Wright State University Lake Campus/University Physics Volume 1/Equations

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Lake 2018-1 Phy1060 Phy1120 Phy2400

Introduction

metric prefixes

da	h	k	M	G	T	P	E	Z	Y
deca	hecto	kilo	mega	giga	tera	peta	exa	zetta	yotta
1E+01	1E+02	1E+03	1E+06	1E+09	1E+12	1E+15	1E+18	1E+21	1E+24
d	c	m	µ	n	p	f	a	z	y
deci	centi	milli	micro	nano	pico	femto	atto	zepto	yocto
1E-01	1E-02	1E-03	1E-06	1E-09	1E-12	1E-15	1E-18	1E-21	1E-24

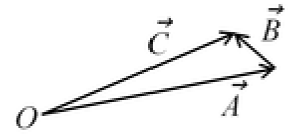
1. The circumference of a circle is $C_\odot = 2\pi r$ and the circle's area is $A_\odot = \pi r^2$ is its area.
2. The surface area of a sphere is $A_\bigcirc = 4\pi r^2$ and sphere's volume is $V_\bigcirc = \frac{4}{3}\pi r^3$
3. 1 kilometer = .621 miles and 1 MPH = 1 mi/hr ≈ .447 m/s
4. Typical air density is 1.2kg/m^3, with pressure 10^5Pa. The density of water is 1000kg/m^3.
5. Earth's mean radius ≈ 6371km, mass ≈ 6×10^{24} kg
6. Universal gravitational constant = G ≈ 6.67×10^{-11} $m^3 \cdot kg^{-1} \cdot s^{-2}$
7. Speed of sound ≈ 340m/s and the speed of light = c ≈ 3×10^8m/s
8. One light-year ≈ 9.5×10^{15}m ≈ 63240AU (Astronomical unit) ... <These 8 equations were added for WSU-L exams>

Units_and_Measurement

The base SI units are mass: kg (kilogram); length: m (meter); time: s (second). Percent error (http://wiki.ubc.ca/index.php?title=Uncertainty_and_Error&oldid=81540) is $(\delta A/A) \times 100\%$

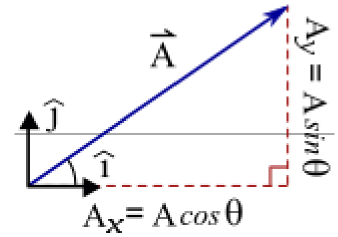
Vectors

Vector $\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$ involves components (A_x,A_y ,A_z) and three orthonormal unit vectors.



□ If $\vec{A}+\vec{B}=\vec{C}$, then $A_x+B_x=C_x$, etc, and vector subtraction is defined by $\vec{B}=\vec{C}-\vec{A}$.

□ The two-dimensional displacement from the origin is $\vec{r}=x\hat{i}+y\hat{j}$. The magnitude is $A\equiv|\vec{A}|=\sqrt{A_x^2+A_y^2}$. The angle (phase) is $\theta=\tan^{-1}(y/x)$.



□ Scalar multiplication $\alpha\vec{A}=\alpha A_x\hat{i}+\alpha A_y\hat{j}+\ldots$

□ Any vector divided by its magnitude is a unit vector and has unit magnitude: $|\hat{V}|=1$ where $\hat{V}\equiv\vec{V}/V$

□ Dot product $\vec{A}\cdot\vec{B}=AB\cos\theta=A_xB_x+A_yB_y+\ldots$ and $\vec{A}\cdot\vec{A}=A^2$

□ Cross product $\vec{A}=\vec{B}\times\vec{C}\Rightarrow A_\alpha=B_\beta C_\gamma-C_\gamma A_\beta$ where (α,β,γ) is any cyclic permutation of (x,y,z), i.e., (α,β,γ) represents either (x,y,z) or (y,z,x) or (z,x,y).

□ Cross-product magnitudes obey $A=BC\sin\theta$ where θ is the angle between $\vec{B}$ and $\vec{C}$, and $\vec{A}\perp\{\vec{B},\vec{C}\}$ by the right hand rule.

