

Relationship between Power Spectrum and Autocorrelation Function

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

AutoCorrelation Function

N Gaussian random variables

Definition

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{XX}(\omega) e^{+j\omega t} d\omega = A[R_{XX}(t, t + \tau)]$$

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} E \left[\frac{1}{2T} \int_{-T}^{+T} X(t_1) e^{+j\omega t_1} dt_1 \int_{-T}^{+T} X(t_2) e^{-j\omega t_2} dt_2 \right]$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} E[X(t_1)X(t_2)] e^{-j\omega(t_2-t_1)} dt_2 dt_1$$

AutoCorrelation and Expectation

N Gaussian random variables

Definition

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} E[X(t_1)X(t_2)] e^{-j\omega(t_2-t_1)} dt_2 dt_1$$

$$E[X(t_1)X(t_2)] = R_{XX}(t_1, t_2)$$

$$S_{XX}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XX}(t_1, t_2) e^{-j\omega(t_2-t_1)} dt_2 dt_1$$

