

# K-Map (2B)

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# K-Map 4 variables (1)

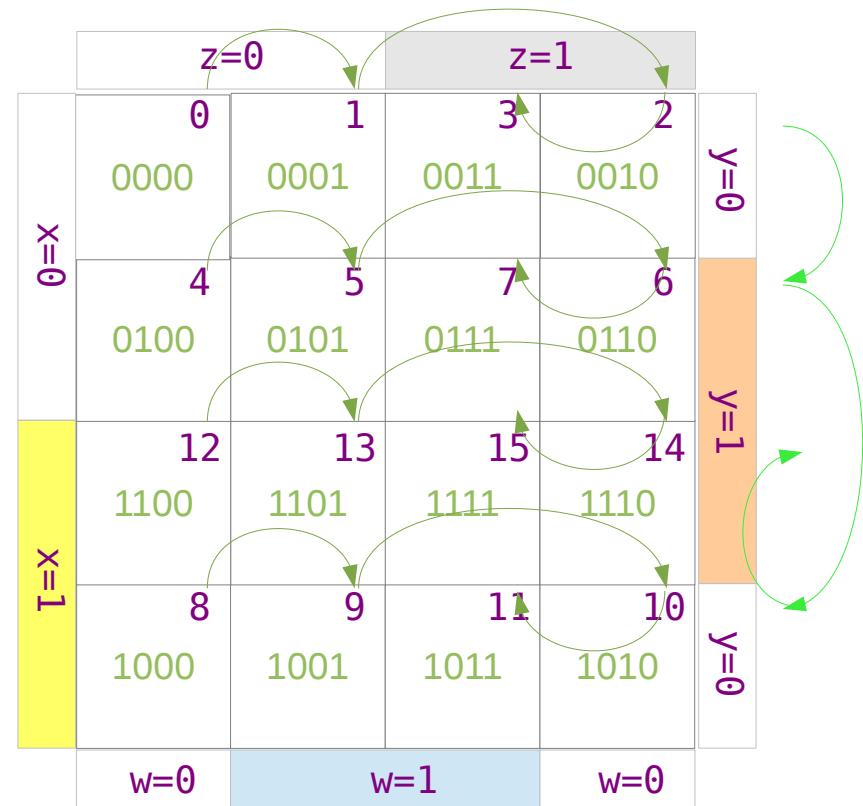
index					minterms
0	0	0	0	0	$\bar{x}\bar{y}\bar{z}\bar{w}$
1	0	0	0	1	$\bar{x}\bar{y}\bar{z}w$
2	0	0	1	0	$\bar{x}\bar{y}z\bar{w}$
3	0	0	1	1	$\bar{x}\bar{y}zw$
4	0	1	0	0	$\bar{x}y\bar{z}\bar{w}$
5	0	1	0	1	$\bar{x}y\bar{z}w$
6	0	1	1	0	$\bar{x}yz\bar{w}$
7	0	1	1	1	$\bar{x}yzw$
8	1	0	0	0	$x\bar{y}\bar{z}\bar{w}$
9	1	0	0	1	$x\bar{y}\bar{z}w$
10	1	0	1	0	$x\bar{y}z\bar{w}$
11	1	0	1	1	$x\bar{y}zw$
12	1	1	0	0	$xy\bar{z}\bar{w}$
13	1	1	0	1	$xy\bar{z}w$
14	1	1	1	0	$xyz\bar{w}$
15	1	1	1	1	$xyzw$

		z=0		z=1	
		w=0	w=1		w=0
		00	01	11	10
y=x	0=0	0000	0001	0011	0010
	1=1	0100	0101	0111	0110
y=0	1=1	1100	1101	1111	1110
	0=0	1000	1001	1011	1010

# K-Map 4 variables (2)

index                      minterms

0	0	0	0	0	$\bar{x}\bar{y}\bar{z}\bar{w}$
1	0	0	0	1	$\bar{x}\bar{y}\bar{z}w$
2	0	0	1	0	$\bar{x}\bar{y}z\bar{w}$
3	0	0	1	1	$\bar{x}\bar{y}zw$
4	0	1	0	0	$\bar{x}y\bar{z}\bar{w}$
5	0	1	0	1	$\bar{x}y\bar{z}w$
6	0	1	1	0	$\bar{x}yz\bar{w}$
7	0	1	1	1	$\bar{x}yzw$
8	1	0	0	0	$x\bar{y}\bar{z}\bar{w}$
9	1	0	0	1	$x\bar{y}\bar{z}w$
10	1	0	1	0	$x\bar{y}z\bar{w}$
11	1	0	1	1	$x\bar{y}zw$
12	1	1	0	0	$xy\bar{z}\bar{w}$
13	1	1	0	1	$xy\bar{z}w$
14	1	1	1	0	$xyz\bar{w}$
15	1	1	1	1	$xyzw$



