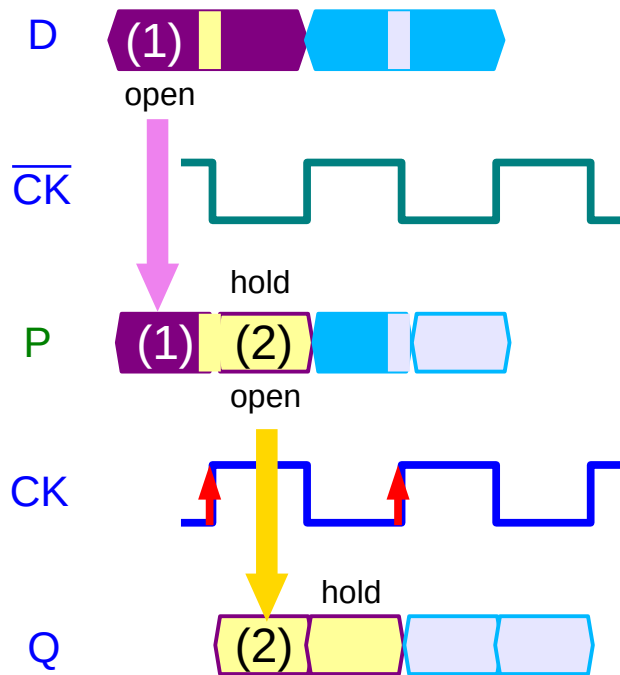


Slave D Latch

Master-Slave D FlipFlop – open and hold

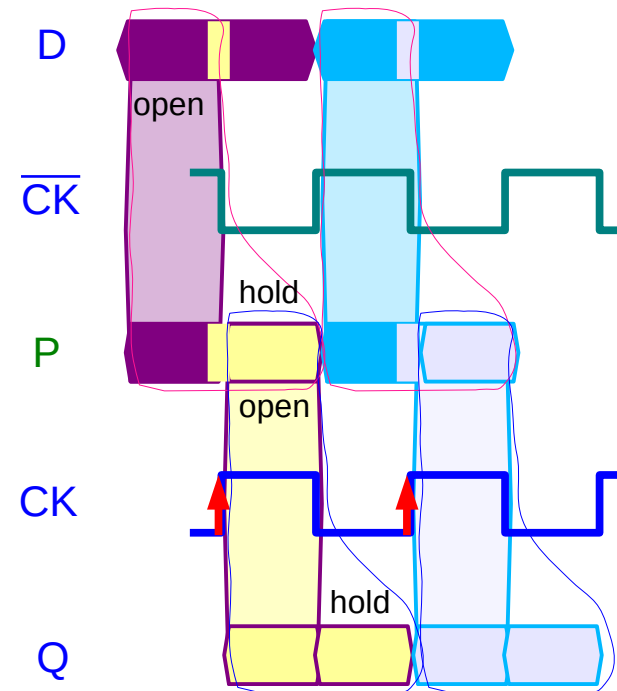
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

Master D Latch



Slave D Latch

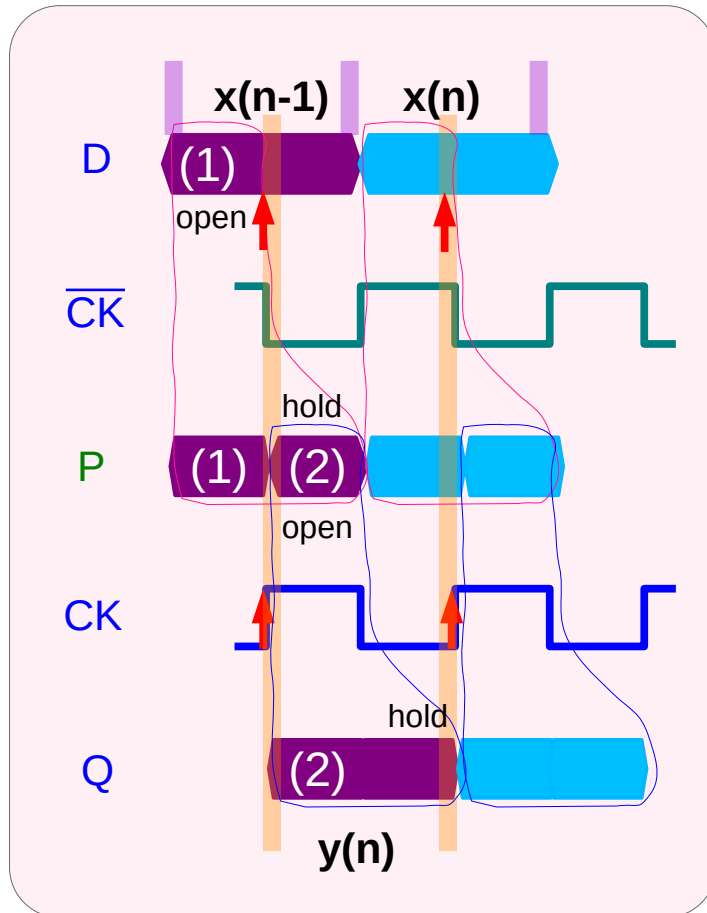
Master D Latch



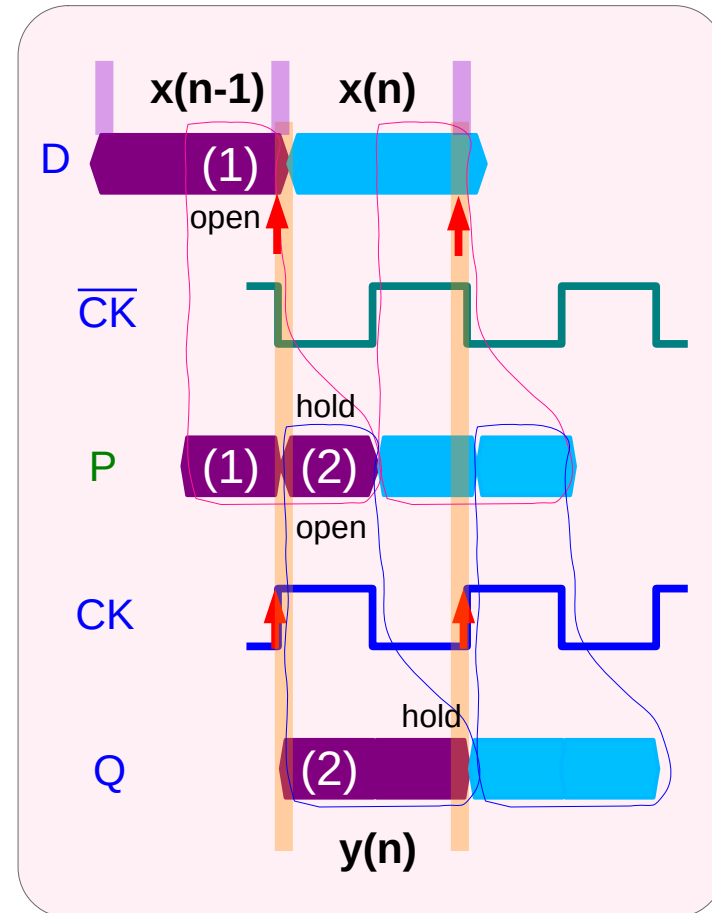
Slave D Latch

Master-Slave D FlipFlop – typical vs. ideal timing (1)

Typical Timing

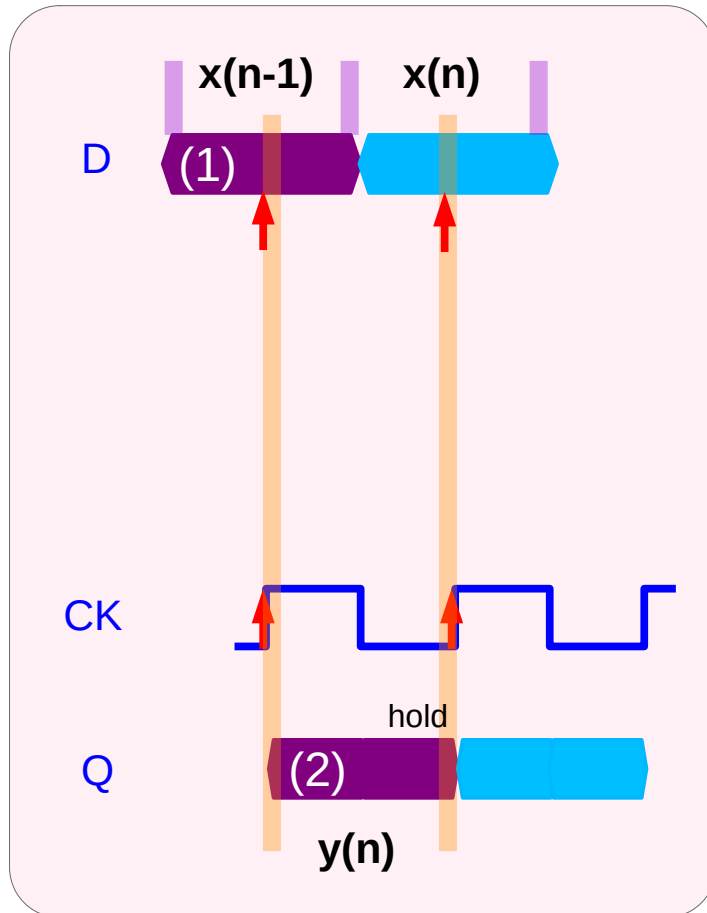


Ideal Timing

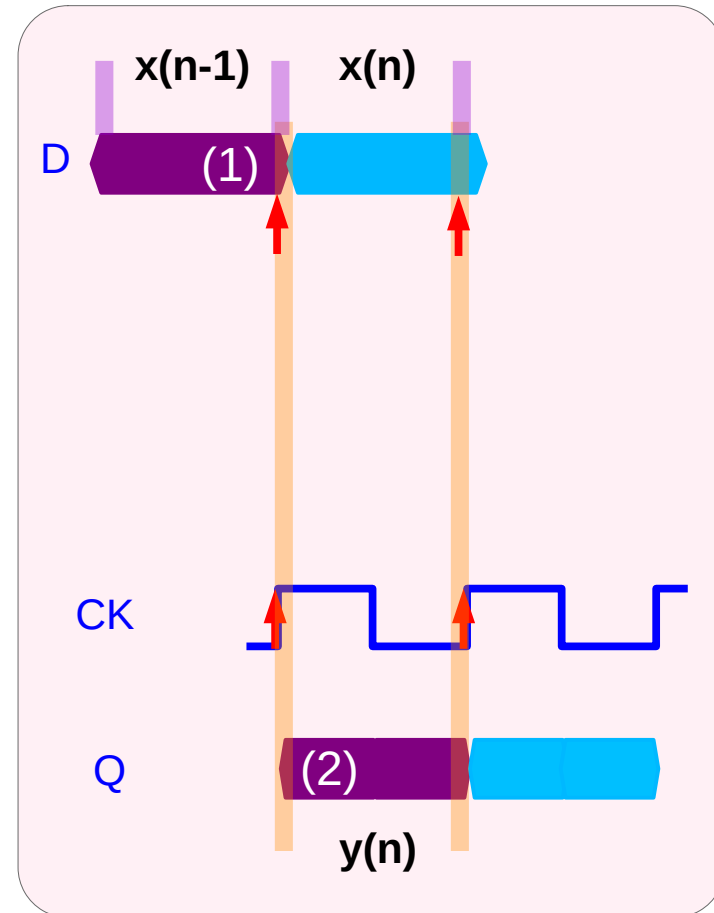


Master-Slave D FlipFlop – typical vs. ideal timing (2)

