Multi-dimensional Arrays (1A)

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Assumption

assume that

value(c) returns the hexadecimal number
that is obtained by printf(“%p”, c),
when the variable c contains
an address as its value

```
#include <stdio.h>
int main(void) {
  int c[3] ;
  printf ("c= %p \n", &c);
}
```

c= 0x7fffd923487c

type(c) can be determined
by the warning messsage of printf(“%d”, c),
when the variable c contains
an address as its value

```
#include <stdio.h>
int main(void) {
  int c[3] ;
  printf ("c= %d \n", &c);
}
```

```
t.c: In function ‘main’:
t.c:5:16: warning: format ‘%d’ expects argument of type ‘int’,
but argument 2 has type ‘int (*)[3]’ [-Wformat=]
   printf ("c= %d \n", &c);
```

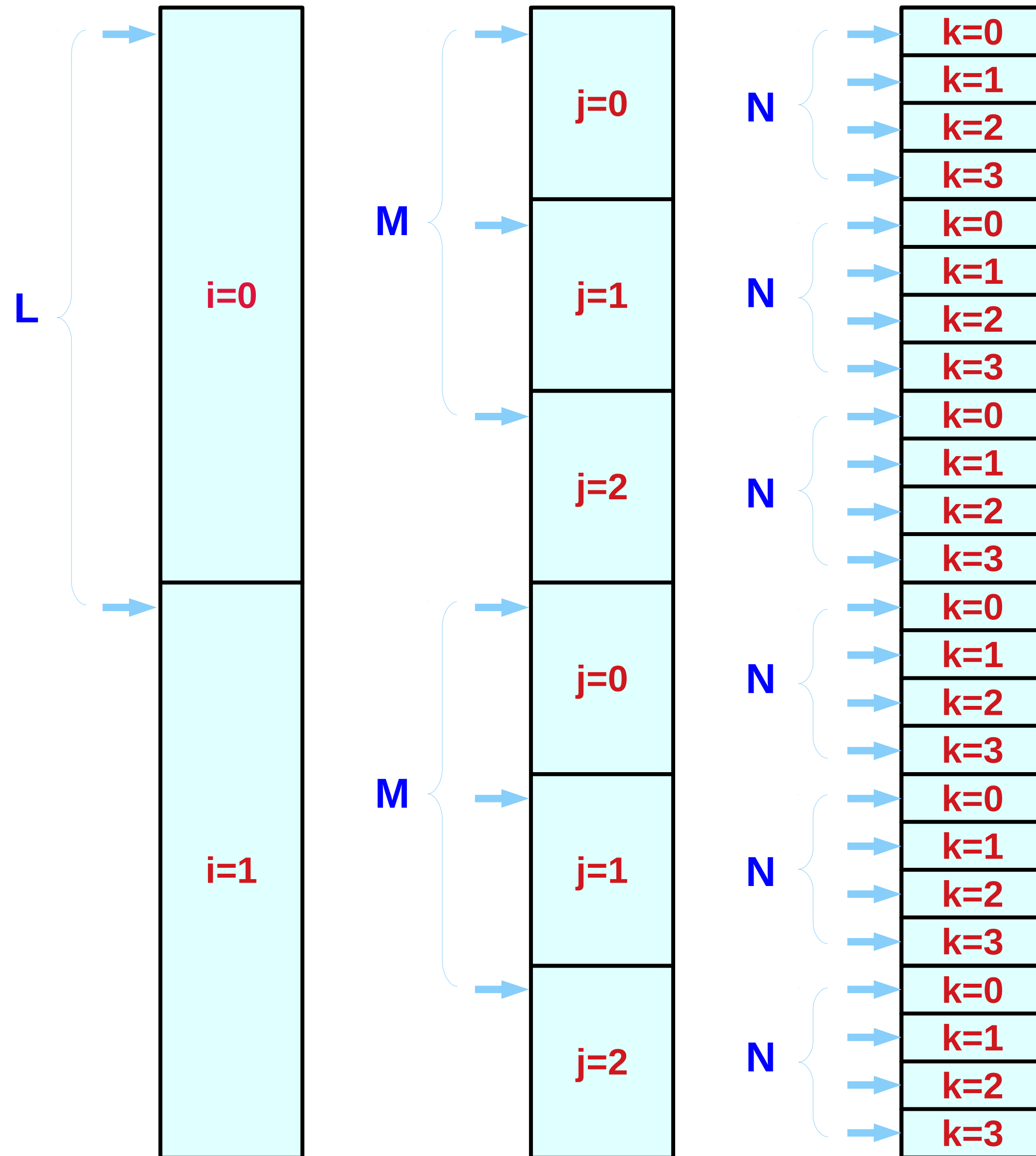
3-d Array Index

c [**i**][**j**][**k**]

interpret as recursive indirections
interpret as hierarchical sub-arrays

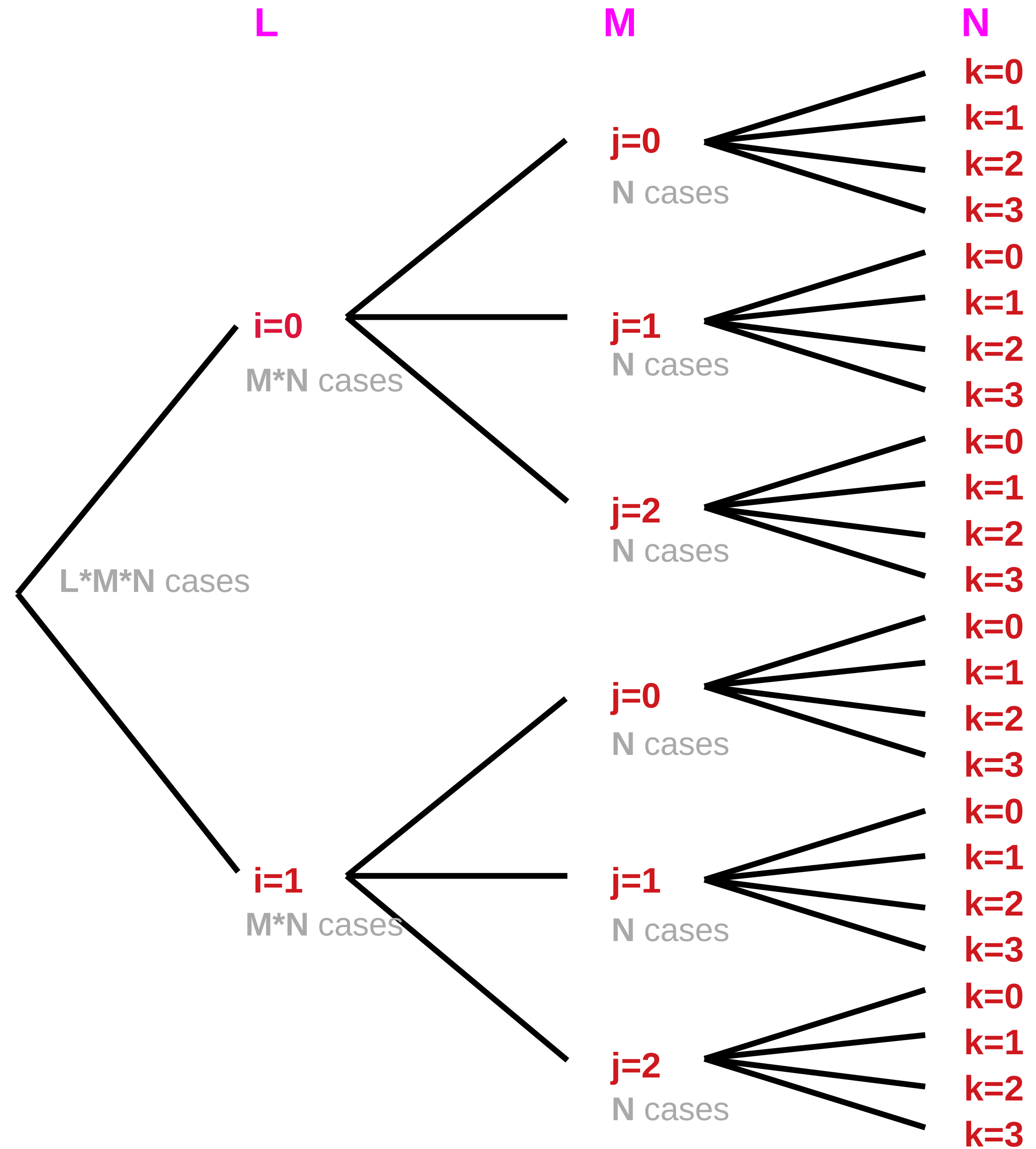
L, M, N – the number of index values

```
int     c [L][M][N];

        c [i][j][k]
```



