Digital Signal Octave Codes (0B)

- Aliasing and Folding Frequencies

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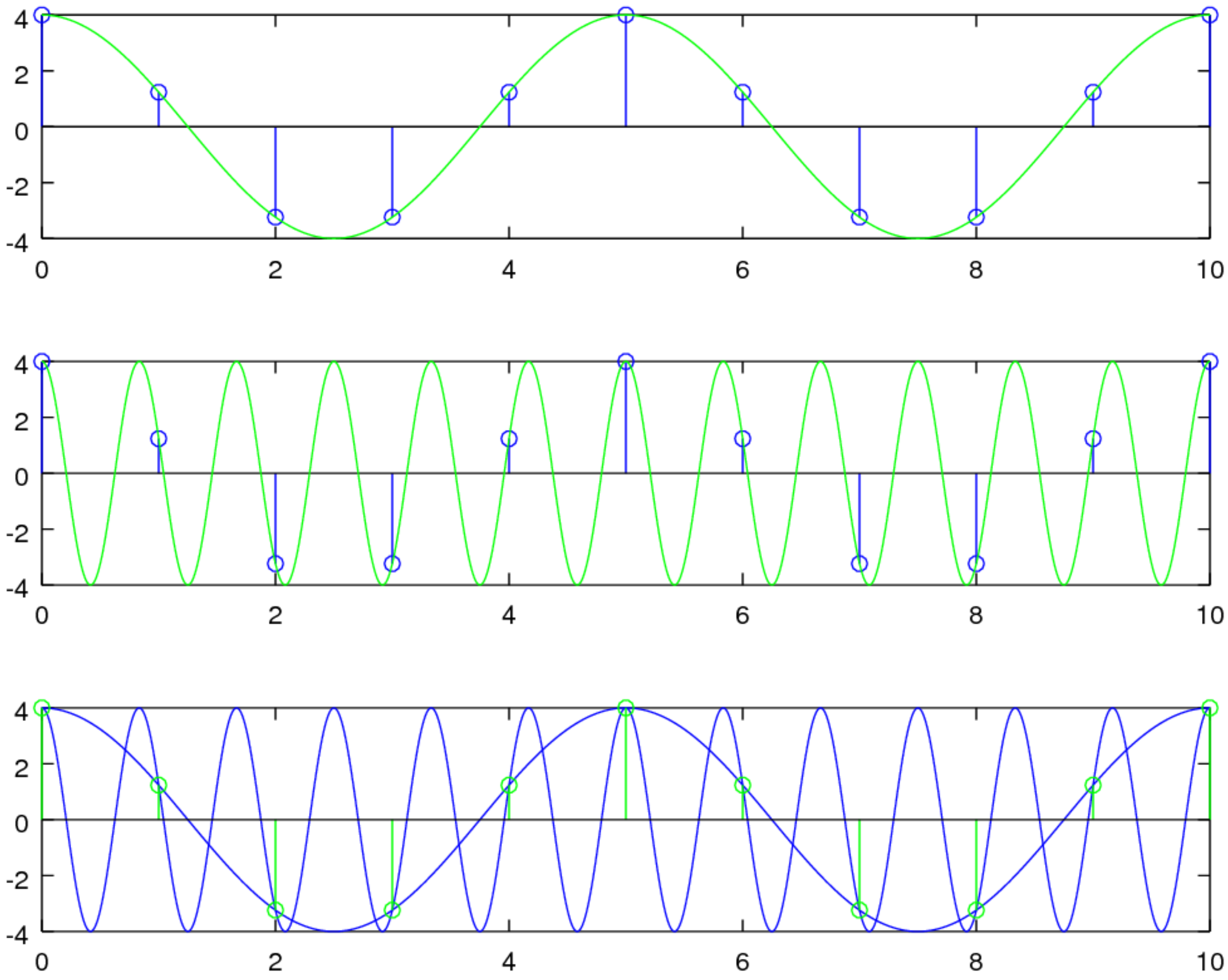
Based on
M.J. Roberts, Fundamentals of Signals and Systems
S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed
S.D. Stearns, Digital Signal Processing with Examples in MATLAB

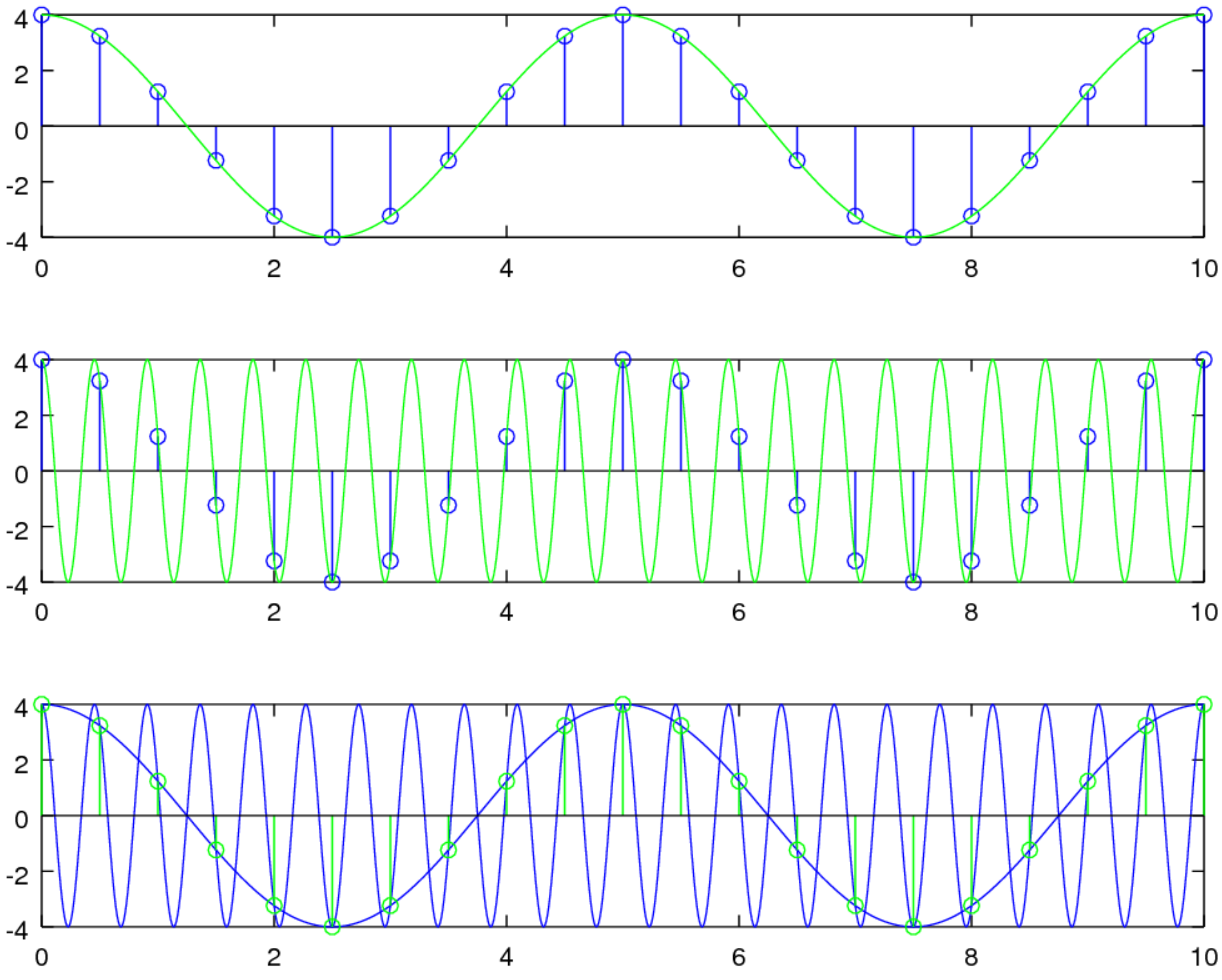
Aliasing Condition Examples




```
clf
n = [0:1:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(6/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

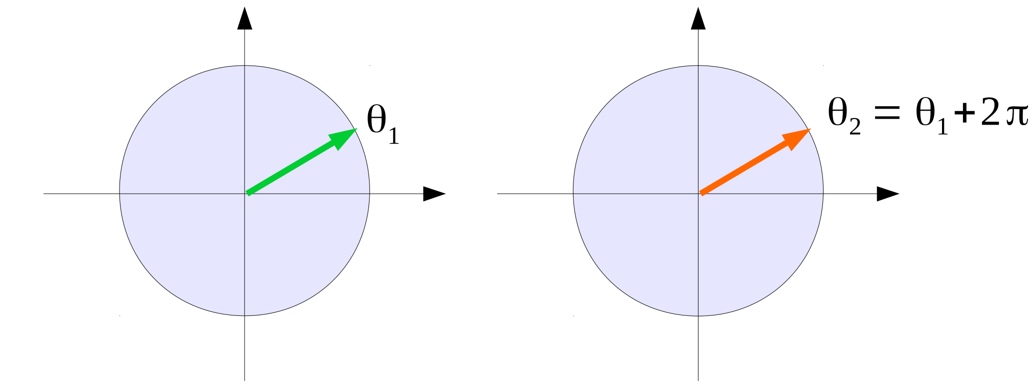
Aliasing Condition Examples




```
clf
n = [0:0.5:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(11/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(11/5)*t);

subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

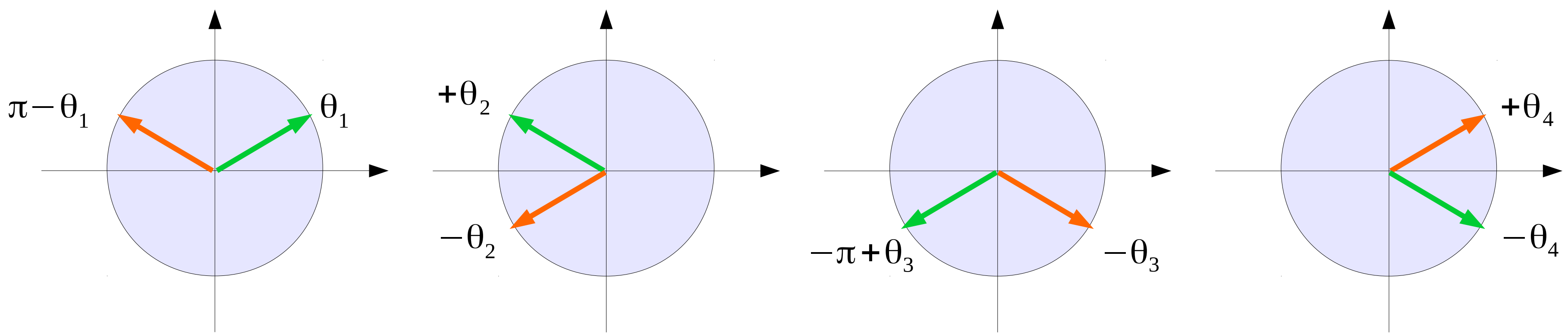
M.J. Roberts, Fundamentals of Signals and Systems

Identical Sine values and Cosine Values



$$\omega_1 t - \omega_2 t = 2\pi$$

periodic condition



$\omega_1 t + \omega_2 t = +\pi$	$\omega_1 t + \omega_2 t = 0$	$\omega_1 t + \omega_2 t = -\pi$	$\omega_1 t + \omega_2 t = 0$
$\sin(\omega_1 t) = \sin(\omega_2 t)$	$-\sin(\omega_1 t) = \sin(\omega_2 t)$	$\sin(\omega_1 t) = \sin(\omega_2 t)$	$-\sin(\omega_1 t) = \sin(\omega_2 t)$
$-\cos(\omega_1 t) = \cos(\omega_2 t)$	$\cos(\omega_1 t) = \cos(\omega_2 t)$	$-\cos(\omega_1 t) = \cos(\omega_2 t)$	$\cos(\omega_1 t) = \cos(\omega_2 t)$

Identical Sine values and Cosine Values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

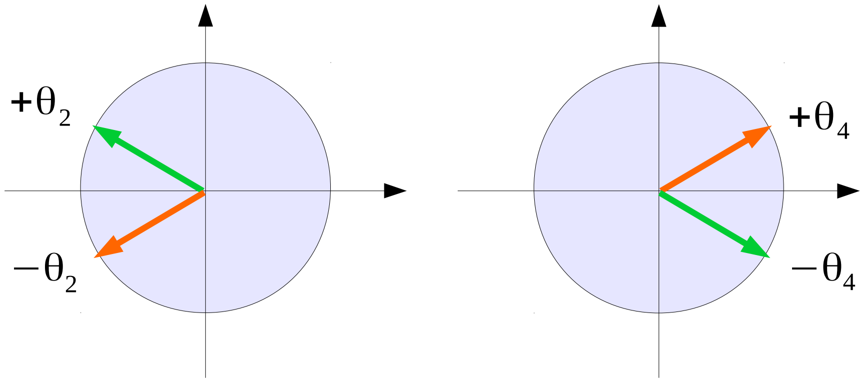
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Angles of identical trigonometric values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