Typical Timing

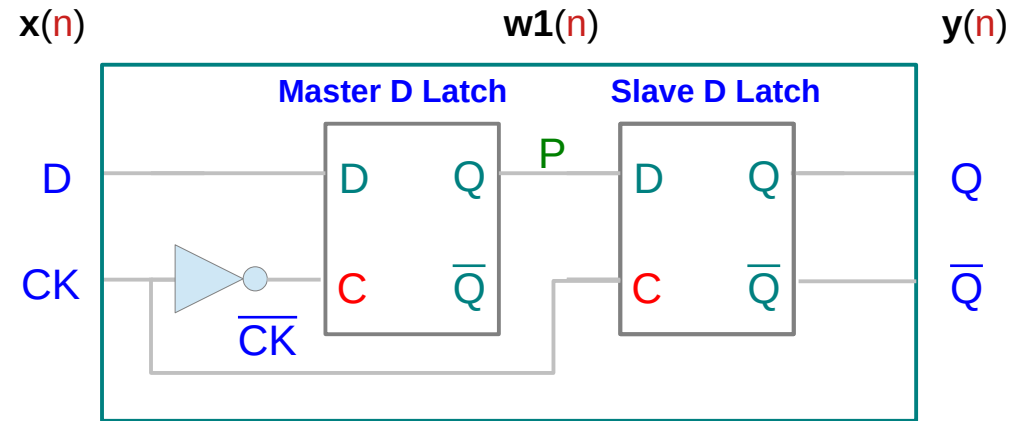
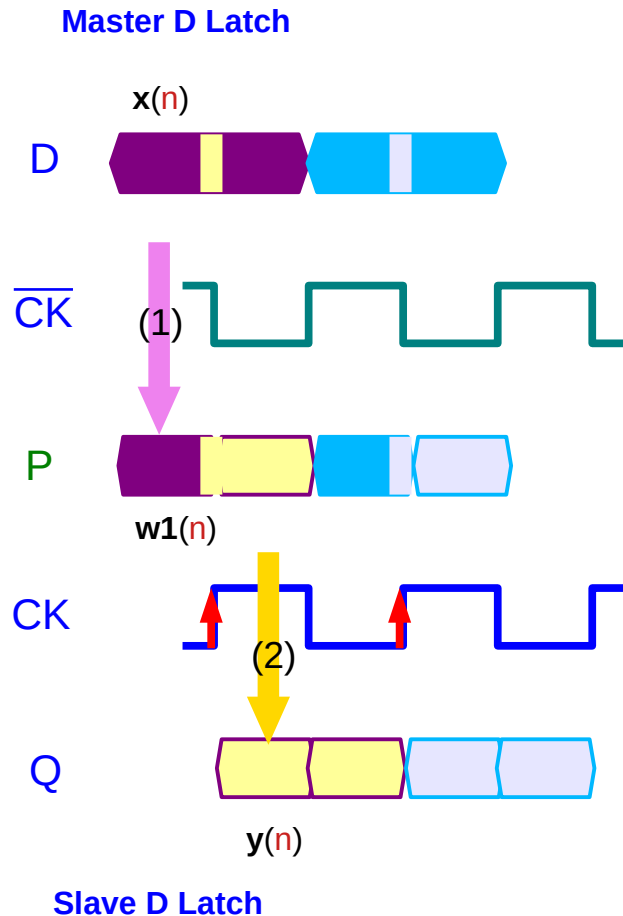


Ideal Timing



Master-Slave D FlipFlop – Rising Edge Sampling

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



(1) the current input $x(n)$ gets stored in the master latch

(2) the current content $w1(n)$ is clocked out to the output $y(n)$

Fixed point representation

fractional numbers

Floating Point Representation

Fixed Point Representation

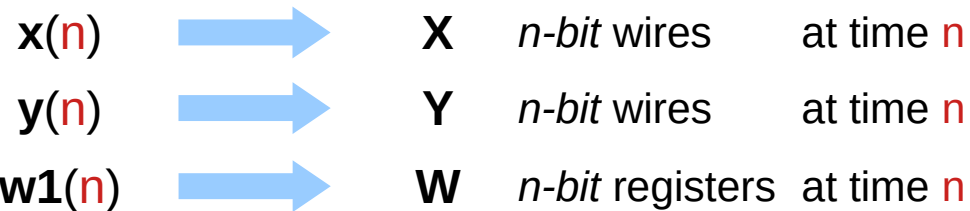
integer + implicit fixed scaling factor

a 2's complement number $(11110101)_2 = -11$

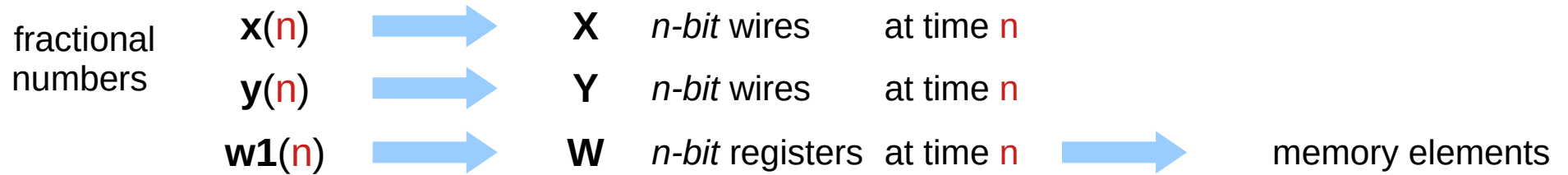
could represent $-11 \cdot 2^{-3} = -88$,
 $-11 \cdot 2^{-5} = -0.34375$
 $-11 \cdot 2^{-12} = -0.002685546875$

with implied scaling factors -3, -5, -12

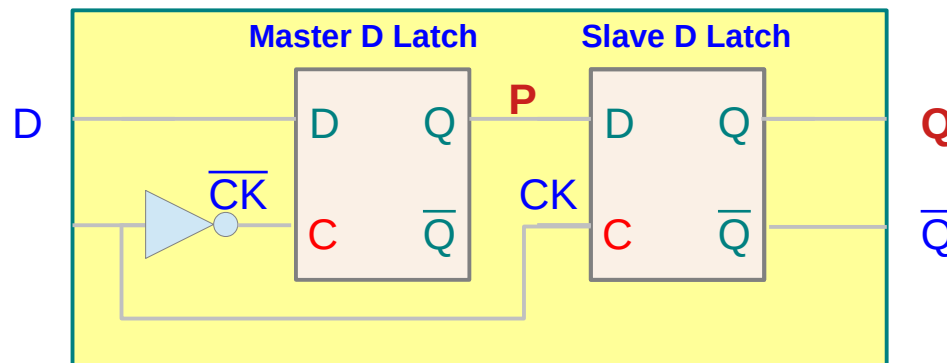
fractional numbers



Memory Elements

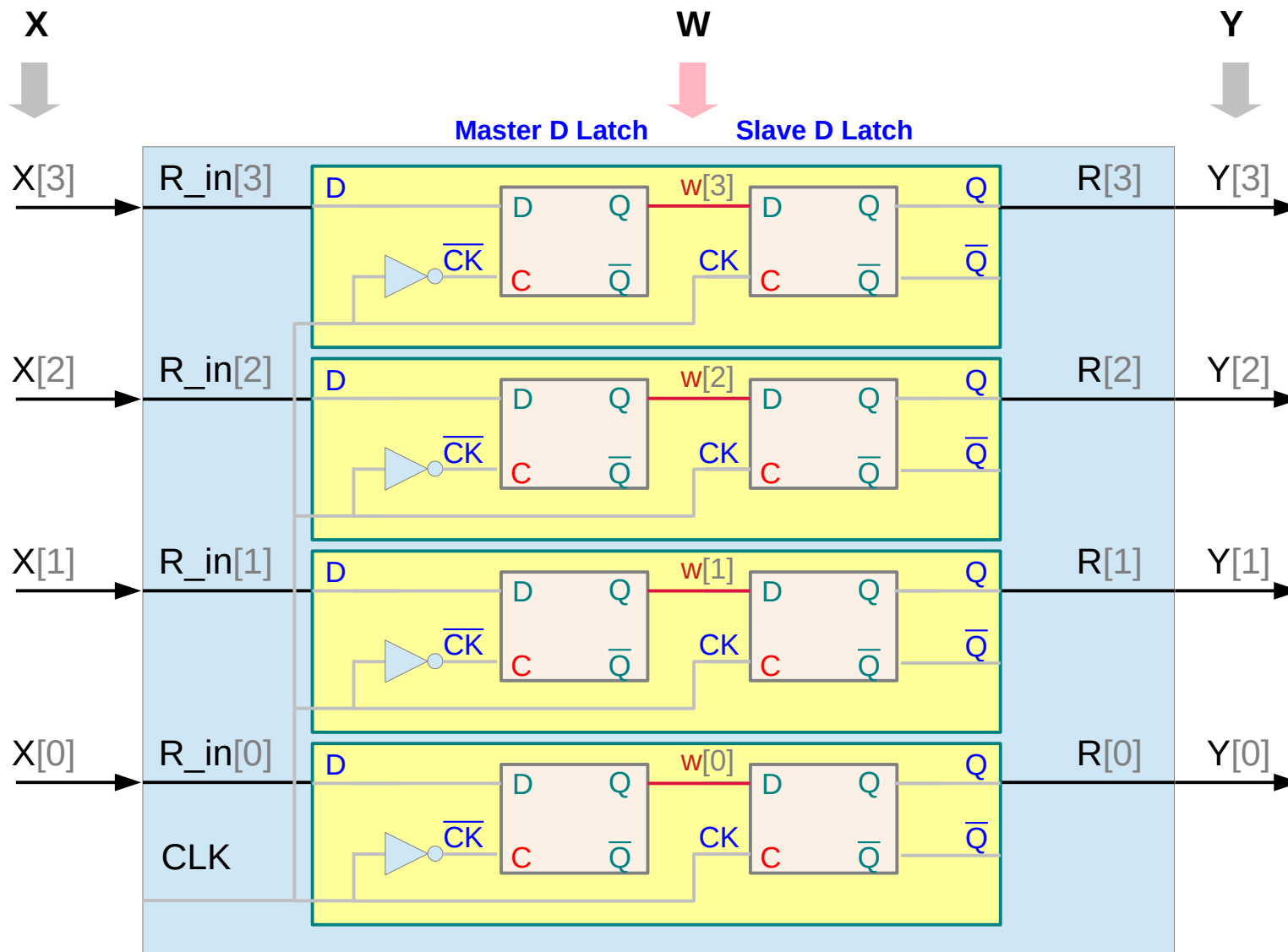


1. using the **P** outputs of **master latches** as the name for **memory elements**
2. using the **Q** outputs of **flipflops** as the name for **memory elements**