Index value tree – all possible combinations

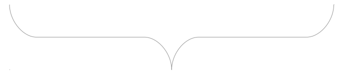
```
int     c [L][M][N];

        c [i][j][k]
```



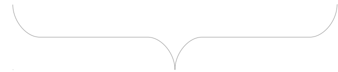
The number of elements of subarrays

```
int     c [L][M][N];

        c [i][j][k]
```

c .... L*M*N elements
c[i] .... M*N elements
c[i][j] .... N elements



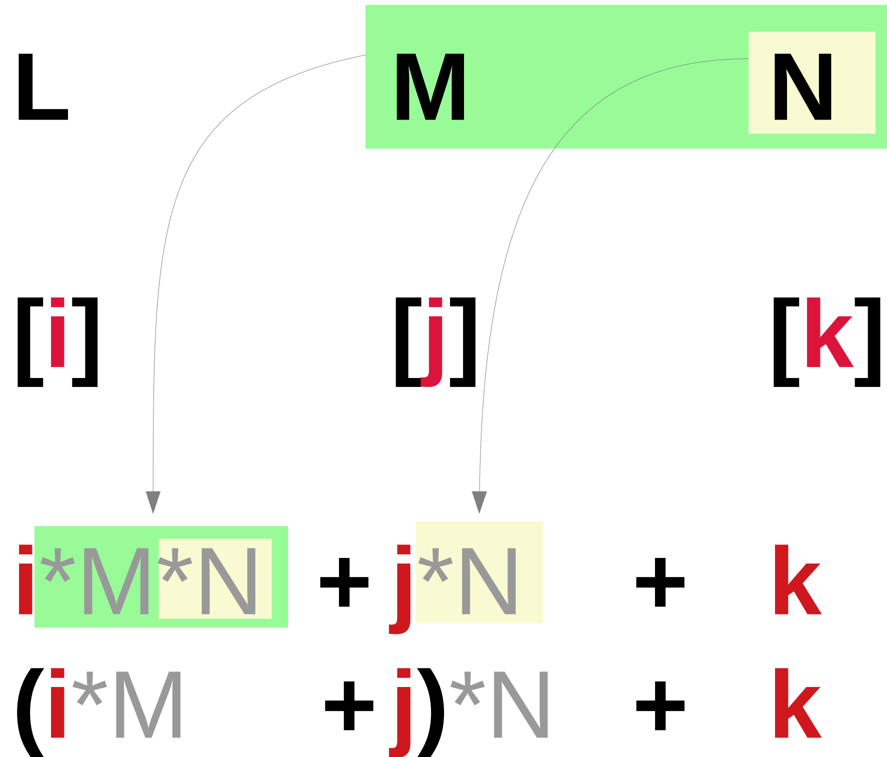
array names



covering elements

From a **3-d** index to a **1-d** index

```
int     c [L][M][N];

        c [i][j][k]
```



i*M*N, j*N, k – index offset values

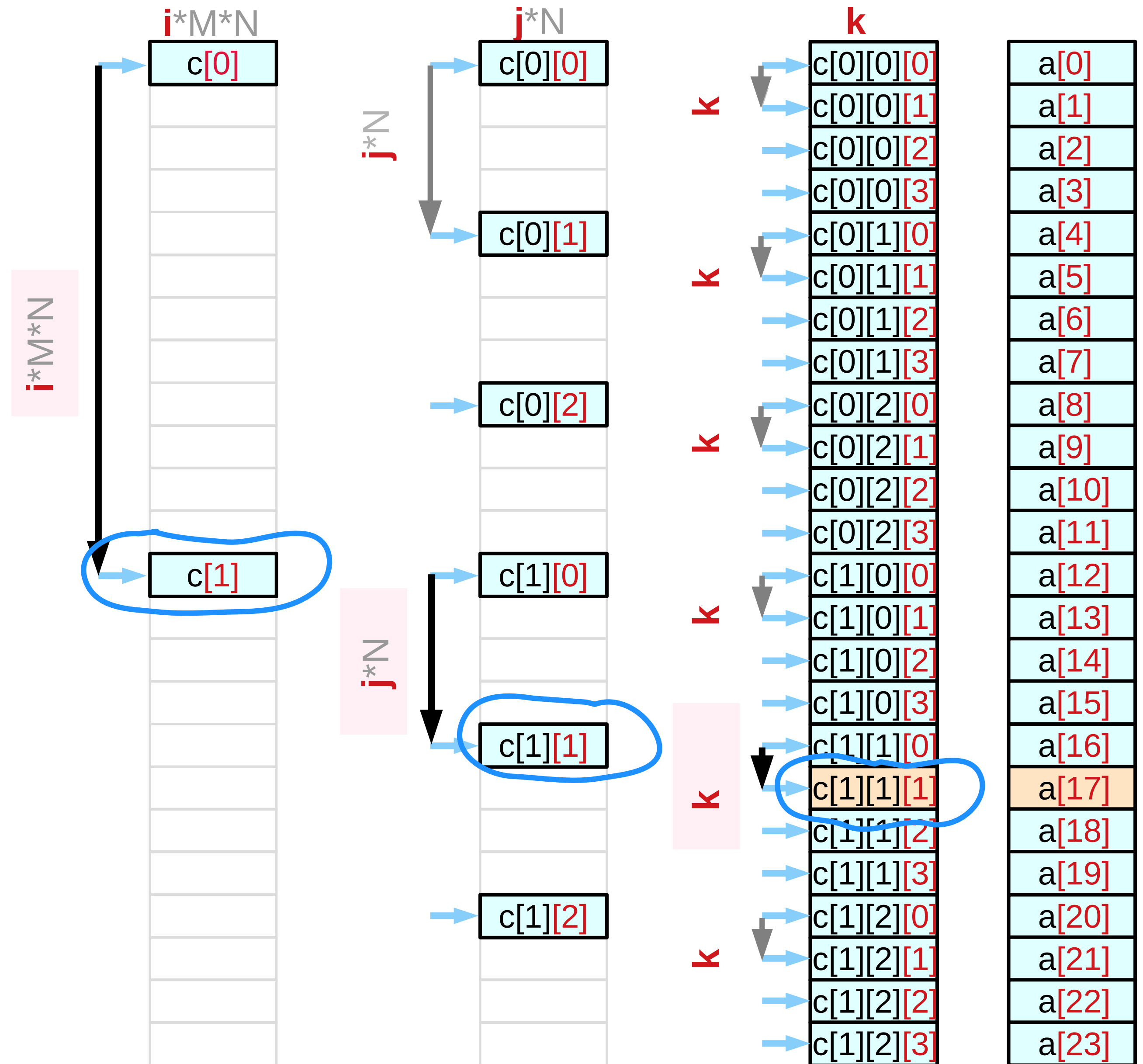
int c [L][M][N];

c [i][j][k]

c [1][1][1]

i=1	j=1	k=1

a [(1*3 + 1)*4 + 1]

