Time Responses (H.1)
Transient Characteristics

20150606

Understanding Control Systems + wikiversity
Understanding Control Systems using CAS + wikiversity

* Time Response

First Order System

$$\boxed{G(s) = \frac{a}{s+a}}$$

pole : $-a$ 한개

stable → $-a < 0$

$a > 0$

time constant $\frac{1}{a} = \tau$

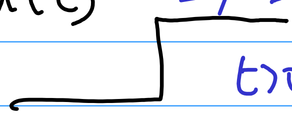
$$\frac{Y(s)}{X(s)} = \frac{a}{s+a}$$

$(s+a)Y(s) = a\,X(s)$

$$\boxed{y' + a\,y(t) = a\,x(t)}$$

Unit step response $y(t)$?

$x(t) = u(t)$ ⇒ 1



$t>0$

$$\boxed{y' + a\,y(t) = 1 \cdot a} \quad (t>0)$$

$y_h = c_1 e^{-at}$

$y_p = 1$

$y_p = A$

$A' + aA = a$

$A = 1$

$y = y_h + y_p = c_1 e^{-at} + 1$

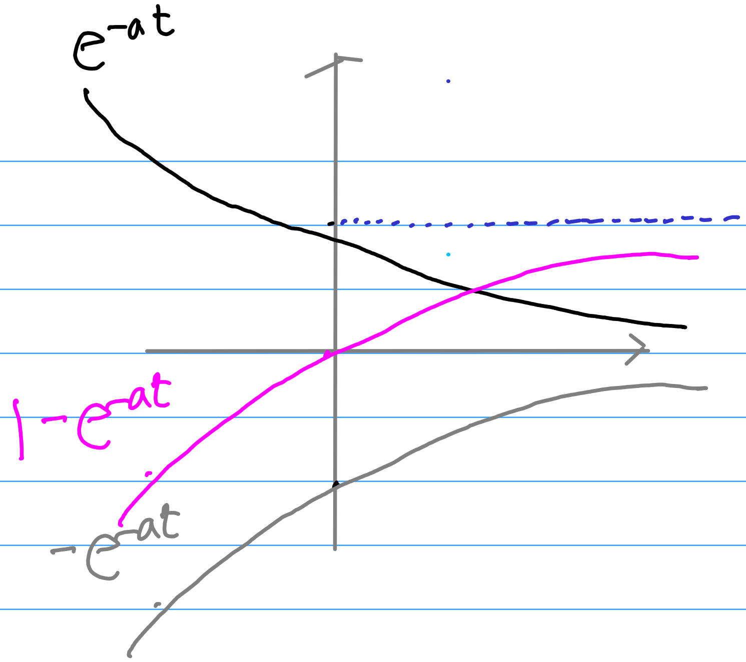
$$\boxed{y(0^+) = 0}$$

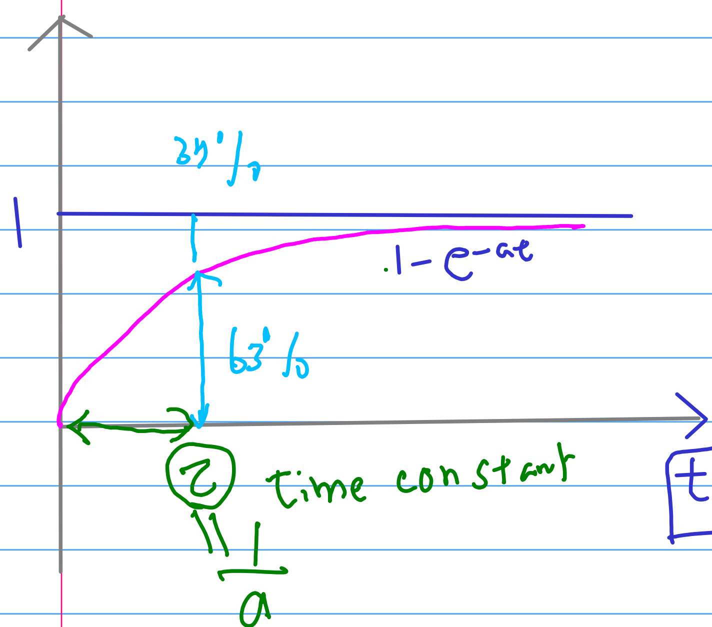
$c_1 e^0 + 1 = 0$

$c_1 = -1$

$$\boxed{y = 1 - e^{-at}}$$

$c(t) = 1 - e^{-at}$





$\boxed{t>0}$

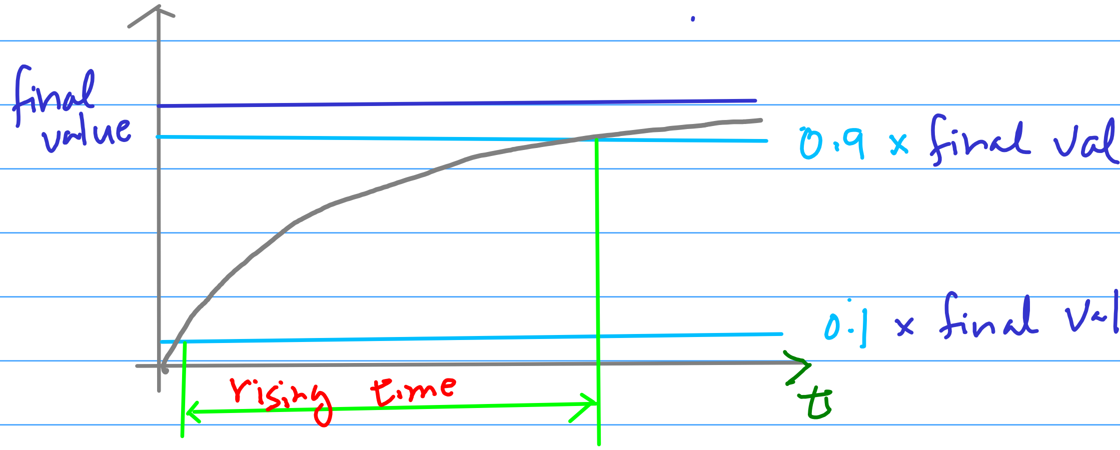
$c(t) = 1 - e^{-at}$

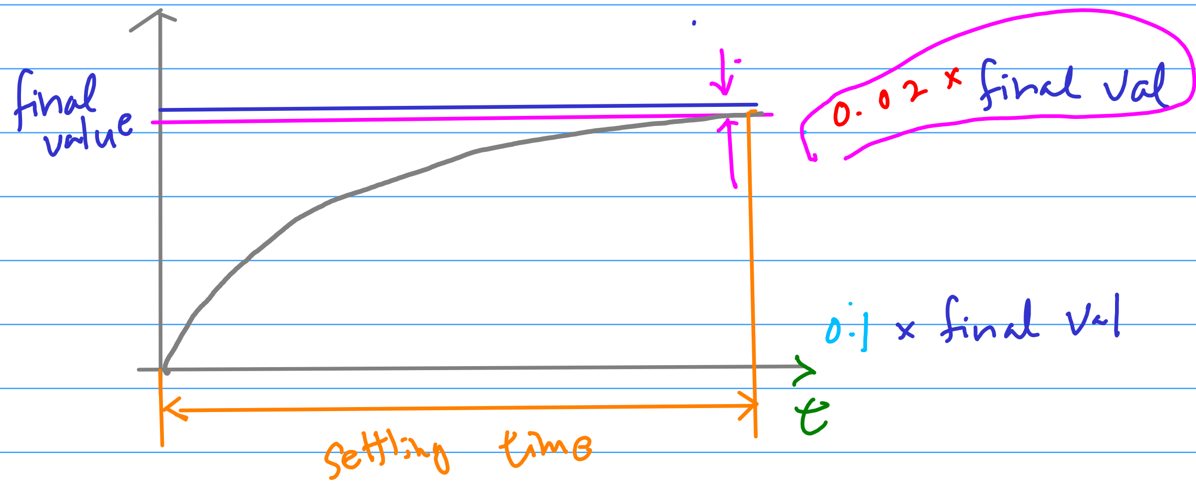
$c\left(\frac{1}{a}\right) = 1 - e^{-1}$