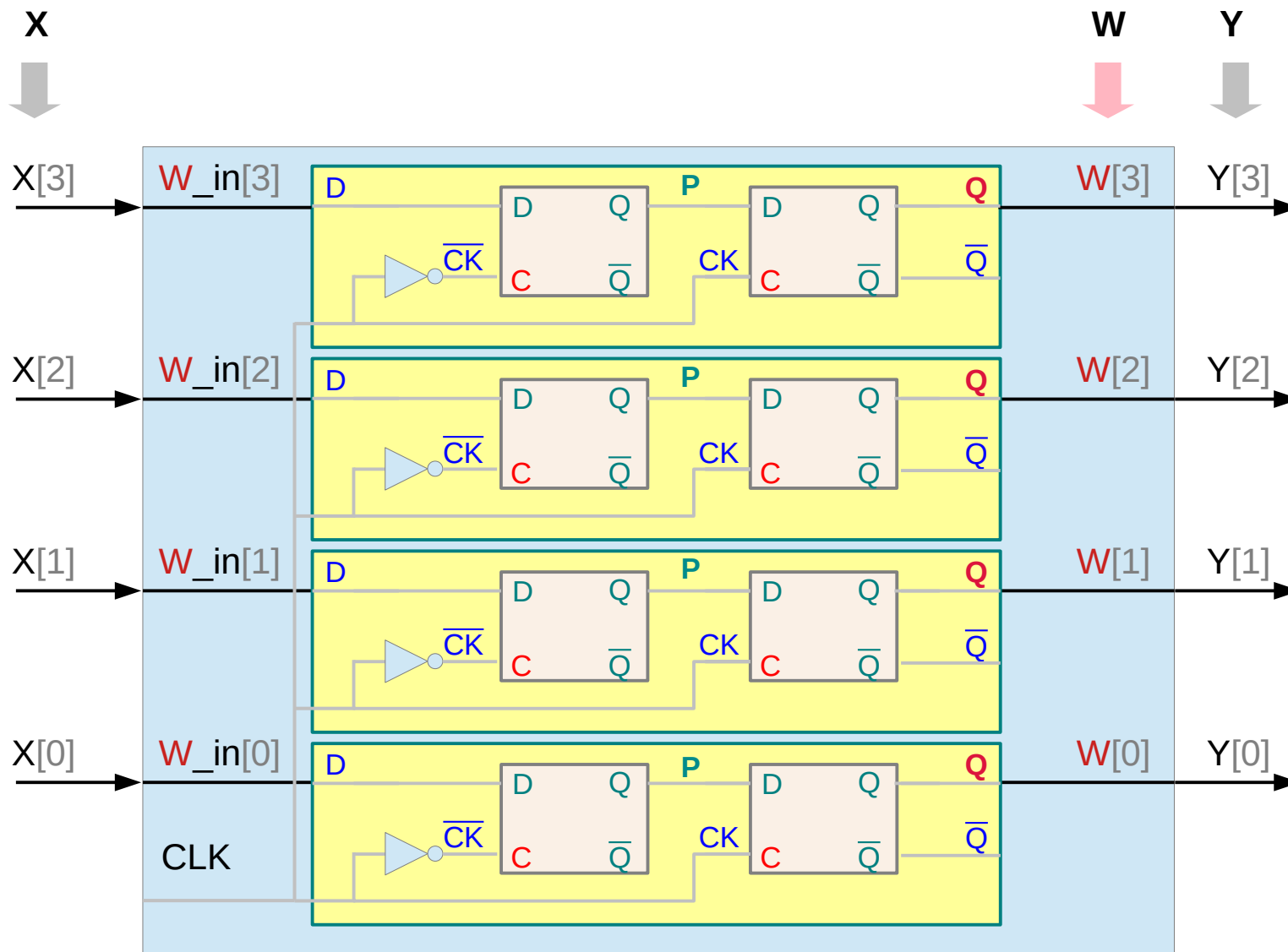


D FlipFlop using Master and Slave D Latches

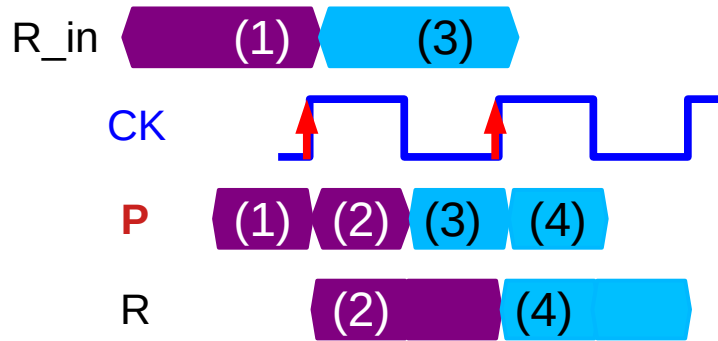
(1) Using master latch outputs P



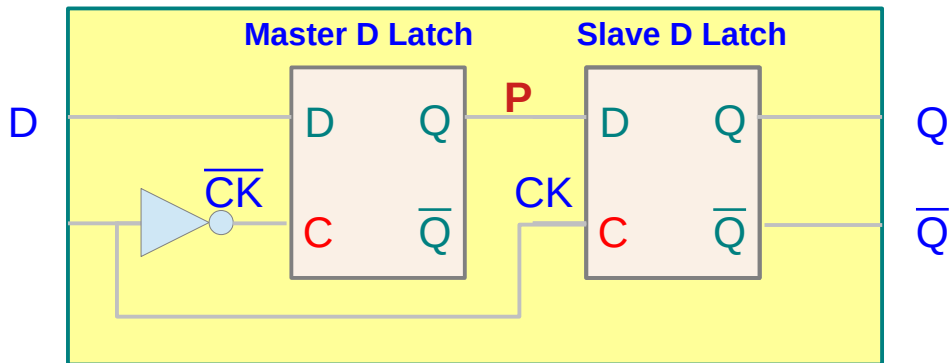
(2) Using flipflop outputs Q



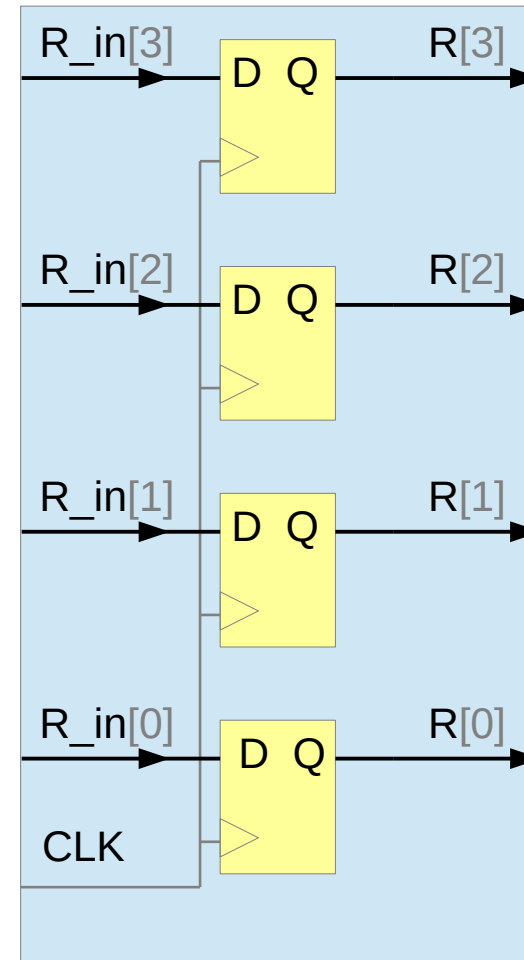
(1) 4-bit Register Using master latch outputs **P**



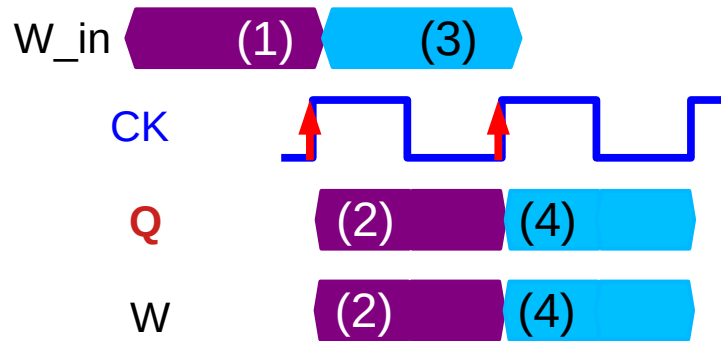
D FlipFlop using Master and Slave D Latches



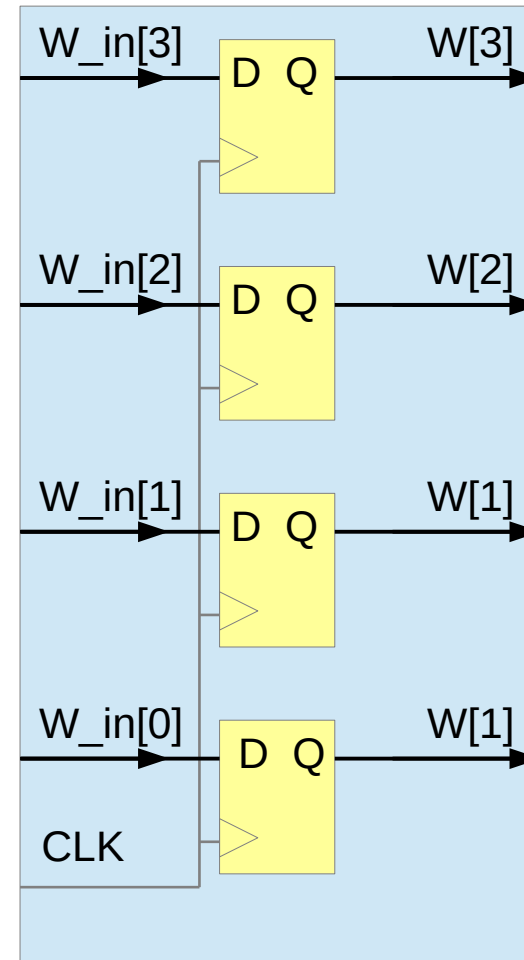
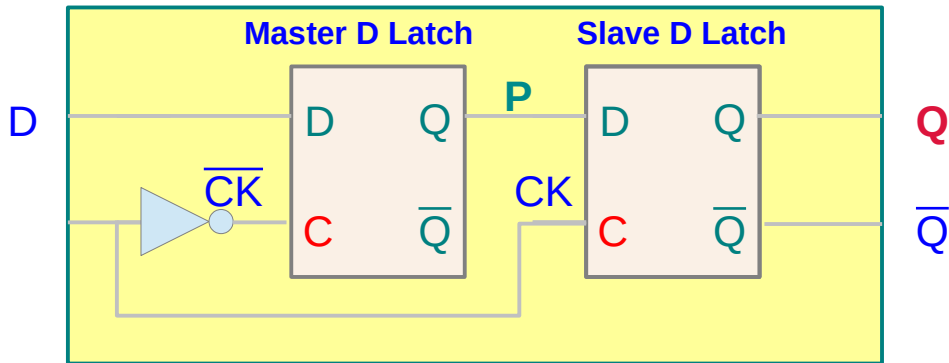
4-bit Register using 4 D flipflops



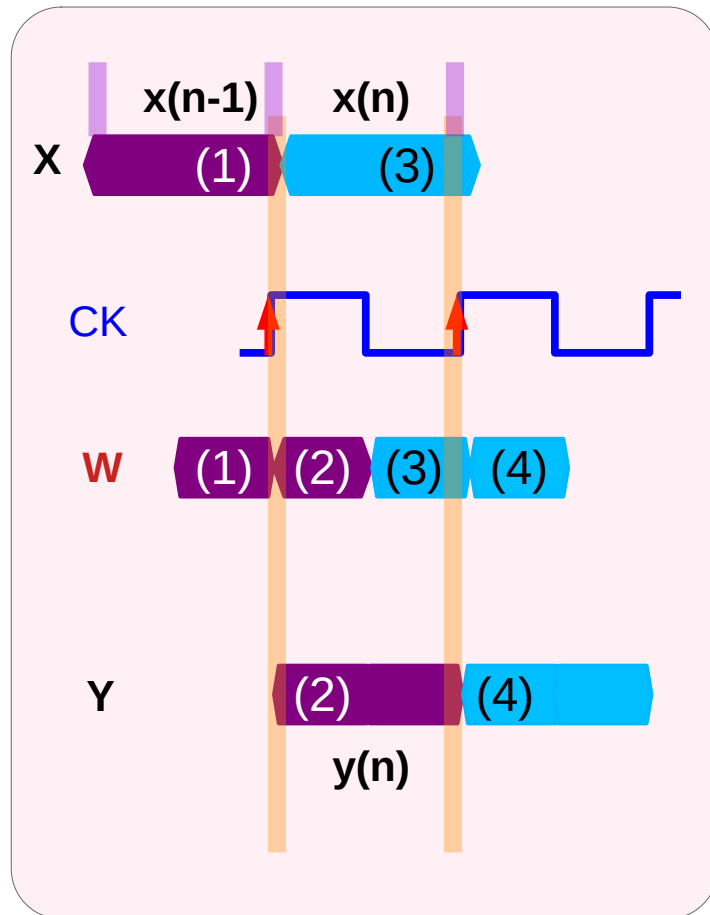
(2) 4-bit Register Using flipflop outputs Q