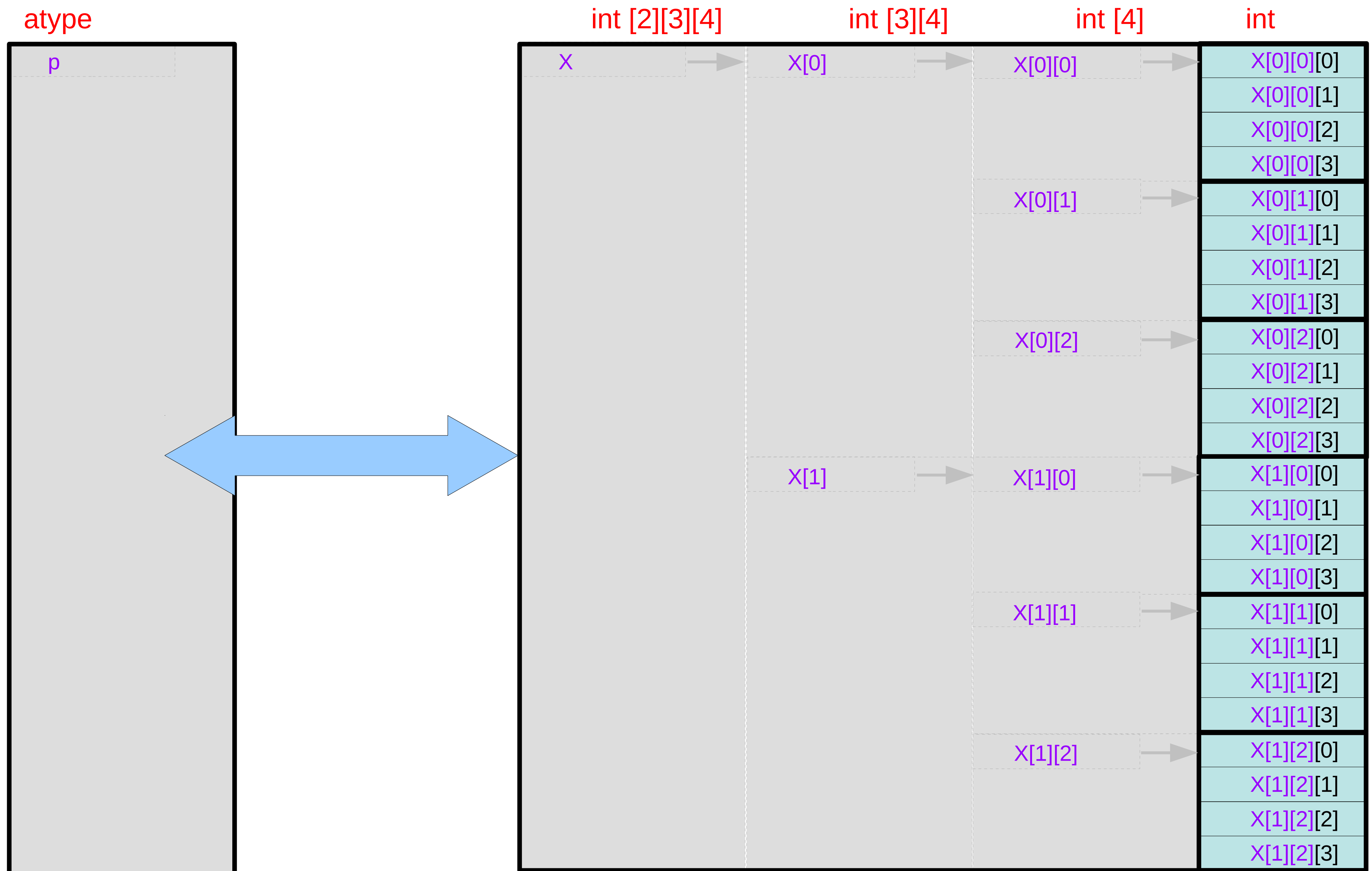


Abstract data type examples in int p[2][3][4];

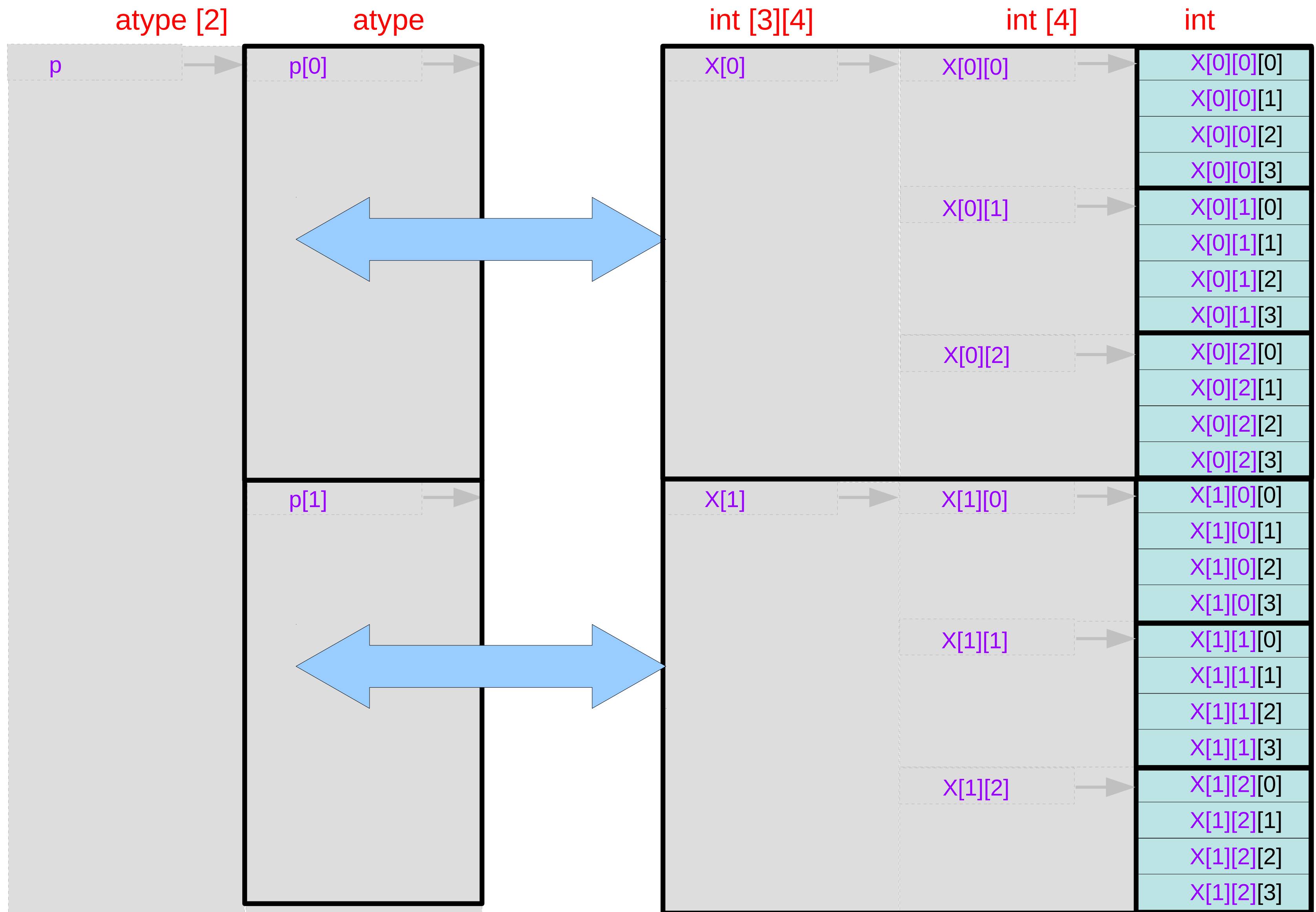
```
int p[2][3][4];
```

```
(1)  atype p;             where atype = int [2][3][4]
(2)  atype p[2];          where atype = int [3][4]
(3)  atype p[2][3];       where atype = int [4]
(4)  atype p[2][3][4];    where atype = int
```