$\|$

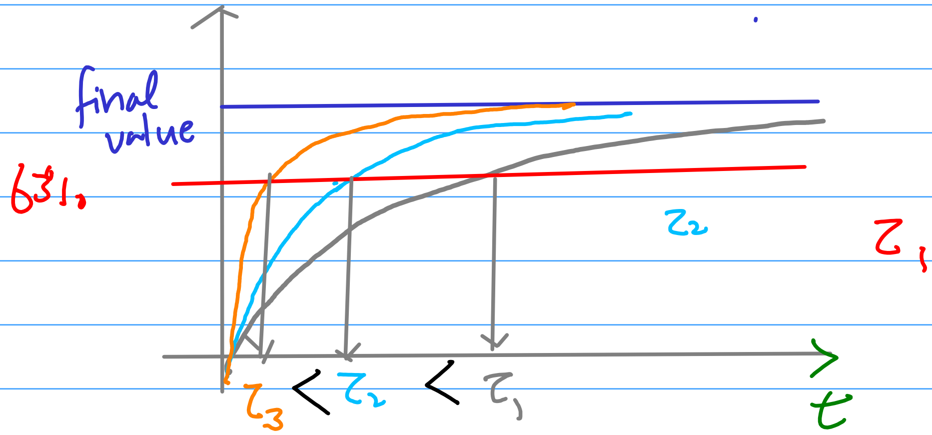
τ time constant

rising time & settling time





$$4\tau = \frac{4}{a}$$

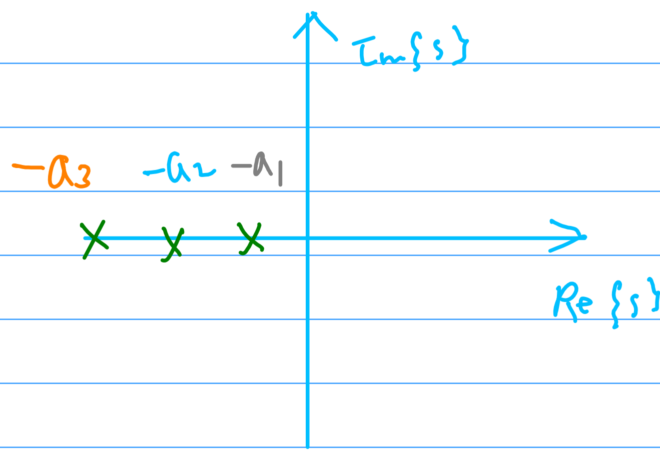


$$\frac{1}{a_3} < \frac{1}{a_2} < \frac{1}{a_1}$$

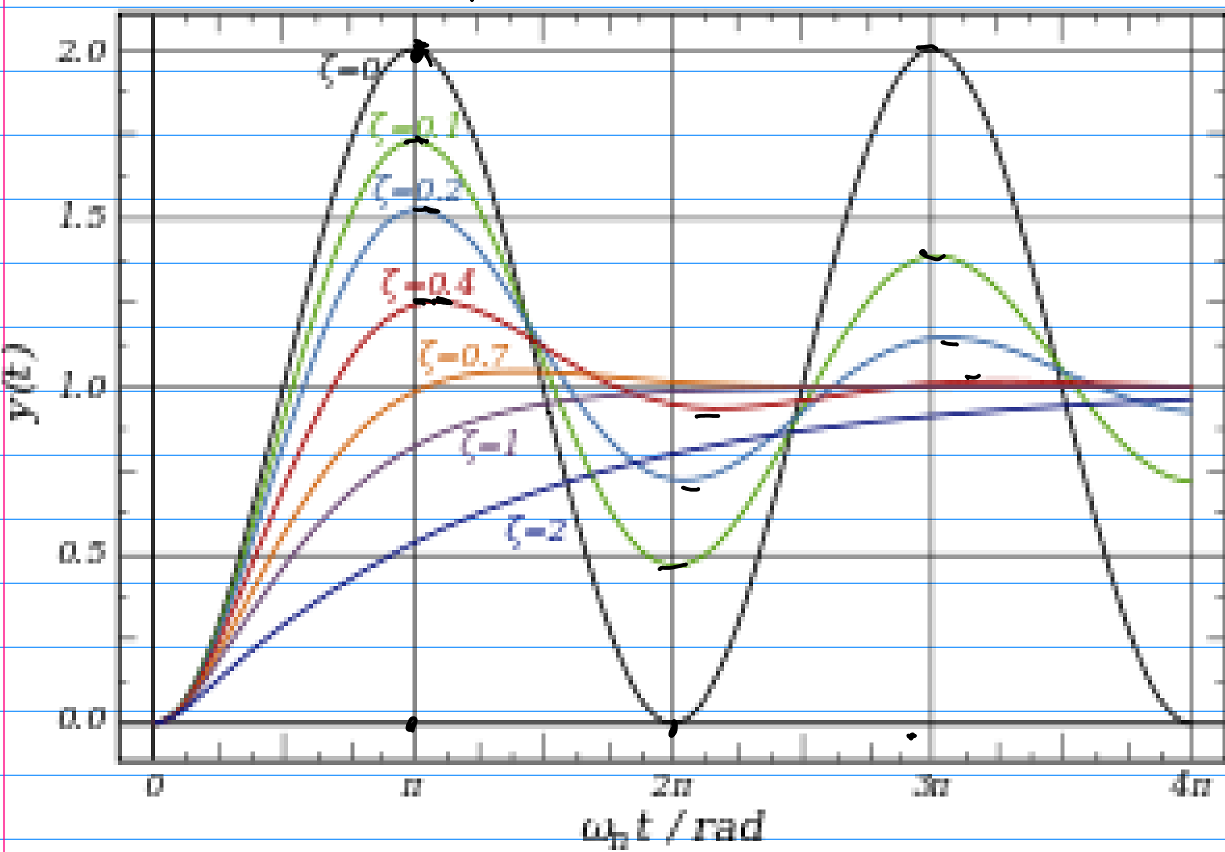
$$a_3 > a_2 > a_1$$

Transfer function

$$\longrightarrow \boxed{\frac{a}{s+a}} \longrightarrow \quad c(t) = 1 - e^{-at}$$



peak 되는 time ? $c'(t) = 0$



$$c(t) = 1 - Ae^{-\sigma t}\sin(\omega t + \theta)$$

$$\sin(\alpha+\beta) = \sin\alpha\,\boxed{\cos\beta} + \cos\alpha\,\boxed{\sin\beta}$$

$$\cos(\alpha+\beta) = \cos\alpha\,\boxed{\cos\beta} - \sin\alpha\,\boxed{\sin\beta}$$

$$\frac{1}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\left[\underbrace{\left(\sqrt{1-\zeta^2}\right)}_{\sin\theta}\boxed{\cos\left(\sqrt{1-\zeta^2}\,\omega_n t\right)} + \underbrace{(\zeta)}_{\cos\theta}\boxed{\sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right)}\right]$$

$$= \frac{1}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right) \qquad \cos\theta = \zeta$$

$$r(t) \longrightarrow \boxed{G(s)} \longrightarrow c(t)$$

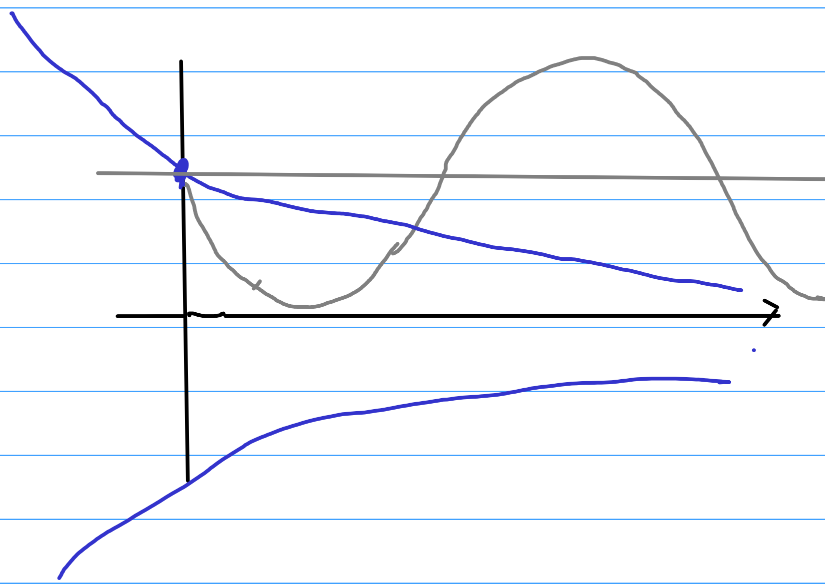
$$C(s) = G(s)R(s) = G(s)\frac{1}{s}$$

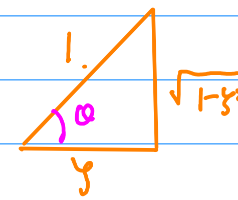
$$= \frac{\omega_n^2}{s(s-s_1)(s-s_2)} = \frac{1}{s} - \frac{s}{(s^2+2\zeta\omega_n s+\omega_n^2)} - \frac{2\zeta\omega_n}{(s^2+2\zeta\omega_n s+\omega_n^2)}$$

$$\boxed{c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_n t}\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)}$$

$$c(0) = 1 - A\sin(\theta) = 0 \qquad A = \frac{1}{\sin\theta}$$

$$\cdot\ c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$





$\cos\theta = \zeta$

$\sin\theta = \sqrt{1-\zeta^2}$

$\cos\theta = \zeta$

$\sin\theta = \sqrt{1-\zeta^2}$

$$\cdot\ c(0) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n 0} \sin\left(\sqrt{1-\zeta^2}\,\omega_n 0 + \theta\right)$$

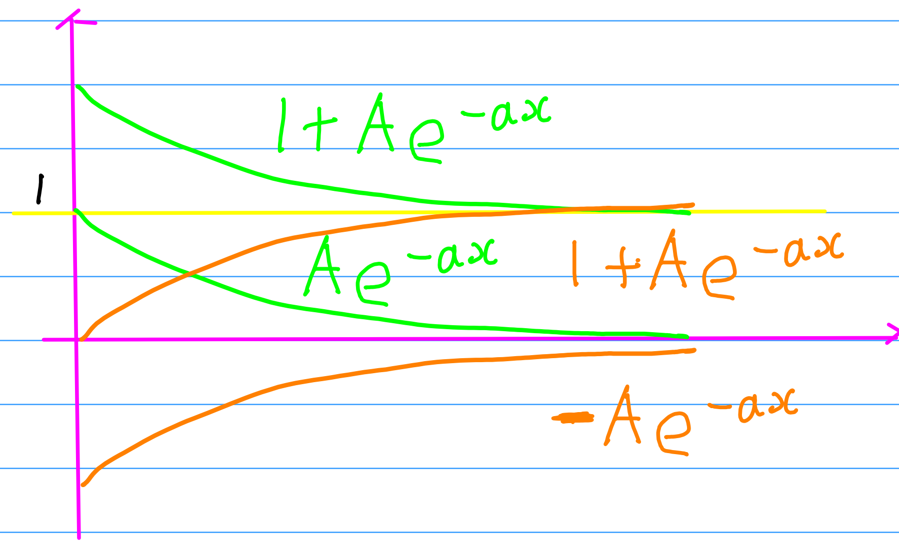
$$= 1 - \frac{1}{\sqrt{1-\zeta^2}} \sin\theta$$

$$= 0$$

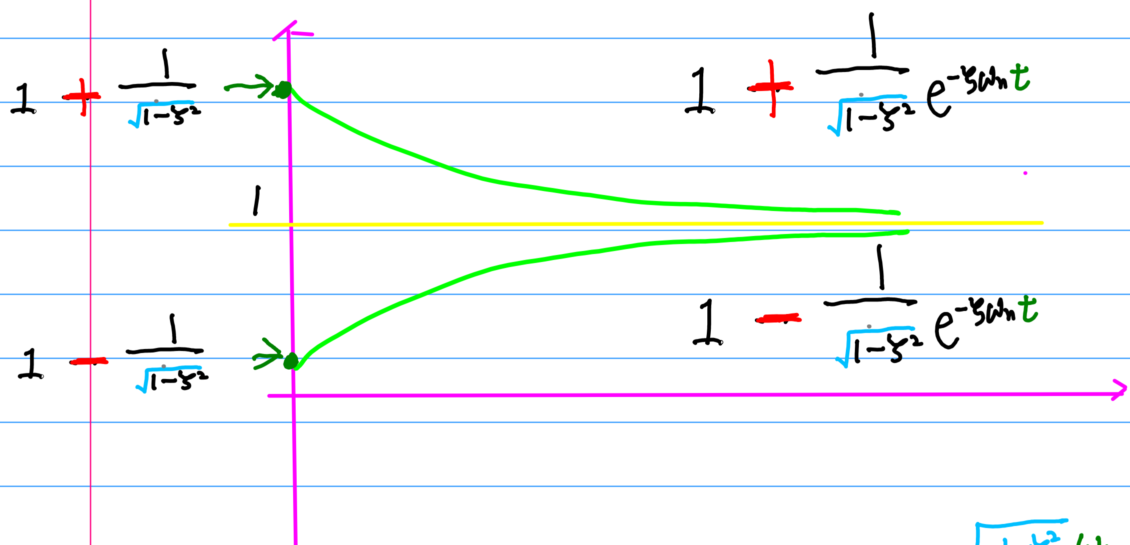
output 0에 해당값이 0이다

envelopes

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$

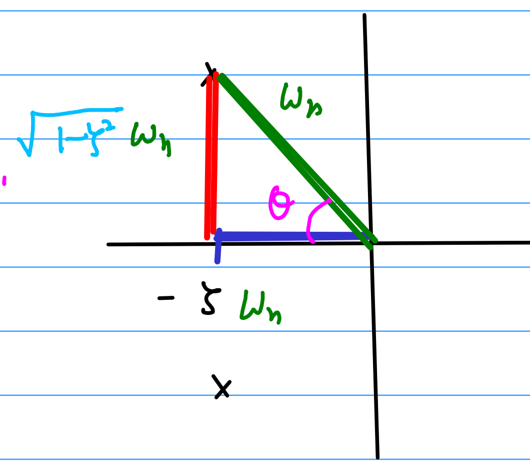


$A>0$



$\cos\theta = \zeta$

$\sin\theta = \sqrt{1-\zeta^2}$



finding $c'(t)$

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$

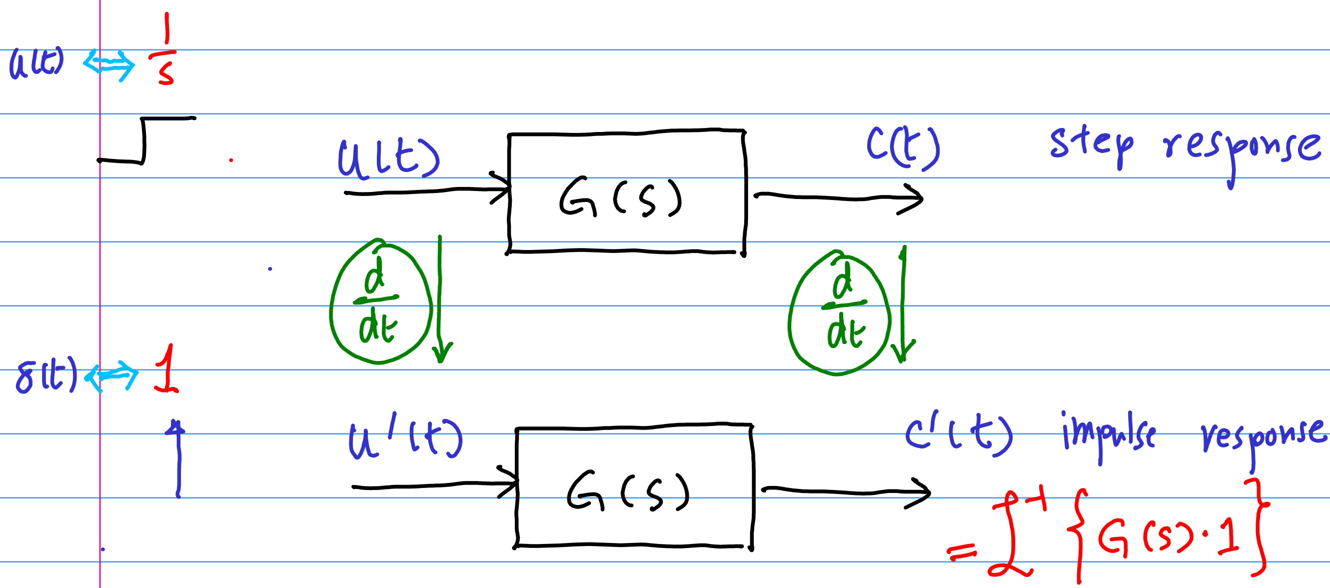
$$c'(t) = -\frac{1}{\sqrt{1-\zeta^2}} \Big\{ -\zeta\omega_n e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$
$$\sqrt{1-\zeta^2}\,\omega_n e^{-\zeta\omega_n t} \cos\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right) \Big\}$$