# K-Map 3 variables (1)

index

0  
1  
2  
3  
4  
5  
6  
7

0	0	0	0
1	0	0	1
2	0	1	0
3	0	1	1
4	1	0	0
5	1	0	1
6	1	1	0
7	1	1	1

minterms

$\bar{x}\bar{y}\bar{z}$   
 $\bar{x}\bar{y}z$   
 $\bar{x}y\bar{z}$   
 $\bar{x}yz$   
 $x\bar{y}\bar{z}$   
 $x\bar{y}z$   
 $xy\bar{z}$   
 $xyz$

	x	y	z		
		00	01	11	10
0		0	1	3	2
1		4	5	7	6

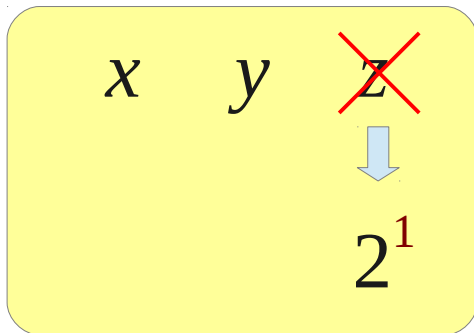
	y=0		y=1	
	z=0	z=1	z=0	z=1
0=x	0	1	3	2
1=x	4	5	7	6

# K-Map 3 variables (2)

index				minterms	
0	0	0	0	$\bar{x}\bar{y}\bar{z}$	} $\bar{x}\bar{y}$
1	0	0	1	$\bar{x}\bar{y}z$	
2	0	1	0	$\bar{x}y\bar{z}$	} $\bar{x}y$
3	0	1	1	$\bar{x}yz$	
4	1	0	0	$x\bar{y}\bar{z}$	} $x\bar{y}$
5	1	0	1	$x\bar{y}z$	
6	1	1	0	$xy\bar{z}$	} $xy$
7	1	1	1	$xyz$	

$$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z = \bar{x}\bar{y}(\bar{z}+z) = \bar{x}\bar{y}$$

a group of 2 minterms



	y=0	y=1	
z=0	00	z=1	
z=0	10	z=0	

		00	01	11	10	
0	0	1	3	2		
1	4	5	7	6		

$\bar{x}\bar{y}$     $\bar{x}y$

$x\bar{y}$     $xy$

# K-Map 3 variables (3)

index

0  
1  
2  
3  
4  
5  
6  
7

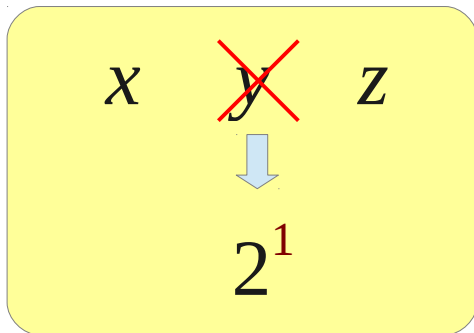
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

minterms

$\bar{x}\bar{y}\bar{z}$  —  $\bar{x}\bar{z}$   
 $\bar{x}\bar{y}z$  —  $\bar{x}z$   
 $\bar{x}y\bar{z}$  —  
 $\bar{x}yz$  —  
 $x\bar{y}\bar{z}$  —  $x\bar{z}$   
 $x\bar{y}z$  —  $xz$   
 $xy\bar{z}$  —  
 $xyz$  —

$$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z = \bar{x}\bar{z}(\bar{y}+y) = \bar{x}\bar{z}$$

a group of 2 minterms



y=0		y=1	
z=0	z=1	z=0	z=1

	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$\bar{x}\bar{z}$     $\bar{x}z$