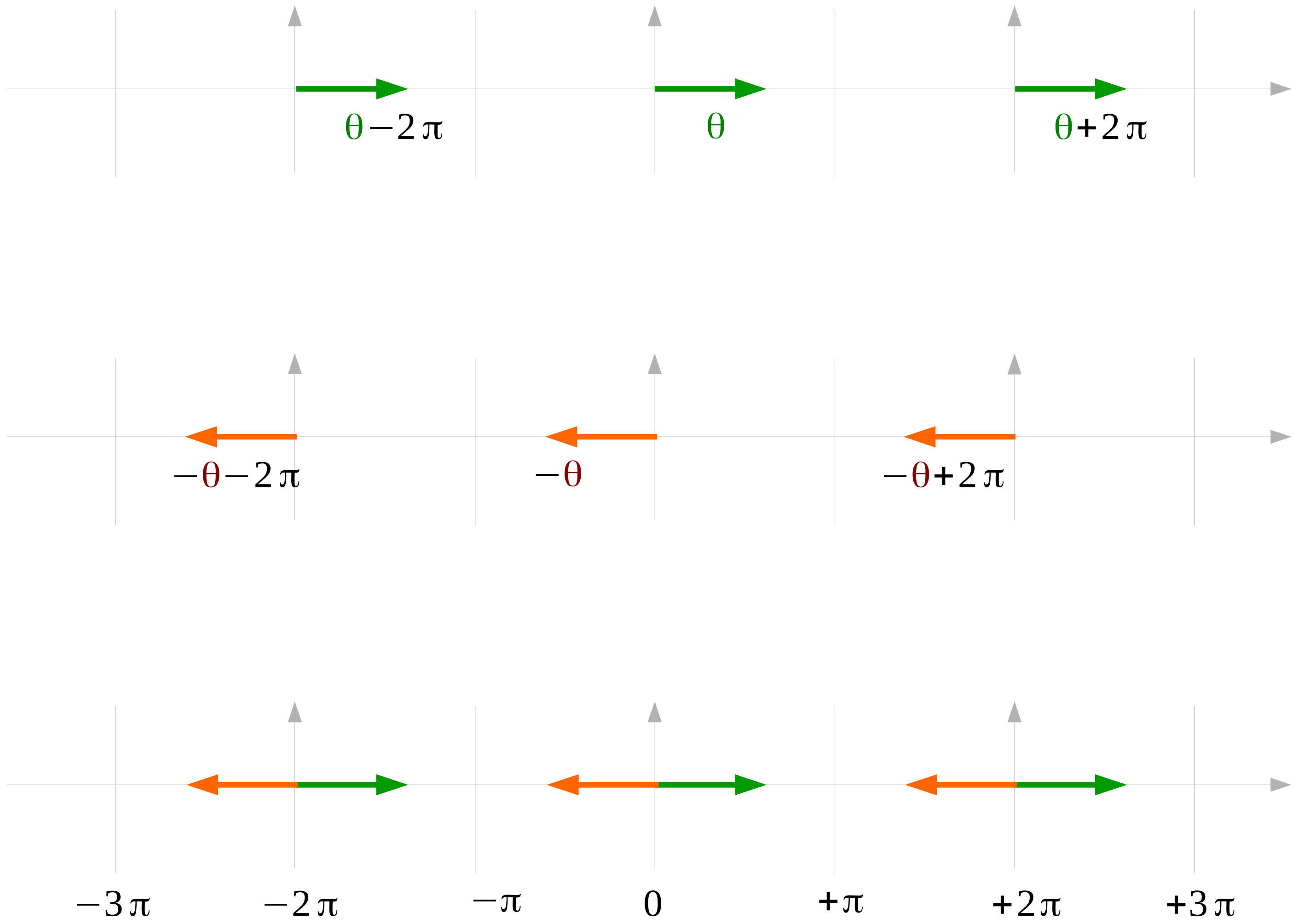
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



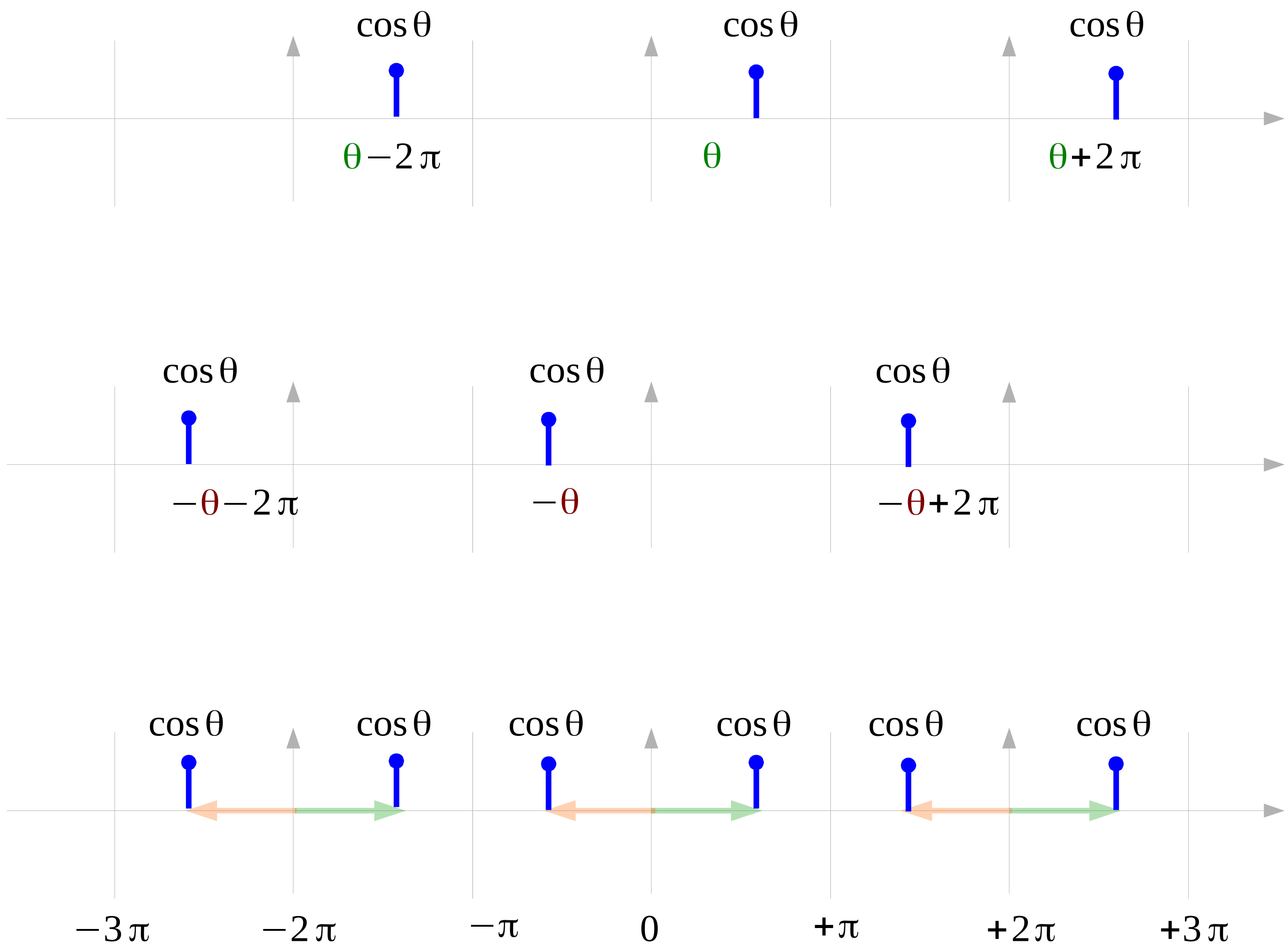
Identical Cosine Values

periodic condition

$\omega_1 t - \omega_2 t = 2n\pi$

$\sin(\omega_1 t) = \sin(\omega_2 t)$

$\cos(\omega_1 t) = \cos(\omega_2 t)$



$\omega_1 t + \omega_2 t = 2n\pi$

$-\sin(\omega_1 t) = \sin(\omega_2 t)$

$\cos(\omega_1 t) = \cos(\omega_2 t)$

Identical Absolute Sine values

periodic condition

$$\omega_1 t - \omega_2 t = 2n\pi$$

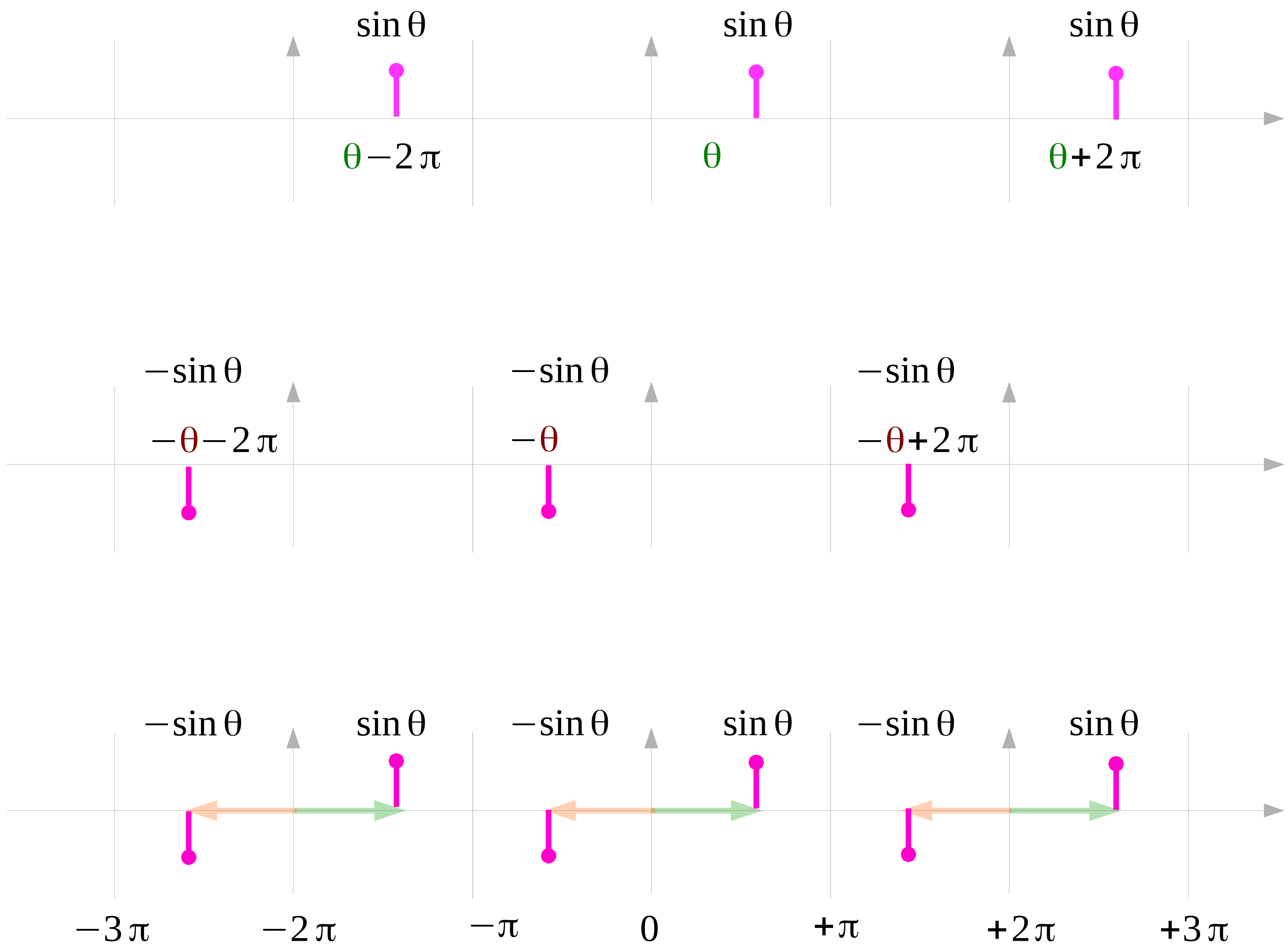
$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$