D FlipFlop using Master and Slave D Latches



(1) Timing diagrams with master latch outputs **P**



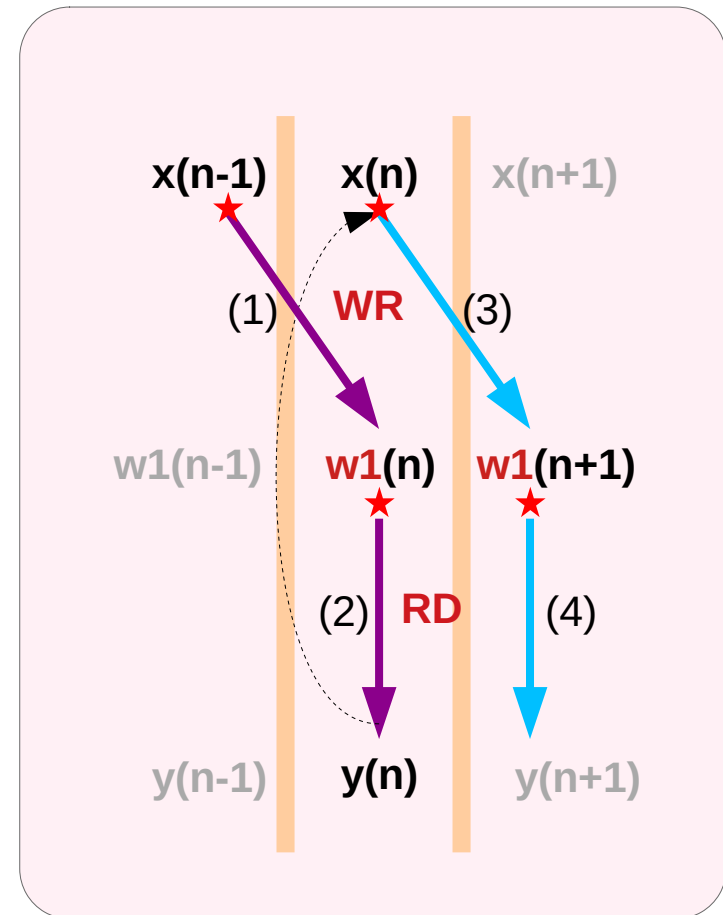
Hardware model

data centered view

Input

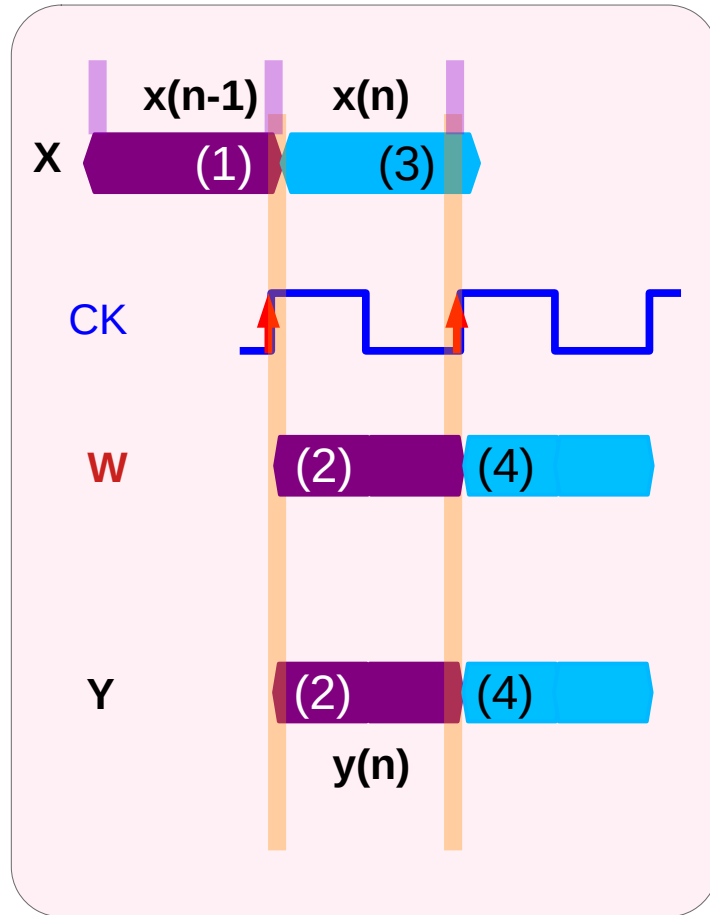
Internal State

Output



The same **DSP C** model

(2) Timing diagrams with flipflop outputs Q



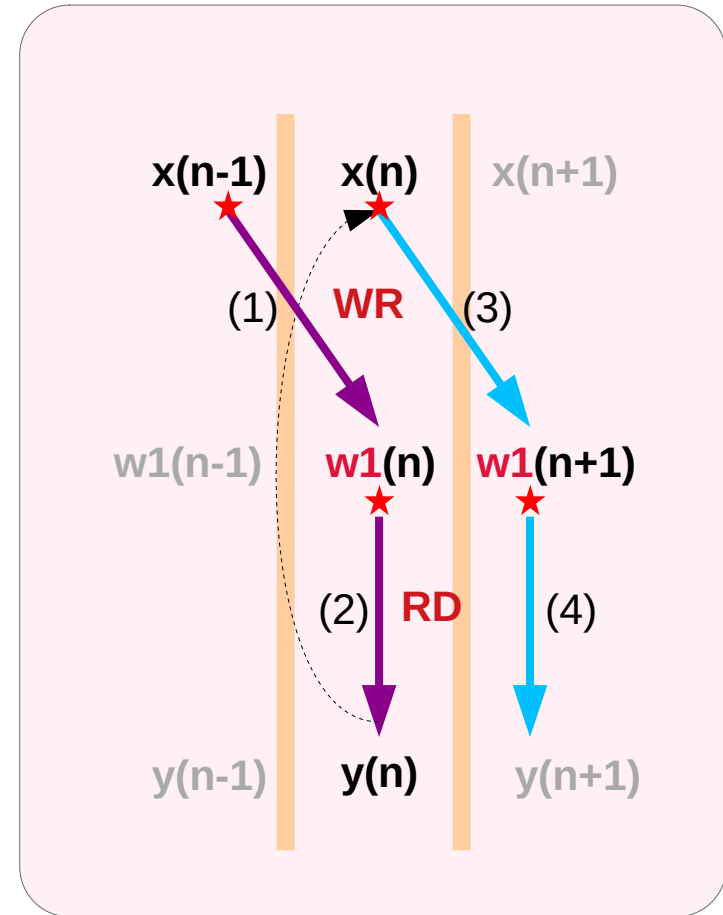
Hardware model

data centered view

Input

Simplified Internal State

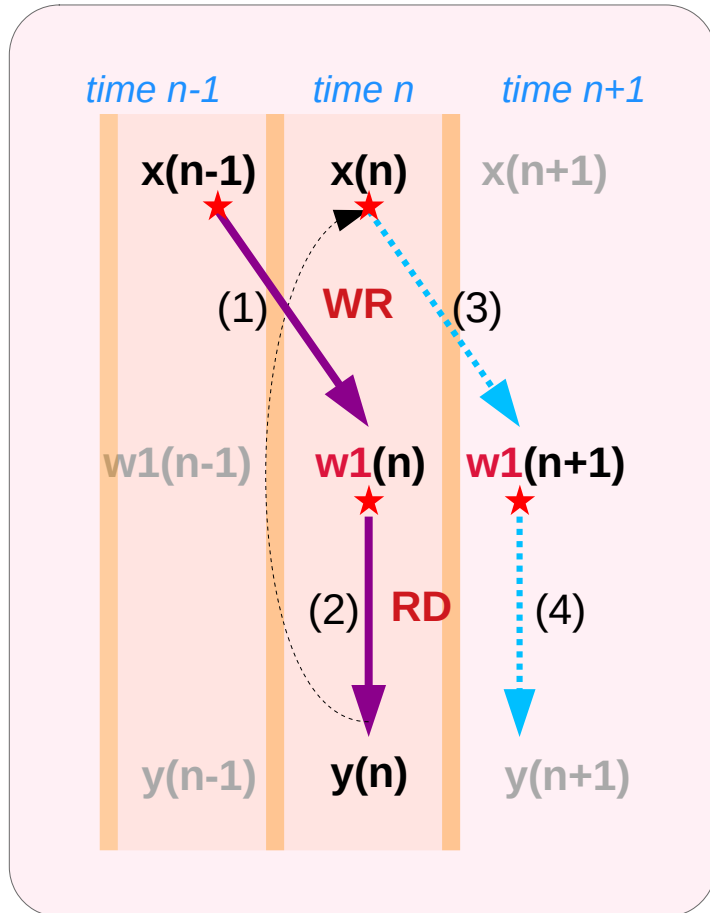
Output



The same DSP C model

Data centered operations

data centered operations



DSP C model

operation spans two time steps (n-1, n)

- (1) $w1(n) = x(n-1)$ *WR* $w1(n)$
 (2) $y(n) = w1(n)$ *RD* $w1(n)$

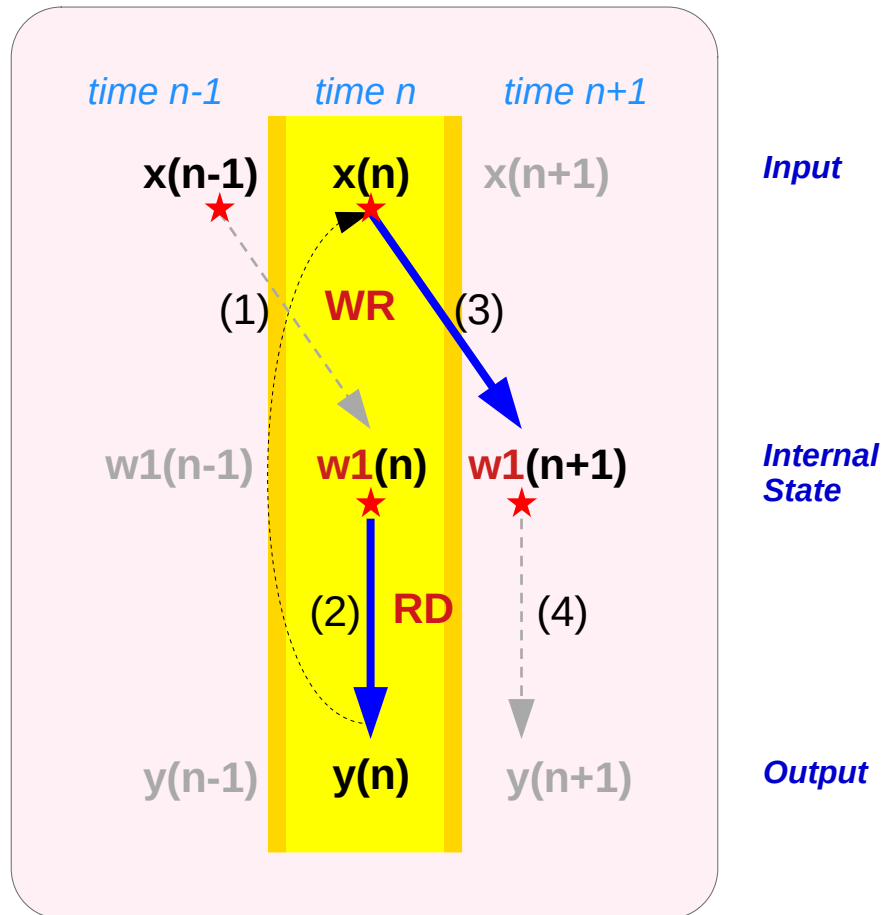
first **WR**, then **RD**
 must avoid **RAW (read after write)** hazards

here, because the written data is read out,
 there is no **RAW (read after write)** hazards

$$\begin{array}{l}
 \underline{y(n-1) = w1(n-1)} \quad \underline{w1(n) = w1(n-1)} \\
 \underline{w1(n) = x(n-1)} \\
 \underline{y(n) = w1(n)} \quad \underline{w1(n+1) = w1(n)} \\
 \underline{w1(n+1) = x(n)} \\
 \underline{y(n+1) = w1(n+1)} \quad \underline{w1(n+2) = w1(n+1)} \\
 \underline{w1(n+2) = x(n+1)} \\
 \underline{y(n+2) = w1(n+2)}
 \end{array}$$

Time centered operations

time centered operations



DSP C model

operation spans only one time step (n)

- (2) $y(n) = w1(n)$ *RD* $w1(n)$
 (3) $w1(n+1) = x(n)$ *WR* $w1(n)$

first **RD**, then **WR**
 no **RAW (read after write)** hazards

| | |
|--------------------|---------------------|
| $y(n-1) = w1(n-1)$ | $w1(n) = w1(n-1)$ |
| $w1(n) = x(n-1)$ | $w1(n+1) = w1(n)$ |
| $y(n) = w1(n)$ | $w1(n+2) = w1(n+1)$ |
| $w1(n+1) = x(n)$ | $w1(n+2) = w1(n+1)$ |
| $y(n+1) = w1(n+1)$ | $w1(n+2) = w1(n+1)$ |
| $w1(n+2) = x(n+1)$ | $w1(n+2) = w1(n+1)$ |
| $y(n+2) = w1(n+2)$ | $w1(n+2) = w1(n+1)$ |

Data vs. time centered operations (1)

data centered operations

time centered operations

| | | | | |
|---|---------------------|---|---------------------|-------------------|
| $y(n-1) = w1(n-1)$ <i>output from $w1(n-1)$</i> <hr/> $w1(n) = x(n-1)$ <i>input to $w1(n)$</i> | $w1(n) = w1(n-1)$ | $y(n-1) = w1(n-1)$ <i>output from $w1(n-1)$</i> <hr/> $w1(n) = x(n-1)$ <i>input to $w1(n)$</i> | $w1(n) = w1(n-1)$ | } <i>time n-1</i> |
| $y(n) = w1(n)$ <i>output from $w1(n)$</i> <hr/> $w1(n+1) = x(n)$ <i>input to $w1(n+1)$</i> | $w1(n+1) = w1(n)$ | $y(n) = w1(n)$ <i>output from $w1(n)$</i> <hr/> $w1(n+1) = x(n)$ <i>input to $w1(n+1)$</i> | $w1(n+1) = w1(n)$ | |
| $y(n+1) = w1(n+1)$ <i>output from $w1(n+1)$</i> <hr/> $w1(n+2) = x(n+1)$ <i>input to $w1(n+2)$</i> | $w1(n+2) = w1(n+1)$ | $y(n+1) = w1(n+1)$ <i>output from $w1(n+1)$</i> <hr/> $w1(n+2) = x(n+1)$ <i>input to $w1(n+2)$</i> | $w1(n+2) = w1(n+1)$ | } <i>time n+1</i> |
| $y(n+2) = w1(n+2)$ <i>output from $w1(n+2)$</i> | | $y(n+2) = w1(n+2)$ <i>output from $w1(n+2)$</i> | | |

- (1) $w1(n) = x(n-1)$ *WR $w1(n)$*
 (2) $y(n) = w1(n)$ *RD $w1(n)$*

- (2) $y(n) = w1(n)$ *RD $w1(n)$*
 (1) $w1(n) = x(n)$ *WR $w1(n)$*

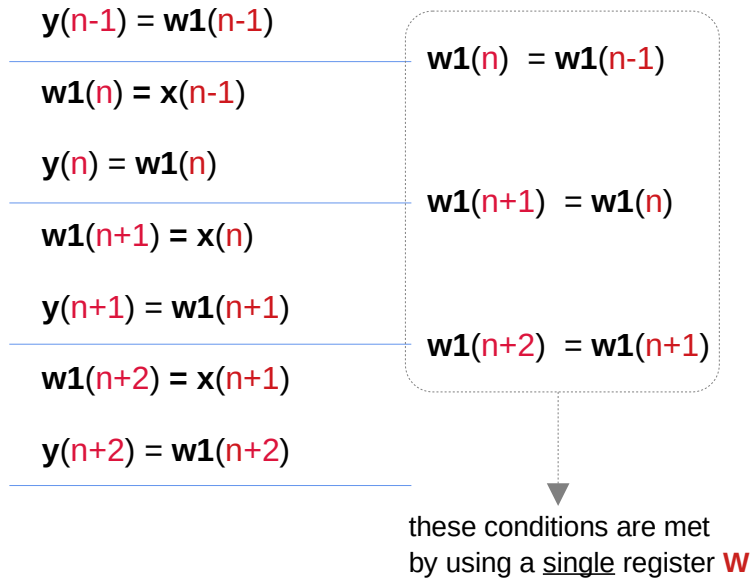
| | |
|----------|---------------------------|
| $W1 = X$ | <i>WR $W1$</i> |
| $Y = W1$ | <i>RD $W1$</i> |

| | |
|----------|---------------------------|
| $Y = W1$ | <i>RD $W1$</i> |
| $W1 = X$ | <i>WR $W1$</i> |