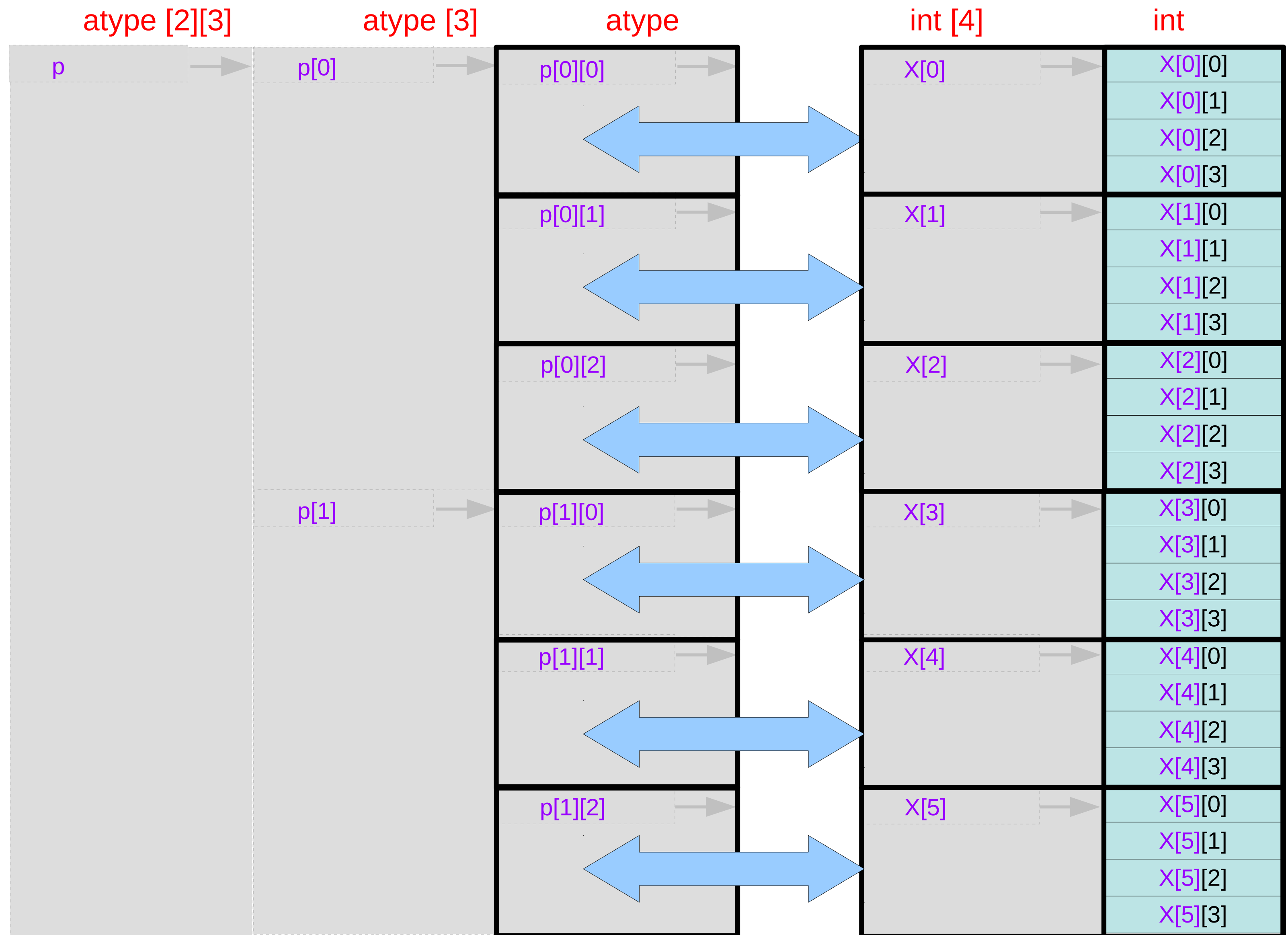
(1) atype p; where atype = int [2][3][4]



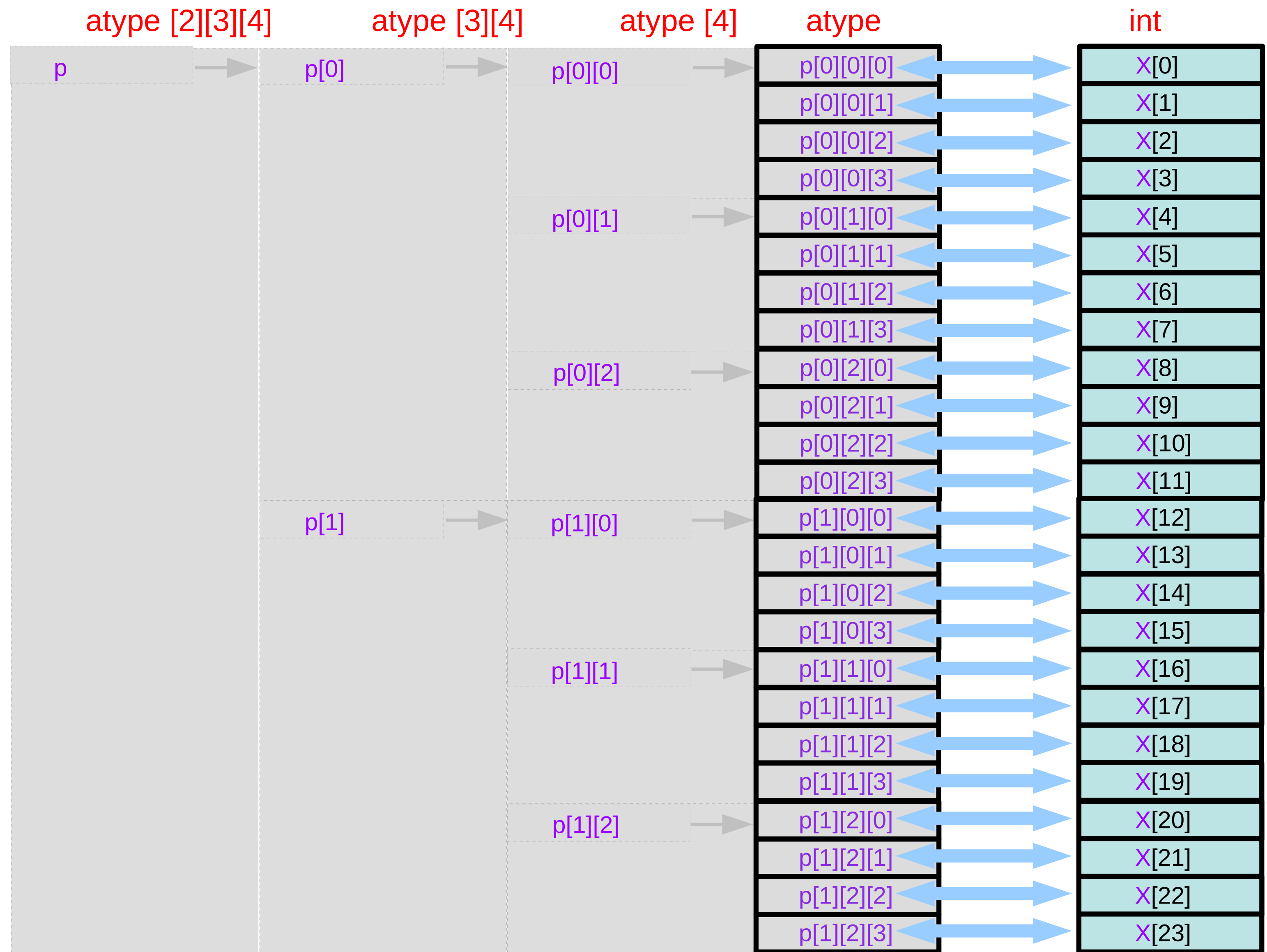
(2) atype p[2]; where atype = int [3][4]



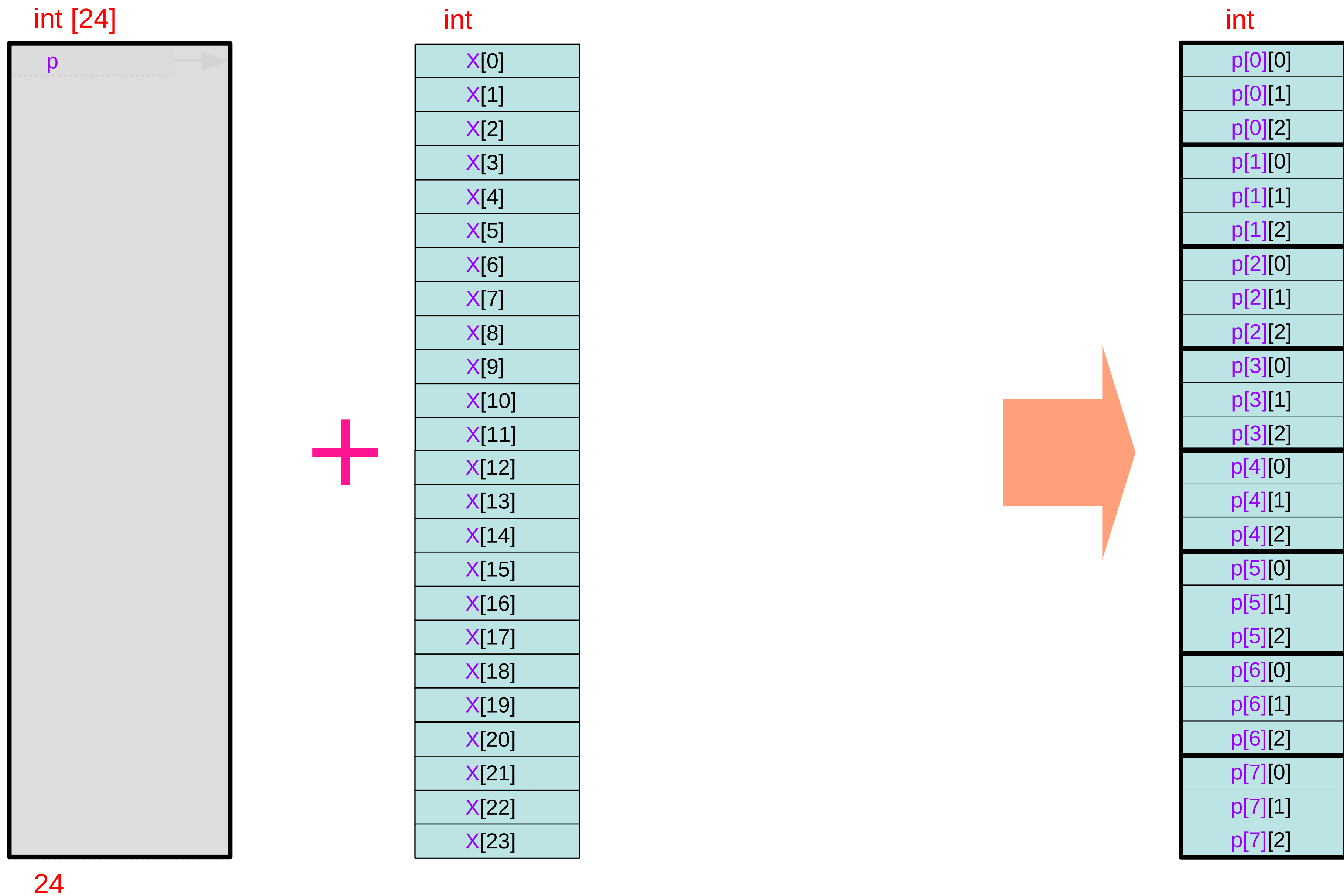
(3) atype p[2][3]; where atype = int [4]