another view

$$c(t) \leftarrow \boxed{\text{step response } \quad 0<\zeta<1}$$

$$c'(t) \leftarrow \boxed{\text{impulse response } \quad 0<\zeta<1}$$

finding c'(t) – impulse response



$$G(s) = \frac{\omega_n^2}{(s^2+2\zeta\omega_n s + \omega_n^2)} = \frac{\omega_n^2}{(s^2+2\zeta\omega_n s + \zeta^2\omega_n^2) - \zeta^2\omega_n^2 + \omega_n^2}$$

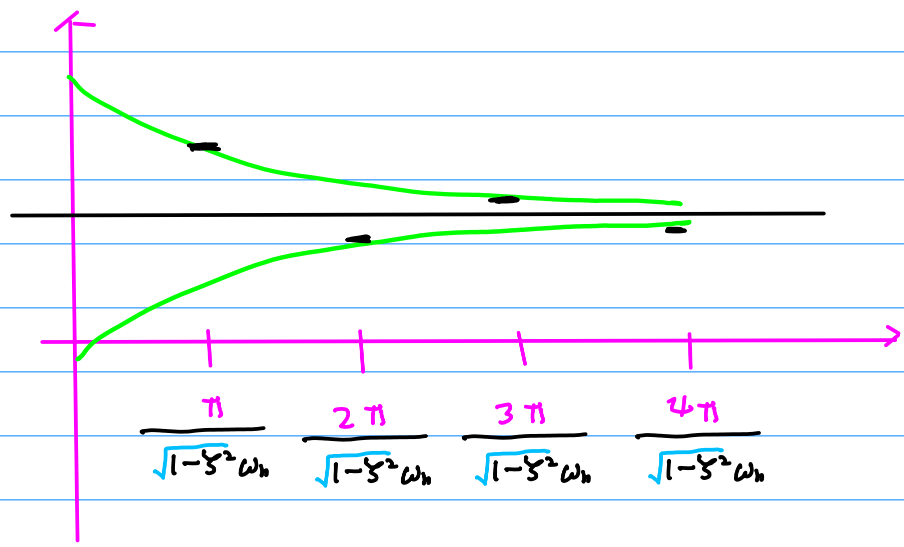
$$= \frac{\left(\sqrt{1-\zeta^2}\,\omega_n\right)\frac{\omega_n}{\sqrt{1-\zeta^2}}}{(s+\zeta\omega_n)^2 + \left(\sqrt{1-\zeta^2}\,\omega_n\right)^2} \Longleftrightarrow$$

$$c'(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t\right)$$

finding t $\quad c'(t) = 0$ $\quad$ (maxima / minima)

$$c'(t) = \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\underbrace{\sqrt{1-\zeta^2}\,\omega_n t}_{= n\pi}\right) = 0$$

$$\sqrt{1-\zeta^2}\,\omega_n t = n\pi \qquad t = \frac{n\pi}{\sqrt{1-\zeta^2}\,\omega_n}$$



$\cos\theta = \zeta$

$\sin\theta = \sqrt{1-\zeta^2}$

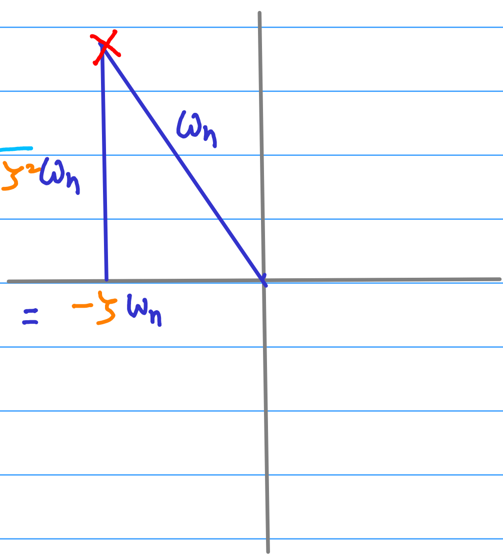
Peak times $\dot{c}(t) = 0$

$$t = \frac{1\cdot\pi}{\sqrt{1-\zeta^2}\,\omega_n} \quad \frac{2\cdot\pi}{\sqrt{1-\zeta^2}\,\omega_n} \quad \frac{3\cdot\pi}{\sqrt{1-\zeta^2}\,\omega_n}$$

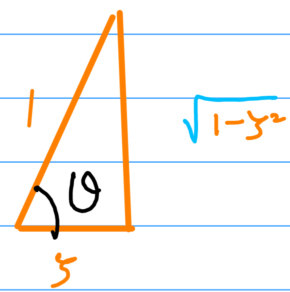
$$\frac{1\cdot\pi}{\omega_d} \quad \frac{2\cdot\pi}{\omega_d} \quad \frac{3\cdot\pi}{\omega_d}$$