Spectrum Representation

periodic condition

$\omega_1 t - \omega_2 t = 2n\pi$

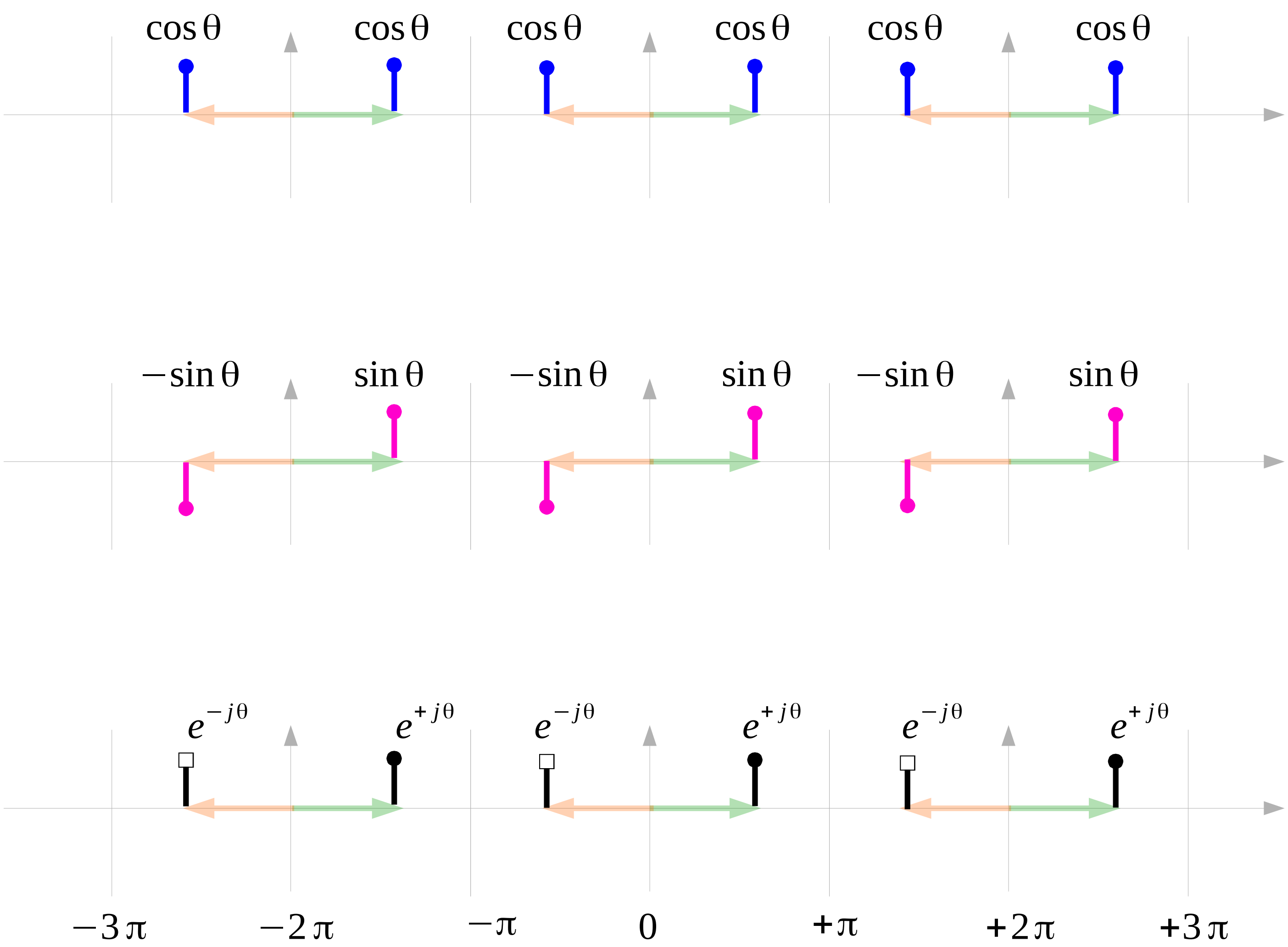
$\sin(\omega_1 t) = \sin(\omega_2 t)$

$\cos(\omega_1 t) = \cos(\omega_2 t)$

$\omega_1 t + \omega_2 t = 2n\pi$

$-\sin(\omega_1 t) = \sin(\omega_2 t)$

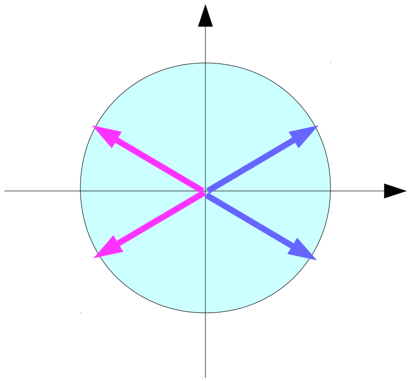
$\cos(\omega_1 t) = \cos(\omega_2 t)$



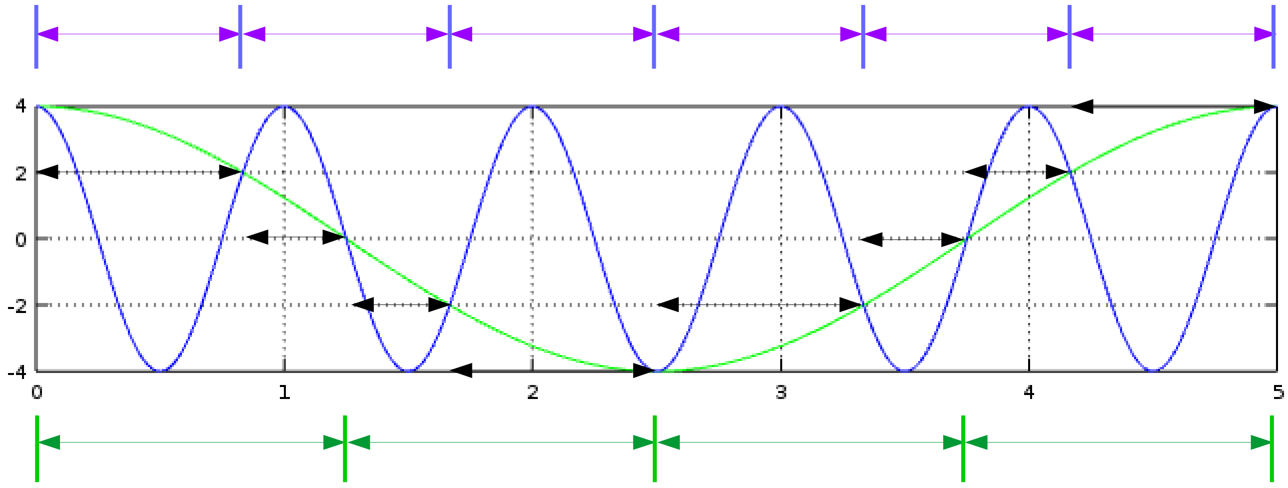
$\cos(2\pi f_1 t) = \cos(2\pi f_2 t)$

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

$$\omega_1 t - \omega_2 t = 2n\pi$$
$$\omega_1 t + \omega_2 t = 2n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = n \end{cases} \qquad \begin{cases} t = \frac{5}{6}n \\ t = \frac{5}{4}n \end{cases}$$



$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \cdots$ $\quad \omega_1 t + \omega_2 t = 2n\pi$

$\cos(\omega_1 t) = \cos(\omega_2 t)$

$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \cdots$ $\quad \omega_1 t - \omega_2 t = 2n\pi$

$\cos(\omega_1 t) = \cos(\omega_2 t)$

Example Frequency Pairs

$f_1 = \frac{2}{5}$	$2\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{3}{5}$	$3\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{4}{5}$	$4\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{5}{5}$	$5\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{6}{5}$	$6\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{7}{5}$	$7\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$
$f_1 = \frac{8}{5}$	$8\ cycles/5\,sec$	⟷	$f_2 = \frac{1}{5}$	$1\ cycles/5\,sec$

Identical Cosine Value Conditions

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

$\omega_1 t + \omega_2 t = 2n\pi$ $\omega_1 t - \omega_2 t = 2n\pi$

Plotting the same valued **cosine** samples

```
clf
t = [0:500]/100;

n1 = 0: 5/2 : 5;
```

$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

$\omega_1 t - \omega_2 t = 2n\pi$

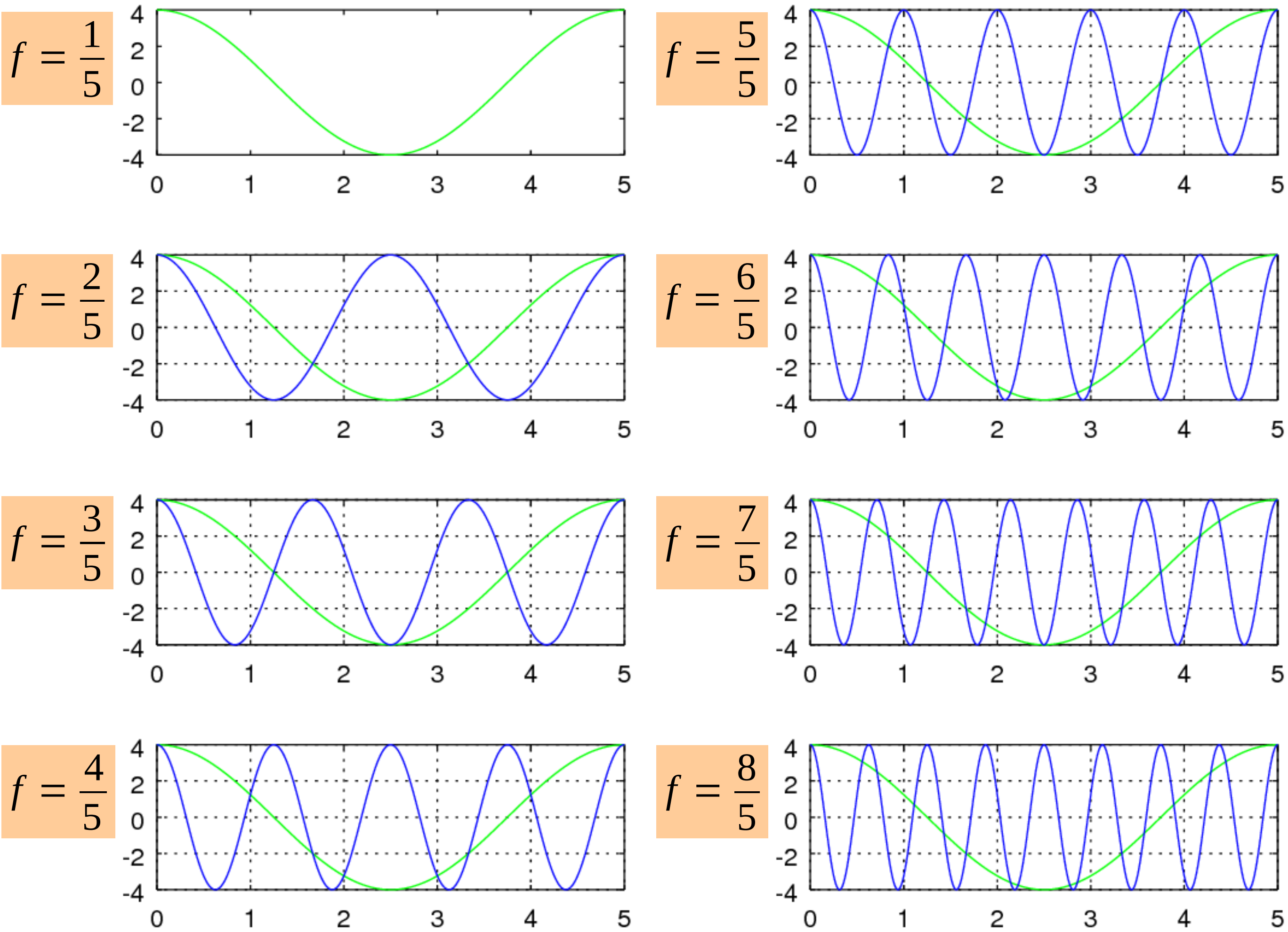
```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);

y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

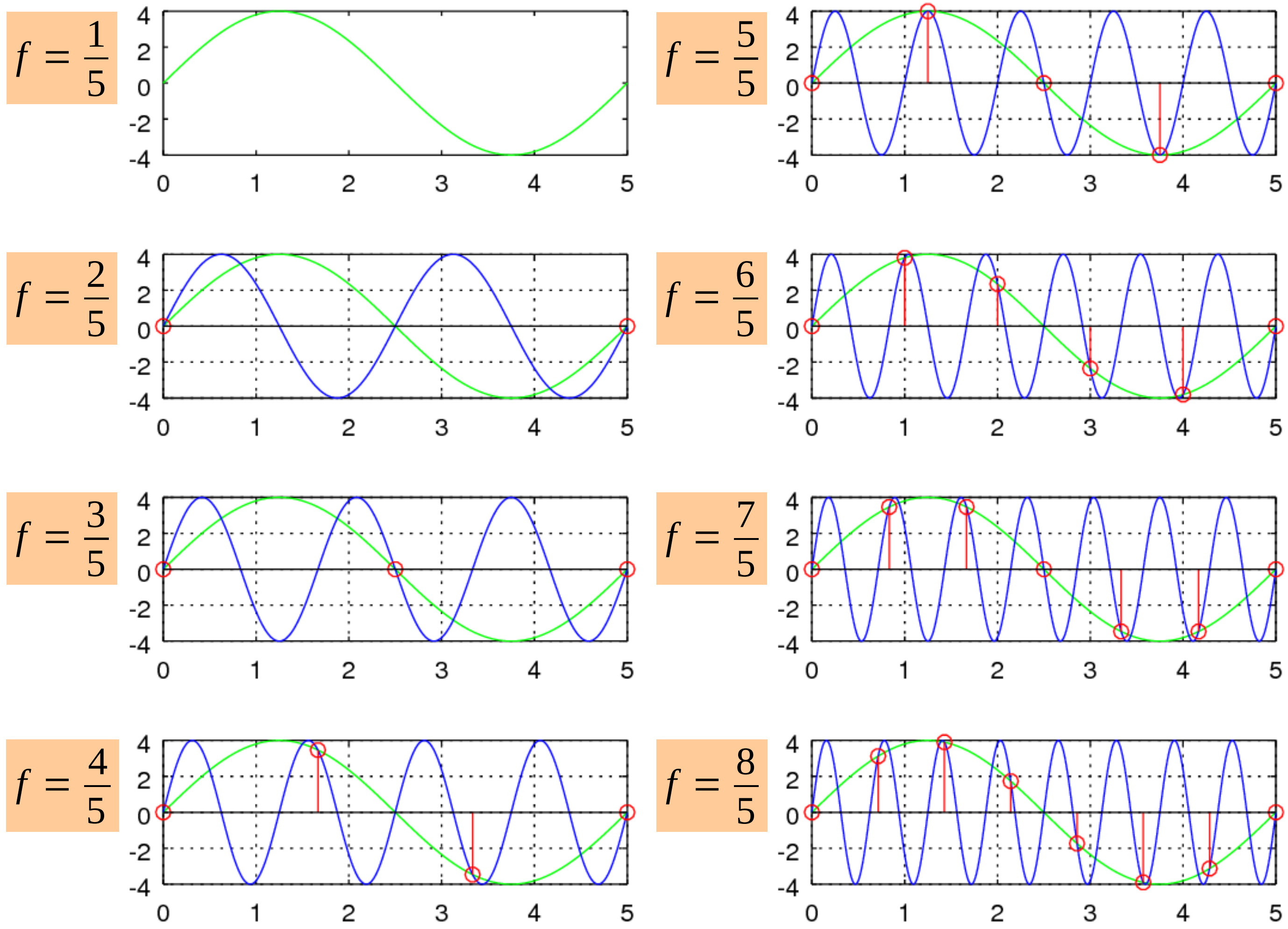
```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