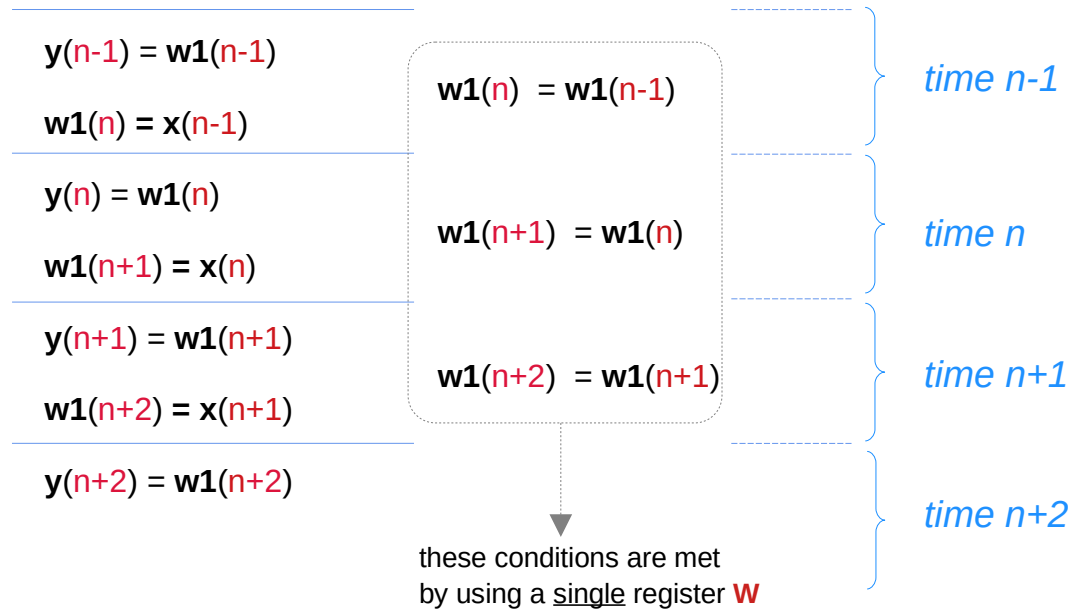
← *Register Transfer Level*

Data vs. time centered operations (2)

data centered operations



time centered operations



Data centered operations

data centered operations

in the Register Transfer Level

| | | | |
|---|---------------------|---|------------|
| $y(n-1) = w1(n-1)$ <small>output $y(n-1)$ from $w1(n-1)$</small> $w1(n) = x(n-1)$ <small>input $x(n-1)$ to $w1(n)$</small> | $w1(n) = w1(n-1)$ | $Y = W1$ <small>output $y(n-1)$ from $W1$</small> $W1 = X$ <small>input $x(n-1)$ to $W1$</small> | } time n-1 |
| $y(n) = w1(n)$ <small>output $y(n)$ from $w1(n)$</small> $w1(n+1) = x(n)$ <small>input $x(n)$ to $w1(n+1)$</small> | $w1(n+1) = w1(n)$ | $Y = W1$ <small>output $y(n)$ from $W1$</small> $W1 = X$ <small>input $x(n)$ to $W1$</small> | |
| $y(n+1) = w1(n+1)$ <small>output $y(n+1)$ from $w1(n+1)$</small> $w1(n+2) = x(n+1)$ <small>input $x(n+1)$ to $w1(n+2)$</small> | $w1(n+2) = w1(n+1)$ | $Y = W1$ <small>output $y(n+1)$ from $W1$</small> $W1 = X$ <small>input $x(n+1)$ to $W1$</small> | } time n+1 |
| $y(n+2) = w1(n+2)$ <small>output $y(n+2)$ from $w1(n+2)$</small> | | $Y = W1$ <small>output $y(n+2)$ from $W1$</small> | |
| (1) $w1(n) = x(n-1)$ $WR\ w1(n)$ (2) $y(n) = w1(n)$ $RD\ w1(n)$ | | $W1 = X$ $WR\ W1$ $Y = W1$ $RD\ W1$ | |

Time centered operations

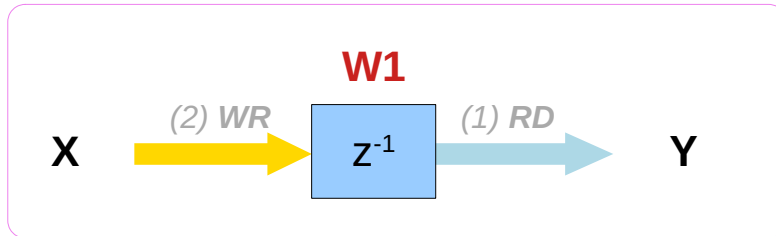
time centered operations

in the Register Transfer Level

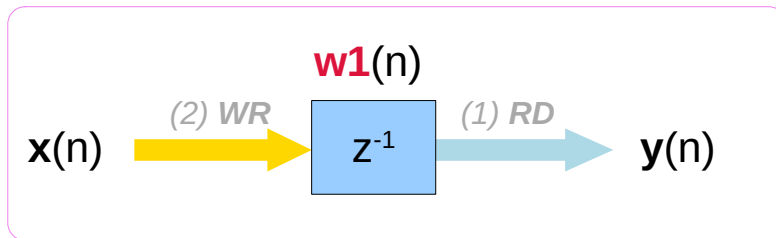
| | | |
|--|---|---|
| $y(n-1) = w1(n-1)$ <small>output from $w1(n-1)$</small> $w1(n) = x(n-1)$ <small>input to $w1(n)$</small> $w1(n) = w1(n-1)$ | $Y = W1$ <small>output $y(n-1)$ from $W1$</small> $W1 = X$ <small>input $x(n-1)$ to $W1$</small> | <div style="display: flex; align-items: center; justify-content: center;"> } time n-1 </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> } time n </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> } time n+1 </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> } time n+2 </div> |
| $y(n) = w1(n)$ <small>output from $w1(n)$</small> $w1(n+1) = x(n)$ <small>input to $w1(n+1)$</small> $w1(n+1) = w1(n)$ | $Y = W1$ <small>output $y(n)$ from $W1$</small> $W1 = X$ <small>input $x(n)$ to $W1$</small> | |
| $y(n+1) = w1(n+1)$ <small>output from $w1(n+1)$</small> $w1(n+2) = x(n+1)$ <small>input to $w1(n+2)$</small> $w1(n+2) = w1(n+1)$ | $Y = W1$ <small>output $y(n+1)$ from $W1$</small> $W1 = X$ <small>input $x(n+1)$ to $W1$</small> | |
| $y(n+2) = w1(n+2)$ <small>output from $w1(n+2)$</small> | $Y = W1$ <small>output $y(n+2)$ from $W1$</small> | |
| (2) $y(n) = w1(n)$ <i>RD</i> $w1(n)$ (1) $w1(n) = x(n)$ <i>WR</i> $w1(n)$ | $Y = W1$ <i>RD</i> $W1$ $W1 = X$ <i>WR</i> $W1$ | |

Simultaneous RD and WR actions

Register Transfer Level



DSP C Model



at time n

$$y(n) = w1(n)$$

$$w1(n+1) = x(n)$$

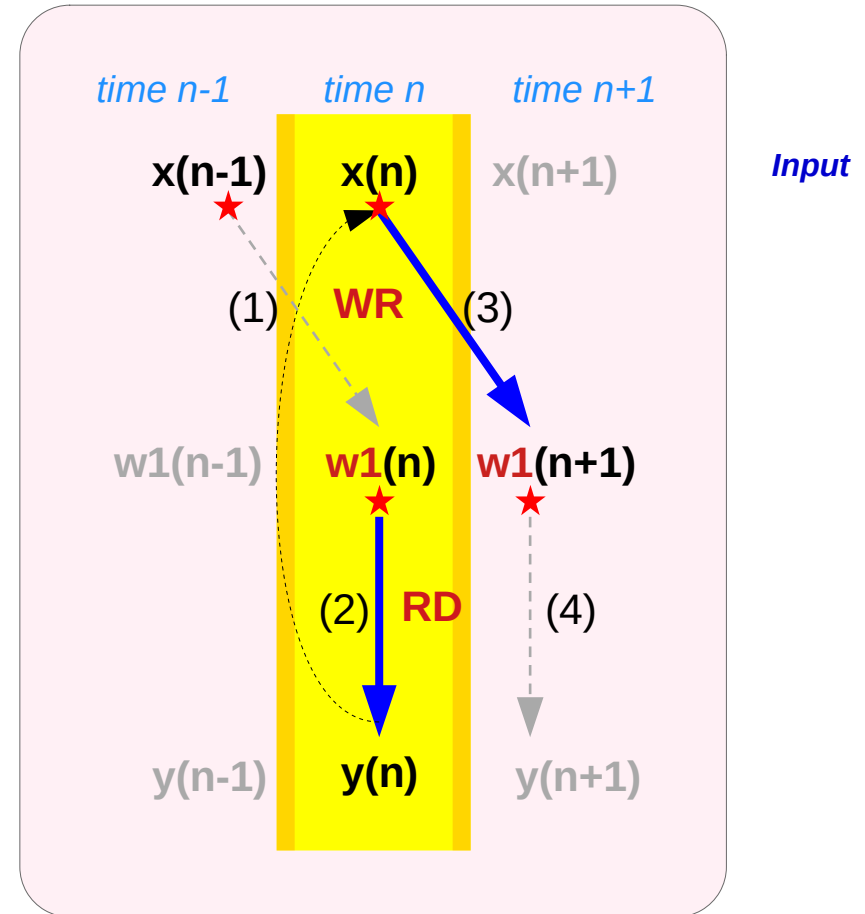
at time $n+1$

$$y(n+1) = w1(n+1)$$

$$w1(n+2) = x(n+1)$$

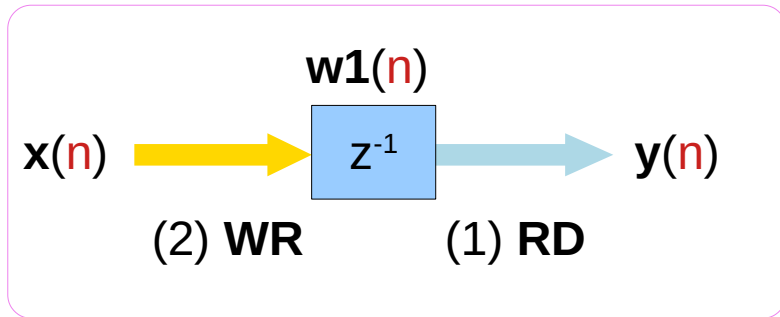
- (1) RD
- (2) WR

time centered operations



DSP C model

Simultaneous **RD** and **WR** actions



| | | |
|-----------------|-----------|------------|
| current content | $w1(n)$ | $= x(n-1)$ |
| next content | $w1(n+1)$ | $= x(n)$ |

at time n

$$\begin{aligned} y(n) &= w1(n) \\ w1(n+1) &= x(n) \end{aligned}$$



at time $n+1$

$$\begin{aligned} y(n+1) &= w1(n+1) \\ w1(n+2) &= x(n+1) \end{aligned}$$

read internal state

update internal state

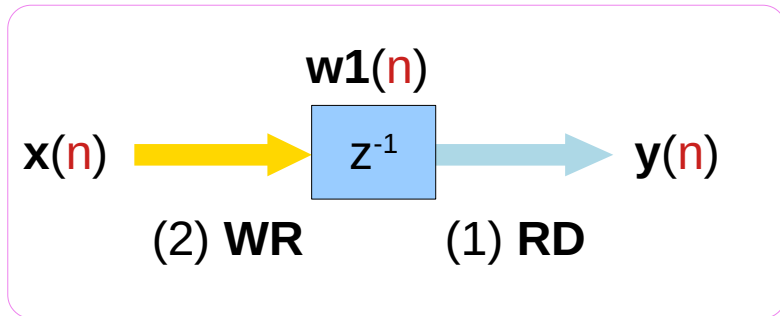
at time n ,

- 1) the content of the register $w1(n)$
becomes the output $y(n)$
- 2) the input $x(n)$ is saved and
becomes the new content $w1(n+1)$