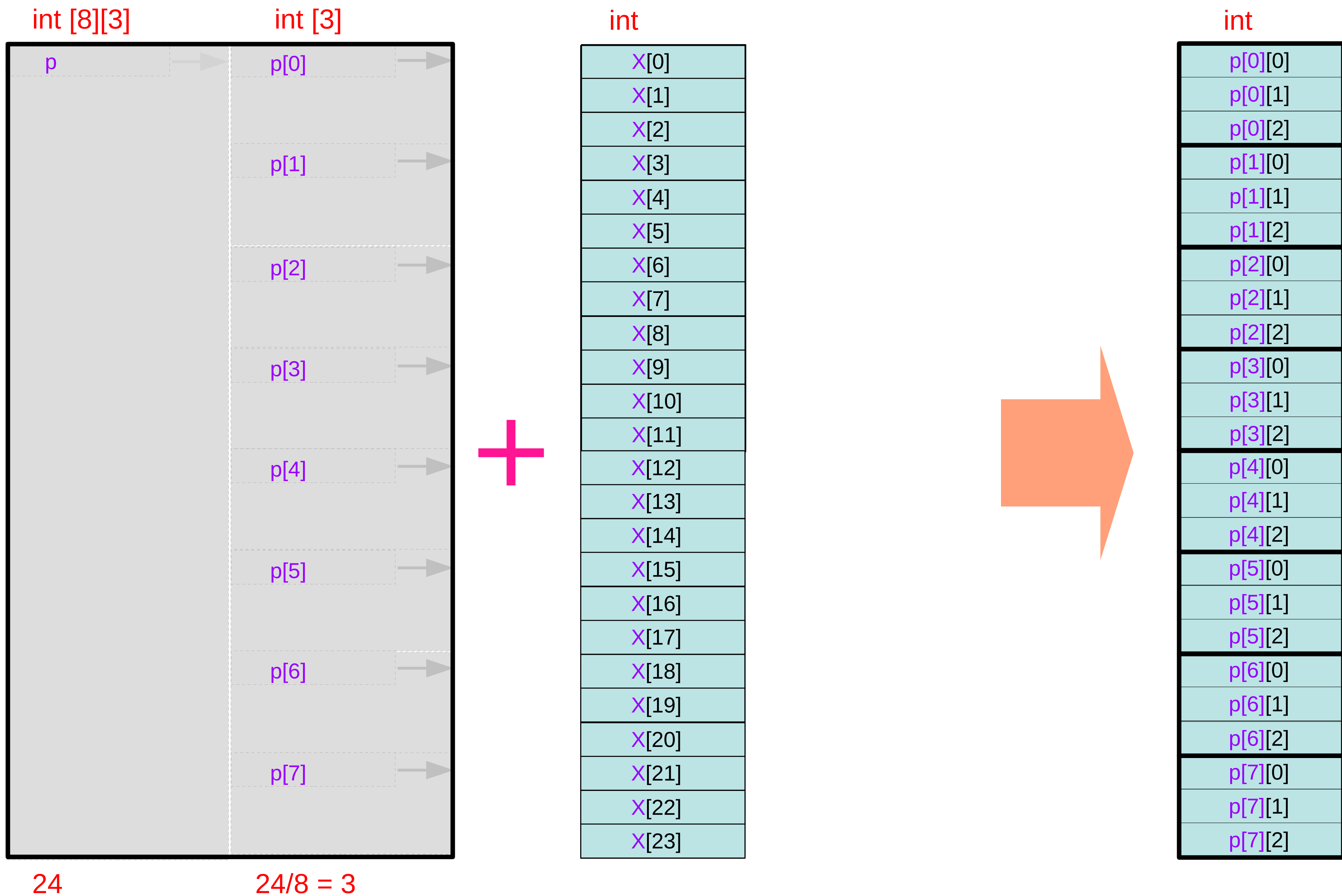
(4) atype p[2][3][4]; where atype = int



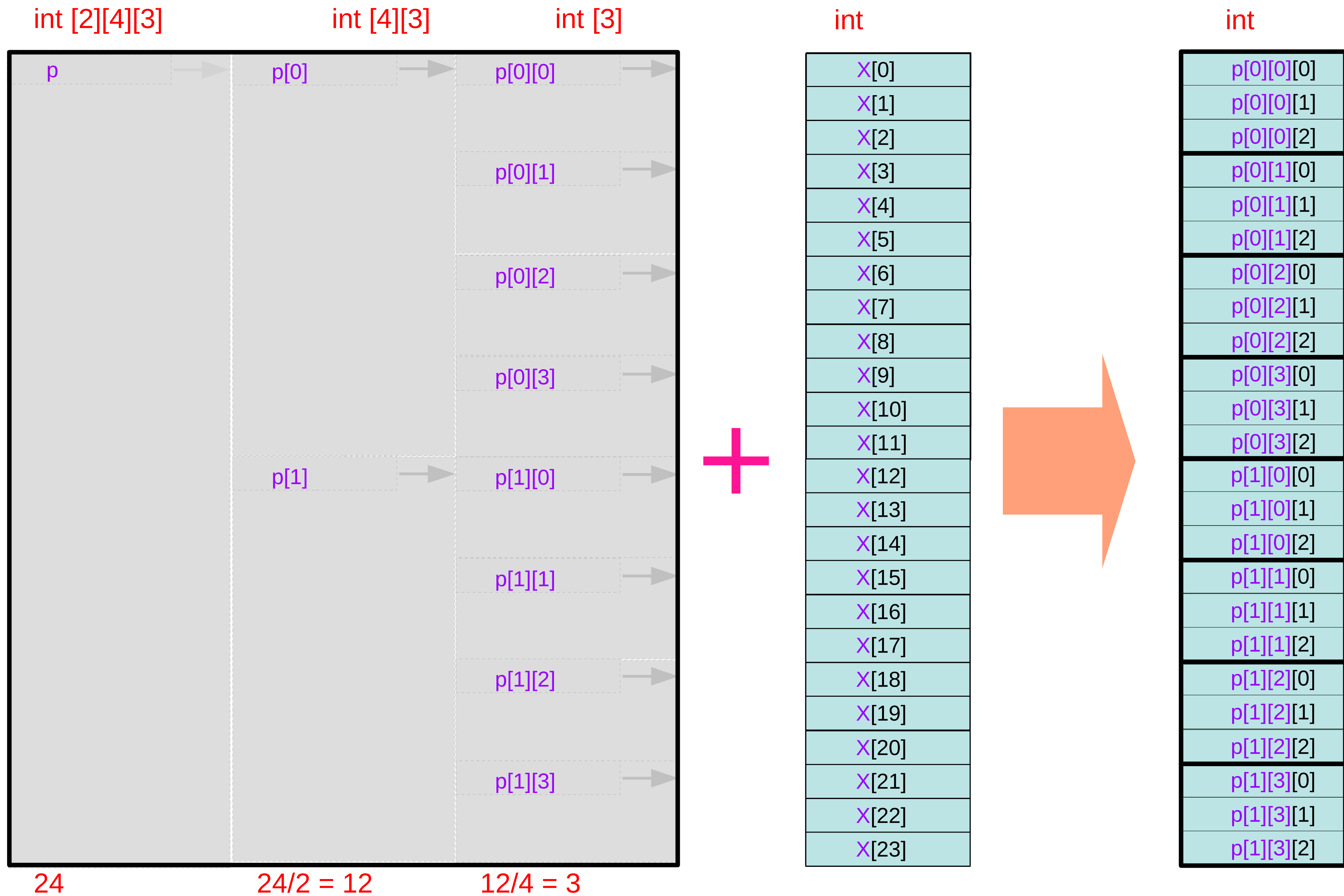
Indexing in int p[24]