Graphs of cos(2π(n/5)t) & cos(2π(1/5)t)




```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

Cosine values at $2\pi f_1 t + 2\pi f_2 t = 2n\pi$

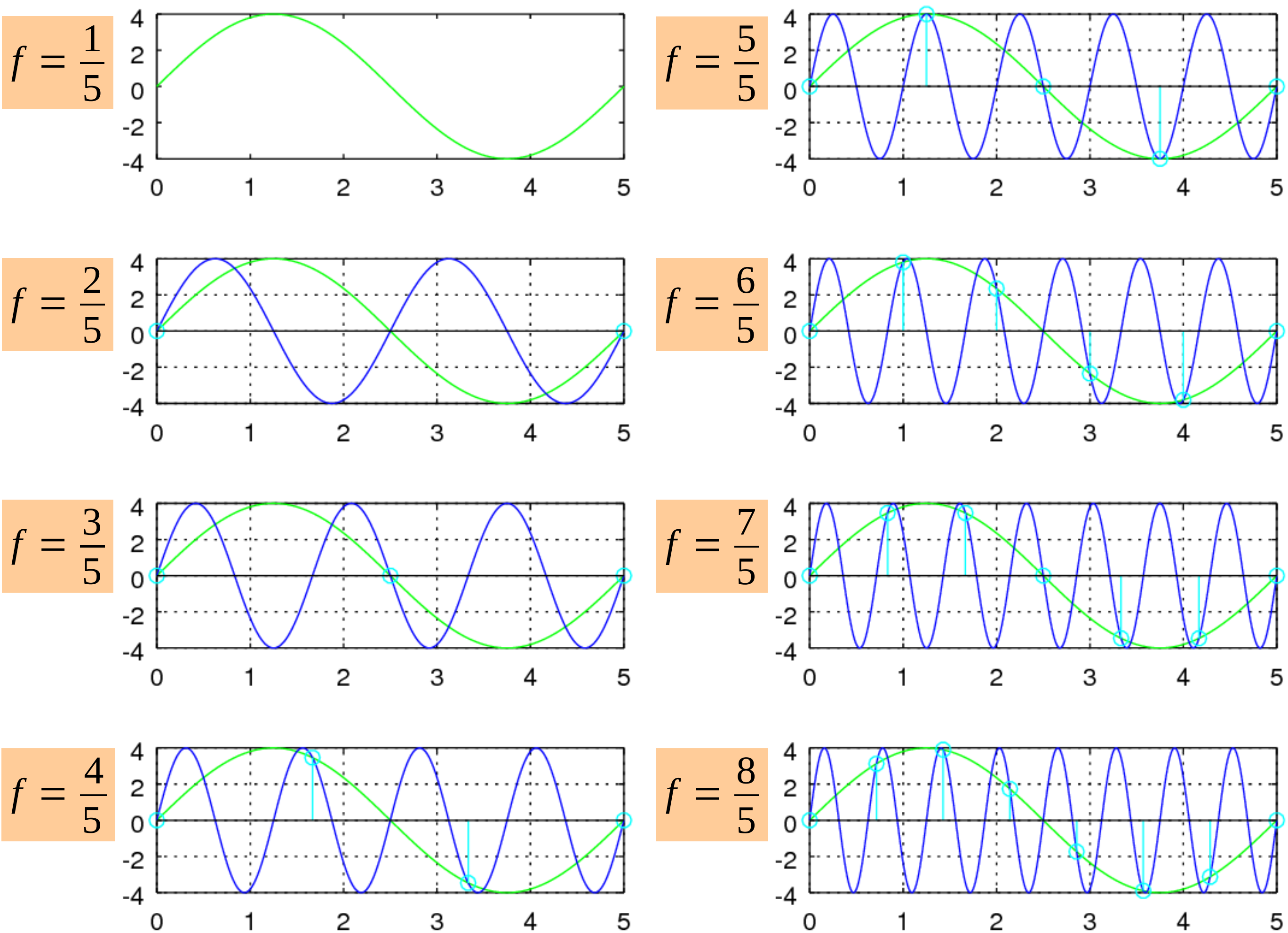


$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

Cosine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

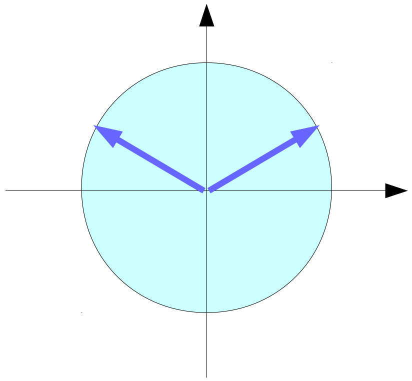
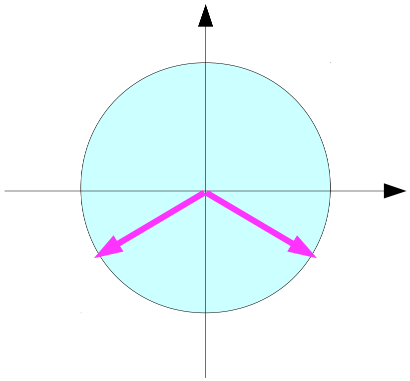


$\omega_1 t - \omega_2 t = 2n\pi$

```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

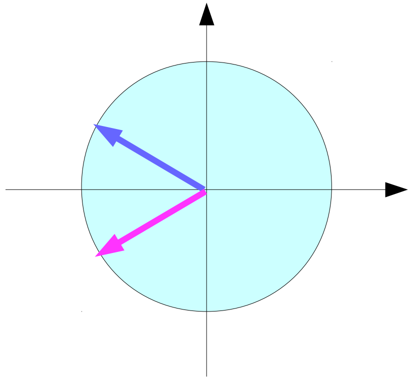
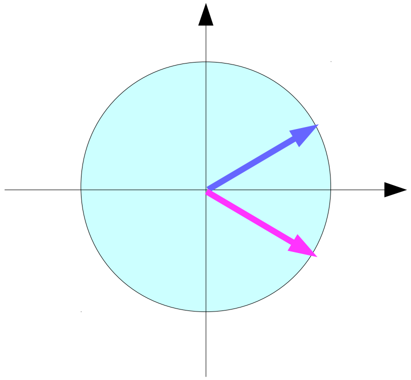
```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

$2\pi f_1 t + 2\pi f_2 t = 2n\pi$, $(2n+1)\pi$ conditions

$$\omega_1 t + \omega_2 t = (2n+1)\pi$$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

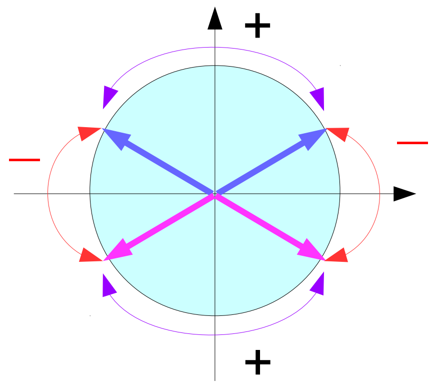
$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$\sin(2\pi f_1 t) = \pm\sin(2\pi f_2 t)$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

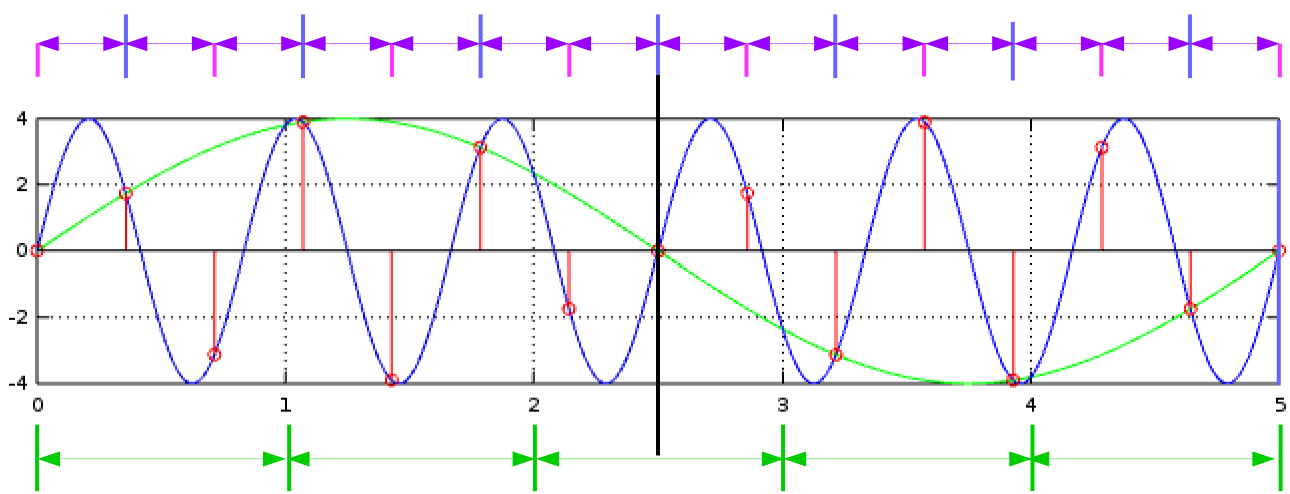
$$\omega_1 t - \omega_2 t = 2n\pi$$
$$\omega_1 t + \omega_2 t = n\pi$$

$$\pm\sin(\omega_1 t) = \sin(\omega_2 t)$$



$$\begin{cases} \frac{6}{5}t + \frac{1}{5}t = \frac{n}{2} \\ \frac{6}{5}t - \frac{1}{5}t = n \end{cases}$$

$$\begin{cases} t = \frac{5}{14}n \\ t = \frac{5}{5}n \end{cases}$$



$\frac{5}{14}, \frac{10}{14}, \frac{15}{14}, \cdots$ $\quad \omega_1 t + \omega_2 t = n\pi$

$$\pm\sin(\omega_1 t) = \sin(\omega_2 t)$$

$1,\ 2,\ 3,\ \cdots$ $\quad \omega_1 t - \omega_2 t = 2n\pi$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

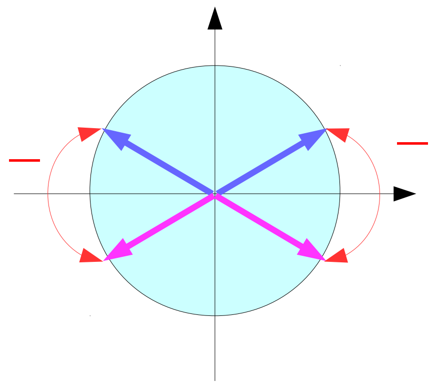
$\sin(2\pi f_1 t) = -\sin(2\pi f_2 t)$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

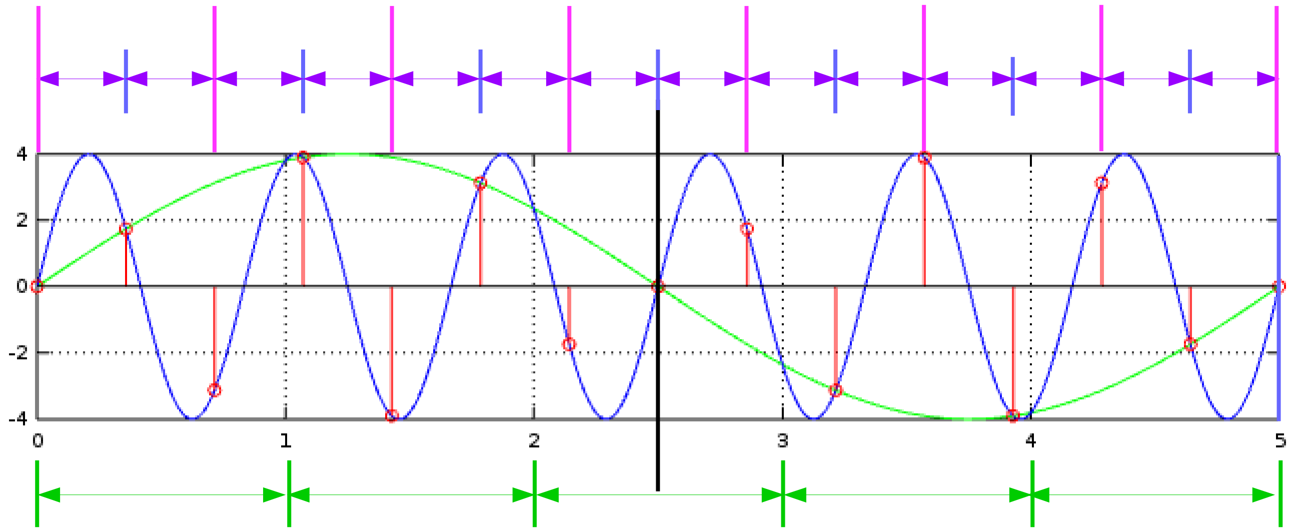
$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\omega_1 t + \omega_2 t = 2n\pi$$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$



$$\begin{cases} \frac{6}{5}t + \frac{1}{5}t = n \\ \frac{6}{5}t - \frac{1}{5}t = n \end{cases} \qquad \begin{cases} t = \frac{5}{7}n \\ t = \frac{5}{5}n \end{cases}$$



$\frac{5}{7}, \frac{10}{7}, \frac{15}{7}, \cdots$ $\qquad \omega_1 t + \omega_2 t = 2n\pi$

$$-\sin(\omega_1 t) = \sin(\omega_2 t)$$

$1, \ 2, \ 3, \ \cdots$ $\qquad \omega_1 t - \omega_2 t = 2n\pi$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

Aliasing Condition Examples

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

Plotting the same valued **sine** samples