$$\sqrt{1-\zeta^2}\,\omega_n = \omega_d$$

$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$



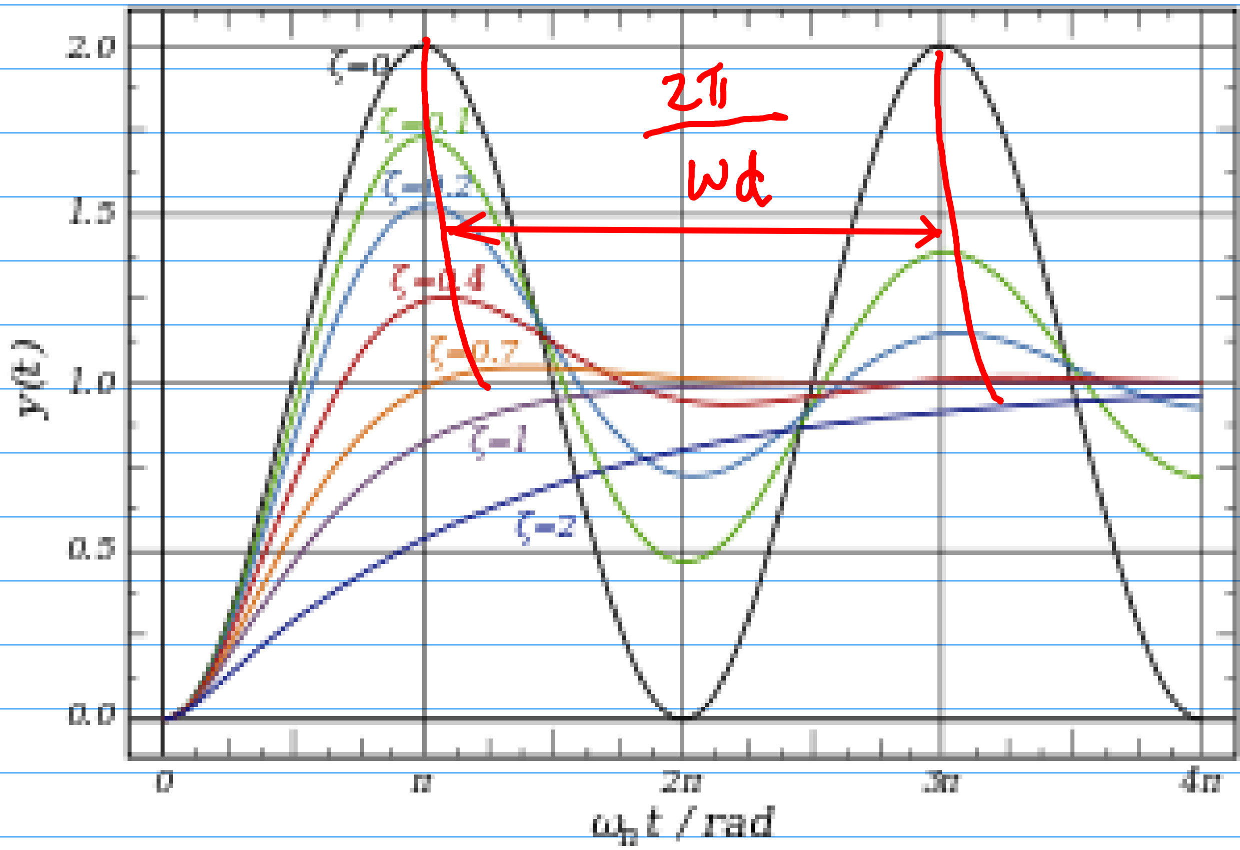
$-\sigma_d = -\zeta\omega_n$



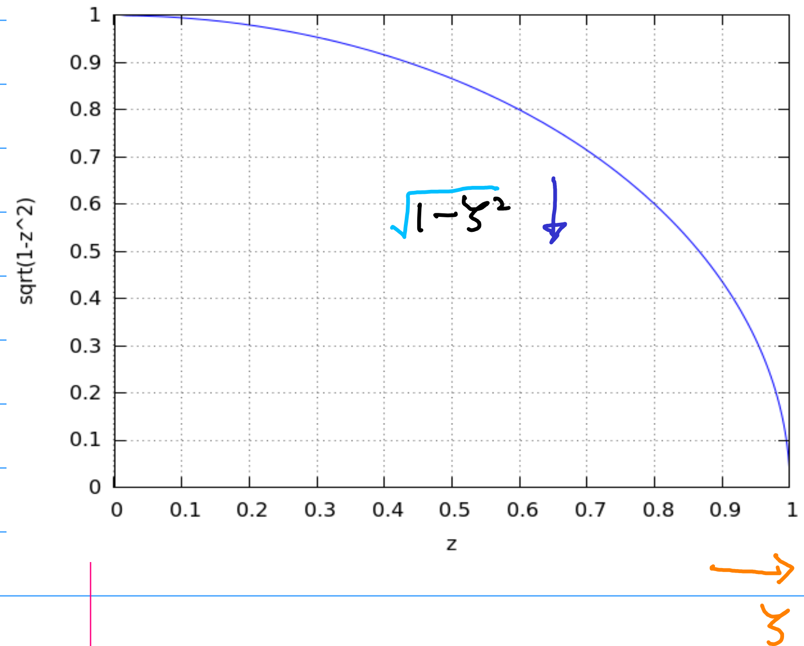
$\cos\theta = \zeta$

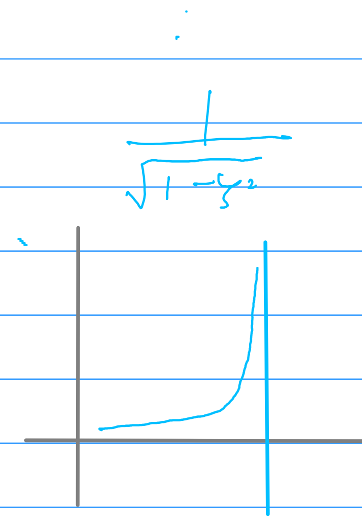
$\sin\theta = \sqrt{1-\zeta^2}$

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

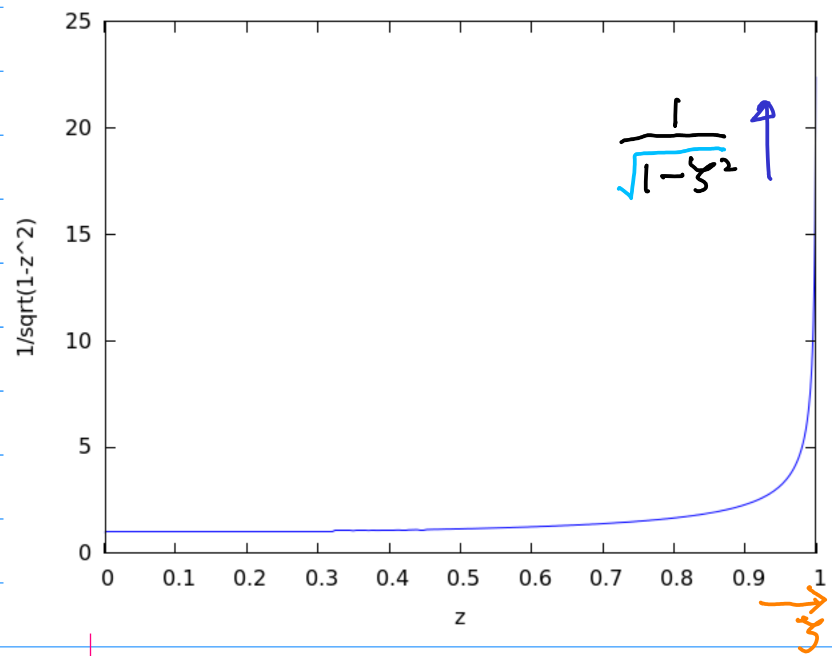


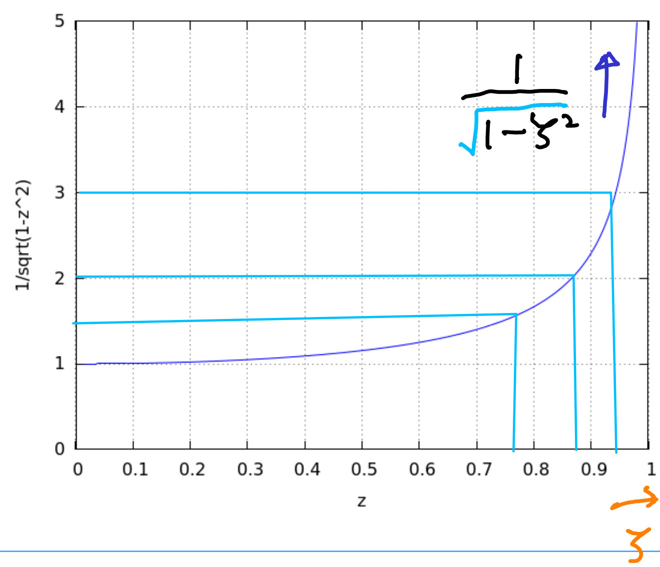
$$\frac{1}{\sqrt{1-\zeta^2}} \qquad 0<\zeta<1$$





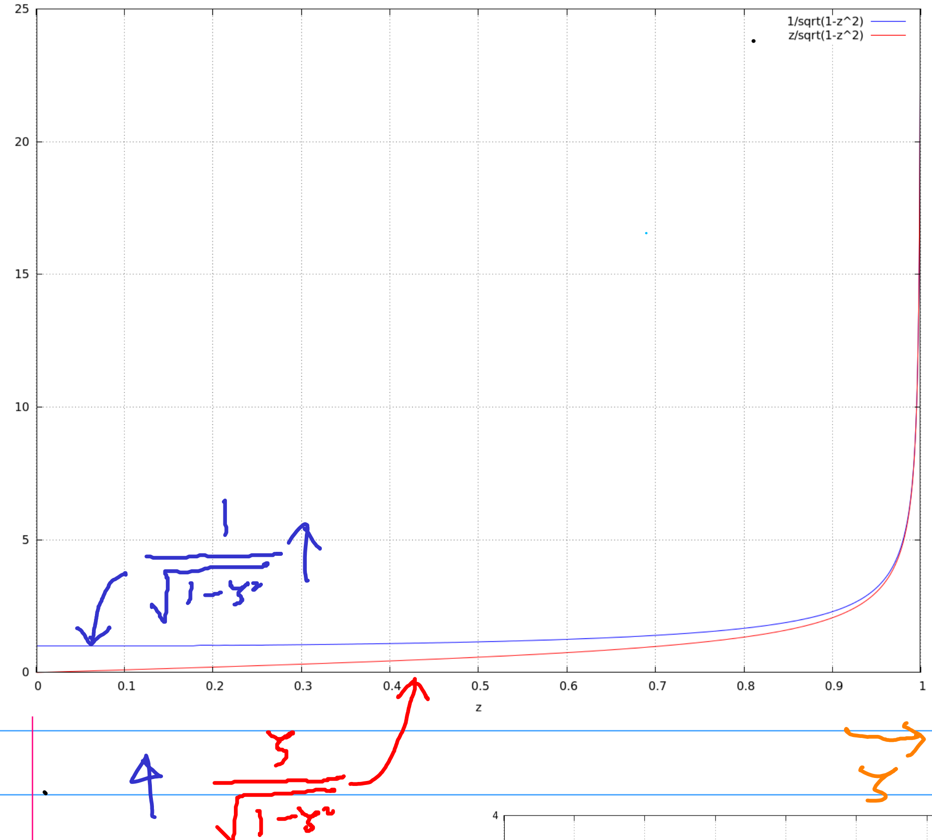
$$\zeta \to 1 \qquad \frac{1}{\sqrt{1-\zeta^2}} \to \infty$$

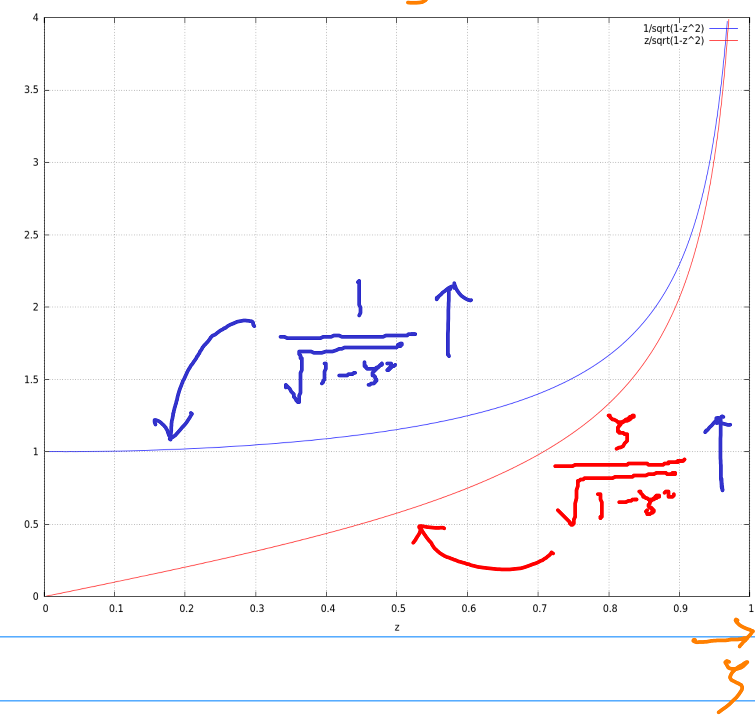




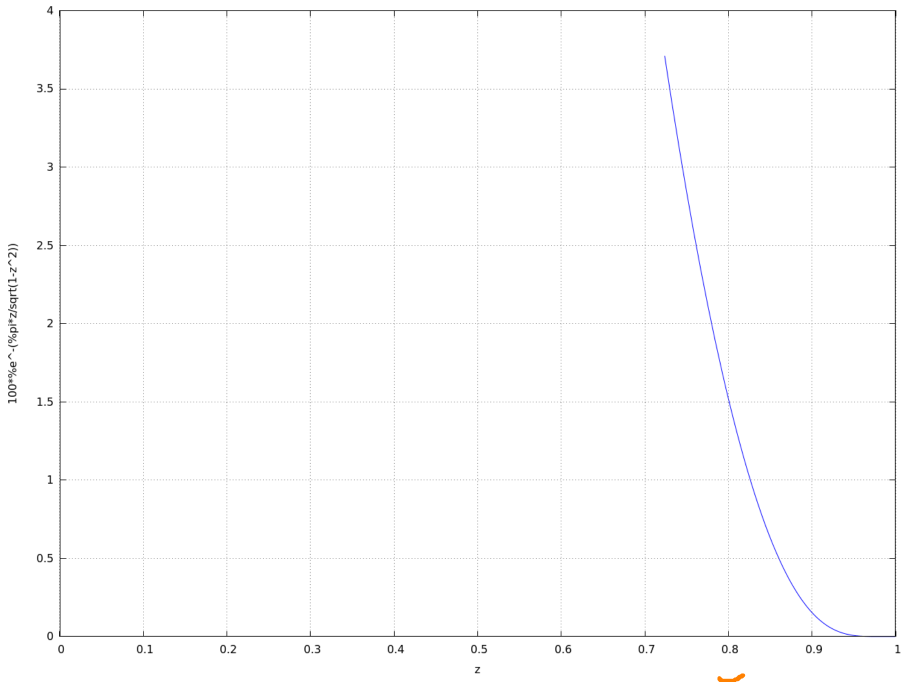
$$t = \frac{n\pi}{\sqrt{1-\zeta^2}\,\omega_n}$$

$$\frac{1}{\sqrt{1-\zeta^2}} \quad \text{v.s.} \quad \frac{\zeta}{\sqrt{1-\zeta^2}} \qquad 0<\zeta<1$$





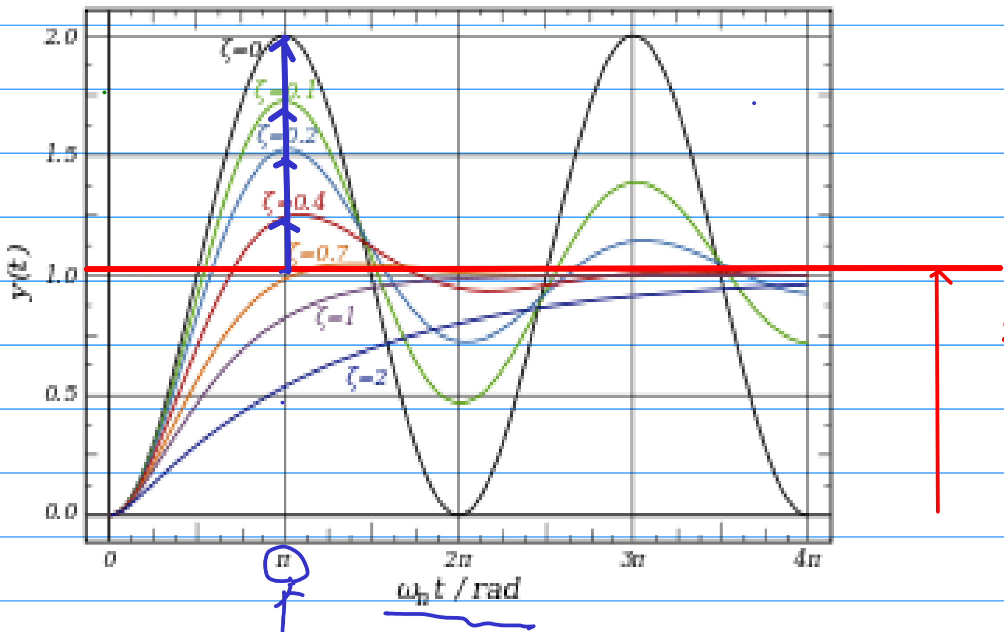
$$e^{-\frac{\zeta \cdot \pi}{\sqrt{1-\zeta^2}}}$$