$x\bar{z}$     $xz$

# K-Map 3 variables (4)

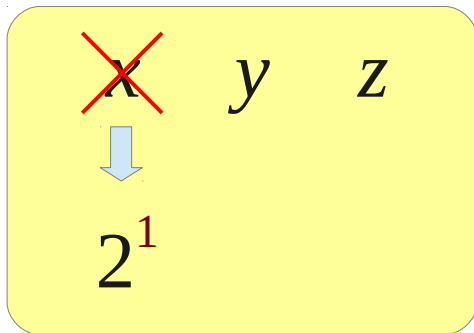
index

minterms

0	<del>0</del>	<del>0</del>	<del>0</del>	$\bar{x}\bar{y}\bar{z}$	$\bar{y}\bar{z}$
1	<del>0</del>	<del>0</del>	1	$\bar{x}\bar{y}z$	$\bar{y}z$
2	<del>0</del>	1	<del>0</del>	$\bar{x}y\bar{z}$	$y\bar{z}$
3	<del>0</del>	1	1	$\bar{x}yz$	$yz$
4	<del>1</del>	<del>0</del>	<del>0</del>	$x\bar{y}\bar{z}$	
5	<del>1</del>	<del>0</del>	1	$x\bar{y}z$	
6	1	1	<del>0</del>	$xy\bar{z}$	
7	1	1	1	$xyz$	

$$\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} = \bar{y}\bar{z}(\bar{x}+x) = \bar{y}\bar{z}$$

a group of 2 minterms



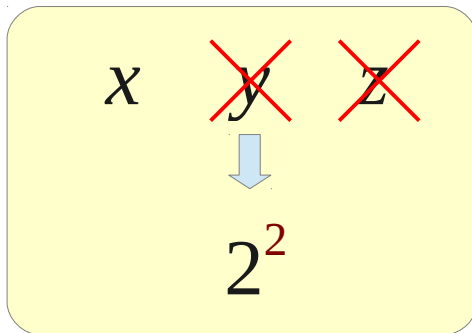
		y=0		y=1	
		z=0		z=1	
		00	01	11	10
0	x=0	0	1	3	2
1	x=1	4	5	7	6
		$\bar{y}\bar{z}$	$\bar{y}z$	$yz$	$y\bar{z}$



# K-Map 3 variables (5)

index				minterms	
0	0	0	0	$\bar{x}\bar{y}\bar{z}$	}
1	0	0	1	$\bar{x}\bar{y}z$	
2	0	1	0	$\bar{x}y\bar{z}$	
3	0	1	1	$\bar{x}yz$	
4	1	0	0	$x\bar{y}\bar{z}$	}
5	1	0	1	$x\bar{y}z$	
6	1	1	0	$xy\bar{z}$	
7	1	1	1	$xyz$	

a group of 4 minterms

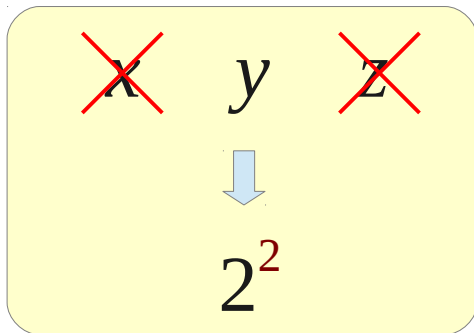


		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x	0	0	1	3	2
	1	4	5	7	6

# K-Map 3 variables (5)

index				minterms
0	0	0	0	$\bar{x}\bar{y}\bar{z}$
1	0	0	1	$\bar{x}\bar{y}z$
2	0	1	0	$\bar{x}y\bar{z}$
3	0	1	1	$\bar{x}yz$
4	1	0	0	$x\bar{y}\bar{z}$
5	1	0	1	$x\bar{y}z$
6	1	1	0	$xy\bar{z}$
7	1	1	1	$xyz$

a group of 4 minterms

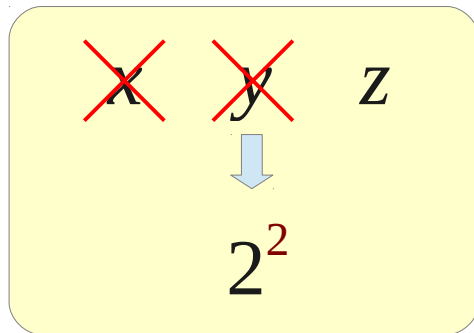


		y=0		y=1	
		z=0		z=1	
		00	01	11	10
x	0	0	1	3	2
	1	4	5	7	6
		$\bar{y}$		y	

# K-Map 3 variables (5)

index		minterms
0	0 0 0	$\bar{x}\bar{y}\bar{z}$ — $\bar{z}$
1	0 0 1	$\bar{x}\bar{y}z$ — $z$
2	0 1 0	$\bar{x}y\bar{z}$
3	0 1 1	$\bar{x}yz$
4	1 0 0	$x\bar{y}\bar{z}$
5	1 0 1	$x\bar{y}z$
6	1 1 0	$xy\bar{z}$
7	1 1 1	$xyz$