```
clf
t = [0:500]/100;

n1 = 0: 5/2 : 5;
```

$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

$\omega_1 t - \omega_2 t = 2n\pi$

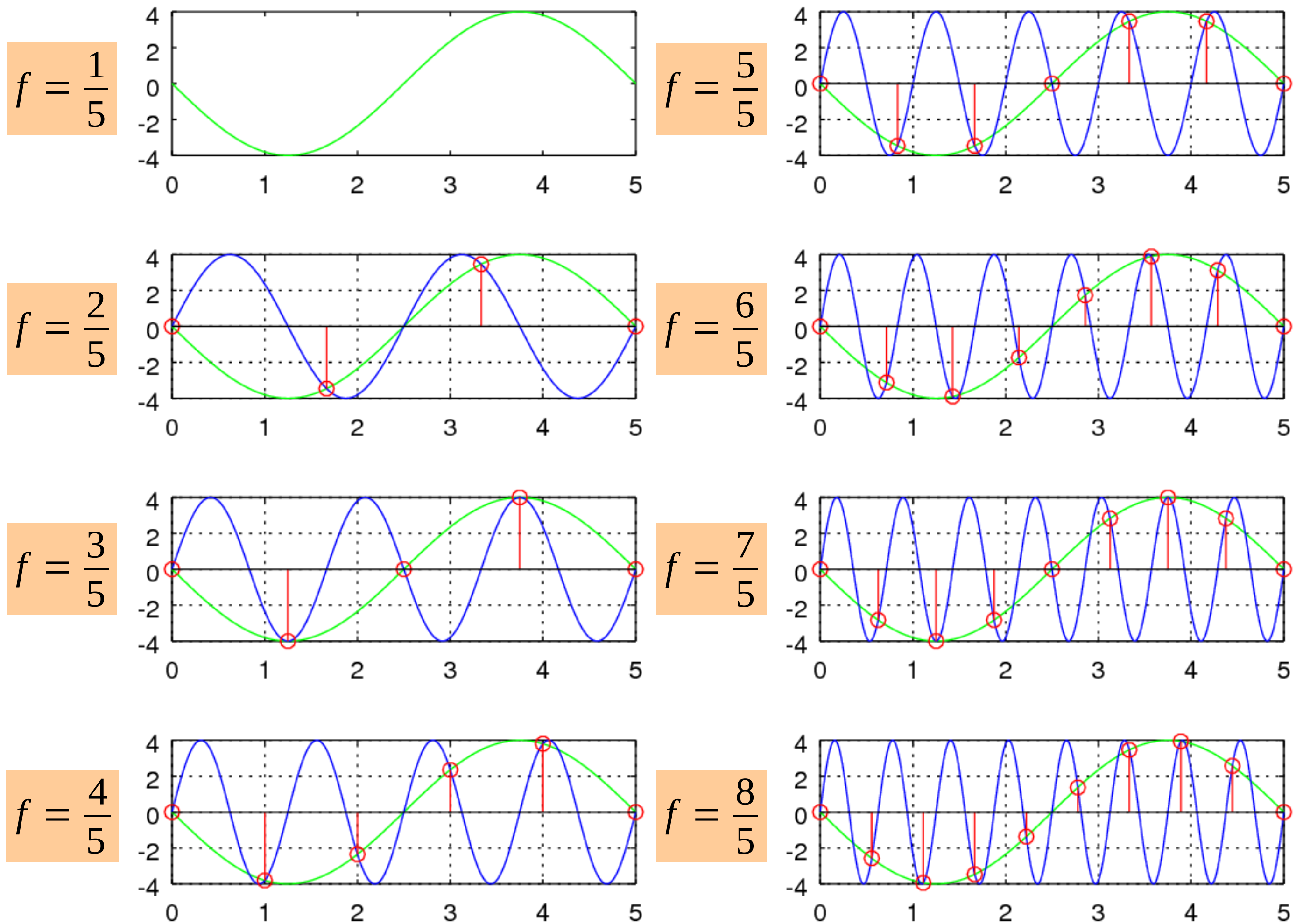
```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
yt1 =–4*sin(2*pi*(1/5)*t);
yt2 = 4*sin(2*pi*(2/5)*t);
yt3 = 4*sin(2*pi*(3/5)*t);
yt4 = 4*sin(2*pi*(4/5)*t);
yt5 = 4*sin(2*pi*(5/5)*t);
yt6 = 4*sin(2*pi*(6/5)*t);
yt7 = 4*sin(2*pi*(7/5)*t);
yt8 = 4*sin(2*pi*(8/5)*t);

y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

Sine values at $2\pi f_1 t + 2\pi f_2 t = 2n\pi$

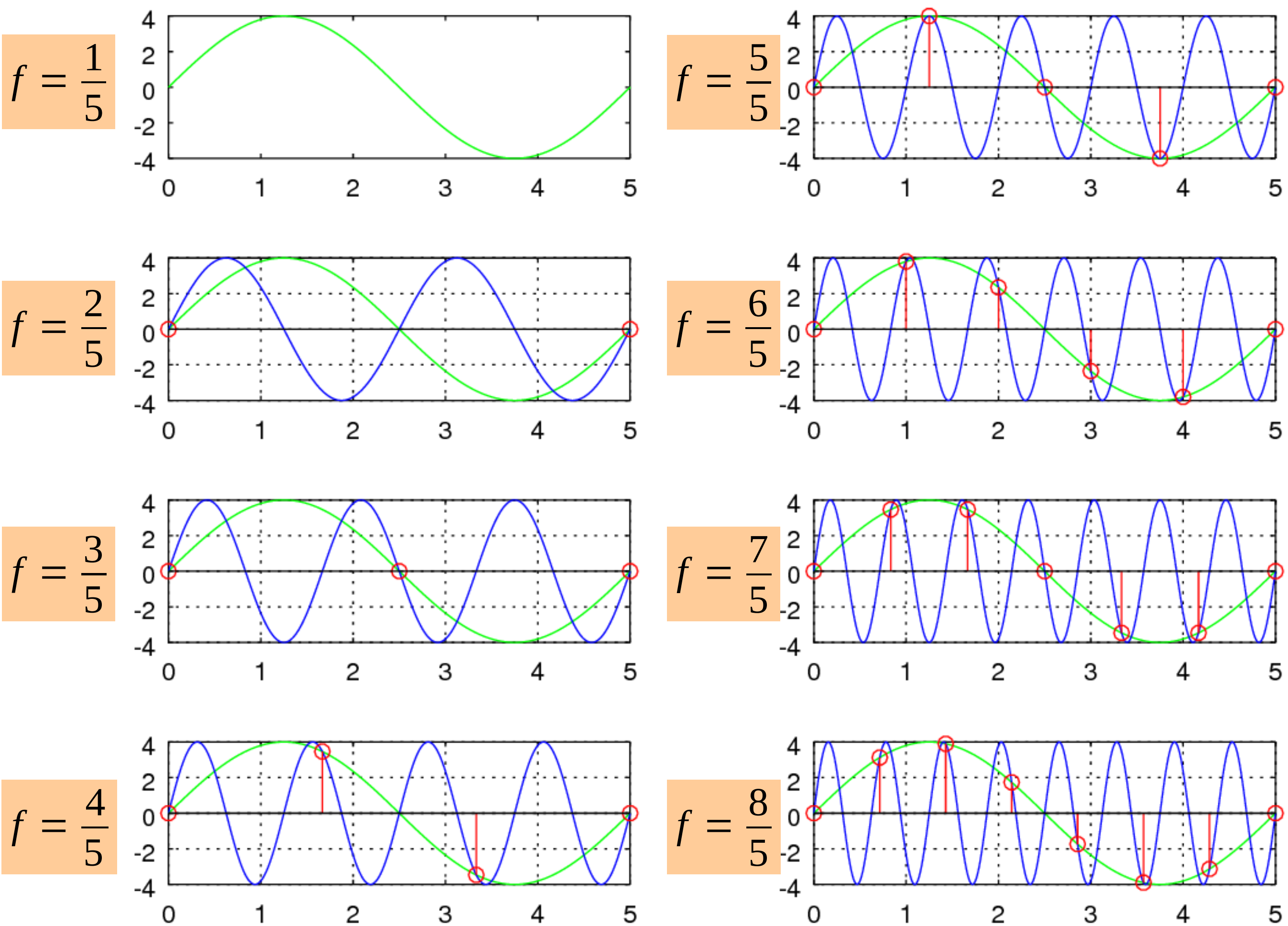


$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);

yt1 =–4*sin(2*pi*(1/5)*t);
```

Sine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

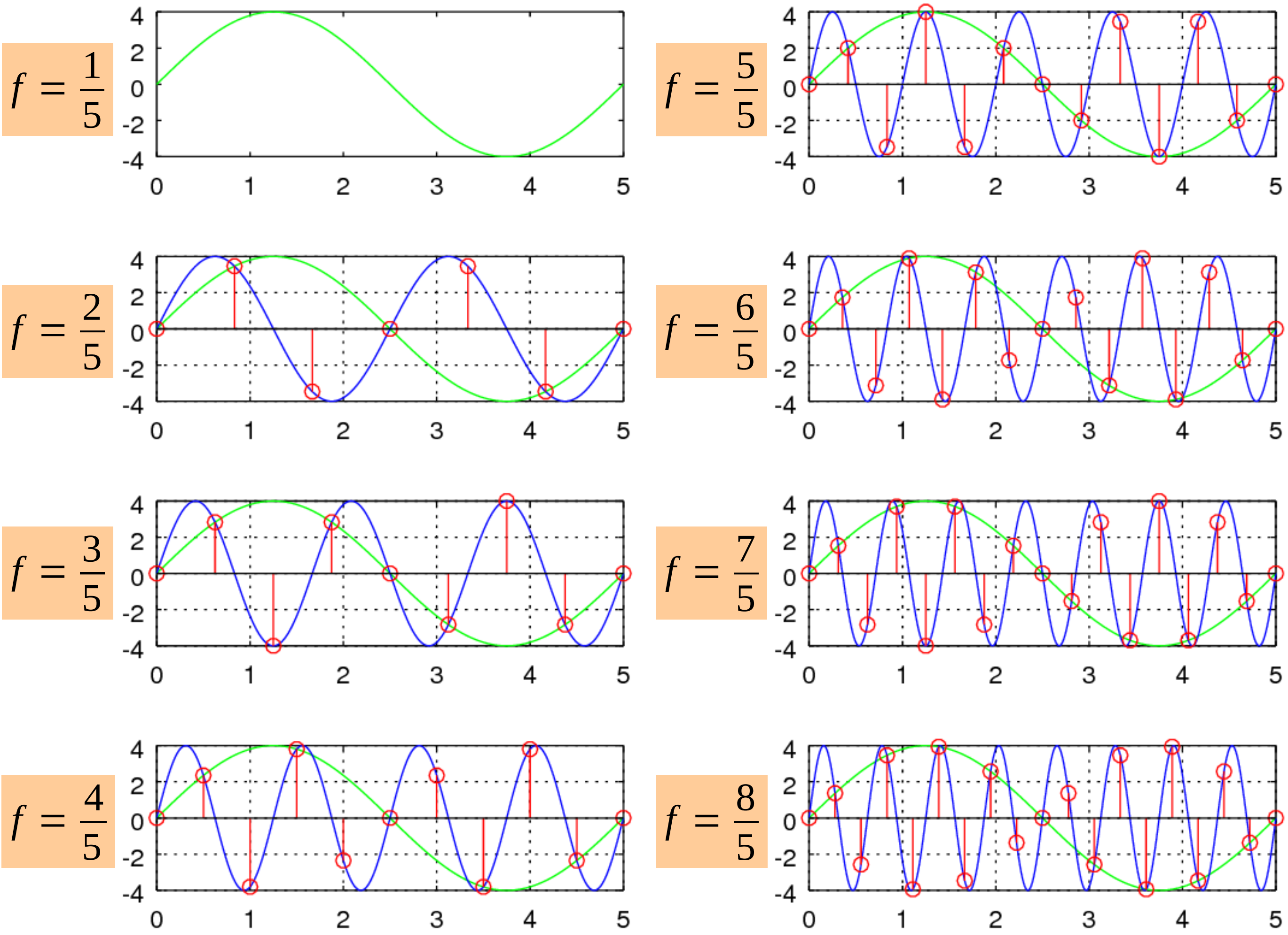


$\omega_1 t - \omega_2 t = 2n\pi$