$\zeta \longrightarrow$

$\boxed{\text{Overshoot}}$

$$\boxed{\%OS} = \frac{c(T_p) - c(\infty)}{c(\infty)} \times 100$$



steady state value

$c(\infty)$

peak time $\boxed{T_p = \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n}}$

$\cos\theta = \zeta$

$\theta = \cos^{-1}\zeta$

$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$

$T_p = \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n}$

$$e^{-\zeta\omega_n t} \Rightarrow e^{-\zeta\cancel{\omega_n}\frac{\pi}{\sqrt{1-\zeta^2}\cancel{\omega_n}}} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\sqrt{1-\zeta^2}\,\omega_n t \Rightarrow \cancel{\sqrt{1-\zeta^2}}\,\cancel{\omega_n}\frac{\pi}{\cancel{\sqrt{1-\zeta^2}}\,\cancel{\omega_n}} = \pi$$

$$c(T_p) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sin\left(\pi + \cos^{-1}\zeta\right)$$

$$= 1 + \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sin\left(\cos^{-1}\zeta\right)$$

$\boxed{\%OS} = f(\zeta) \downarrow \quad \zeta \uparrow$

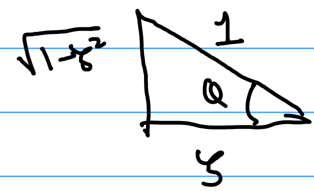
$$c(T_p) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sin\left(\pi + \cos^{-1}\zeta\right)$$

$$= 1 + \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sin\left(\cos^{-1}\zeta\right)$$

$$= 1 + \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sin\left(\theta\right)$$

$\zeta^2 + \left(\sqrt{1-\zeta^2}\right)^2 = 1$

$$= 1 + \frac{1}{\sqrt{1-\zeta^2}} e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sqrt{1-\zeta^2}$$



$$= 1 + e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$\zeta = \cos\theta$

$\sqrt{1-\zeta^2} = \sin\theta$

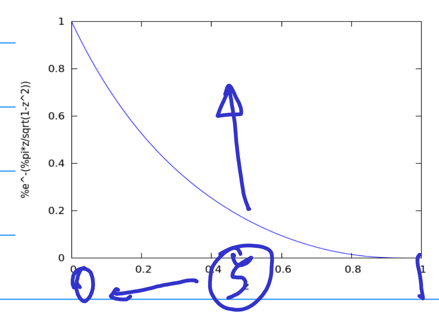
$$\boxed{\%OS} = \frac{c(T_p) - c(\infty)}{c(\infty)} \times 100$$

$c(\infty) = 1$

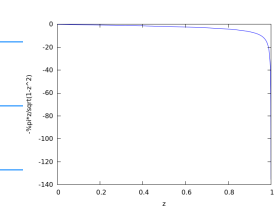
step response

$$= \frac{c(T_p) - 1}{1} \times 100$$

$$= e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$



∴ 0.S는 ζ에만 의존한다.



$$\zeta = g(\%OS)$$

$$\%OS = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100$$

$$\frac{\%OS}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\ln\left(\frac{\%OS}{100}\right) = \left(-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)$$

$$\boxed{\ln^2\left(\frac{\%OS}{100}\right)} = +\frac{\zeta^2\pi^2}{1-\zeta^2}$$

$$(1-\zeta^2)\,\square = \zeta^2\pi^2$$

$$\square - \square\zeta^2 = \zeta^2\pi^2$$

$$\square = (\square + \pi^2)\zeta^2$$

$$\zeta^2 = \frac{\square}{\square + \pi^2}$$

$$\zeta = \frac{\sqrt{\square}}{\sqrt{\square + \pi^2}}$$

$$\zeta = \frac{\sqrt{\ln^2\left(\frac{\%OS}{100}\right)}}{\sqrt{\ln^2\left(\frac{\%OS}{100}\right) + \pi^2}}$$

$$= \frac{\left|\ln\left(\frac{\%OS}{100}\right)\right|}{\sqrt{\ln^2\left(\frac{\%OS}{100}\right) + \pi^2}}$$

$$= \frac{-\ln\left(\frac{\%OS}{100}\right)}{\sqrt{\ln^2\left(\frac{\%OS}{100}\right) + \pi^2}}$$

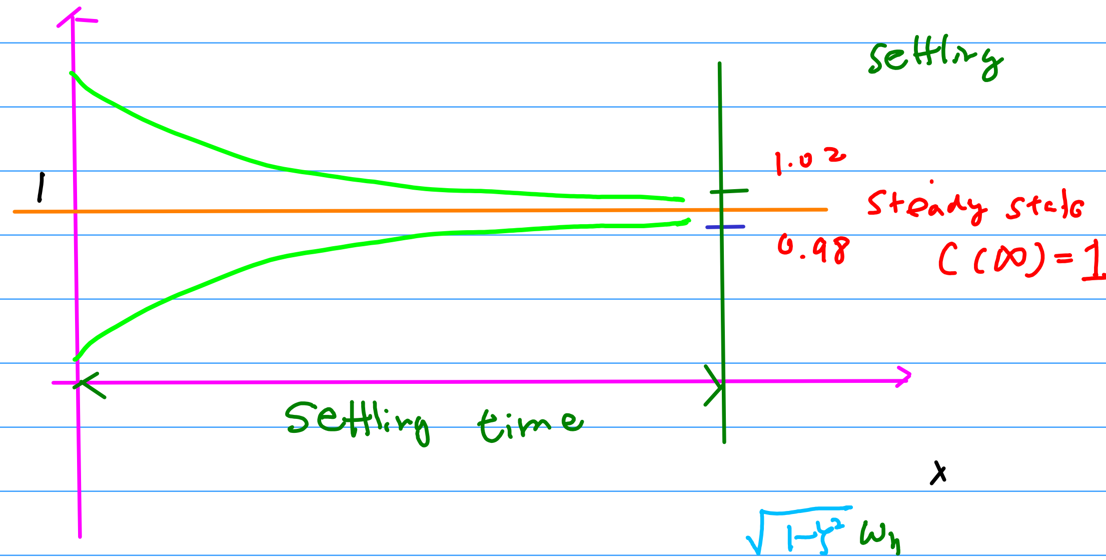
$$0 \leq \%OS \leq 100$$

$$0 \leq \frac{\%OS}{100} \leq 1$$

$$\ln\left(\frac{\%OS}{100}\right) < 0$$

Settling time

Settling Time T_s는 steady state 값의 2% 오차내에 진입하는 시간.



$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$

$$1 - c(t) = \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \boxed{\sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)}$$

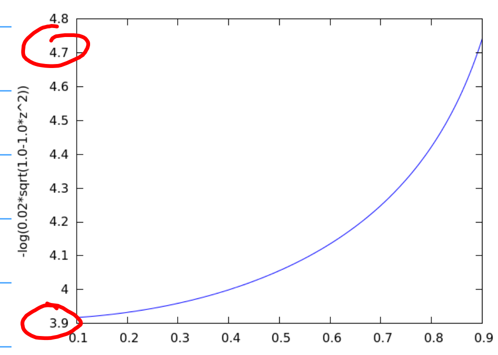
$$-1 < \square \leq +1$$

$$\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} = 0.02$$

$$e^{-\zeta\omega_n t} = 0.02 \cdot \sqrt{1-\zeta^2}$$

$$-\zeta\omega_n t = \ln\left(0.02 \cdot \sqrt{1-\zeta^2}\right)$$

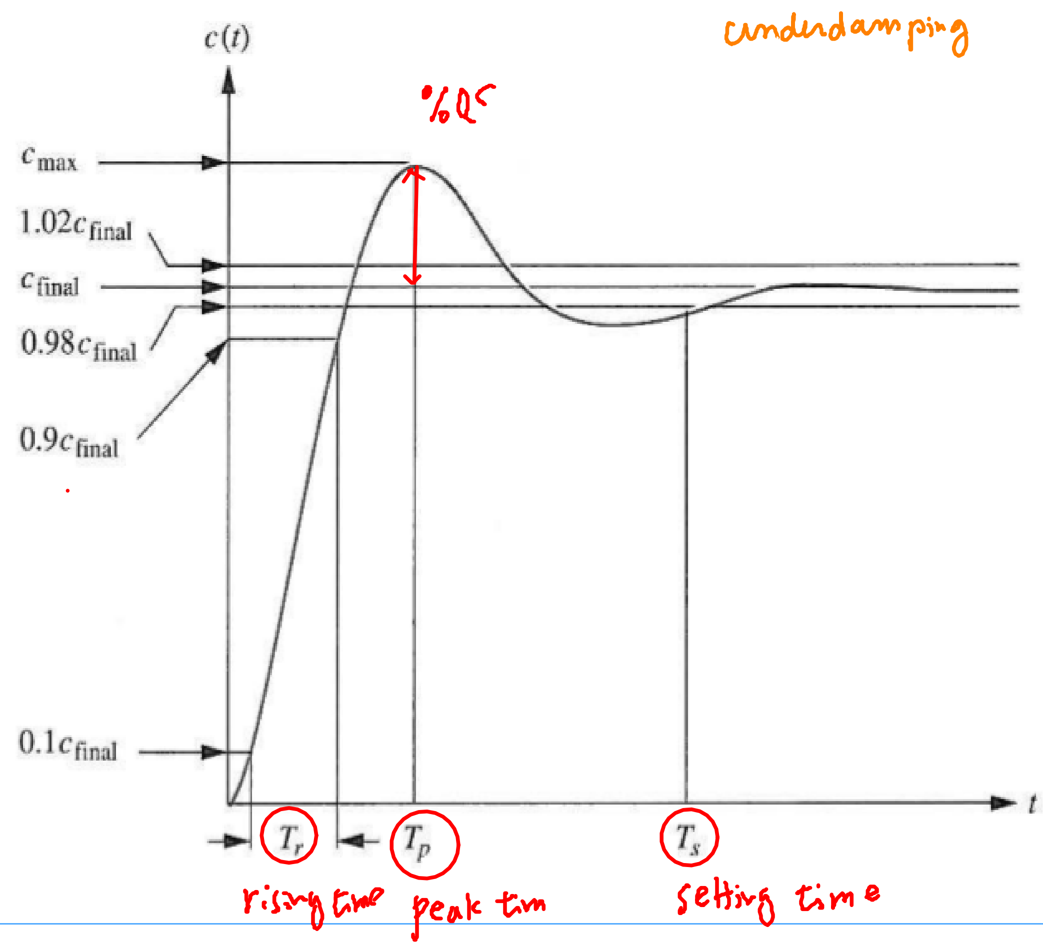
$$t = -\frac{\ln\left(0.02 \cdot \sqrt{1-\zeta^2}\right)}{\zeta\omega_n} = \frac{4}{\zeta\omega_n}$$



$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

ζ : damping ratio $\boxed{0<\zeta<1}$

ω_n : natural frequency



① Peak Time $\quad T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{\omega_d}$

② Percent Overshoot $\quad \%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}}$$

③ Settling Time $\quad T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{\sigma_d}$

④ Rising Time $\quad T_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$

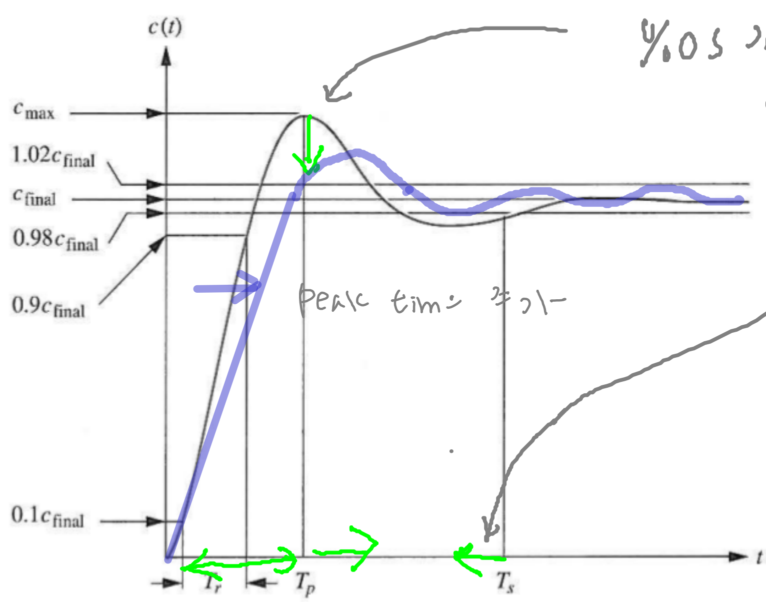
damping ratio만 증가되는경우 $\zeta \uparrow$ $0 < \zeta < 1$ critical damping 쪽으로 이동

① Peak Time $T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{\omega_d}$ $\uparrow$

② Percent Overshoot $\%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$ $\downarrow$

③ Settling Time $T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{\sigma_d}$ $\downarrow$

④ Rising Time $T_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$ $\uparrow$



%OS 가 작아져서 빨리 settle된다

peak time 증가

$\zeta \to 1$ critical damp같이 된다.

