

# Logic Background (1A)

---

Copyright (c) 2014 - 2018 Young W. Lim.

Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is based on 6.825:

Techniques in Artificial Intelligence - courses - MIT

<https://courses.csail.mit.edu/6.825/>

Please send corrections (or suggestions) to [youngwlim@hotmail.com](mailto:youngwlim@hotmail.com).

This document was produced by using LibreOffice.

# Proposition : etymology

---

From Old French, from Latin *prōpositiō*  
 (“a **proposing**, design, theme, case”).

**The content of an assertion**

that may be taken as being **true** or **false**

and is considered abstractly without reference to  
the linguistic sentence that constitutes the assertion.

# Predicate : etymology

From Middle French predicate (French prédicat),  
from post-classical Late Latin praedicatum (“**thing said of a subject**”),  
a noun use of the neuter past participle of praedicare (“**proclaim**”)

From Latin predicātus, perfect passive participle of praedicō,  
from prae + dicō (“**declare, proclaim**”), from dicō (“say, tell”).

# Proposition

---

In Aristotelian logic a proposition is a particular kind of sentence, one which affirms or denies a **predicate** of a subject.

In formal logic a proposition is considered as objects of a formal language.

A formal language begins with different types of **symbols**.

# Predicate

(grammar) The part of the sentence (or clause) which states something about the subject or the object of the sentence.

<u>The dog</u> <u>barked very loudly</u>
<i>subject</i> <i>predicate</i>

(logic) A term of a statement, where the statement may be true or false depending on whether the thing referred to by the values of the statement's variables has the property signified by that (predicative) term.

# Propositional Logic

**Propositional logic** includes only

- operators and
  - propositional constants
- as symbols in its language.

The propositions in this language are

- propositional constants  
considered atomic propositions
- composite propositions  
recursive application of operators to propositions

# Predicate Logic

**Predicate logic** include

- variables,
- operators,
- predicate and
- function symbols, and
- quantifiers

as symbols in their languages.



## A Formal Language

**Syntax** – legal expressions

**Semantics** – the meaning of legal expressions

**Proof System** – a way of manipulating syntactic expressions  
to get another syntactic expressions

Multiple Percepts → Conclusions

Current State, Operators → Next State Properties

# Propositional Logic

**Sentences** (WFFs : Well Formed Formulas)

T and F are **sentences**

**Propositional variables** are **sentences** (A, B, C, ...)

If A and B are sentences, the followings are also sentences

(A),  $\neg A$ ,  $A \wedge B$ ,  $A \vee B$ ,  $A \rightarrow B$ ,  $A \leftrightarrow B$

# Precedence of Connectives

$\neg$  highest

$\wedge$

$\vee$

$\rightarrow$

$\leftrightarrow$  lowest

$$A \vee B \wedge C = A \vee (B \wedge C)$$

$$A \wedge B \rightarrow C \vee D = (A \wedge B) \rightarrow (C \vee D)$$

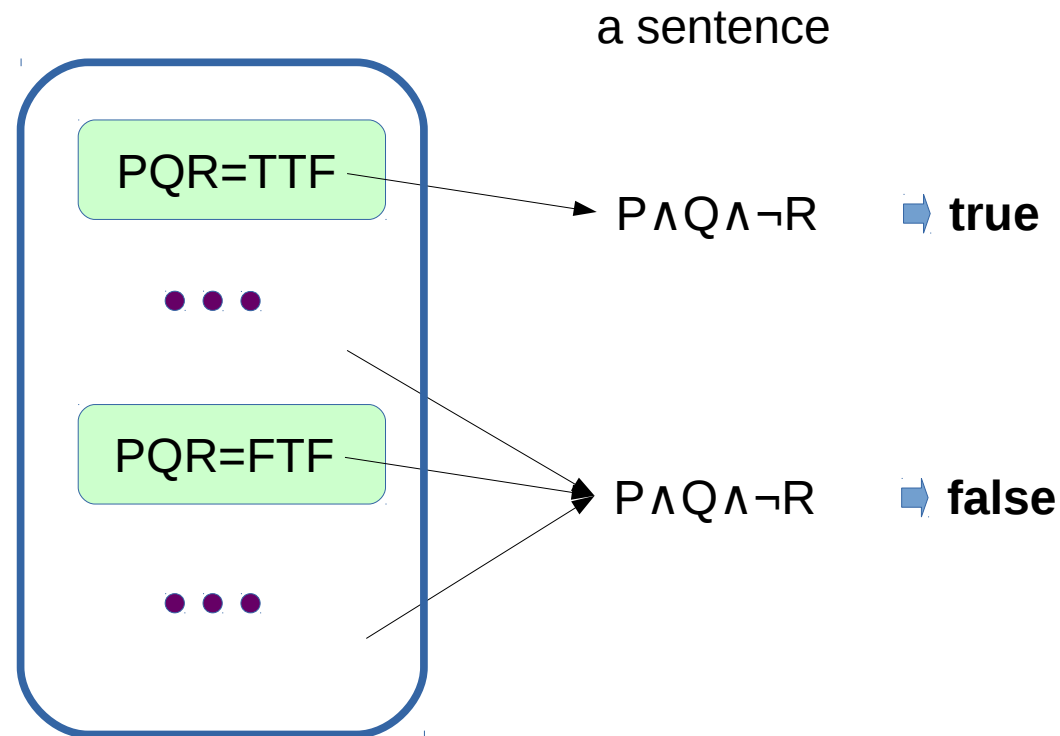
$$A \rightarrow B \vee C \leftrightarrow D = (A \rightarrow (B \vee C)) \leftrightarrow D$$