RD access of $w1(n) = x(n-1)$

WR access of $w1(n+1) = x(n)$

Current content $w1(n)$ and current input $x(n)$

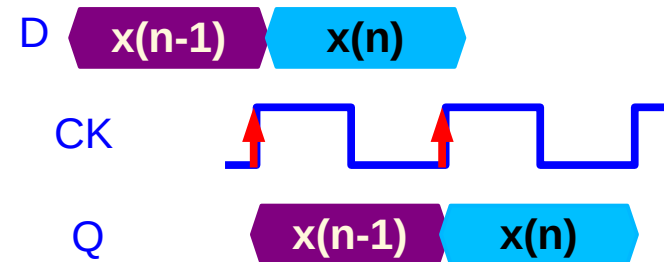
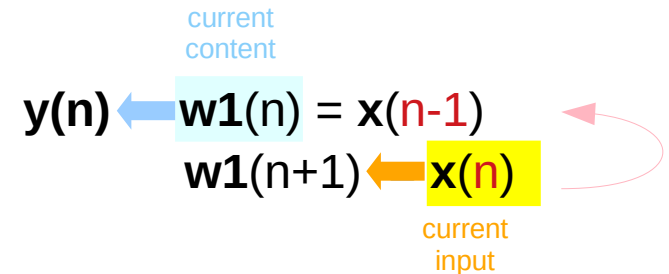


| | | |
|-----------------|-----------|------------|
| current content | $w1(n)$ | $= x(n-1)$ |
| next content | $w1(n+1)$ | $= x(n)$ |

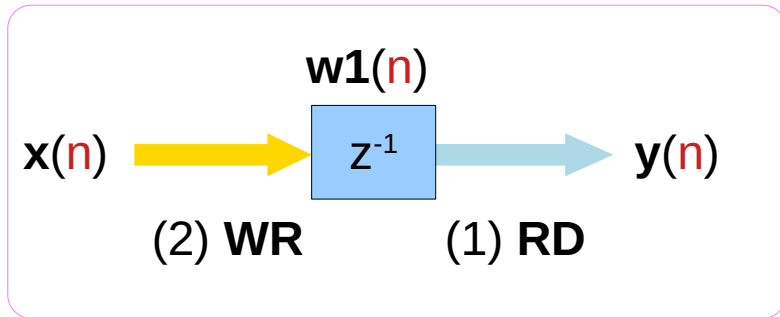
a register holding the previous input sample $x(n-1)$

- (1) the **current content** $x(n-1)$ is clocked out to the output
- (2) the **current input** $x(n)$ gets stored in the register

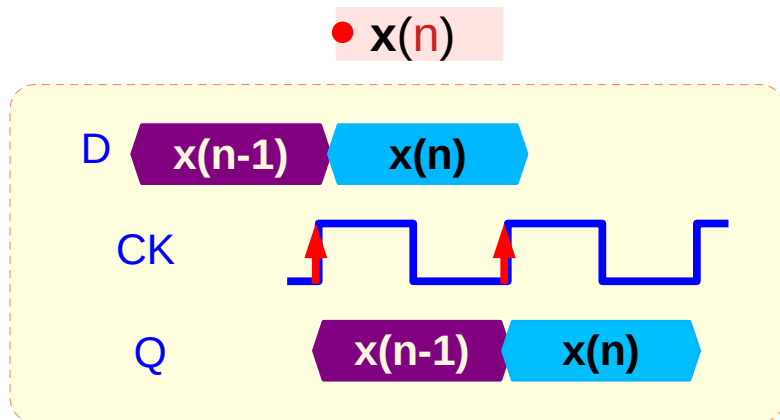
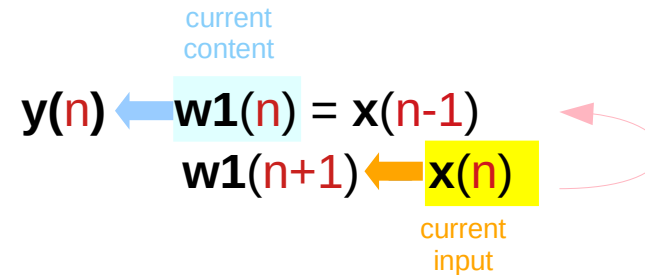
It will be held for one sampling instant and become the output at the next time $n+1$



Current content $w1(n)$ and current input $x(n)$



| | | |
|-----------------|-----------|------------|
| current content | $w1(n)$ | $= x(n-1)$ |
| next content | $w1(n+1)$ | $= x(n)$ |



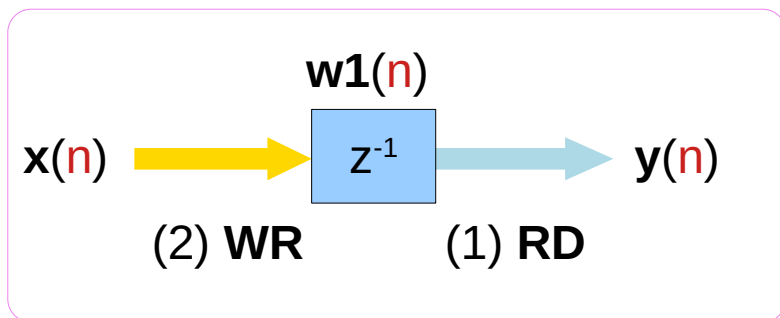
• $w1(n)$

• $y(n)$

*simulate a clocked hardware
ignoring delay constraints in hardware*

zero-delay simulation

Delay element modeling



| | |
|--------------------------------|-----------------------------------|
| $w1(n) \longrightarrow y(n)$ | $y(n) = w1(n)$ (1) RD old w1 |
| $x(n) \longrightarrow w1(n+1)$ | $w1(n+1) = x(n)$ (2) WR new w1 |

The content of the delay register at time n as the **internal state** of the filter by

internal state at time n

$$w1(n) = x(n-1)$$

internal state at time $n+1$

$$w1(n+1) = x(n)$$

output at time n

$$y(n) = w1(n)$$

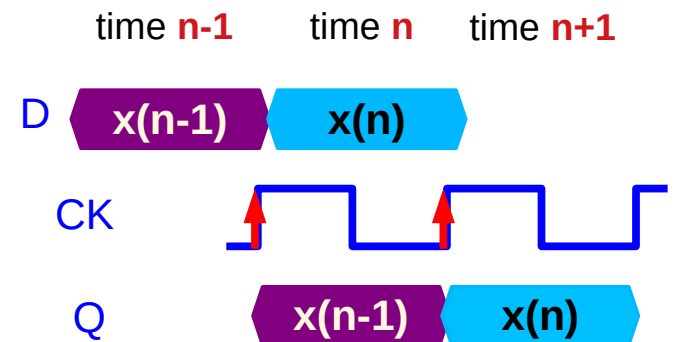
RD before WR

WAR (Write after Read) Access

WR at time $n-1$

WR at time n

RD at time n



*simulate a clocked hardware
ignoring delay constraints in hardware*

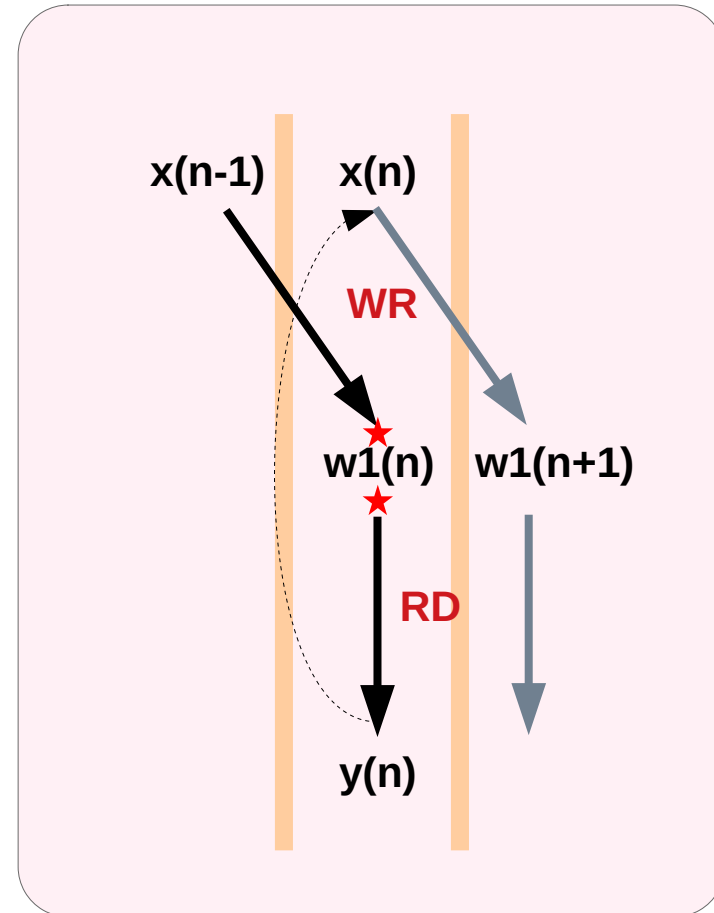
zero-delay simulation

WAR (Write after Read)

| | | |
|--------------------|--------|--------|
| $y(n) = w1(n)$ | (1) RD | old w1 |
| $w1(n+1) = x(n)$ | (2) WR | new w1 |
| $y(n+1) = w1(n+1)$ | (1) RD | old w1 |
| $w1(n+2) = x(n+1)$ | (2) WR | new w1 |

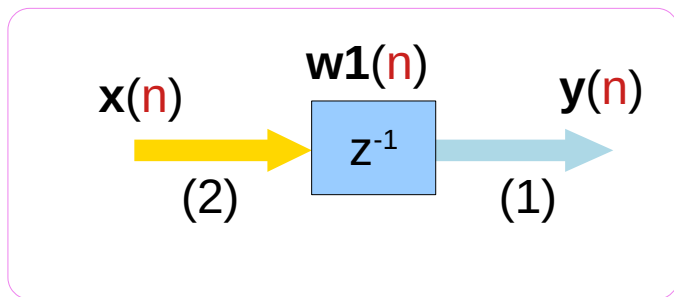
| | | |
|--------------------|--------|--------|
| $w1(n+1) = x(n)$ | (2) WR | new w1 |
| $y(n) = w1(n)$ | (1) RD | old w1 |
| $w1(n+2) = x(n+1)$ | (2) WR | new w1 |
| $y(n+1) = w1(n+1)$ | (1) RD | old w1 |

WAR (Write after Read) Violation



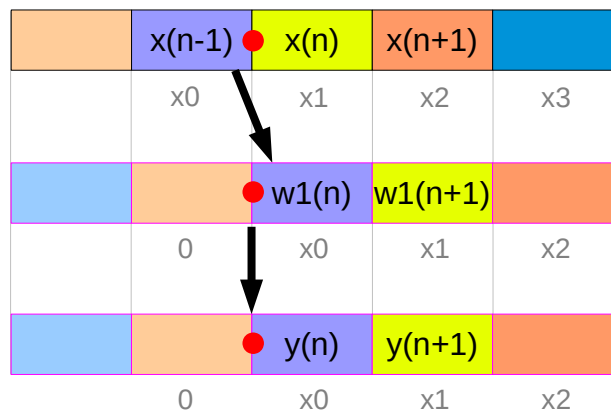
DSP C model

Single Delay



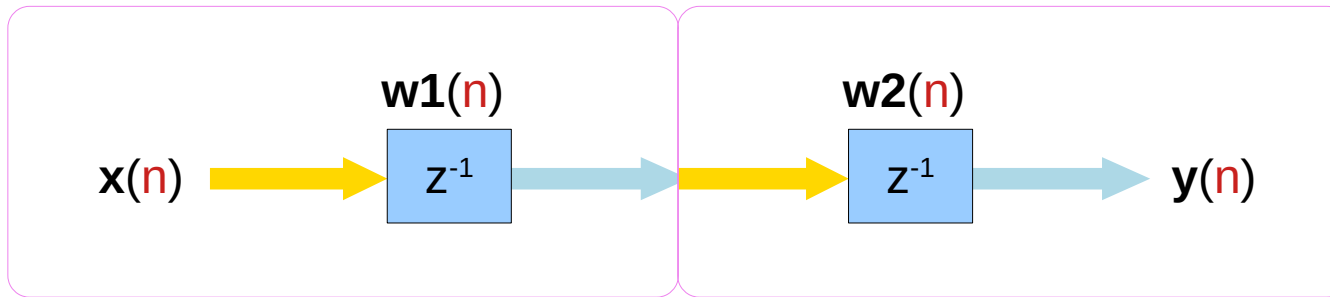
$$y(n) = w1(n) \quad (1) \text{ RD } \rightarrow$$

$$w1(n+1) = x(n) \quad (2) \text{ WR } \rightarrow$$



| n | $x(n)$ | $w1(n)$ | $y(n)$ |
|-----|--------|---------|--------|
| 0 | x_0 | 0 | 0 |
| 1 | x_1 | x_0 | x_0 |
| 2 | x_2 | x_1 | x_1 |
| 3 | x_3 | x_2 | x_2 |
| 4 | x_4 | x_3 | x_3 |