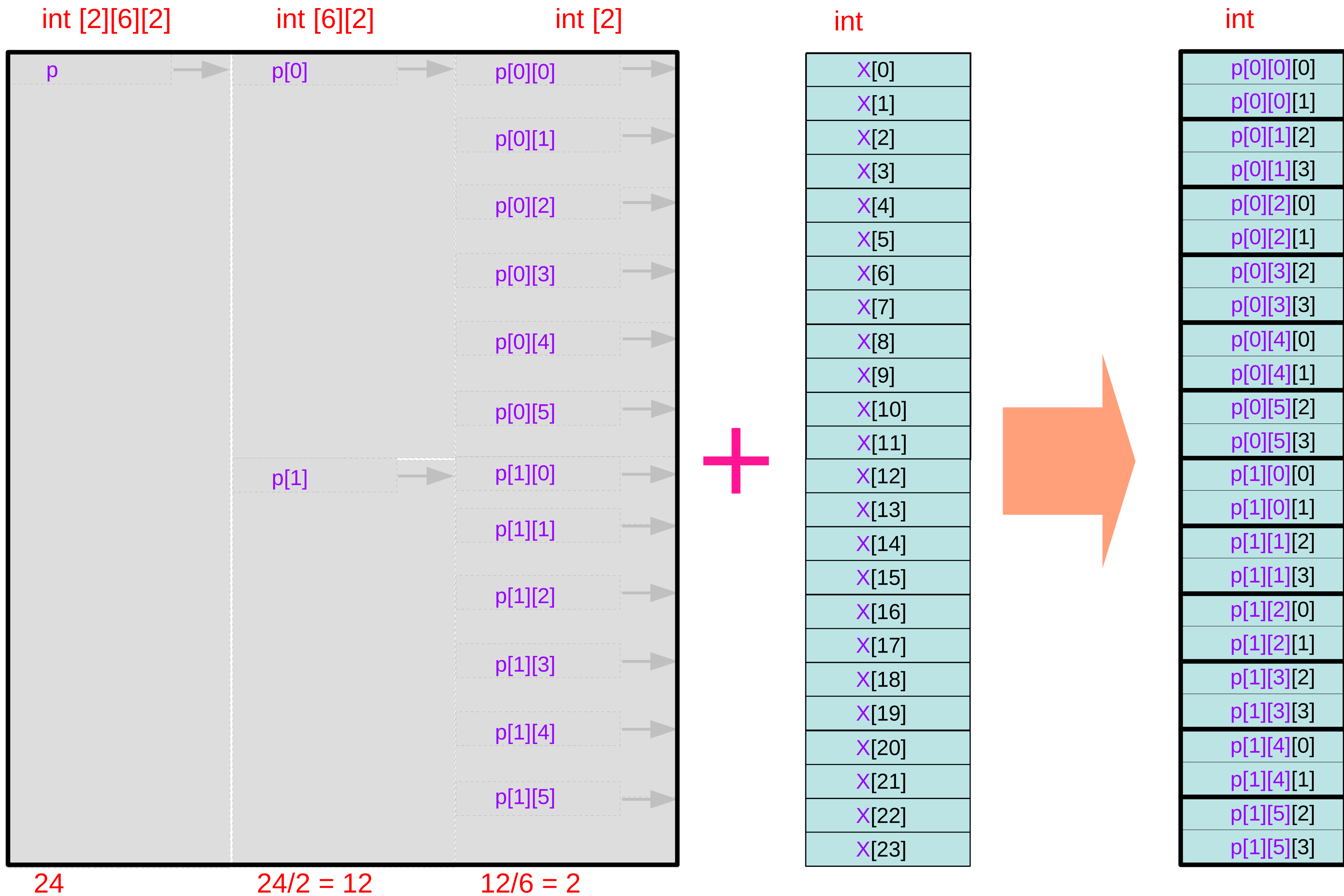
Indexing in int p[8][3]



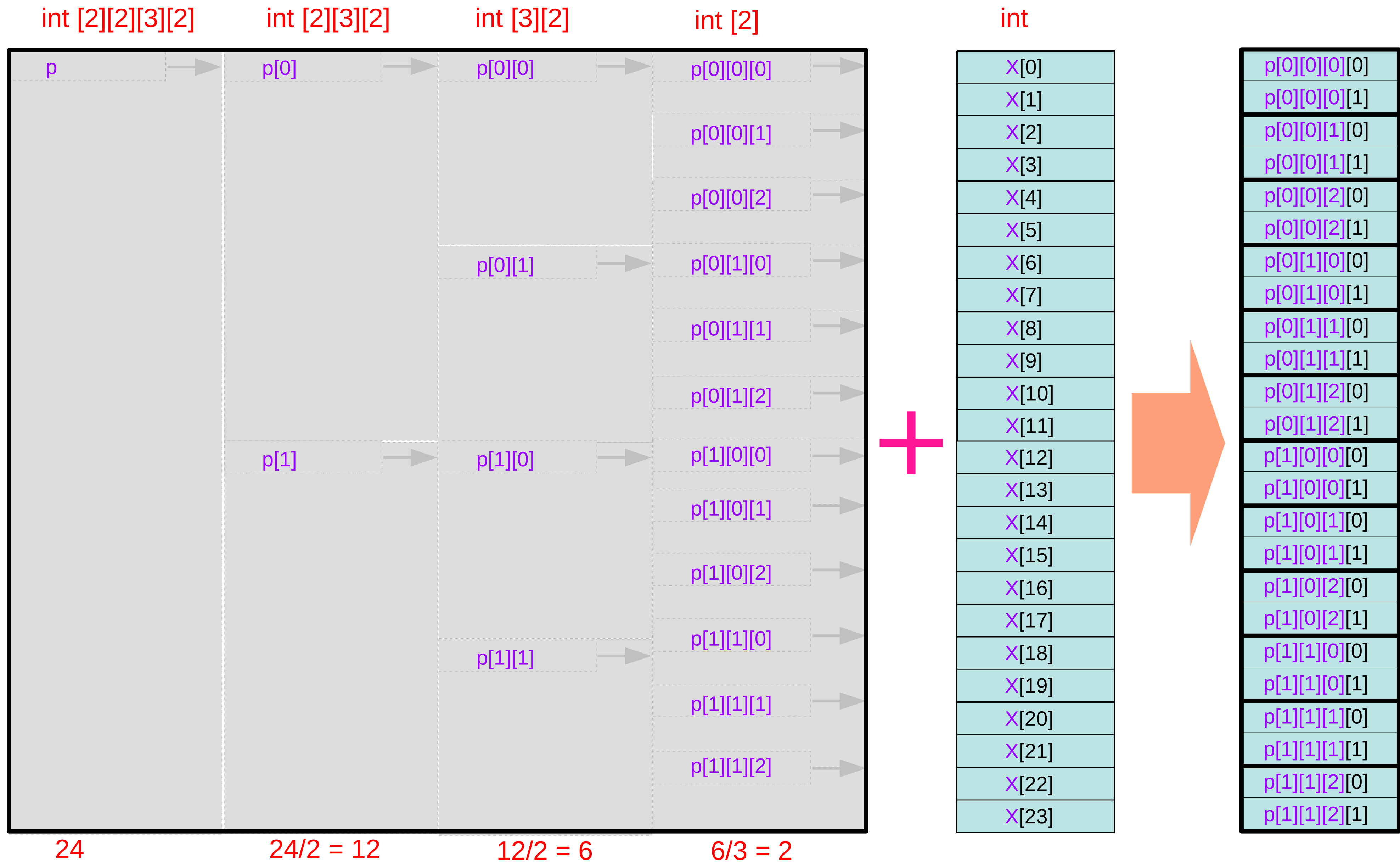
Indexing in int p[2][4][3]