ωn 만 증가되는 경우

$\omega_n \uparrow$ $\qquad$ $\omega_n > 0$

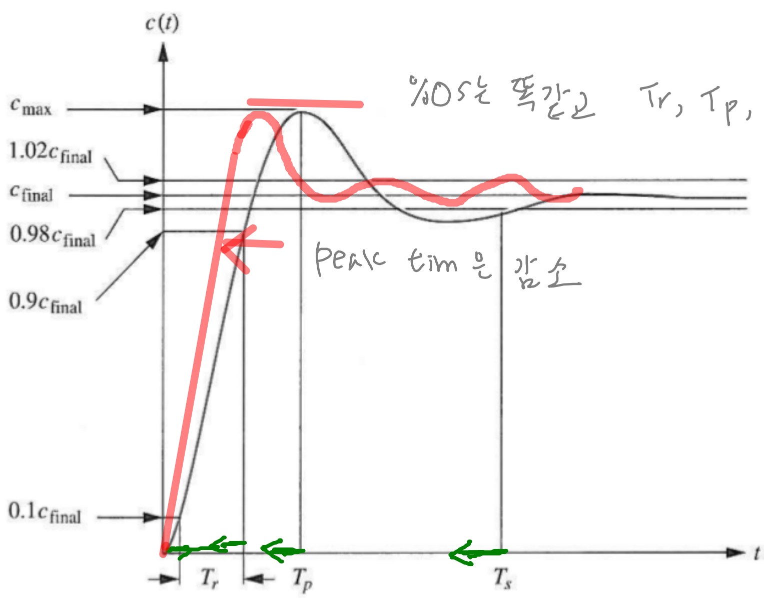
① Peak Time $\qquad T_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{\pi}{\omega_d}$ $\quad\downarrow$

② Percent Overshoot $\qquad \%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$ $\quad\leftrightarrow$

③ Settling Time $\qquad T_s = \dfrac{4}{\zeta\omega_n} = \dfrac{4}{\sigma_d}$ $\quad\downarrow$

④ Rising Time $\qquad T_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$ $\quad\downarrow$

%OS는 똑같고 Tr, Tp, Ts 빨라진다



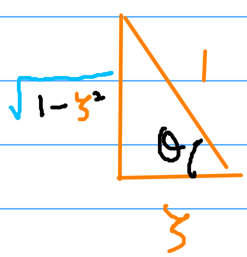
2nd Order System's Complex Conjugate Poles

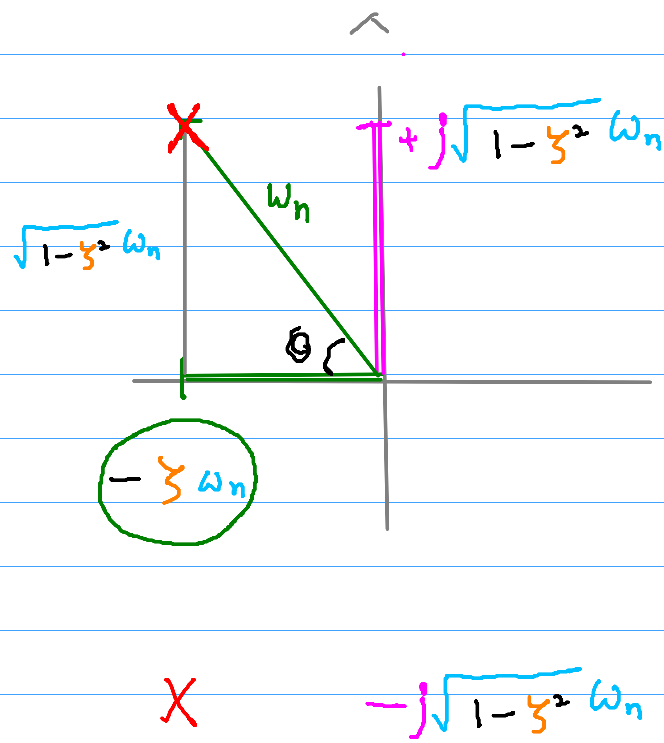
$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ transfer function

T.func 분모를 0으로 만드는 s : pole

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$ $0 < \zeta < 1$ ..... 부족감쇠

$$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n$$

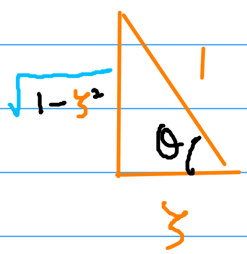


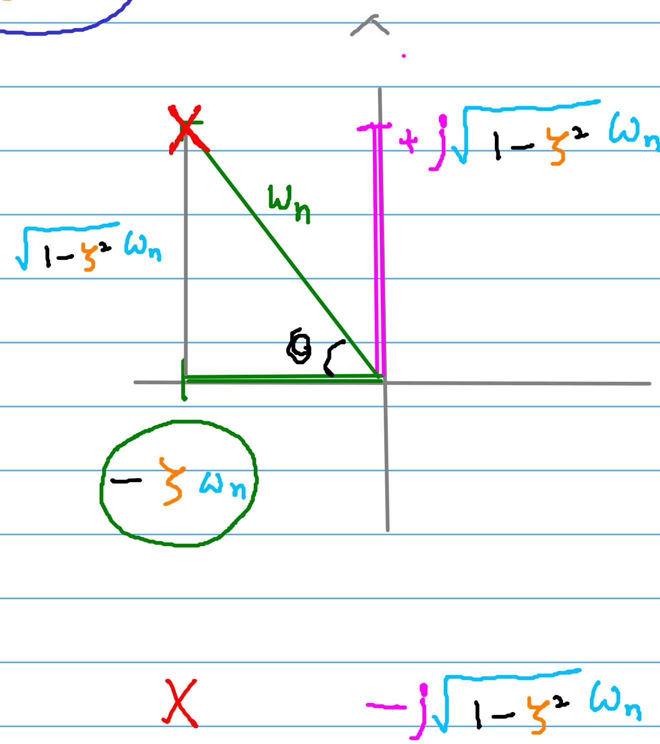


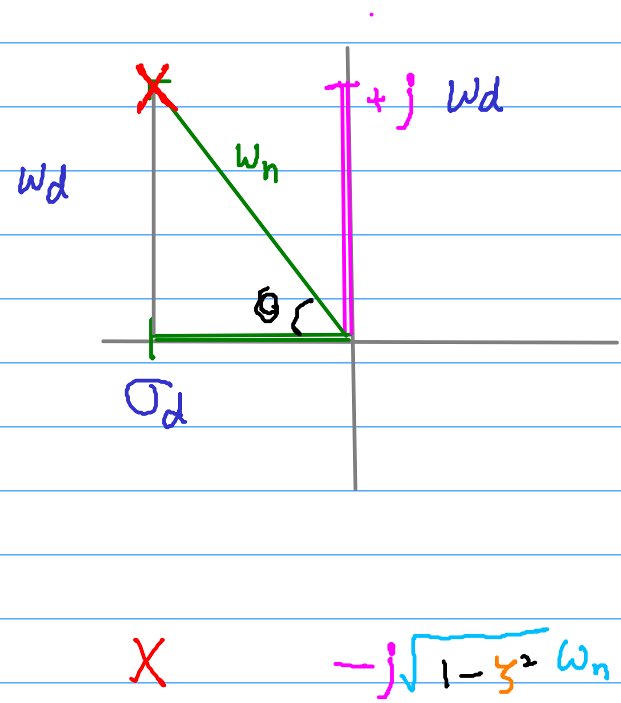
σ_d & ω_d

$$\sigma_d \pm j\,\omega_d$$

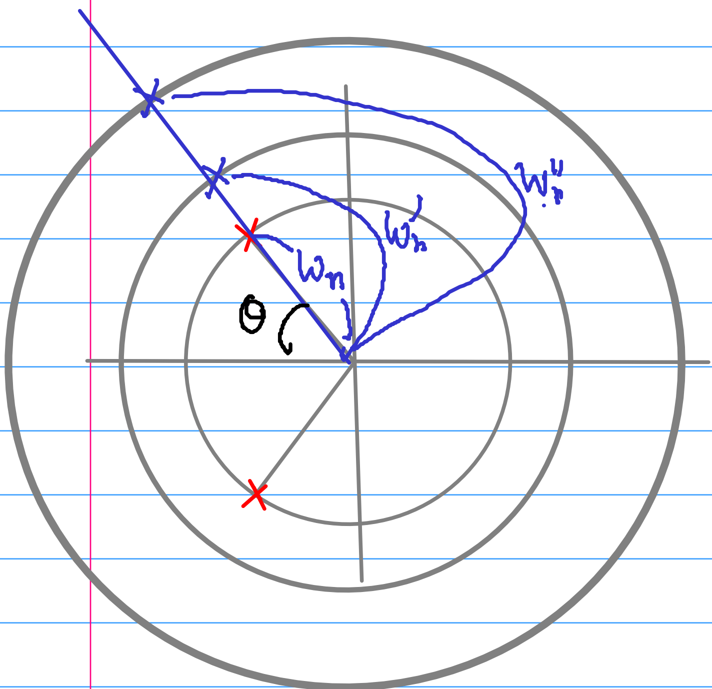
$$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n$$







root locus when ω_n and ζ change

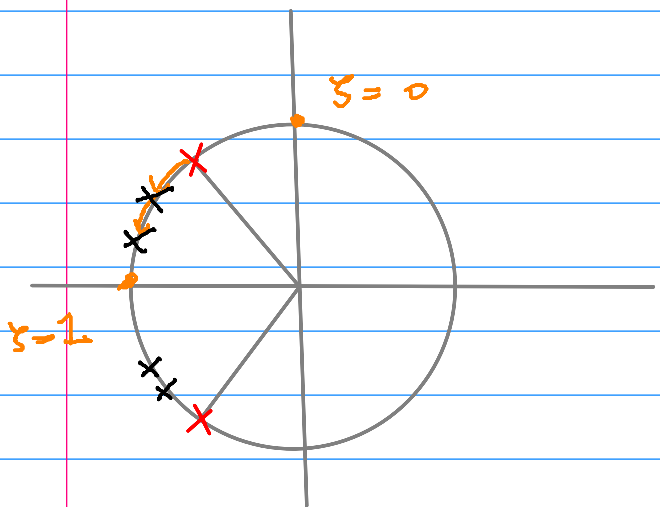


$\cos\theta = \zeta$ fixed

$\omega_n < \omega_n' < \omega_n''$

ζ fixed

ω_n ↑



$s = \pm j\,\omega_n \quad (\zeta = 0)$

$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n$

$\zeta = 0$ pure imaginary

real $\zeta = 1$

ζ ↑

ω_n fixed

$s = -\zeta\omega_n \quad (\zeta = 1)$

$= -\omega_n$

ζ ↑ $\quad$ σ_d ↑

ω_n ↑ $\quad$ ω_d fixed

real part가
운의 실수 영향으로

① Peak Time $\qquad T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_d}$ ⟷ 똑같다.

