

Stationarity

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

- 1 First-Order Stationary Processes
- 2 Autocorrelation and Crosscorrelation Functions

First Order Stationary

N Gaussian random variables

Definition

if the first order density function does not change with a shift in time origin

$$f_X(x_1; t_1) = f_X(x_1; t_1 + \Delta)$$

must be true for any time t_1 and any real number Δ if $X(t)$ is to be a first-order stationary

Consequences of stationarity

N Gaussian random variables

Definition

$f_X(x, t_1)$ is independent of t_1

the process mean value is a constant

$$m_X(t) = \bar{X} = \text{constant}$$

the process mean value

N Gaussian random variables

Definition

$$m_X(t) = \bar{X} = \text{constant}$$

$$m_X(t_1) = \int_{-\infty}^{\infty} x f_X(x; t_1) dx$$

$$m_X(t_2) = \int_{-\infty}^{\infty} x f_X(x; t_2) dx$$

$$m_X(t_1) = m_X(t_1 + \Delta)$$

Second-Order Stationary Process

N Gaussian random variables

Definition

if the second order density function does not change with a shift in time origin

$$f_X(x_1, x_2; t_1, t_2) = f_X(x_1, x_2; t_1 + \Delta, t_2 + \Delta)$$

must be true for any time t_1, t_2 and any real number Δ if $X(t)$ is to be a second-order stationary

Auto-correlation function

$$R_{XX}(t, t + \tau) = E[X(t)X(t + \tau)] = R_{XX}(\tau)$$

N^{th} -order Stationary Processes

N Gaussian random variables

Definition

if the second order density function does not change with a shift in time origin

$$f_X(x_1, \dots, x_N; t_1, \dots, t_N) = f_X(x_1, \dots, x_N; t_1 + \Delta, \dots, t_N + \Delta)$$

must be true for any time t_1, \dots, t_N and any real number Δ if $X(t)$ is to be a second-order stationary

Wide Sense Stationary Process

N Gaussian random variables

Definition

$$m_X(t) = \bar{X} = \text{constant}$$

$$E[X(t)X(t+\tau)] = R_{XX}(\tau)$$

The properties of autocorrelation functions (1)

 N Gaussian random variables

Definition

$$|R_{XX}(\tau)| \leq R_{XX}(0)$$

$$R_{XX}(-\tau) = R_{XX}(\tau)$$

$$R_{XX}(0) = E[X^2(t)]$$

$$P[|X(t+\tau) - X(t)| > \varepsilon] = \frac{2}{\varepsilon^2} (R_{XX}(0) - R_{XX}(\tau))$$

The properties of autocorrelation functions (2)

N Gaussian random variables

Definition

if $X(t) = \bar{X} + N(t)$ where $N(t)$ is WSS, is zero-mean, and has autocorrelation function $R_{NN}(\tau) \rightarrow 0$ as $|\tau| \rightarrow \infty$, then

$$\lim_{|\tau| \rightarrow \infty} R_{XX}(\tau) = \bar{X}^2$$

if $X(t)$ is mean square periodic, i.e, there exists a $T \neq 0$ such that $E[(X(t+T) - X(t))^2] = 0$ for all t , then $R_{XX}(t)$ will have a periodic component with the same period

$R_{XX}(\tau)$ cannot have an arbitrary shape