```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);

yt1 =+4*sin(2*pi*(1/5)*t);
```

Sine values at $2\pi f_1 t + 2\pi f_2 t = n\pi$

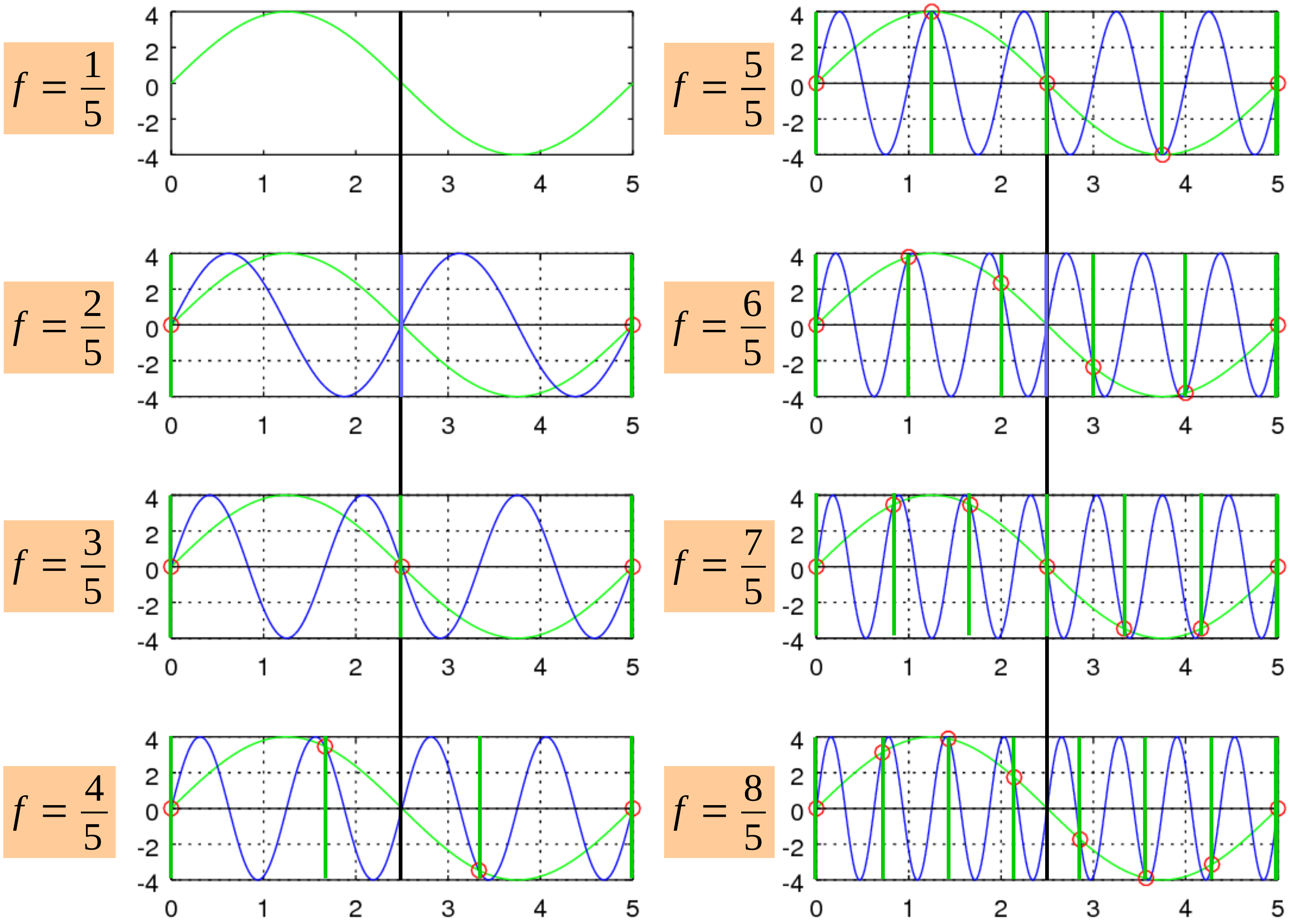


$\omega_1 t + \omega_2 t = 2n\pi$

```
n2 = 0: (1/2)5/3 : 5;
n3 = 0: (1/2)5/4 : 5;
n4 = 0: (1/2)5/5 : 5;
n5 = 0: (1/2)5/6 : 5;
n6 = 0: (1/2)5/7 : 5;
n7 = 0: (1/2)5/8 : 5;
n8 = 0: (1/2)5/9 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

Sine values at $2\pi f_1 t - 2\pi f_2 t = 2n\pi$

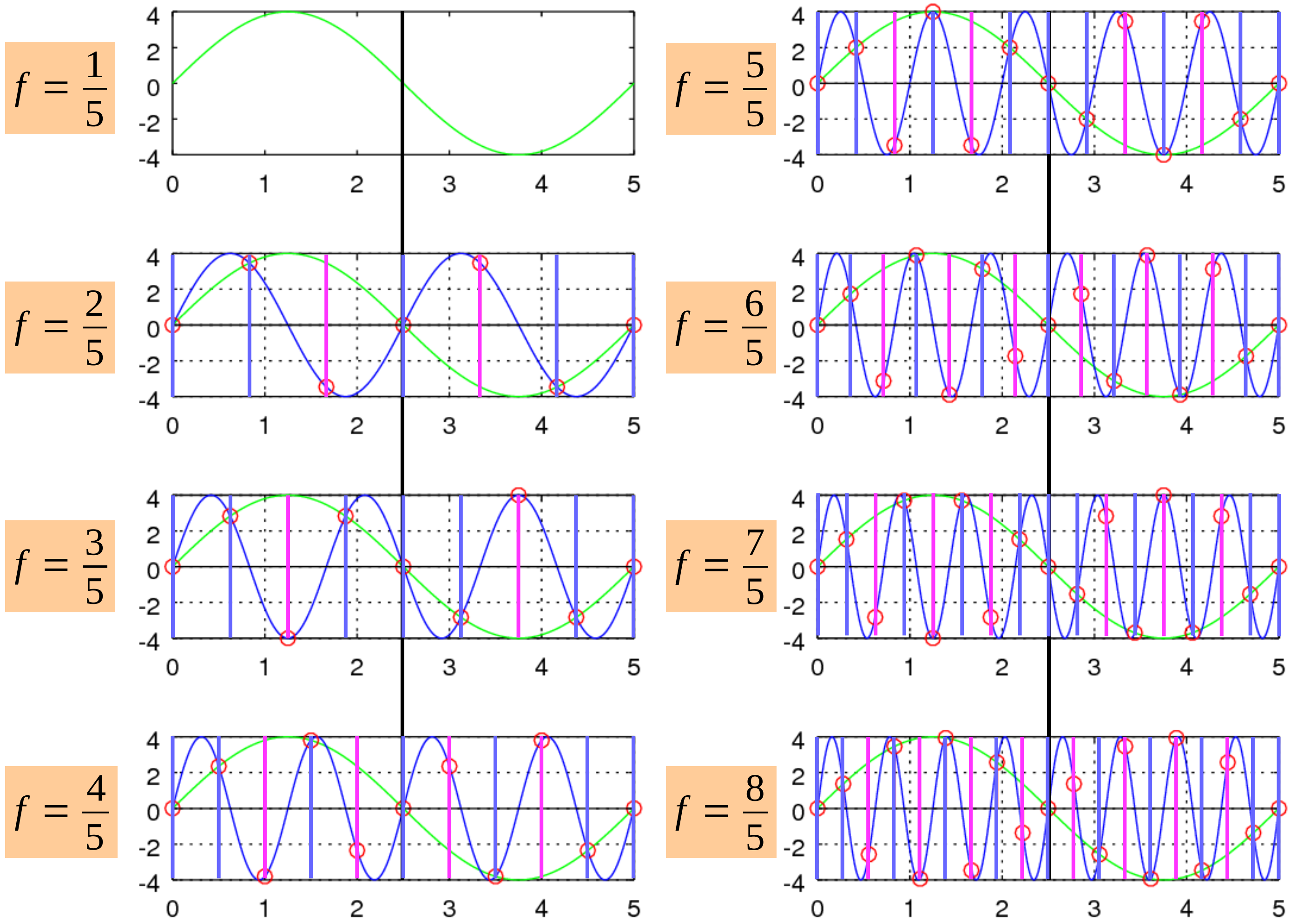


$\omega_1 t - \omega_2 t = 2n\pi$

n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;

y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);

Aliasing Condition Examples (1)