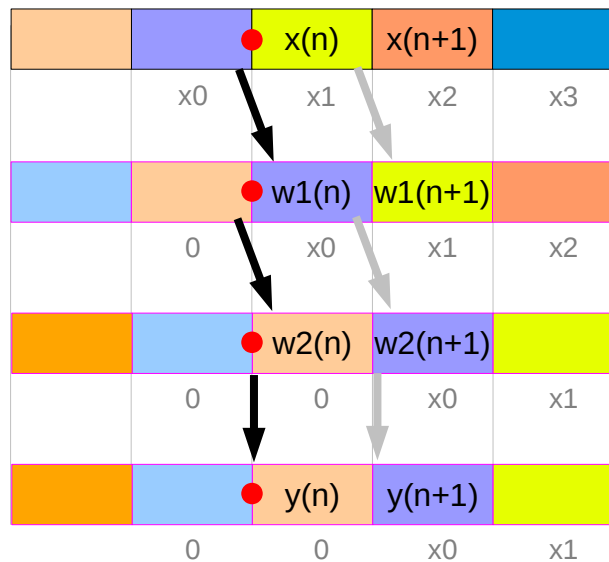
Double Delay



$$y(n) = w2(n)$$

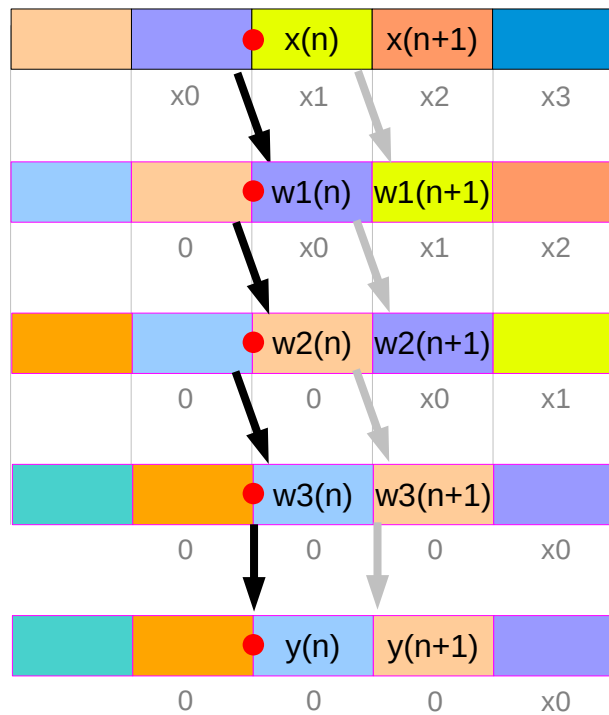
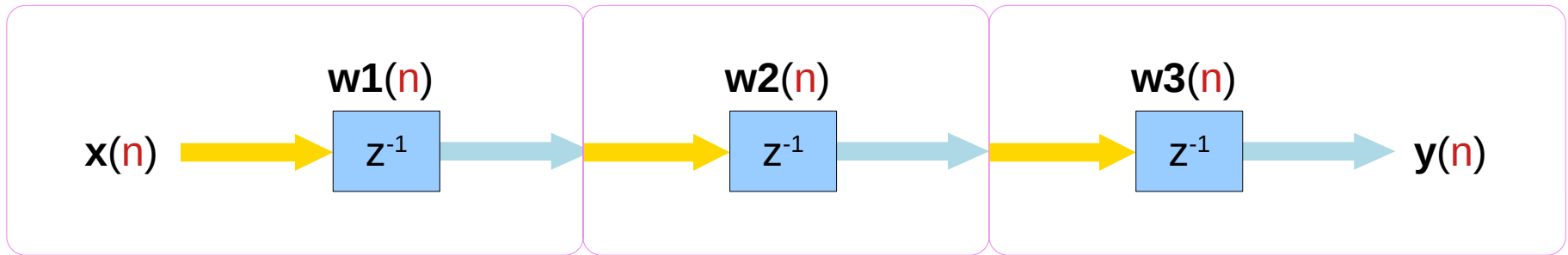
$$w2(n+1) = w1(n)$$

$$w1(n+1) = x(n)$$



| n | $x(n)$ | $w1(n)$ | $w2(n)$ | $y(n)$ |
|-----|--------|---------|---------|--------|
| 0 | x_0 | 0 | 0 | 0 |
| 1 | x_1 | x_0 | 0 | 0 |
| 2 | x_2 | x_1 | x_0 | x_0 |
| 3 | x_3 | x_2 | x_1 | x_1 |
| 4 | x_4 | x_3 | x_2 | x_2 |

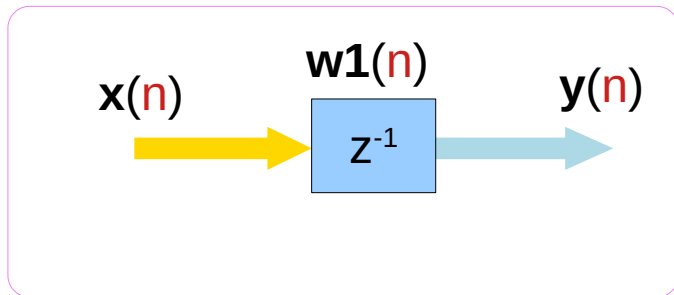
Triple Delay



| n | $x(n)$ | $w1(n)$ | $w2(n)$ | $w3(n)$ | $y(n)$ |
|-----|--------|---------|---------|---------|--------|
| 0 | x_0 | 0 | 0 | 0 | 0 |
| 1 | x_1 | x_0 | 0 | 0 | 0 |
| 2 | x_2 | x_1 | x_0 | 0 | 0 |
| 3 | x_3 | x_2 | x_1 | x_0 | x_0 |
| 4 | x_4 | x_3 | x_2 | x_1 | x_1 |

$$\begin{aligned}
 y(n) &= w3(n) \\
 w3(n+1) &= w2(n) \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n)
 \end{aligned}$$

Single Delay – IO Equations



single delay

| | |
|------------------|--------|
| $y(n) = w1(n)$ | output |
| $w1(n+1) = x(n)$ | input |

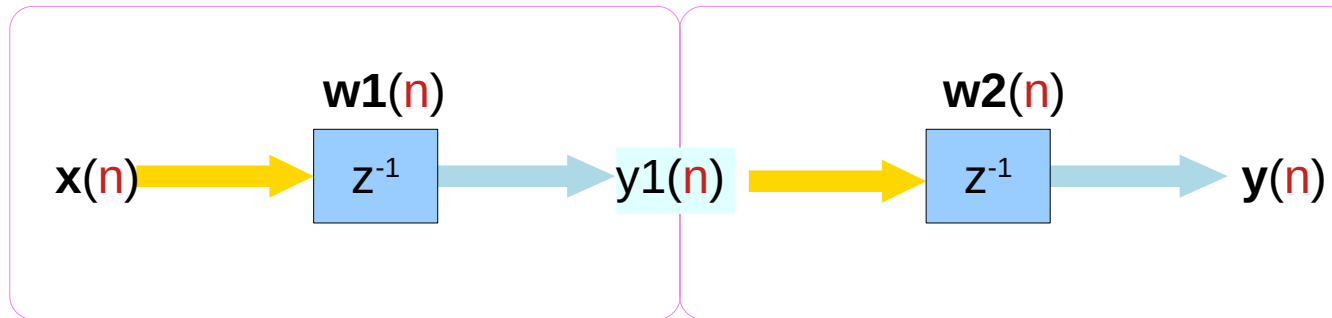
double delay

| | |
|-------------------|--------|
| $y(n) = w2(n)$ | output |
| $w2(n+1) = w1(n)$ | |
| $w1(n+1) = x(n)$ | input |

triple delay

| | |
|-------------------|--------|
| $y(n) = w3(n)$ | output |
| $w3(n+1) = w2(n)$ | |
| $w2(n+1) = w1(n)$ | |
| $w1(n+1) = x(n)$ | input |

Double Delay – IO Equations



$$y1(n) = w1(n)$$

$$w1(n+1) = x(n)$$

$$y(n) = w2(n)$$

$$w2(n+1) = y1(n)$$

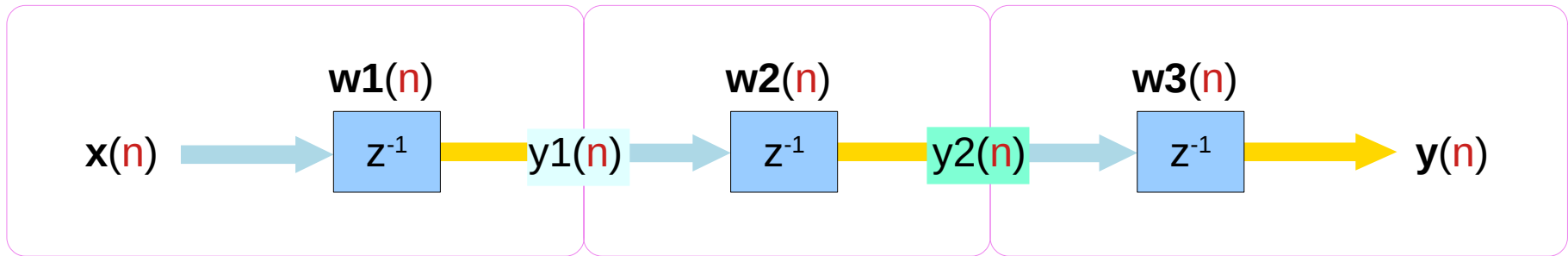
$$y1(n) = w1(n)$$

$$w2(n+1) = y1(n)$$

$$w2(n+1) = w1(n)$$

| | |
|-------------------|--------|
| $y(n) = w2(n)$ | output |
| $w2(n+1) = w1(n)$ | |
| $w1(n+1) = x(n)$ | input |

Triple Delay – IO Equations



$$y1(n) = w1(n)$$

$$w1(n+1) = x(n)$$

$$y2(n) = w2(n)$$

$$w2(n+1) = y1(n)$$

$$y(n) = w3(n)$$

$$w3(n+1) = y2(n)$$

$$y1(n) = w1(n)$$

$$w2(n+1) = y1(n)$$

$$w2(n+1) = w1(n)$$

$$y2(n) = w2(n)$$

$$w3(n+1) = y2(n)$$

$$w3(n+1) = w2(n)$$

$$y(n) = w3(n) \quad \text{output}$$

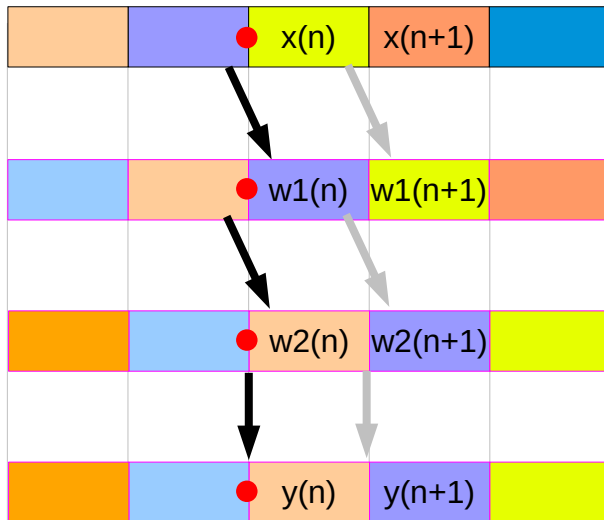
$$w3(n+1) = w2(n)$$

$$w2(n+1) = w1(n)$$

$$w1(n+1) = x(n) \quad \text{input}$$

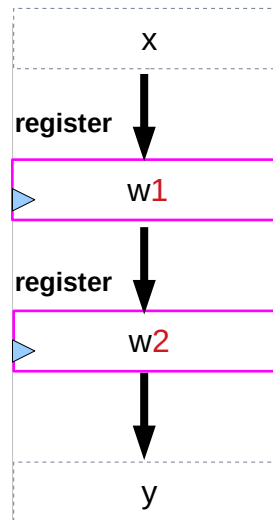
Delay C Model

Timing Chart



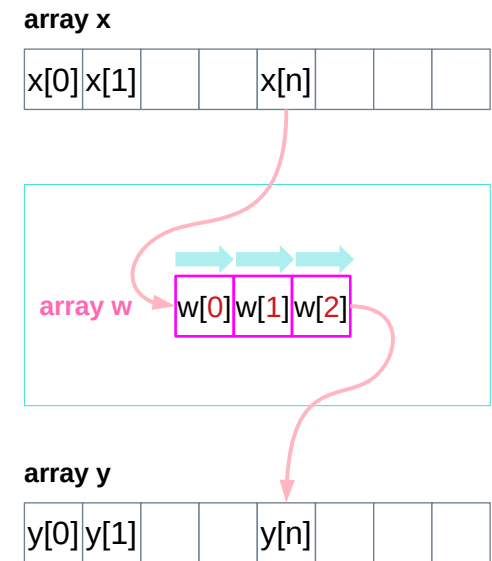
$$\begin{aligned}
 y(n) &= w2(n) \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n)
 \end{aligned}$$

Register Transfer



$$\begin{aligned}
 y &= w2 \\
 w2 &= w1 \\
 w1 &= x
 \end{aligned}$$

DSP C Model for simulation



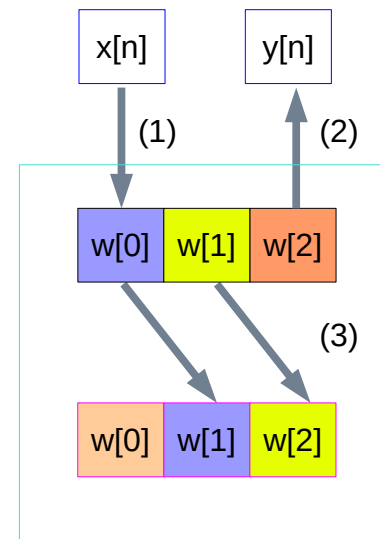
$$\begin{aligned}
 y[n] &= w[2] \\
 w[0] &= x[n] \\
 w[2] &= w[1] \\
 w[1] &= w[0]
 \end{aligned}$$