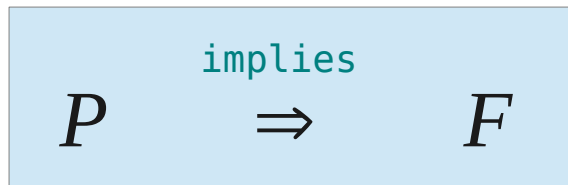
a group of 4 minterms



		y=0		y=1	
		z=0	z=1		z=0
		00	01	11	10
x	0	0	1	3	2
	1	4	5	7	6
		$\bar{z}$	z		

# Implicant

F takes the value 1, whenever P equals 1



assuming  $P$  is a product term in a sum of products

$$f(x, y, z, w) = xy + yz + w$$

general, reduced

$$(x=1, y=1)$$

$$(x=1, y=1, z=1)$$

$$(x=1, y=1, z=1, w=1)$$



$$xy$$

$$xyz$$

$$xyzw$$

$\Rightarrow$

$\Rightarrow$

$\Rightarrow$

$$f(x, y, z, w) = xy + yz + w$$

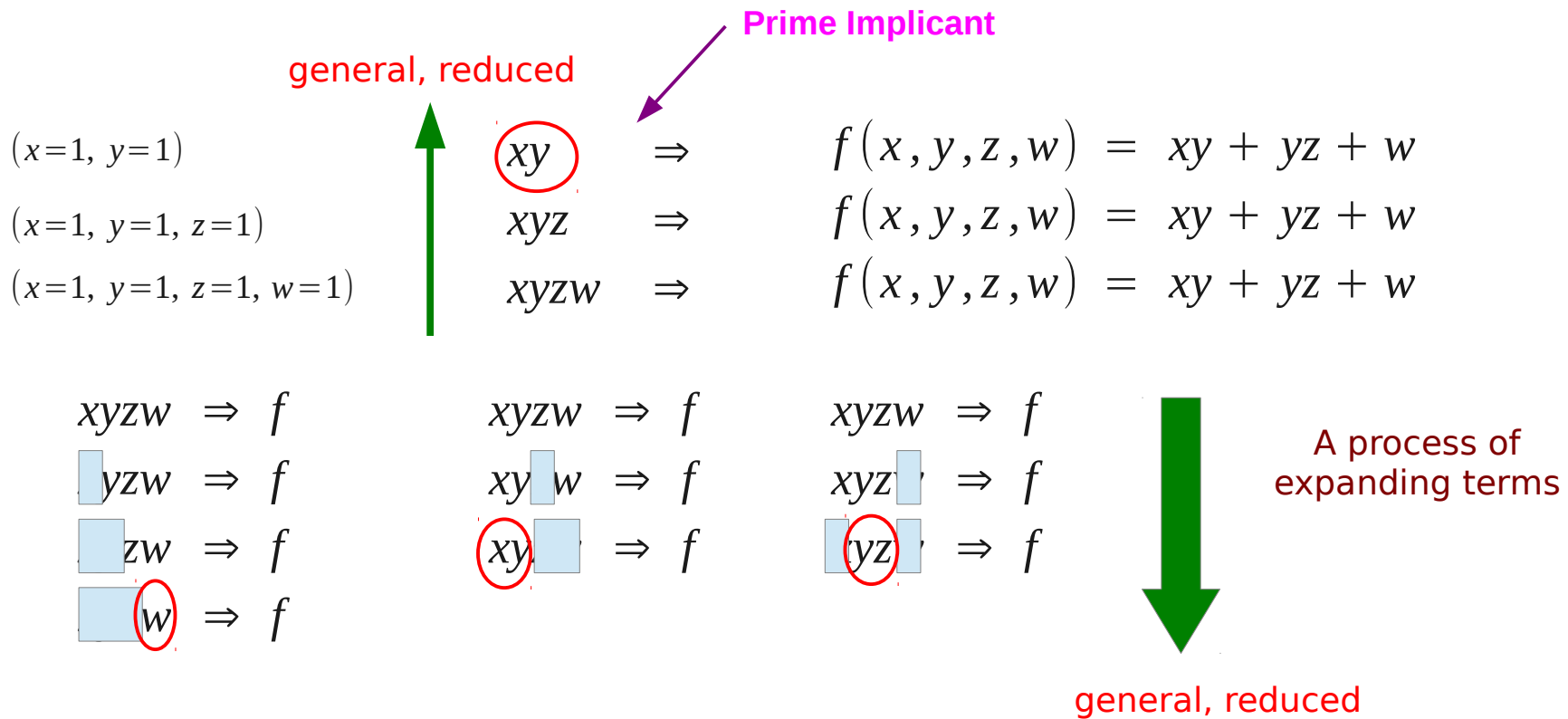
$$f(x, y, z, w) = xy + yz + w$$

$$f(x, y, z, w) = xy + yz + w$$

# Prime Implicant

**Prime Implicant:** An implicant that is minimal

The removal of any literal from P results in a non-implicant for F



# Expanding Terms

$$xyzw \Rightarrow f$$

$$\boxed{x}yzw \Rightarrow f$$

$$\boxed{xy}z \Rightarrow f$$

$$\boxed{xyz} \Rightarrow f$$

$$xyzw \Rightarrow f$$

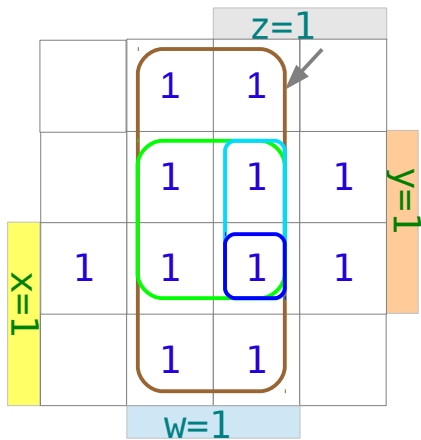
$$xy\boxed{z}w \Rightarrow f$$