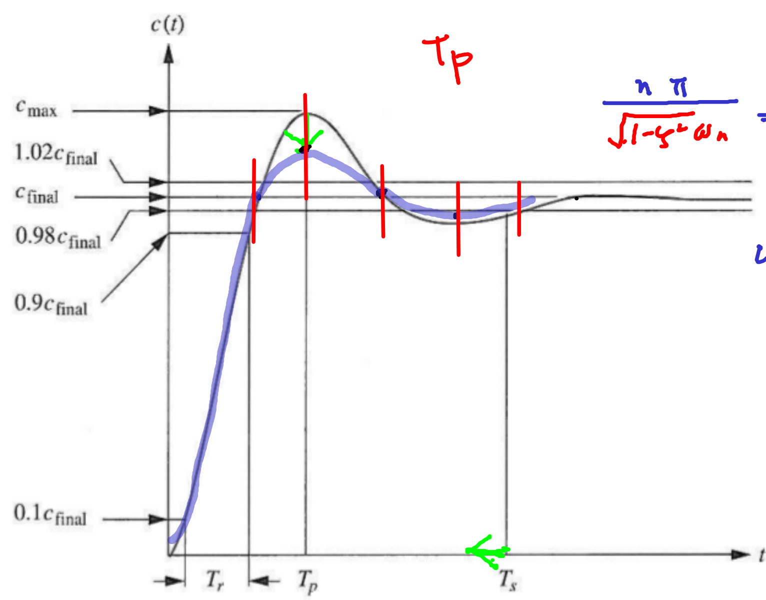
② Percent Overshoot $\qquad \%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100$ ↓

③ Settling Time $\qquad T_s = \dfrac{4}{\sigma_d} = \dfrac{4}{\sigma_d}$ ↓

④ Rising Time $\qquad T_r = \dfrac{2.16\zeta + 0.6}{\omega_n}$? 변동 X

ω_d : fixed

ω_n ↑



$\dfrac{n\pi}{\sqrt{1-\zeta^2}\,\omega_n} = \dfrac{n\pi}{\omega_d}$ 똑같다

ω_n ↑

$\sqrt{1-\zeta^2}\,\omega_n = \omega_d$

$\omega_n = \dfrac{\omega_d}{\sqrt{1-\zeta^2}}$

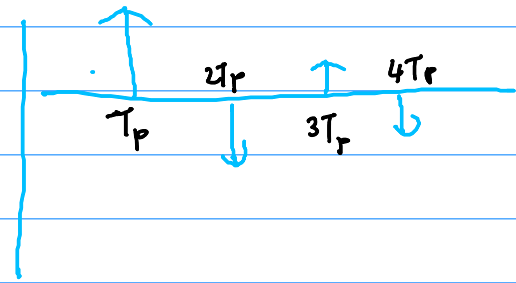
$\dfrac{\zeta}{\omega_n} = \dfrac{\zeta}{\omega_d}\sqrt{1-\zeta^2}$

peak time이 같아서 rise time도 똑같다.
ζ가 증가하면 O.S. 감소

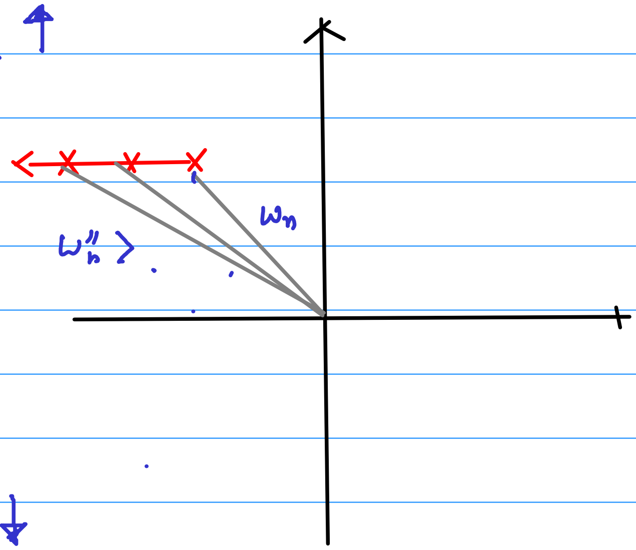
Root Locus

$\boxed{\begin{array}{l}\sigma_d \uparrow \\ \omega_d \text{ fixed}\end{array}}$

$$\boxed{T_p = \frac{\pi}{\sqrt{1-\zeta^2}\,\omega_n}} = \frac{\pi}{\omega_d}$$



$\sigma_d \uparrow$



$\zeta \uparrow$

$\omega_n \uparrow$

$\sigma_d \uparrow$

$\boxed{\omega_d \text{ fixed}}$

$\theta \downarrow$

$\zeta = \cos\theta \uparrow$

$\omega_n \uparrow$

$\boxed{\omega_d \text{ fixed}}$

$$\sqrt{1-\zeta^2}\,\omega_n = \omega_d$$

$0 < \zeta < 1$

$0 < \theta < 90$

$\zeta \downarrow$
$\omega_n \uparrow$

σ_d fixed
$\omega_d \uparrow$

real part가
음의 실수 영향으로

① Peak Time $\qquad T_p = \dfrac{\pi}{\omega_d} = \dfrac{\pi}{\omega_d} \quad \downarrow$

② Percent Overshoot $\qquad \%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100 \quad \uparrow$

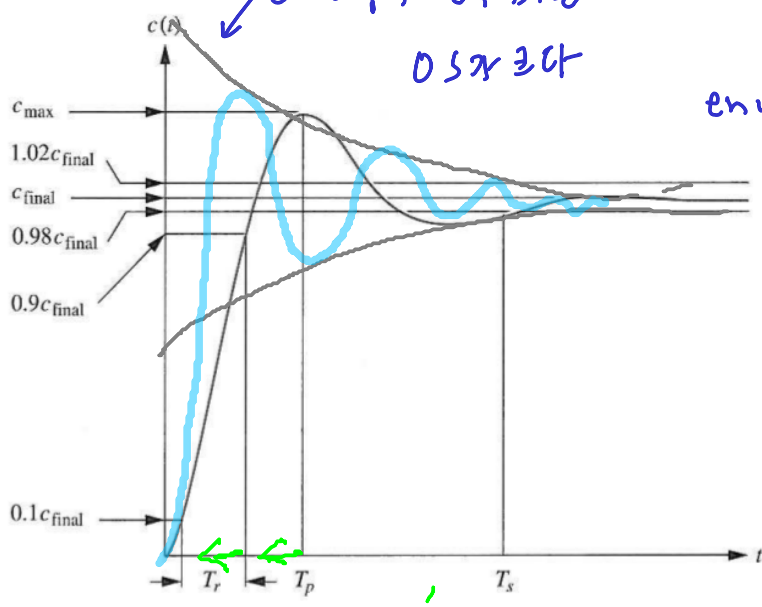
③ Settling Time $\qquad T_s = \dfrac{4}{\sigma_d} = \dfrac{4}{\sigma_d} \quad \leftrightarrow$

④ Rising Time $\qquad T_r = \dfrac{2.16\zeta + 0.6}{\omega_n} \quad \downarrow$ 변동 X

envelop가 큰쪽에서는
OS가 크다

ζ가 감소하면 OS 증가

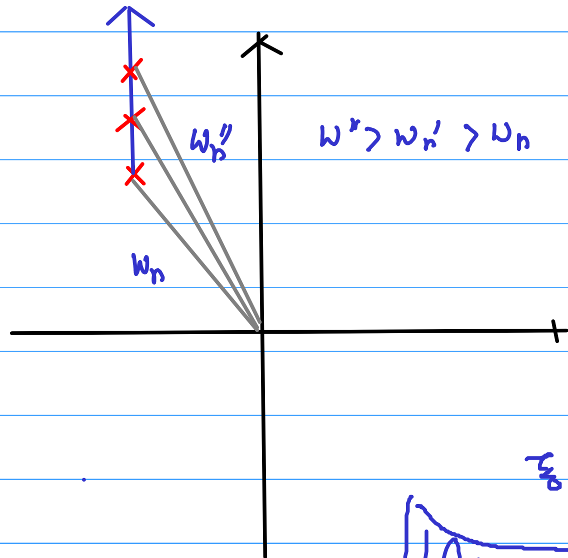
envelope가 같아서 settling time
똑같다.



Root Locus

σ_d fixed
$\omega_d \uparrow$

$\omega_d \uparrow$



$\zeta \downarrow$
$\omega_n \uparrow$
$\boxed{\sigma_d \text{ fixed}}$
$\omega_d \uparrow$

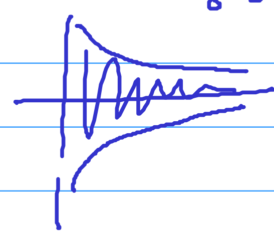
$\theta \uparrow$

$\zeta = \cos\theta \downarrow$

$\omega_n \uparrow$

$\boxed{\sigma_d \text{ fixed}}$

동일한 exp. decay



포락선, the same envelope

$0 < \zeta < 1$

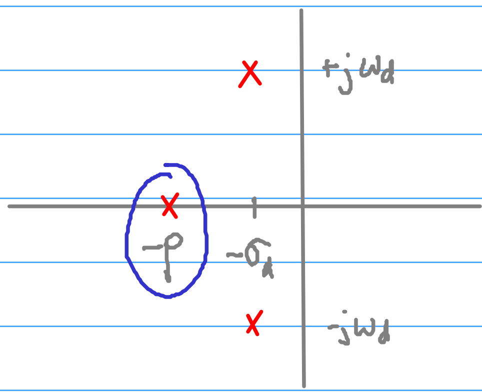
$0 < \theta < 90$

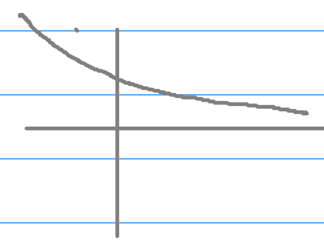
우세극점 dominant pole

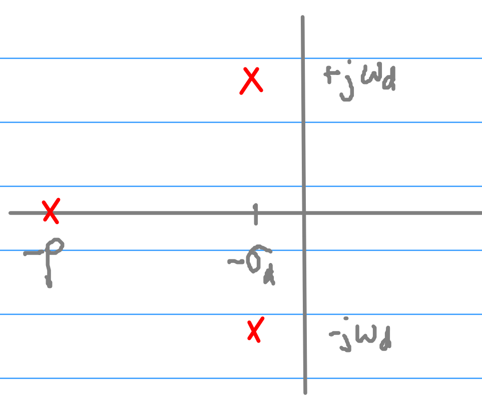
$$G(s) = \frac{1}{(s+p)(s+\sigma_d - j\omega_d)(s+\sigma_d + j\omega_d)}$$

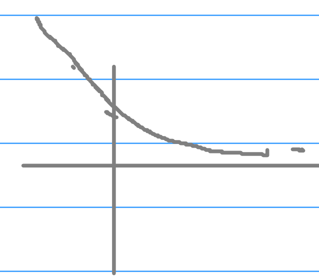
$$= \frac{1}{(s+p)\left((s+\sigma_d)^2 + \omega_d^2\right)}$$