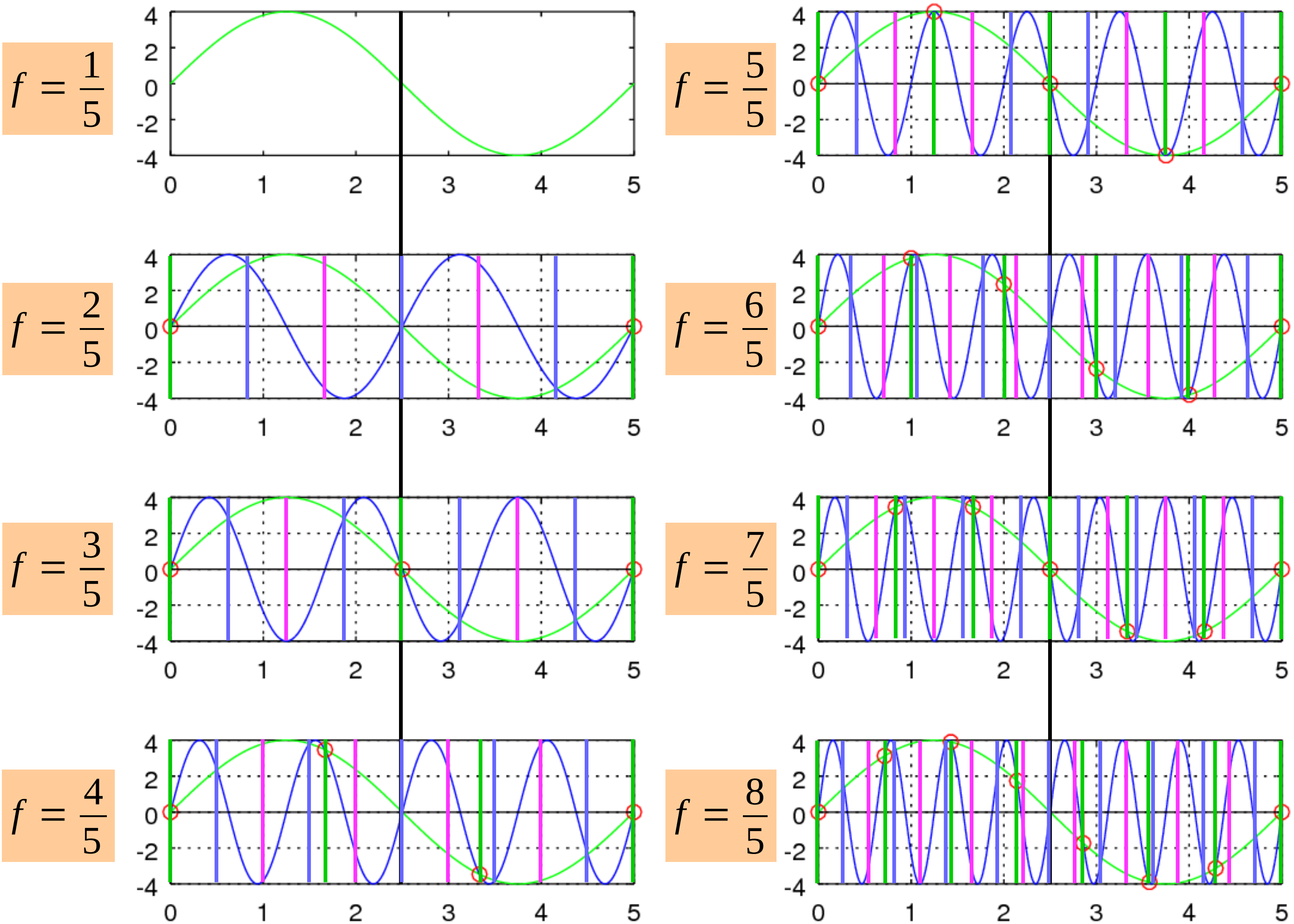


$$\omega_1 t + \omega_2 t = 2 n \pi$$

```
n2 = 0: (1/2)5/3 : 5;
n3 = 0: (1/2)5/4 : 5;
n4 = 0: (1/2)5/5 : 5;
n5 = 0: (1/2)5/6 : 5;
n6 = 0: (1/2)5/7 : 5;
n7 = 0: (1/2)5/8 : 5;
n8 = 0: (1/2)5/9 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

Aliasing Condition Examples (2)

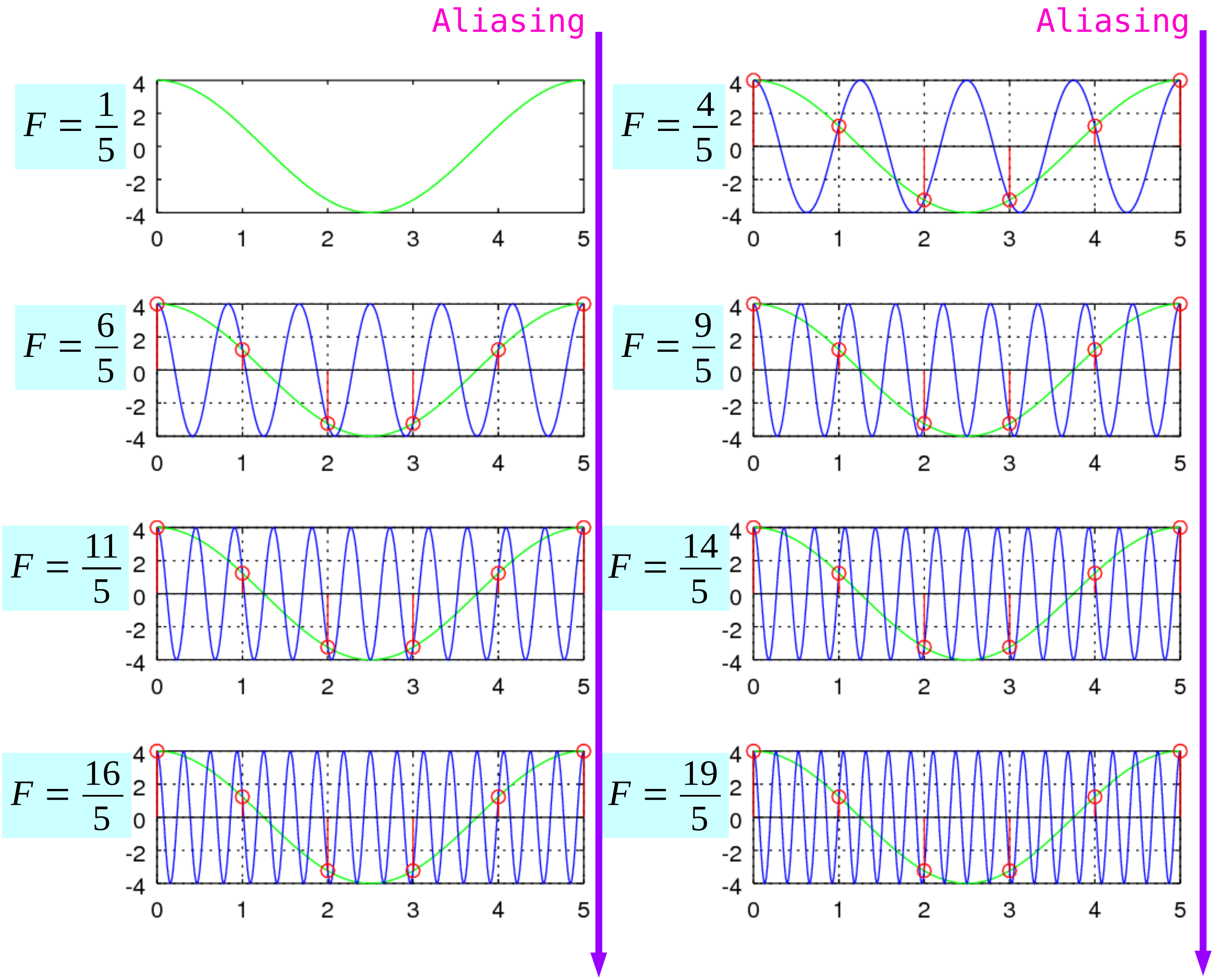


$$\omega_1 t - \omega_2 t = 2n\pi$$

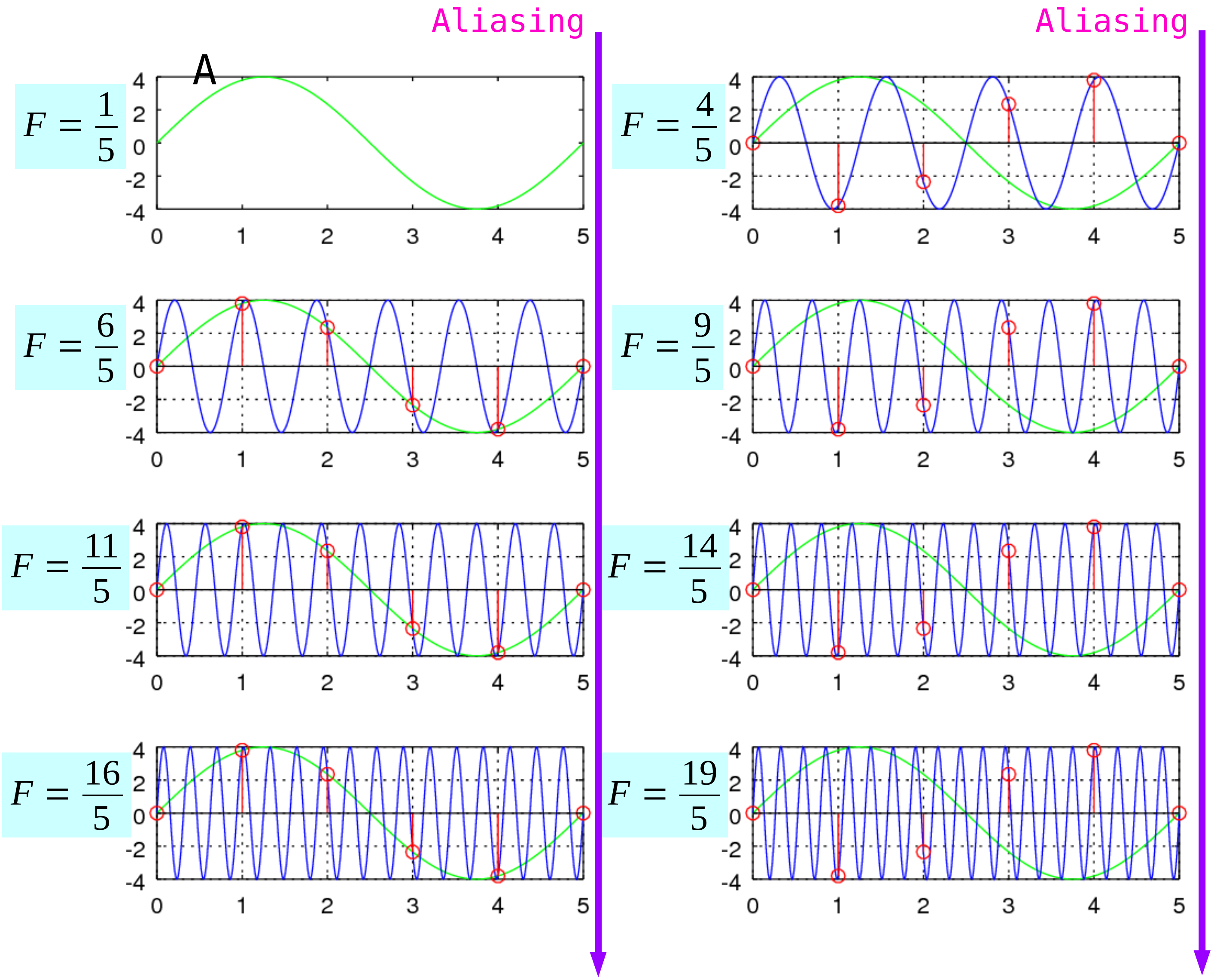
```
n2 = 0: 5/1 : 5;
n3 = 0: 5/2 : 5;
n4 = 0: 5/3 : 5;
n5 = 0: 5/4 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/6 : 5;
n8 = 0: 5/7 : 5;
```

```
y2 = 4*sin(2*pi*(2/5)*n2);
y3 = 4*sin(2*pi*(3/5)*n3);
y4 = 4*sin(2*pi*(4/5)*n4);
y5 = 4*sin(2*pi*(5/5)*n5);
y6 = 4*sin(2*pi*(6/5)*n6);
y7 = 4*sin(2*pi*(7/5)*n7);
y8 = 4*sin(2*pi*(8/5)*n8);
```

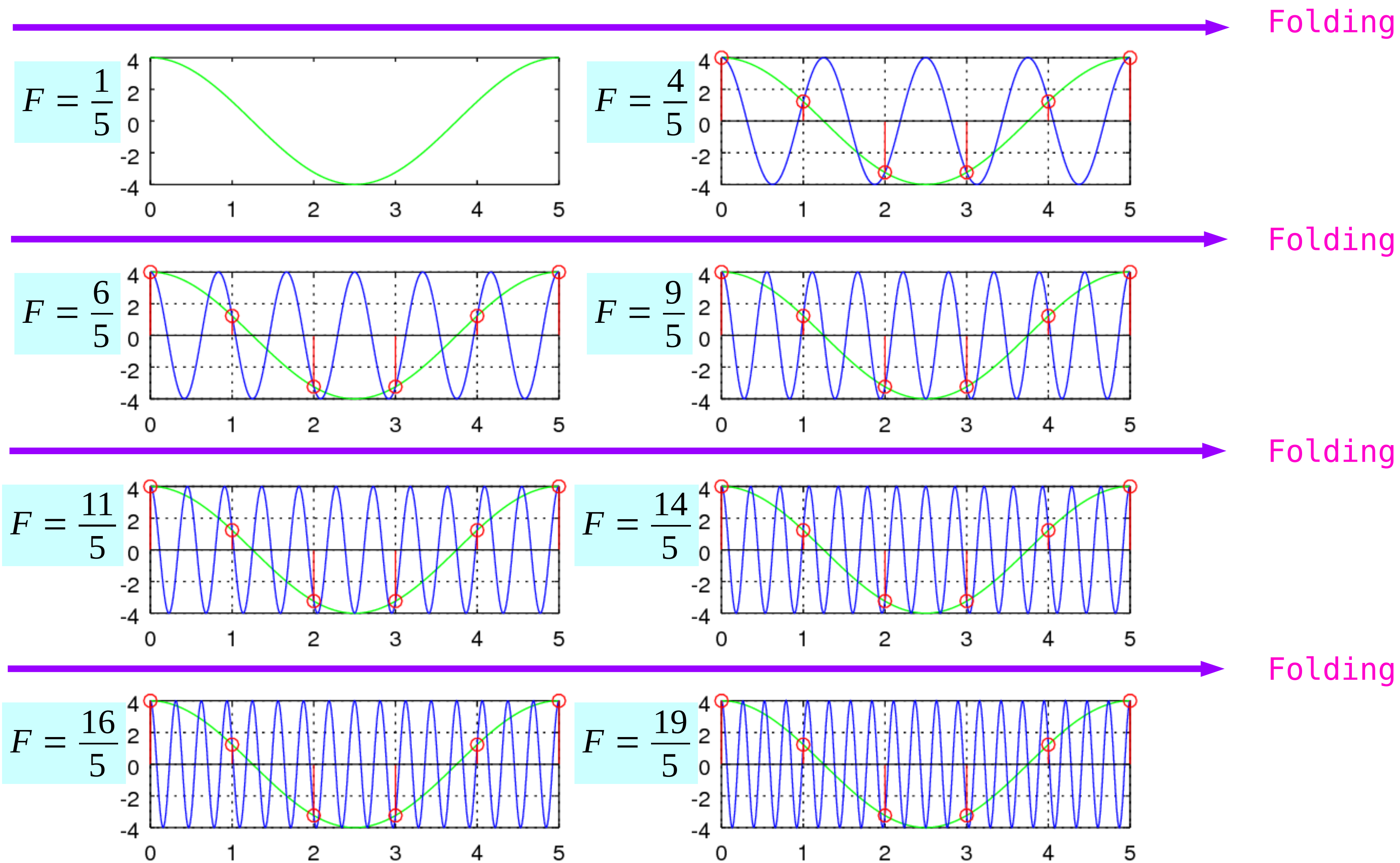
Aliasing – Cosine



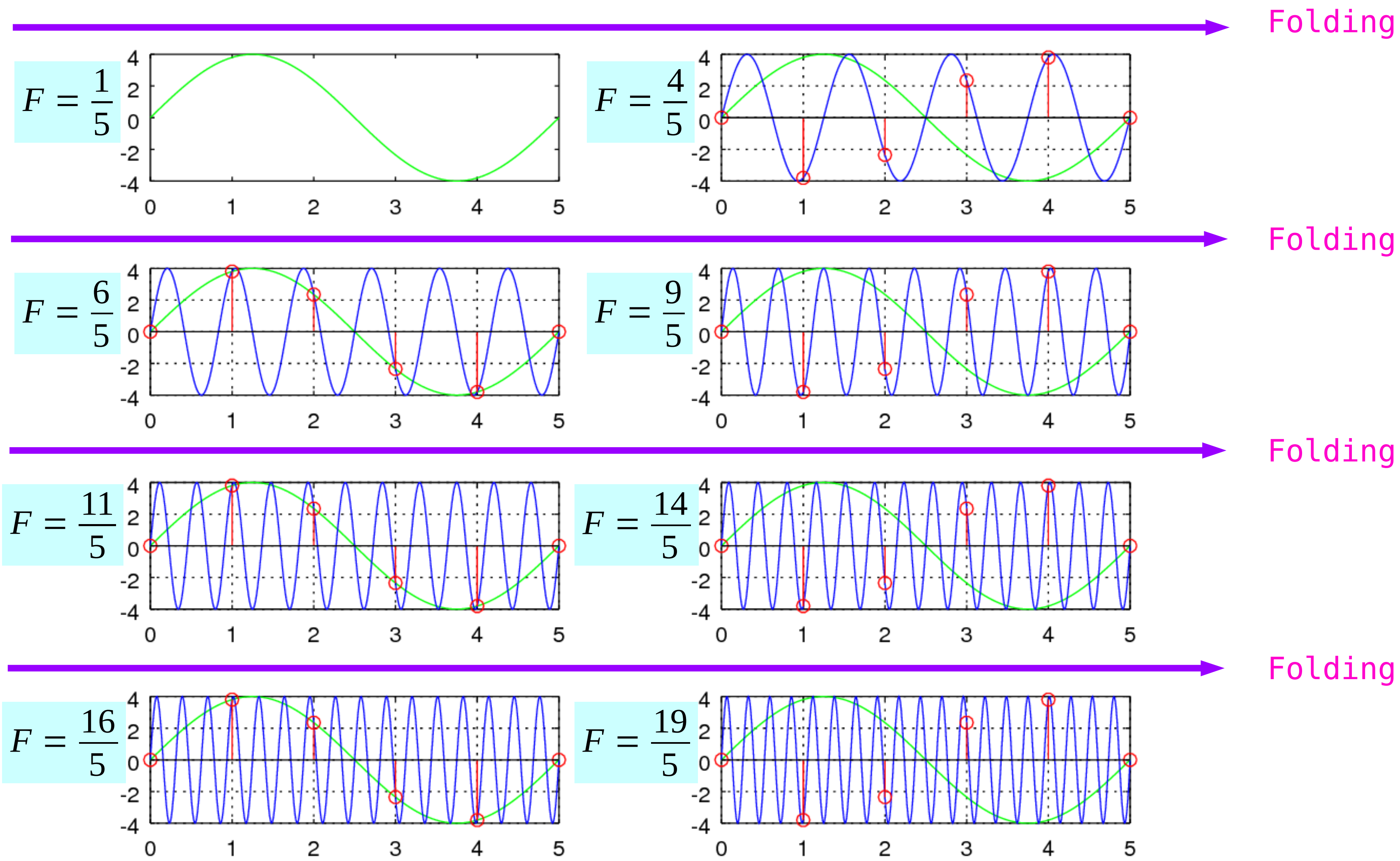
Aliasing – Sine



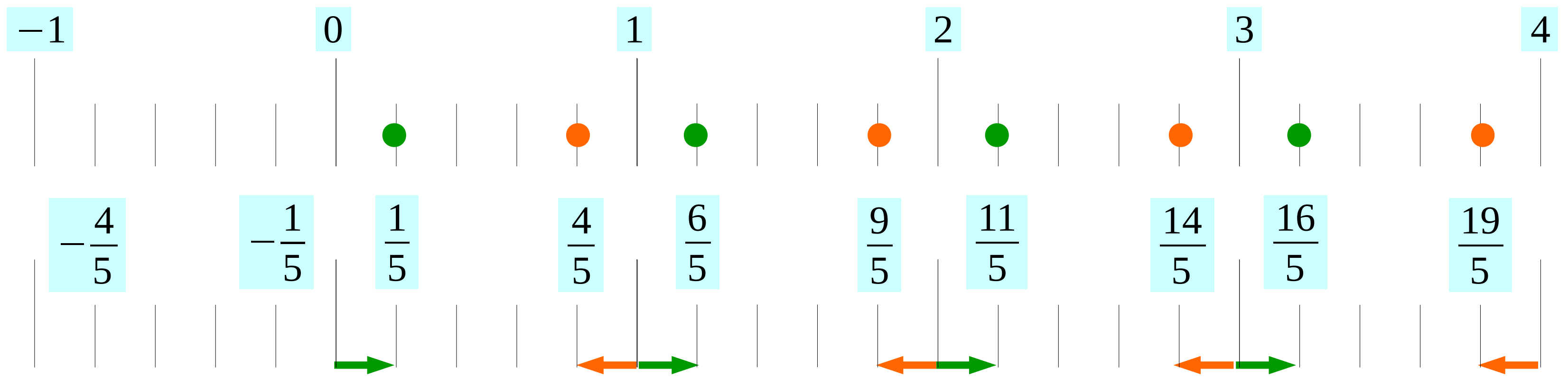
Folding – Cosine



Folding – Sine



Aliasing and Folding



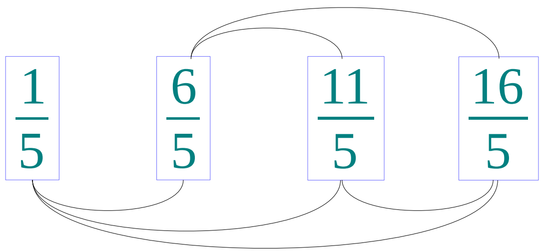
Plotting of Aliasing & Folding Frequencies

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
yt3 = 4*cos(2*pi*(11/5)*t);
yt4 = 4*cos(2*pi*(16/5)*t);
yt5 = 4*cos(2*pi*(4/5)*t);
yt6 = 4*cos(2*pi*(9/5)*t);
yt7 = 4*cos(2*pi*(14/5)*t);
yt8 = 4*cos(2*pi*(19/5)*t);

n1 = 0: 5/5 : 5;
n2 = 0: 5/5 : 5;
n3 = 0: 5/5 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/5 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/5 : 5;
n8 = 0: 5/5 : 5;
```

```
y2 = 4*cos(2*pi*(6/5)*n2);
y3 = 4*cos(2*pi*(11/5)*n2);
y4 = 4*cos(2*pi*(16/5)*n2);
y5 = 4*cos(2*pi*(4/5)*n5);
y6 = 4*cos(2*pi*(9/5)*n5);
y7 = 4*cos(2*pi*(14/5)*n5);
y8 = 4*cos(2*pi*(19/5)*n5);


subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n2, y3, 'r');
```

```
subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n2, y4, 'r');

subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n5, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n5, y7, 'r');

subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n5, y8, 'r');
```