□ Vector identities $c(\mathbf{A}+\mathbf{B})=c\mathbf{A}+c\mathbf{B}$

□ $\mathbf{A}+\mathbf{B}=\mathbf{B}+\mathbf{A}$

□ $\mathbf{A}+(\mathbf{B}+\mathbf{C})=(\mathbf{A}+\mathbf{B})+\mathbf{C}$

□ $\mathbf{A}\cdot\mathbf{B}=\mathbf{B}\cdot\mathbf{A}$

□ $\mathbf{A}\times\mathbf{B}=-\mathbf{B}\times\mathbf{A}$

□ $(\mathbf{A}+\mathbf{B})\cdot\mathbf{C}=\mathbf{A}\cdot\mathbf{C}+\mathbf{B}\cdot\mathbf{C}$

□ $(\mathbf{A}+\mathbf{B})\times\mathbf{C}=\mathbf{A}\times\mathbf{C}+\mathbf{B}\times\mathbf{C}$

□ $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})=\mathbf{B}\cdot(\mathbf{C}\times\mathbf{A})=(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$

□ $\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=(\mathbf{A}\cdot\mathbf{C})\,\mathbf{B}-(\mathbf{A}\cdot\mathbf{B})\,\mathbf{C}$

□ $(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D})=(\mathbf{A}\cdot\mathbf{C})\,(\mathbf{B}\cdot\mathbf{D})-(\mathbf{B}\cdot\mathbf{C})\,(\mathbf{A}\cdot\mathbf{D})$

Motion_Along_a_Straight_Line

Delta as difference $\Delta x=x_f-x_i\to dx\to 0$ in limit of differential calculus.

□ Average velocity $\bar{v}=\Delta x/\Delta t\to v=dx/dt$ (instantaneous velocity)

□ Acceleration $\bar{a}=\Delta v/\Delta t\to a=dv/dt$.

☐ WLOG set $\Delta t = t$ and $\Delta x = x - x_0$ if $t_i = 0$. Then $\Delta v = v - v_0$, and $v(t) = \int_0^t a(t')dt' + v_0$, $x(t) = \int_0^t v(t')dt' + x_0 = x_0 + \bar{v}t$, where $\bar{v} = \frac{1}{t}\int_0^t v(t')dt'$ is the average velocity.

☐ At constant acceleration: $\bar{v} = \frac{v_0+v}{2}$, $v = v_0 + at$, $x = x_0 + v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2a\Delta x$.

☐ For free fall, replace $x \to y$ (positive up) and $a \to -g$, where g = 9.81 m/s^2 at Earth's surface).

Motion_in_Two_and_Three_Dimensions

Instantaneous velocity: $\vec{v}(t) = v_x(t)\hat{i} + v_y(t)\hat{j} + v_z(t)\hat{k} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$

☐ $\vec{v}(t) = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t)-\vec{r}(t)}{\Delta t}$, where $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

☐ Acceleration $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, where $a_x(t) = dv_x/dt = d^2x/dt^2$.

☐ Average values: $\vec{v}_{ave} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}(t_2)-\vec{r}(t_2)}{t_2-t_1}$, and $\vec{a}_{ave} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}(t_2)-\vec{v}(t_2)}{t_2-t_1}$

☐ Free fall time of flight $T_{of} = \frac{2(v_0\sin\theta_0)}{g}$,

☐ Trajectory $y = (\tan\theta_0)x - \left[\frac{g}{2(v_0\cos\theta_0)^2}\right]x^2$,

☐ Range $R = \frac{v_0^2\sin 2\theta_0}{g}$

☐ Uniform circular motion: position $\vec{r}(t)$, velocity $\vec{v}(t) = d\vec{r}(t)/dt$, and acceleration $\vec{a}(t) = d\vec{v}(t)/dt$: $\vec{r} = A\cos\omega t\hat{i} + A\sin\omega t\hat{j}$, $\vec{v} = -A\omega\sin\omega t\hat{i} + A\omega\cos\omega t\hat{j}$, $\vec{a} = -A\omega^2\cos\omega t\hat{i} - A\omega^2\sin\omega t\hat{j}$. Note that if $A = r$ then $|\vec{a}| = a_C = \omega^2 r = v^2/r$ where $v \equiv |\vec{v}| = \omega r$.

☐ Tangential and centripetal acceleration $\vec{a} = \vec{a}_c + \vec{a}_T$ where $a_T = d|\vec{v}|/dt$.

☐ Relative motion: $\vec{r}_{PS} = \vec{r}_{PS'} + \vec{r}_{S'S}$, $\vec{v}_{PS} = \vec{v}_{PS'} + \vec{v}_{S'S}$, $\vec{v}_{PC} = \vec{v}_{PA} + \vec{v}_{AB} + \vec{v}_{BC}$, $\vec{a}_{PS} = \vec{a}_{PS'} + \vec{a}_{S'S}$

This is a transclusion, added two days before Test 1

$$x = x_0 + v_{0x}\Delta t + \frac{1}{2}a_x\Delta t^2 \quad v_x = v_{0x} + a_x\Delta t \quad v_x^2 = v_{x0}^2 + 2a_x\Delta x$$

$$y = y_0 + v_{0y}\Delta t + \frac{1}{2}a_y\Delta t^2 \quad v_y = v_{0y} + a_y\Delta t \quad v_y^2 = v_{x0}^2 + 2a_y\Delta y$$

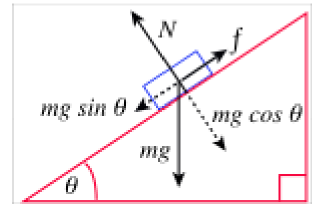
$v^2 = v_0^2 + 2a_x\Delta x + 2a_y\Delta y$ ...in advanced notation this becomes $\Delta(v^2) = 2\vec{a}\cdot\Delta\vec{\ell}$.

