z-Testing

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Outline

Based on

"Understanding Statistics in the Behavioral Sciences" R. R. Pagano
https://en.wikipedia.org/

Sampling distribution

(1) all the values that the statistic can make (2) the probability of getting each value
under assumption that it resulted from chance alone

Null Hypothesis Population

- an actual or theoretical set of population scores
  that would result if the experiment were done on the entire population
  and the independent variable had no effect
- called the null hypothesis population because it is used
  to test the validity of the null hypothesis

Sampling distribution of the mean (1)

- gives all the values a statistic can make, along with the probability of getting each value if sampling is random from the null hypothesis population

Sampling distribution of the mean (2)

- $\mu_{\overline{X}}$
  mean of the sampling distribution of the mean
- $\sigma_{\overline{X}}$
  standard deviation of the sampling distribution of the mean
- $\mu_{\overline{X}} = \mu$
  equal to the mean of the raw-score population
- $\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$
  equal to the standard devivation of the raw-score population divided by $\sqrt{N}$

Sampling distribution of the mean (3)

$\sigma_{\overline{X}} = \sigma/\sqrt{N}$ the Sample Mean Scores

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{N}}$ $\sigma_{\overline{X}} = \sqrt{\frac{\Sigma(\overline{X}-\mu_{\overline{X}})^2}{N}}$

Central limit theorem

- A z-test is any statistical test for which the distribution of the test statistic under the null hypothesis can be approximated by a normal distribution.
- Because of the central limit theorem, many test statistics are approximately normally distributed for large samples.

Critical value

- For each significance level,
  the z-test has a single critical value
- 1.96 for 5% two tailed
- more convenient than the Student's t-test
  which has separate critical values for each sample size.

Approximate tests

- many statistical tests can be conveniently performed as approximate z-tests if the sample size is large or the population variance is known.

- If the population variance is *unknown*
  (and therefore has to be estimated from the sample itself)
  and the sample size is not large ($n < 30$),
  the Student's t-test may be more appropriate.

One-sample location test

- The term z-test is often used to refer specifically to the one-sample location test comparing the mean of a set of measurements to a given constant when the sample variance is known.

z-test conditions

- If the observed data $X_1, ..., X_n$ are
  (1) independent,
  (2) have a common mean μ, and
  (3) have a common variance σ^2,
- then the sample average $\overline{X}$
  has mean μ and variance σ^2/n.

z-test null hypothesis

- The null hypothesis is that
  the mean value of X is a given number μ_0.
- we can use $\overline{X}$ as a test-statistic,
  rejecting the null hypothesis if $X - \mu_0$ is large.

σ^2

- To calculate the standardized statistic $Z = (X - 0)/s$, we need to either know or have an approximate value for σ^2, from which we can calculate $s^2 = \sigma^2/n$.
- In some applications, σ^2 is known, but this is uncommon.

σ^2

- If the sample size is moderate or large,
  we can substitute the sample variance for σ^2,
  giving a plug-in test.
- The resulting test will not be an exact Z-test
  since the uncertainty in the sample variance is not accounted for
- however, it will be a good approximation unless the sample size is small.

z equations

$$z = \frac{X - \mu}{\sigma} \qquad z_{obt} = \frac{\overline{X}_{obt} - \mu_{\overline{X}}}{\sigma_{\overline{X}}}$$

obt

- obt means *obtained*
- subscript obt or opt is an *obtained* value in statistics test
- refers to the actual results *obtained* from the sample data set
- e.g. mean (μ_{obt})and standard deviation (σ_{obt}) compared to the expected population mean

https://answers.yahoo.com/question/index?qid=20090418043924AAGDeK2&guccounter=1&gu

critical region and critical value

- the critical region for rejection of the null hypothesis
  the area under the curve that contains all the values of the statistic that allow rejection of the null hypothesis
- the critical value of a statistic
  the value of the statistic that bounds the critical region