$$xy\boxed{zw} \Rightarrow f$$

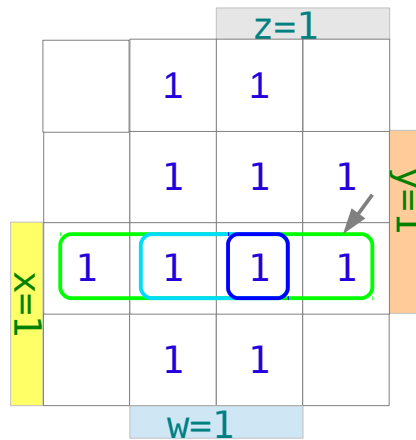
$$xyzw \Rightarrow f$$

$$xyz\boxed{w} \Rightarrow f$$

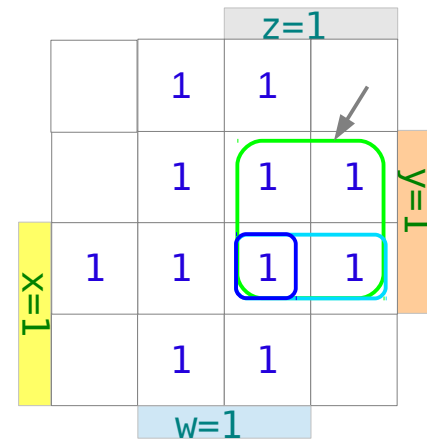
$$\boxed{xyz}w \Rightarrow f$$



Prime Implicant

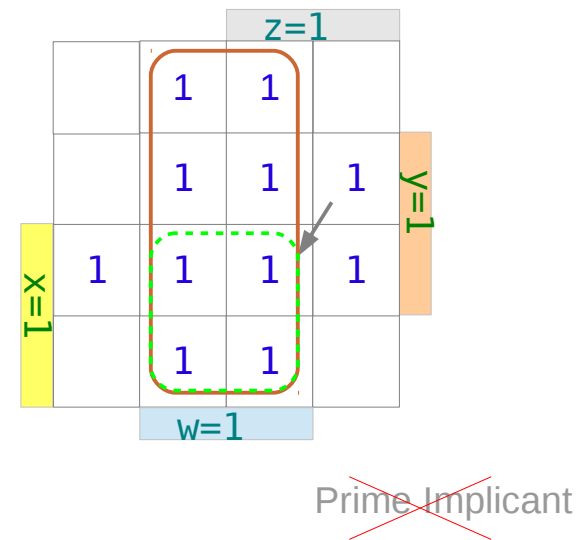
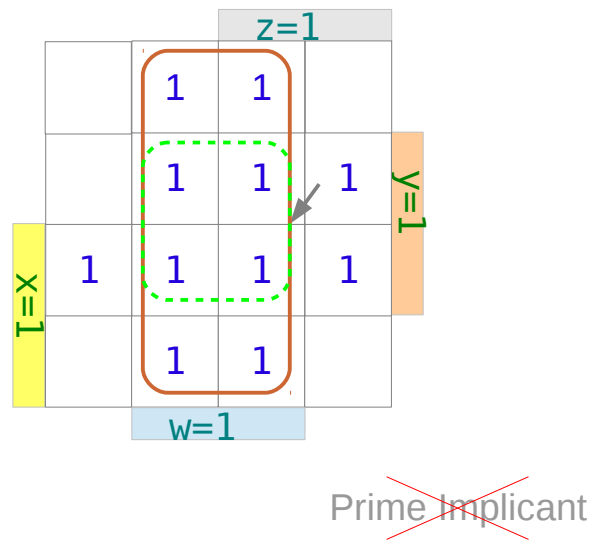
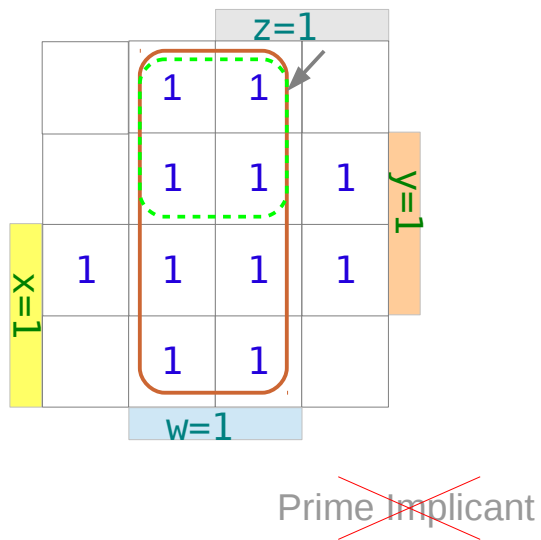
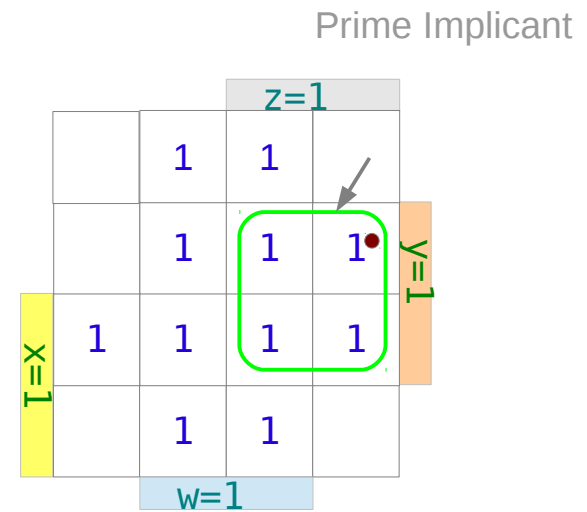
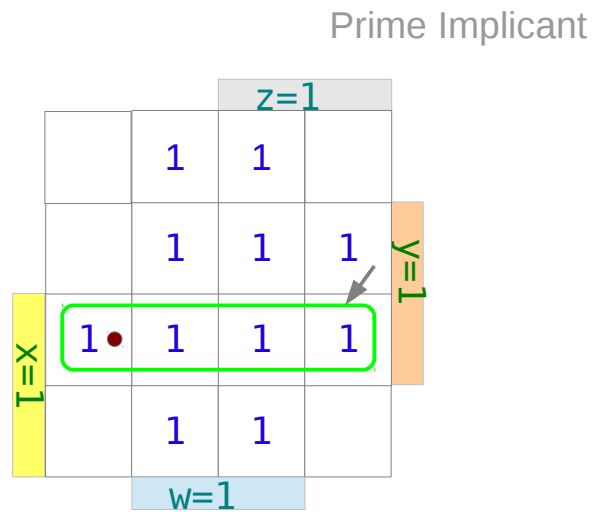
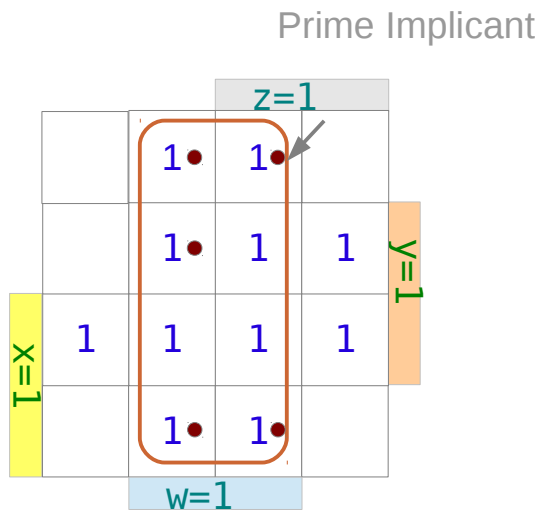