Aliasing Frequencies (1)



$$\frac{1}{5} \quad \frac{6}{5} \quad \frac{11}{5} \quad \frac{16}{5}$$

A+B

$$\frac{7}{5} \quad \frac{12}{5} \quad \frac{17}{5} \quad \frac{22}{5} \quad \frac{27}{5} \Rightarrow \frac{5}{7} \quad \frac{5}{12} \quad \frac{5}{17} \quad \frac{5}{22} \quad \frac{5}{27}$$

A-B

$$\frac{5}{5} \quad \frac{10}{5} \quad \frac{15}{5} \Rightarrow \frac{1}{1} \quad \frac{1}{2} \quad \frac{1}{3} \Rightarrow 1$$

Aliasing Frequencies (2)

$\frac{4}{5}$ $\frac{9}{5}$ $\frac{14}{5}$ $\frac{19}{5}$

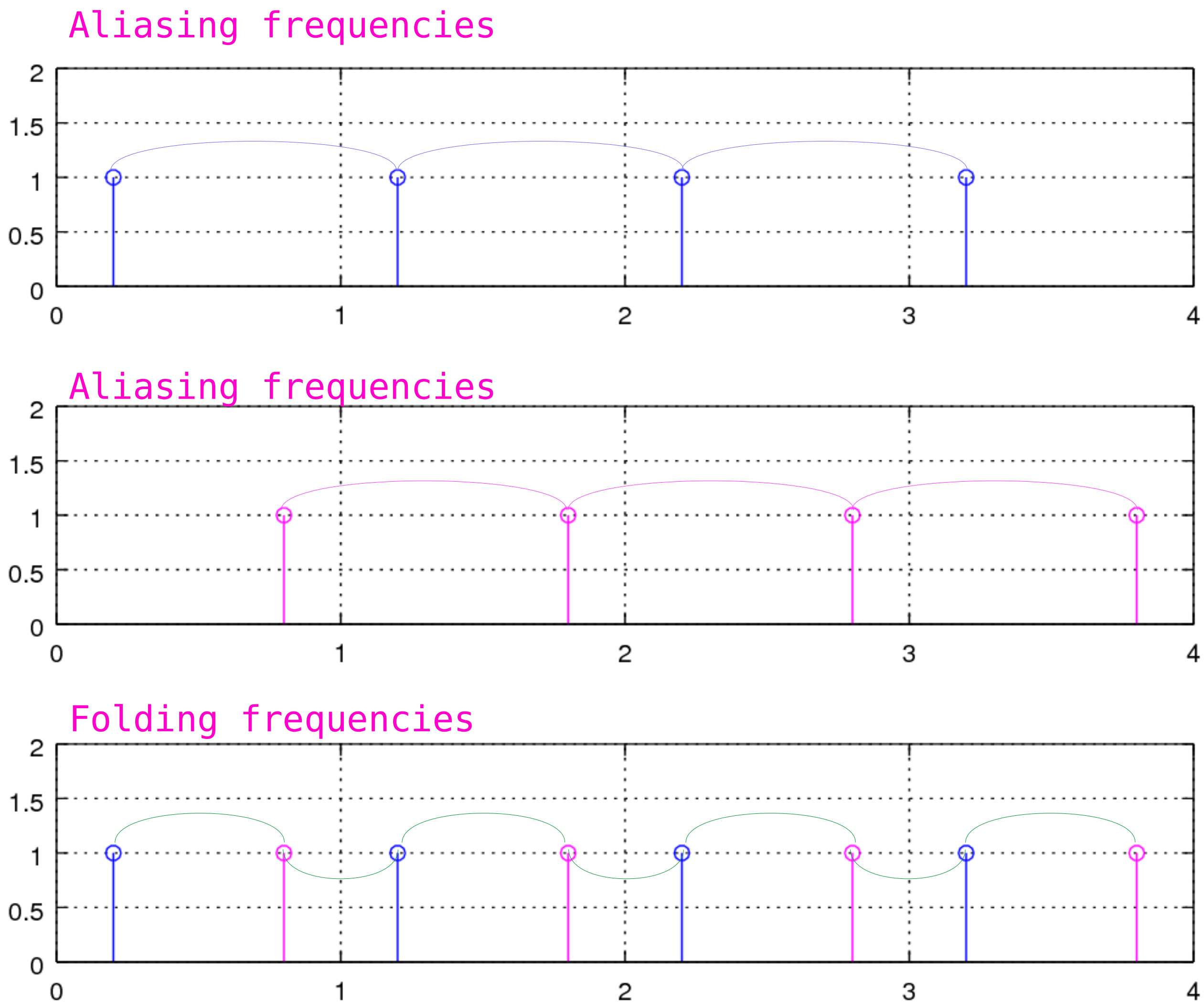
A+B

$\frac{13}{5}$ $\frac{18}{5}$ $\frac{23}{5}$ $\frac{28}{5}$ $\frac{33}{5}$ ⇒ $\frac{5}{13}$ $\frac{5}{18}$ $\frac{5}{23}$ $\frac{5}{28}$ $\frac{5}{33}$

A-B

$\frac{5}{5}$ $\frac{10}{5}$ $\frac{15}{5}$ ⇒ $\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ ⇒ 1

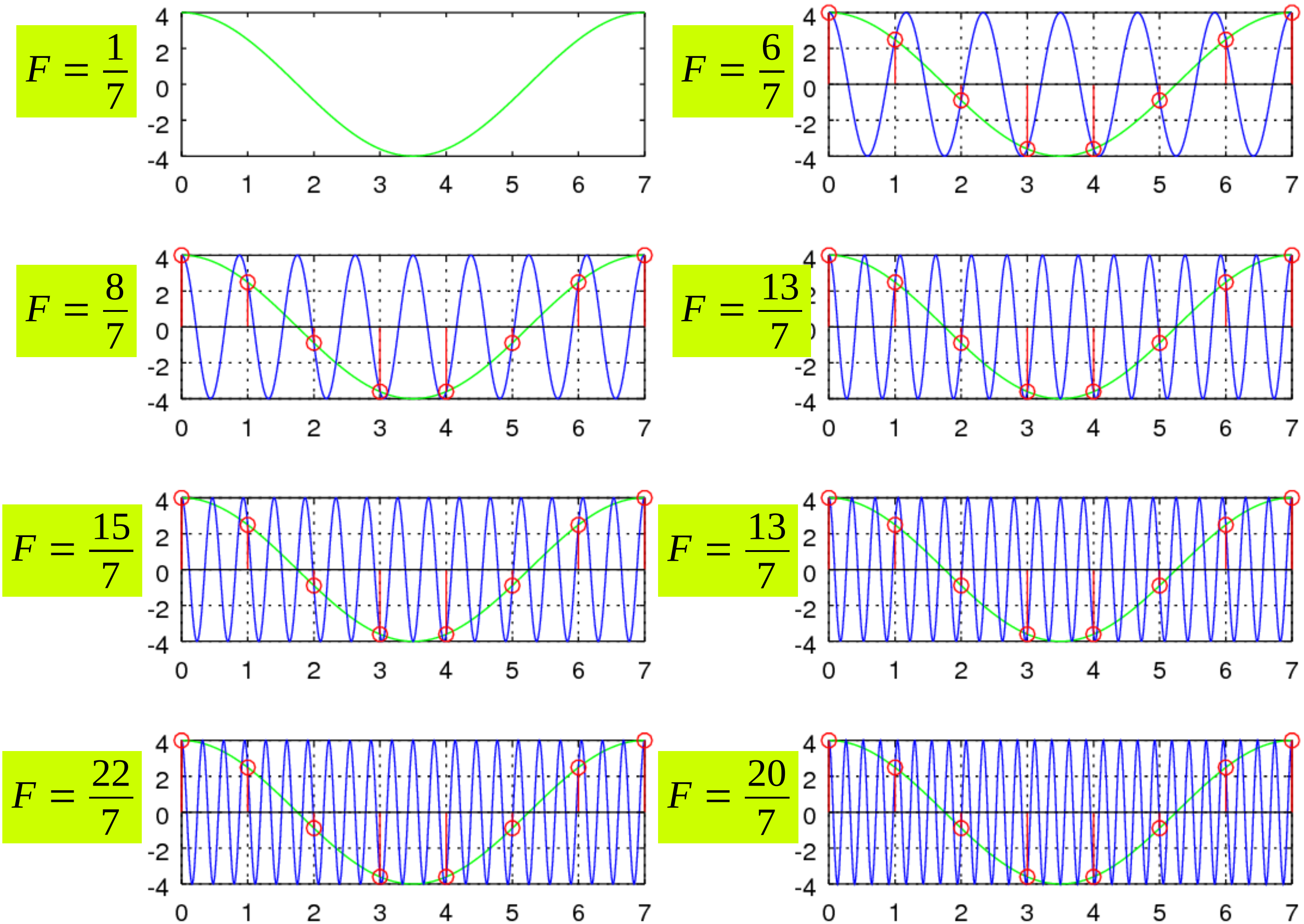
Aliasing and Folding Frequencies




```
n1 = [1/5, 6/5, 11/5, 16/5];
n2 = [4/5, 9/5, 14/5, 19/5];

y1 = [1, 1, 1, 1];
y2 = [1, 1, 1, 1];

subplot(3, 1, 1)
stem(n1, y1, 'b'); grid on;
axis([0, 4, 0, 2]);

subplot(3, 1, 2)
stem(n2, y2, 'm'); grid on;
axis([0, 4, 0, 2]);

subplot(3, 1, 3)
stem(n1, y1, 'b'); hold on;
stem(n2, y2, 'm'); grid on;
axis([0, 4, 0, 2]);
```

J.H. McClellan, et al., Signal Processing First

Graphs of cos(2π(n/7)t) & cos(2π(1/7)t)



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] M.J. Roberts, Fundamentals of Signals and Systems
[4] S.J. Orfanidis, Introduction to Signal Processing
[5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

[6] A “graphical interpretation” of the DFT and FFT, by Steve Mann