In free fall we often set, a_x=0 and a_y= -g. If angle is measured with respect to the x axis:

$v_x = v\cos\theta$ $\quad v_y = v\sin\theta$ $\quad v_{x0} = v_0\cos\theta_0$ $\quad v_{y0} = v_0\sin\theta_0$

Newton's_Laws_of_Motion

Newton's 2$^{\text{nd}}$ Law $m\vec{a} = d\vec{p}/dt = \sum \vec{F}_j$, where $\vec{p} = m\vec{v}$ is momentum, m is mass, and $\sum \vec{F}_j$ is the sum of all forces This sum needs only include external forces because all internal forces cancel by the 3$^{\text{rd}}$ law $\vec{F}_{AB} = -\vec{F}_{BA}$. The 1$^{\text{st}}$ law is that velocity is constant if the net force is zero.

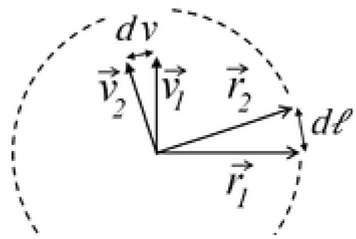


☐ Weight= $\vec{w} = m\vec{g}$.

☐ normal force is a component of the contact force by the surface. If the only forces are contact and weight, $|\vec{N}| = N = mg\cos\theta$ where θ is the angle of incline.

☐ Hooke's law $F = -kx$ where k is the spring constant.

Applications_of_Newton's_Laws



Use for this quiz

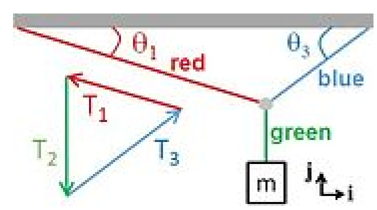
$f_s \le \mu_s N$ and $f_k = \mu_k N$: $f =$ friction, $\mu_{s,k} =$ coefficient of (static,kinetic) friction, $N =$ normal force.

☐ Centripetal force$F_c = mv^2/r = mr\omega^2$ for uniform circular motion. Angular velocity ω is measured in radians per second.

☐ Ideal angle of banked curve: $\tan\theta = v^2/(rg)$ for curve of radius r banked at angle θ.

☐ Drag equation $F_D = \frac{1}{2}C\rho A v^2$ where $C =$ Drag coefficient, $\rho =$ mass density, $A =$ area, $v =$ speed. Holds approximately for large Reynold's number $=$ Re $= \rho v L/\eta$, where $\eta =$dynamic viscosity; $L =$ characteristic length.

☐ Stokes's law models a sphere of radius r at small Reynold's number: $F_s = 6\pi r\eta v$.



The x and y components of the three forces of tension on the small grey circle where the three "massless" ropes meet are:

$$T_{1x} = -T_1\cos\theta_1 \;, \qquad T_{1y} = T_1\sin\theta_1$$
$$T_{2x} = 0 \;, \qquad T_{2y} = -mg$$
$$T_{3x} = T_3\cos\theta_3 \;, \qquad T_{3y} = T_3\sin\theta_3$$

Work_and_Kinetic_Energy

Infinitesimal work done by force: $dW = \vec{F}\cdot d\vec{r} = |\vec{F}|\,|d\vec{r}|\cos\theta$ leads to the path integral $W_{AB} = \int_A^B \vec{F}\cdot d\vec{r}$

☐ Work done from A→B by friction $-f_k|\ell_{AB}|$, gravity $-mg(y_B - y_A)$, and spring $-\frac{1}{2}k\left(x_B^2 - x_A^2\right)$

☐ Work-energy theorem: The work done on a particle is $W_{net} = K_B - K_A$ where kinetic energy $= K = \frac{1}{2}mv^2 = \dfrac{p^2}{2m}$.

☐ Power= $P = dW/dt = \vec{F}\cdot\vec{v}$.

Potential_Energy_and_Conservation_of_Energy

Potential Energy: $\Delta U_{AB} = U_B - U_A = -W_{AB}$; PE at $\vec{r}$ WRT $\vec{r}_0$ is $\Delta U = U(\vec{r}) - U(\vec{r}_0)$