# Semantics

**Meaning** of a sentence : **true** or **false**

**Interpretation** : an assignment of **true** or **false**  
to the **propositional variables**

interpretations



# Models

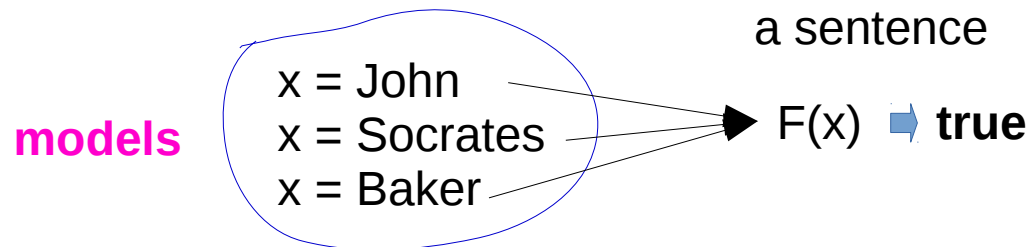
Semantics :

The relationship between **sentences** and **interpretations**

There are some **set of interpretations** that makes **a sentence true** → **models**

An interpretation is a **model** of a sentence  
if the **sentence** is **true** in that **interpretation**

An **interpretation**  $i$  is a **model** of a sentence  $\varphi$  **iff**  $\models_i \varphi$



# Models and Interpretations

models

8 interpretations

P	Q	R	$P \wedge (Q \vee R)$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

T	T	T
T	T	F

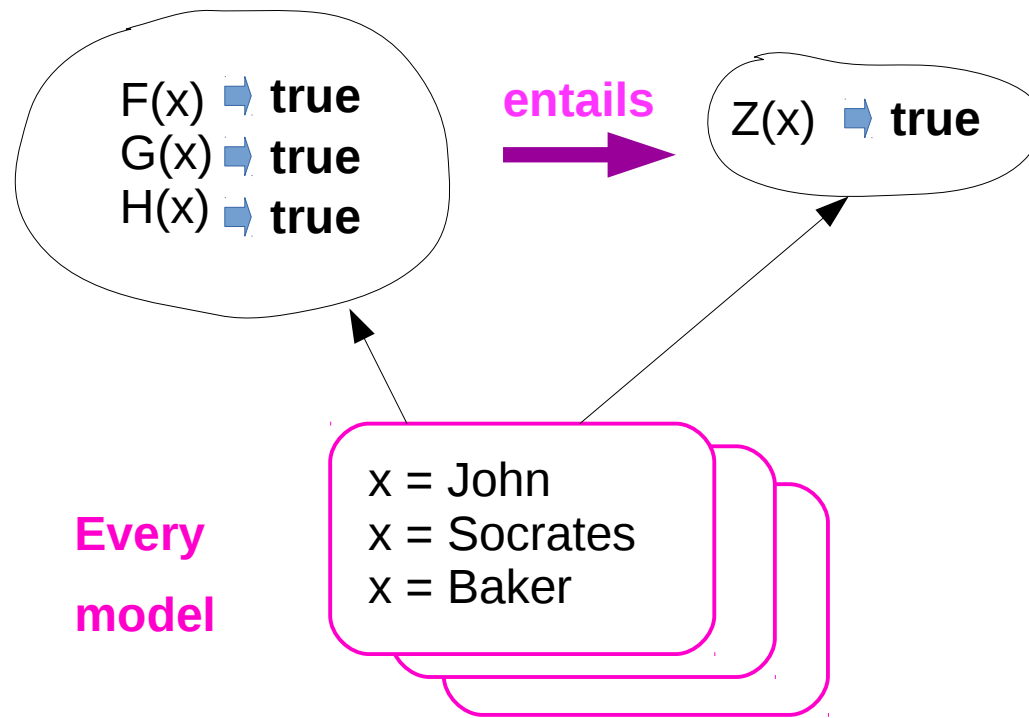
T	T	T
T	F	T

T	F	T
---	---	---

# Entailment

An interpretation  $i$  is a **model** of a sentence  $\varphi$  **iff**  $\models_i \varphi$

A set of sentences KB **entails** a sentence  $\varphi$   
**iff** every **model** of KB is also a **model** of  $\varphi$



## References

- [1] en.wikipedia.org
- [2] en.wiktionary.org
- [3] U. Endriss, “Lecture Notes : Introduction to Prolog Programming”
- [4] <http://www.learnprolognow.org/> Learn Prolog Now!
- [5] [http://www.csupomona.edu/~jrfisher/www/prolog\\_tutorial](http://www.csupomona.edu/~jrfisher/www/prolog_tutorial)
- [6] [www.cse.unsw.edu.au/~billw/cs9414/notes/prolog/intro.html](http://www.cse.unsw.edu.au/~billw/cs9414/notes/prolog/intro.html)
- [7] [www.cse.unsw.edu.au/~billw/dictionaries/prolog/negation.html](http://www.cse.unsw.edu.au/~billw/dictionaries/prolog/negation.html)
- [8] <http://ilppp.cs.lth.se/>, P. Nugues, `An Intro to Lang Processing with Perl and Prolog