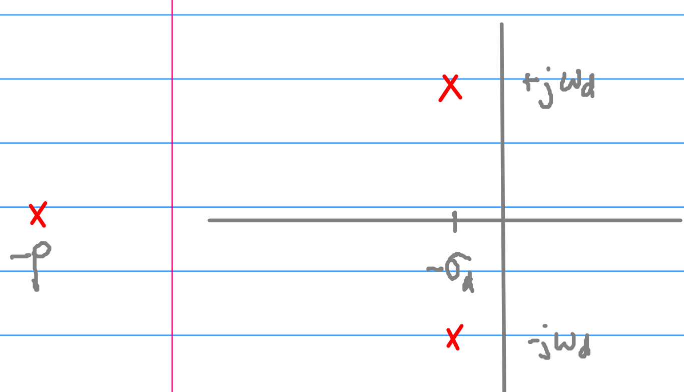
$$\frac{1}{(s+p)} \Longleftrightarrow e^{-pt}$$

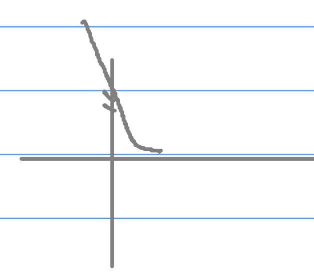












$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin\left(\sqrt{1-\zeta^2}\,\omega_n t + \theta\right)$$

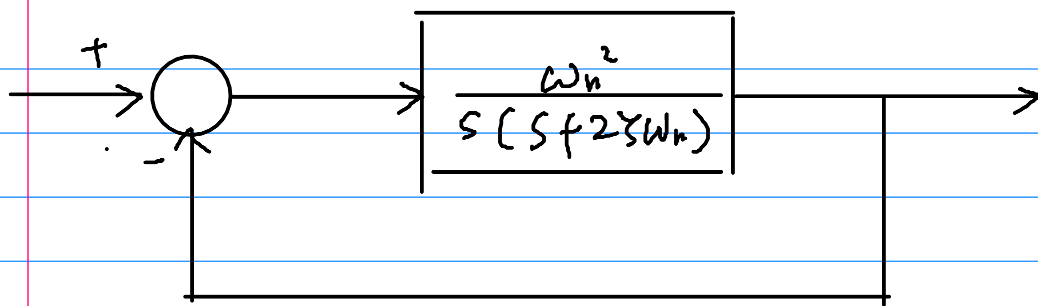
ϕ

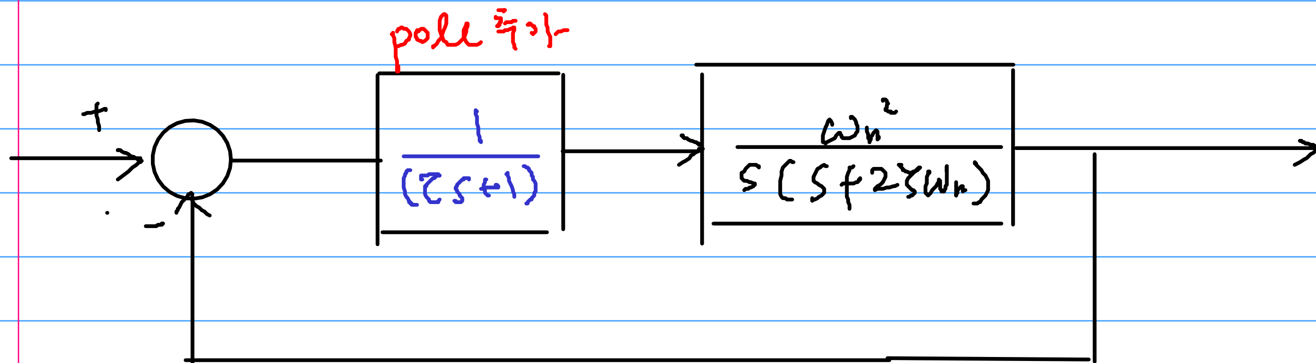
$\cos\theta = \zeta = \frac{1}{2}$

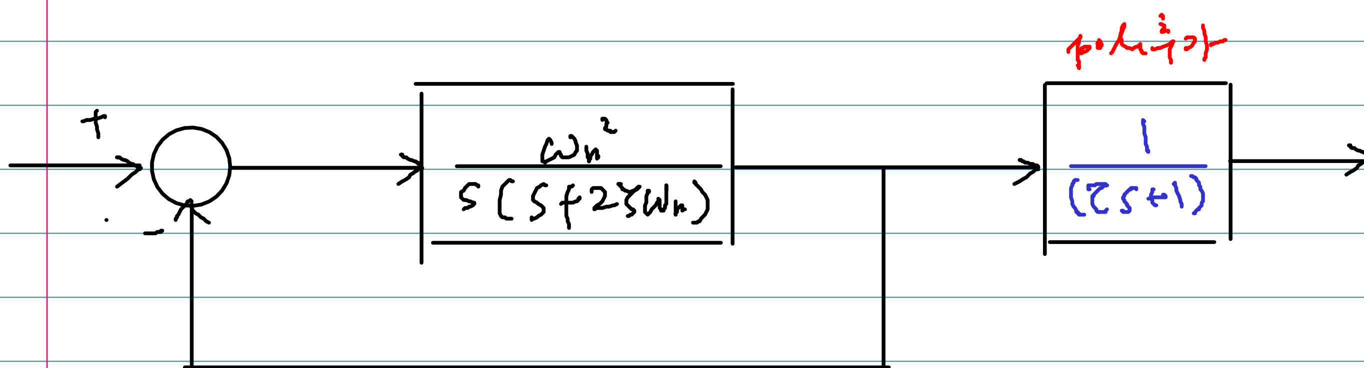
$\zeta = 0.5$

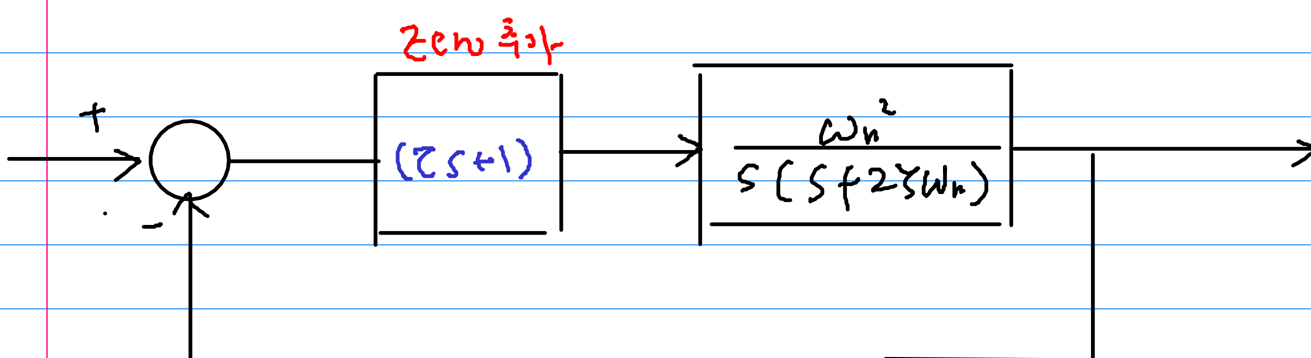
$\omega_n = 2$

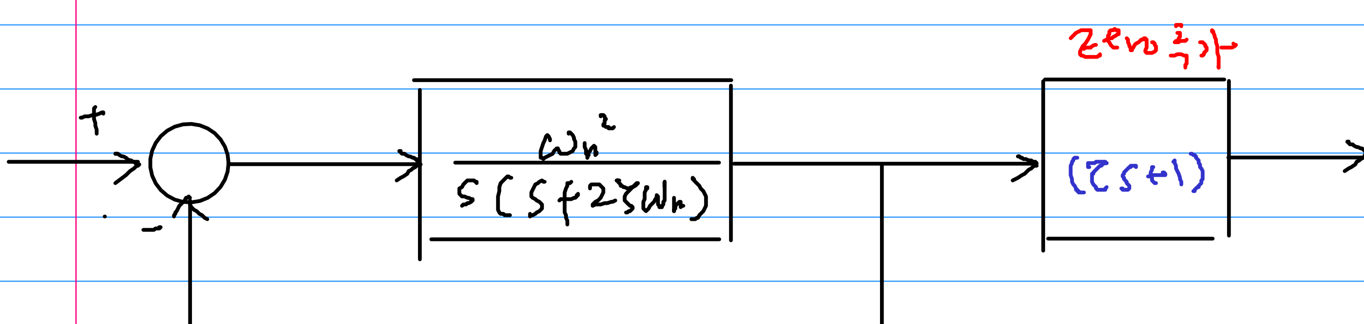
$\theta = \cos^{-1}\frac{1}{2} = 60\,\frac{\pi}{180} = \frac{\pi}{3}$

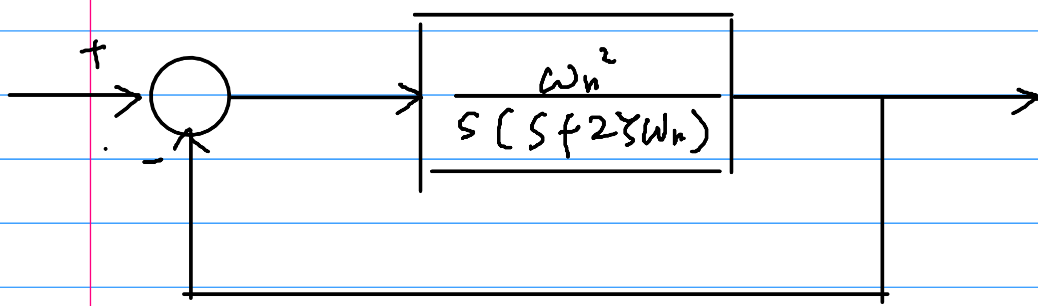

+
-
$\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$


pole 추가
+
-
$\frac{1}{(\tau s+1)}$
$\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$

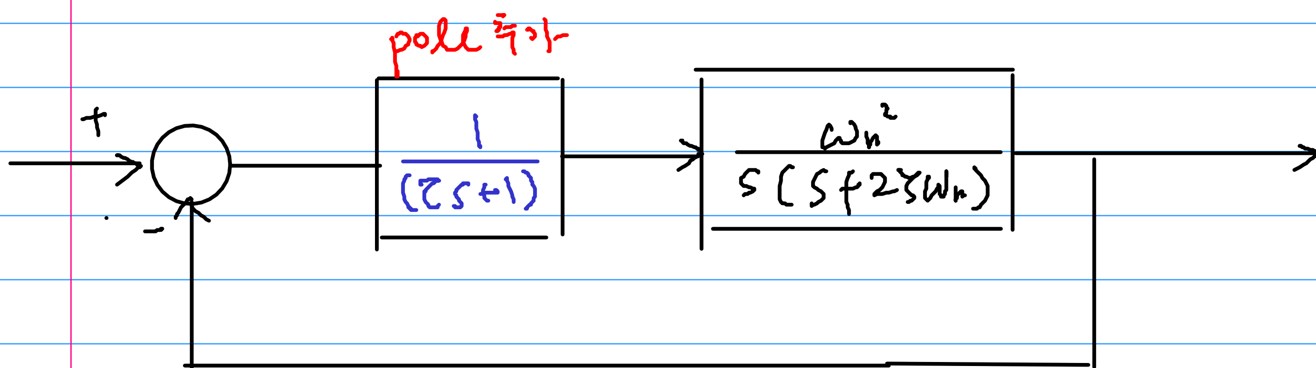

pole 추가
+
-
$\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$
$\frac{1}{(\tau s+1)}$


zero 추가
+
-
$(\tau s+1)$
$\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$


zero 추가
+
-
$\frac{\omega_n^2}{s(s+2\zeta\omega_n)}$
$(\tau s+1)$



$$G(s) = \frac{\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



$$G(s) = \frac{1}{\tau s^2 + (1+2\zeta\omega_n\tau)s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\frac{1}{\tau} \to 0$ $\quad \tau \uparrow$ $\quad \%OS \uparrow$

$T_r \uparrow$

$\omega_n \downarrow$