Indexing in int p[2][6][2]



Indexing in int p[2][2][3][2]



Indexing in muti-dimension arrays

int [24]	int p[8][3]	int p[2][4][3]	int p[2][6][2]	int p[2][2][3][2]
X[0]	p[0][0]	p[0][0][0]	p[0][0][0]	p[0][0][0][0]
X[1]	p[0][1]	p[0][0][1]	p[0][0][1]	p[0][0][0][1]
X[2]	p[0][2]	p[0][0][2]	p[0][1][2]	p[0][0][1][0]
X[3]	p[1][0]	p[0][1][0]	p[0][1][3]	p[0][0][1][1]
X[4]	p[1][1]	p[0][1][1]	p[0][2][0]	p[0][0][2][0]
X[5]	p[1][2]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
X[9]	p[3][0]	p[0][3][0]	p[0][4][1]	[illegible]
X[10]	p[3][1]	p[0][3][1]	p[0][5][2]	p[0][1][2][0]
X[11]	p[3][2]	p[0][3][2]	p[0][5][3]	p[0][1][2][1]
X[12]	p[4][0]	p[1][0][0]	p[1][0][0]	p[1][0][0][0]
X[13]	p[4][1]	p[1][0][1]	p[1][0][1]	p[1][0][0][1]
X[14]	p[4][2]	p[1][0][2]	p[1][1][2]	p[1][0][1][0]
X[15]	p[5][0]	p[1][1][0]	p[1][1][3]	p[1][0][1][1]
X[16]	p[5][1]	p[1][1][1]	p[1][2][0]	p[1][0][2][0]
X[17]	p[5][2]	p[1][1][2]	p[1][2][1]	p[1][0][2][1]
X[18]	p[6][0]	p[1][2][0]	p[1][3][2]	p[1][1][0][0]
X[19]	p[6][1]	p[1][2][1]	p[1][3][3]	p[1][1][0][1]
X[20]	p[6][2]	p[1][2][2]	p[1][4][0]	p[1][1][1][0]
X[21]	p[7][0]	p[1][3][0]	p[1][4][1]	p[1][1][1][1]
X[22]	p[7][1]	p[1][3][1]	p[1][5][2]	p[1][1][2][0]
X[23]	p[7][2]	p[1][3][2]	p[1][5][3]	p[1][1][2][1]

1-d array | 2-d array | 3-d array | 3-d array | 4-d array

Subarray sizes of 24 element multi-dimensional arrays (1)

int p[2][2][3][2]

p sizeof(p)	int [2][2][3][2] = 2*2*3*2 =24 integers
p[0] sizeof(p[0])	int [2][3][2] = 2*3*2 =12 integers
p[0][0] sizeof(p[0][0])	int [3][2] = 3*2 =6 integers
p[0][0][0] sizeof(p[0][0][0])	int [2] = 2 integers
p[0][0][0][0] sizeof(p[0][0][0][0])	int = 1 integers

int p[2][6][2]

p sizeof(p)	int [2][6][2] = 2*6*2 =24 integers
p[0] sizeof(p[0])	int [6][2] = 6*2 =12 integers
p[0][0] sizeof(p[0][0])	int [2] = 2 integers
p[0][0][0] sizeof(p[0][0][0])	int = 1 integers

int p[2][4][3]

p sizeof(p)	int [2][4][3] = 2*4*3 =24 integers
p[0] sizeof(p[0])	int [4][3] = 4*3 =12 integers
p[0][0] sizeof(p[0][0])	int [3] = 3 integers
p[0][0][0] sizeof(p[0][0][0])	int = 1 integers

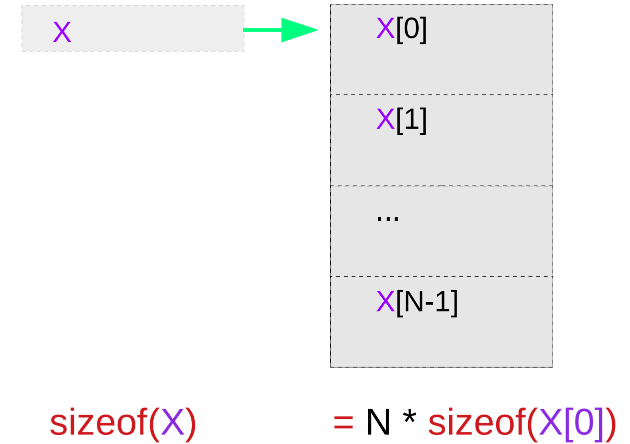
Subarray sizes of 24 element multi-dimensional arrays (1)

int p[8][3]

p sizeof(p)	int [8][3] = 8*3 =24 integers
p[0] sizeof(p[0])	int [3] = 3 integers
p[0][0] sizeof(p[0][0])	int = 1 integers

int p[24]

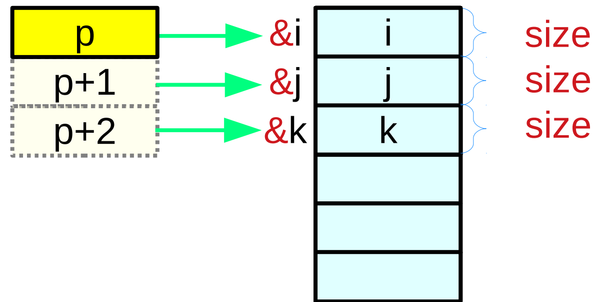
p sizeof(p)	int [24] = 24 integers
p[0] sizeof(p[0])	int = 1 integers



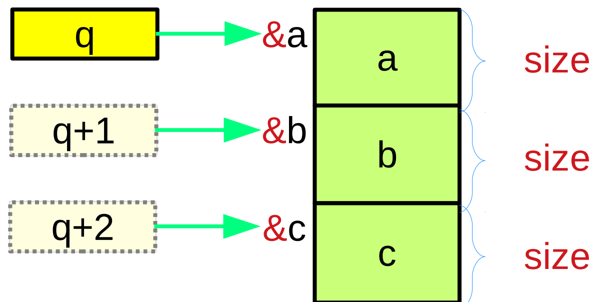
Virtual array pointers in a multi-dimensional array

Pointers to various data types

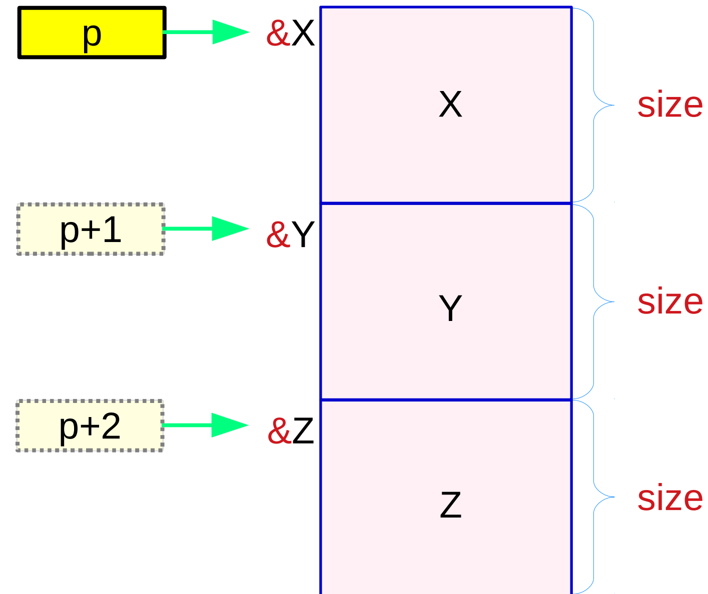
int *p; int i, j, k;



double *q; double a, b, c;