Prime Implicant



Prime Implicant

# Prime Implicant Example



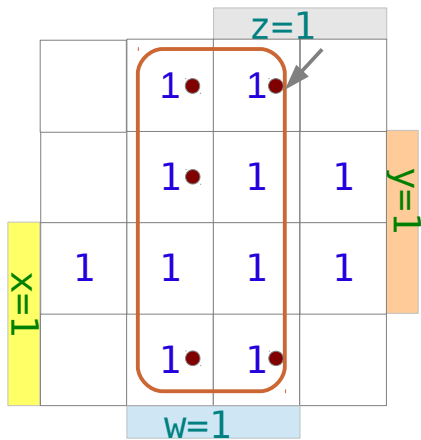
# Essential Prime Implicant

## Essential Prime Implicant:

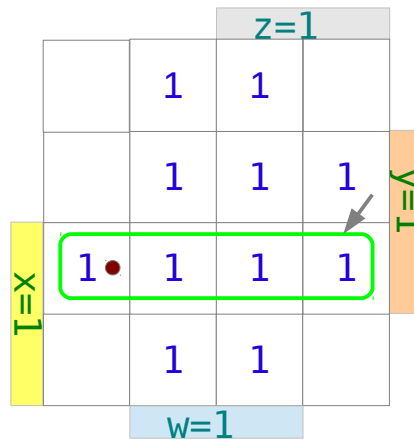
*prime implicants*

that cover an *output of the function*

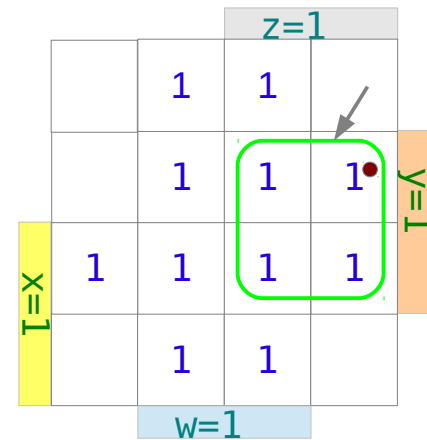
that no combination of other implicants is able to cover



Essential Prime Implicant



Essential Prime Implicant

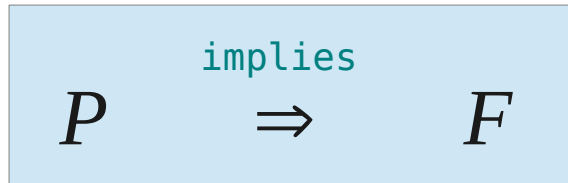


Essential Prime Implicant



# Implicant

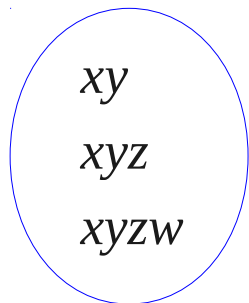
F takes the value 1, whenever P equals 1



assuming  $P$  is a product term in a sum of products

$$f(x, y, z, w) = xy + yz + w$$

General,  
Reduced



implicants

$$\begin{aligned} &\Rightarrow f(x, y, z, w) = xy + yz + w \\ &\Rightarrow f(x, y, z, w) = xy + yz + w \\ &\Rightarrow f(x, y, z, w) = xy + yz + w \end{aligned}$$

# 2's Complement

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# Decimal to Binary (1)

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# Decimal to Binary (2)

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# Laplace Equation

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Decimal

# Laplace Equation

---

# Laplace Equation

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## References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
- [3] M.L. Boas, "Mathematical Methods in the Physical Sciences"