IO Equations for the Triple Delay (1)

$$\begin{aligned}y(n) &= w_2(n) \\w_0(n) &= x(n) \\w_2(n+1) &= w_1(n) \\w_1(n+1) &= w_2(n)\end{aligned}$$

$$D = 2, 1$$

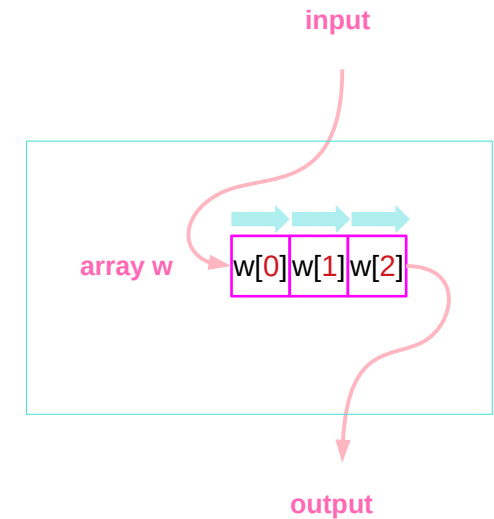
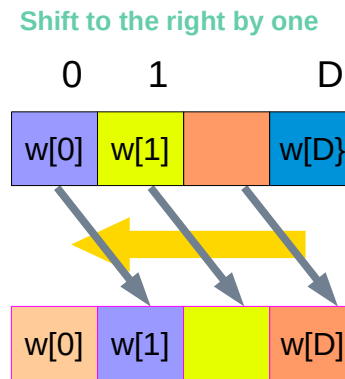
```
y[n] = w[2]           // get the output
w[0] = x[n]           // put the input
w[2] = w[1]           // shift
w[1] = w[0]           // shift
```



delay.c

```
/* delay.c - delay by D time samples */  
/* w[0] = input, w[D] = output */
```

```
void delay(int D, double *w)  
{  
    int i;  
  
    for (i=D; i>=1; i--)  
        w[i] = w[i-1];  
  
    // reverse-order updating  
}
```



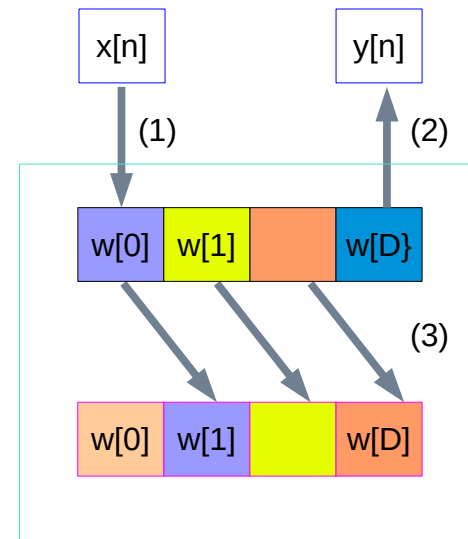
order of execution

$$\begin{aligned} w[D] &= w[D-1] \\ \dots & \dots \\ w[2] &= w[1] \\ w[1] &= w[0] \end{aligned}$$

Using the delay function

```
double *w;  
w = (double *) calloc(D+1, sizeof(double)); // (D+1)-dimensional
```

```
for (n = 0; n < Ntot; n++) {  
    y[n] = w[D]; // (1) write output  
    w[0] = x[n]; // (2) read input  
    delay(D, w); // (3) update delay line  
}
```



Delay Functions

$$y(n) = w_1(n)$$
$$w_1(n+1) = x(n)$$

$$y(n) = w_2(n)$$
$$w_2(n+1) = w_1(n)$$
$$w_1(n+1) = x(n)$$

$$y(n) = w_3(n)$$
$$w_3(n+1) = w_2(n)$$
$$w_2(n+1) = w_1(n)$$
$$w_1(n+1) = x(n)$$

$$y(n) = w_D(n)$$
$$w_0(n) = x(n)$$
$$w_i(n+1) = w_{i-1}(n),$$
$$i = D, D-1, \dots, 2, 1$$

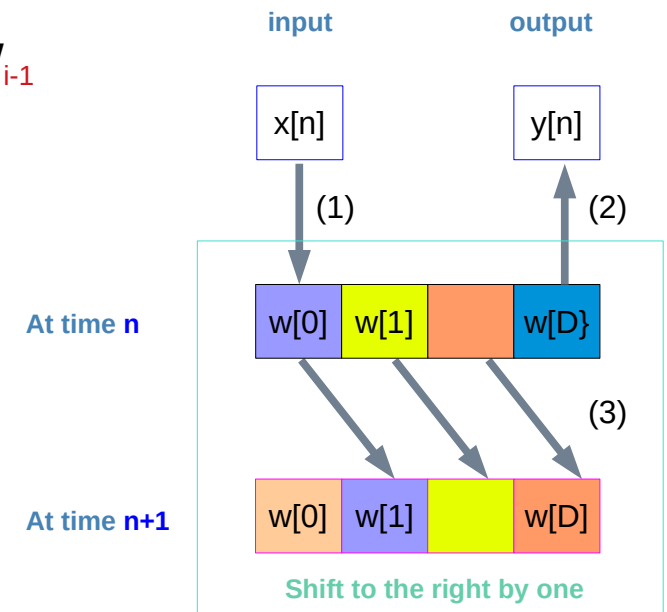
time index : n

memory location : W_i

memory index : i

$$w_i(n+1) = w_{i-1}(n)$$

the current value at w_{i-1}
will become
the next value at w_i



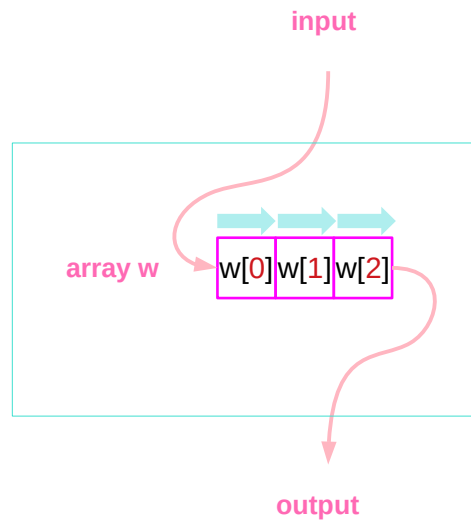
Holding a delayed input sequence

$$w_0(n) = x(n)$$

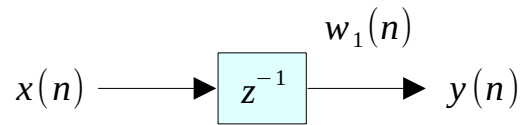
$$w_1(n) = x(n-1) = w_0(n-1)$$

$$w_2(n) = x(n-2) = w_1(n-1)$$

$$w_3(n) = x(n-3) = w_2(n-1)$$



Single Delay (1)



$$w_1(n) = x(n-1) \quad (\text{internal state at time } n)$$

$$w_1(n+1) = x(n) \quad (\text{internal state at time } n+1)$$

$$y(n) = w_1(n)$$

$$w_1(n+1) = x(n)$$

$$y(n+1) = w_1(n+1)$$

$$w_1(n+2) = x(n+1)$$

$$w_1(0) = 0$$

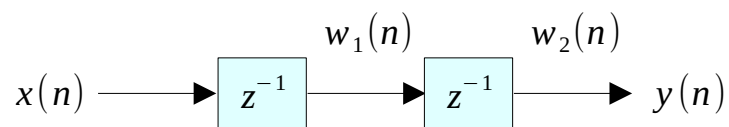
$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample x do:

$$y := w_1$$

$$w_1 := x$$

Double Delay (1)



$$w_2(n) = w_1(n-1) = x((n-1)-1) = x(n-2)$$

$$w_1(n) = x(n-1)$$

$$w_2(n+1) = w_1(n) \quad n \quad x(n) \quad w_1(n) \quad w_2(n) \quad y(n)$$

$$w_1(n+1) = x(n) \quad 0 \quad x_0 \quad 0 \quad 0 \quad 0$$

$$1 \quad x_1 \quad x_0 \quad 0 \quad 0$$

$$y(n) = w_2(n) \quad 2 \quad x_2 \quad x_1 \quad x_0 \quad x_0$$

$$w_2(n+1) = w_1(n) \quad 3 \quad x_3 \quad x_2 \quad x_1 \quad x_1$$

$$w_1(n+1) = x(n) \quad 4 \quad x_4 \quad x_3 \quad x_2 \quad x_2$$

$$w_1(0) = 0$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

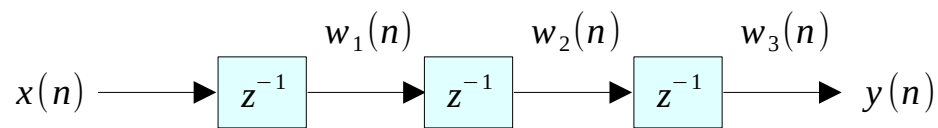
for each input sample x do:

$$y := w_2$$

$$w_2 := w_1$$

$$w_1 := x$$

Triple Delay (1)



$$w_3(n) = w_2(n-1) = w_1(n-2) = x(n-3)$$

$$w_2(n) = w_1(n-1)$$

$$w_1(n) = x(n-1)$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample x do:

$$y := w_3$$

$$w_3 := w_2$$

$$w_2 := w_1$$

$$w_1 := x$$

$$w_3(n+1) = w_2(n) \quad n \quad x(n) \quad w_1(n) \quad w_2(n) \quad y(n)$$

$$w_2(n+1) = w_1(n) \quad 0 \quad x_0 \quad 0 \quad 0 \quad 0$$

$$w_1(n+1) = x(n) \quad 1 \quad x_1 \quad x_0 \quad 0 \quad 0$$

$$2 \quad x_2 \quad x_1 \quad x_0 \quad x_0$$

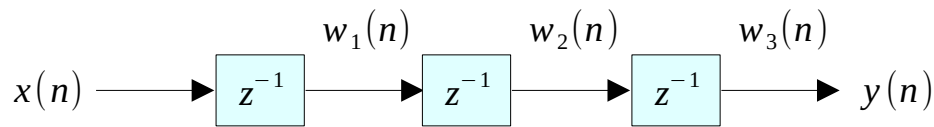
$$y(n) = w_3(n) \quad 3 \quad x_3 \quad x_2 \quad x_1 \quad x_1$$

$$w_3(n+1) = w_2(n) \quad 4 \quad x_4 \quad x_3 \quad x_2 \quad x_2$$

$$w_2(n+1) = w_1(n)$$

$$w_1(n+1) = x(n)$$

D Unit Delay (1)



$$w_i(n) = w_{i-1}(n-1) \quad \text{for } i = 1, 2, \dots, D$$

$$w_3(n+1) = w_2(n) \quad n \quad x(n) \quad w_1(n) \quad w_2(n) \quad y(n)$$

$$w_2(n+1) = w_1(n) \quad 0 \quad x_0 \quad 0 \quad 0 \quad 0$$

$$w_1(n+1) = x(n) \quad 1 \quad x_1 \quad x_0 \quad 0 \quad 0$$

$$2 \quad x_2 \quad x_1 \quad x_0 \quad x_0$$

$$y(n) = w_D(n) \quad 3 \quad x_3 \quad x_2 \quad x_1 \quad x_1$$

$$w_0(n) = x(n) \quad 4 \quad x_4 \quad x_3 \quad x_2 \quad x_2$$

$$w_i(n+1) = w_{i-1}(n)$$

$$i = D, D-1, \dots, 2, 1$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample x do:

$$y := w_D$$

$$w_0 := x$$

$$w_0 := x$$

for $i = D, D-1, \dots, 1$ do:

$$w_i := w_{i-1}$$

for each input sample w_0 do:

for $i = D, D-1, \dots, 1$ do:

$$w_i := w_{i-1}$$

D Unit Delay (1)

```
/* delay.c - delay by D time samples */
void delay(int D, double *w)      w[0] = input, w[D] = output
{
    int i;

    for (i=D; i>=1; i--)          reverse-order updating
        w[i] = w[i-1];

}
```

dot

/ dot.c - dot product of two length-(M+1) vectors */*

```
double dot(int M, double *h, double *w)
```

```
{
```

```
    int i;
```

```
    double y;
```

```
    for (y=0, i=0; i<=M; i++)
```

```
        y += h[i] * w[i];
```

```
    return y;
```

```
}
```

Usage: y = dot(M, h, w);

h = filter vector, w = state vector

M = filter order

compute dot product

$$y = h_0 w_0 + h_1 w_1 + \dots + h_M w_M = [h_0, h_1, \dots, h_M] \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_M \end{bmatrix} = \mathbf{h}^T \mathbf{w}$$

Direct Form

Considering the widely used
Edge triggered
D-type Flip Flops

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2}$$

References

- [1] S. J. Ofranidis , Introduction to Signal Processing