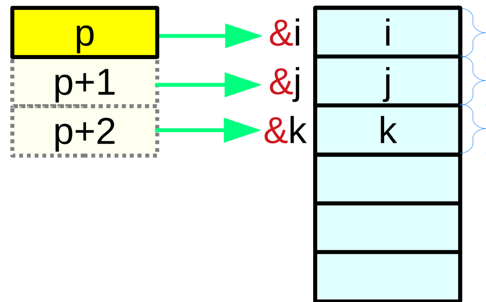
T *p; T X, Y, Z;



pointer abstract data

Pointers to primitive data

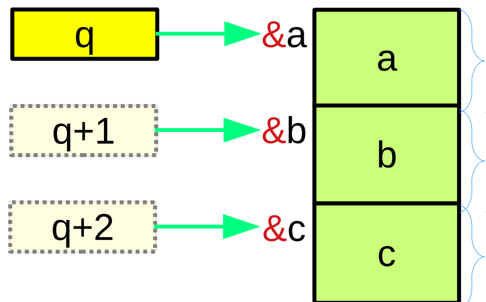
int *p; **int i, j, k;** **sizeof(int) = 4 bytes**



size	=	sizeof(i)	=	sizeof(*p)	≠	sizeof(p)
size	=	sizeof(j)	=	sizeof(*(p+1))	≠	sizeof(p+1)
size	=	sizeof(k)	=	sizeof(*(p+2))	≠	sizeof(p+2)

pointer size
4 or 8 bytes

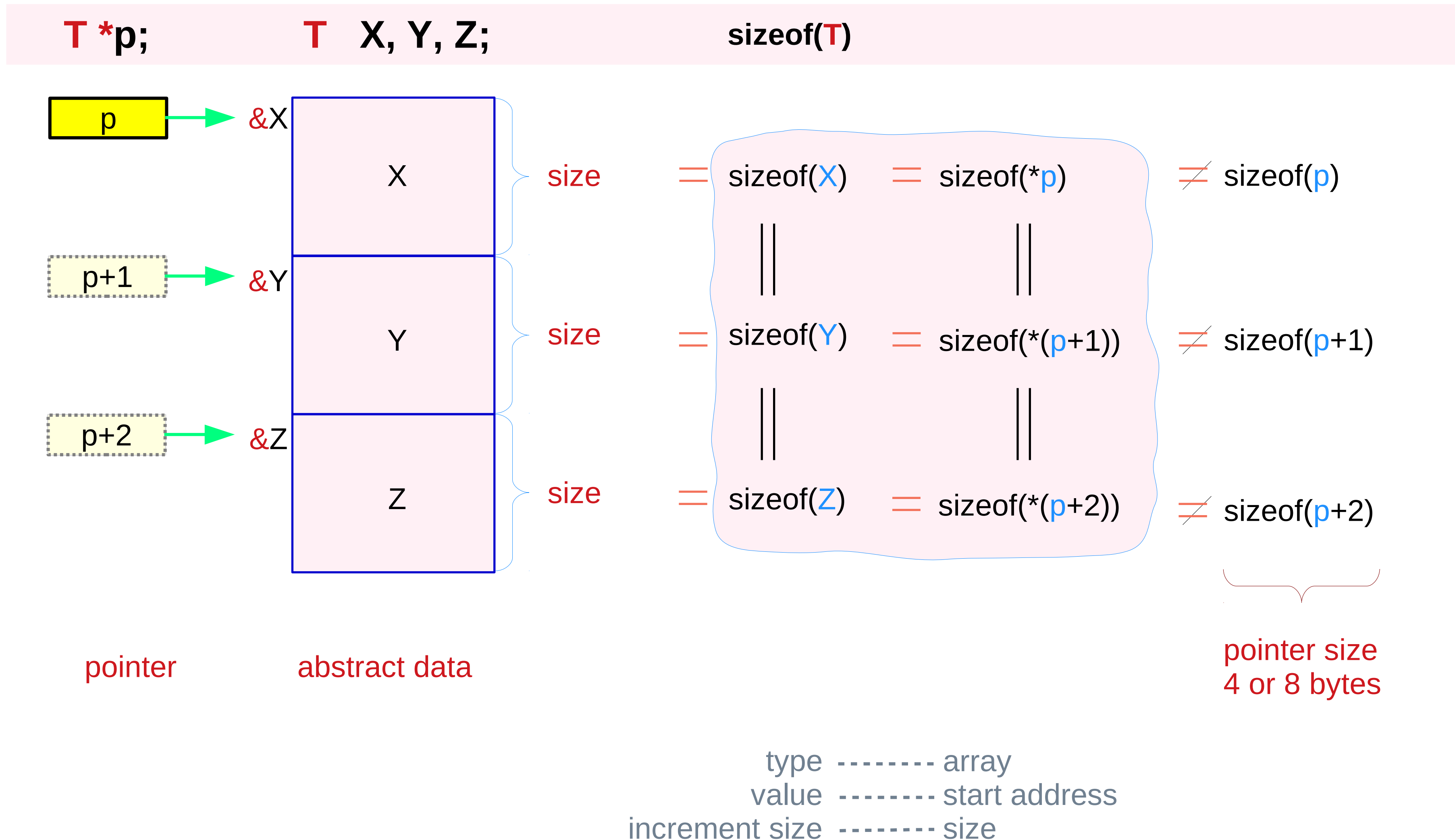
double *q; **double a, b, c;** **sizeof(double) = 8 bytes**



size	=	sizeof(a)	=	sizeof(*q)	≠	sizeof(q)
size	=	sizeof(b)	=	sizeof(*(q+1))	≠	sizeof(q+1)
size	=	sizeof(c)	=	sizeof(*(q+2))	≠	sizeof(q+2)

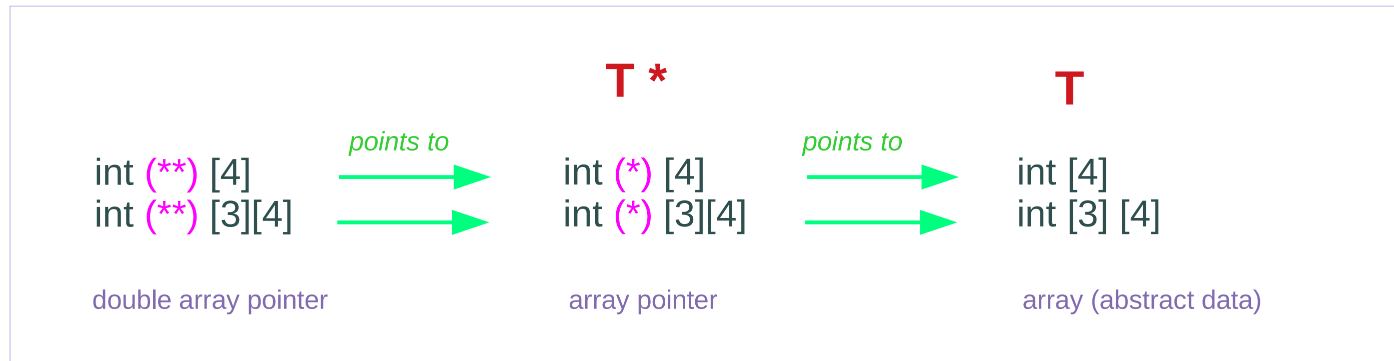
pointer size
4 or 8 bytes

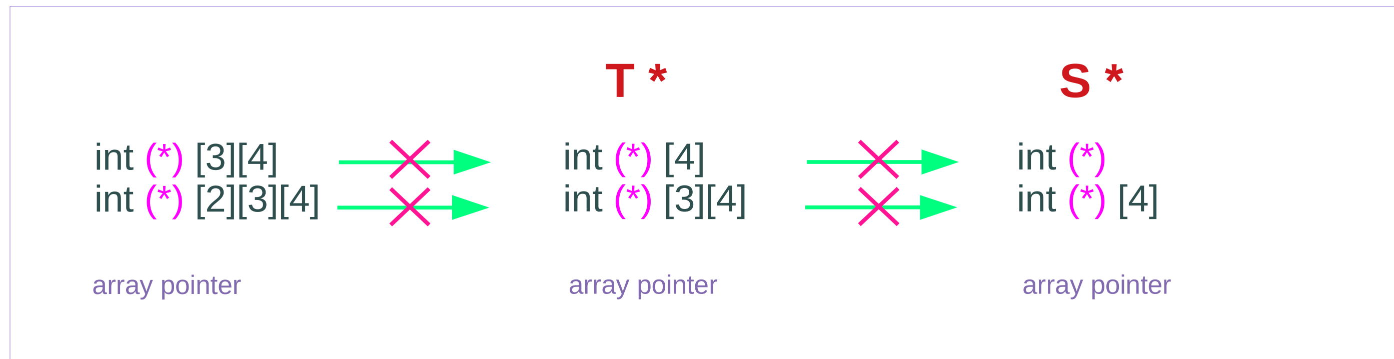
Pointers to abstract data



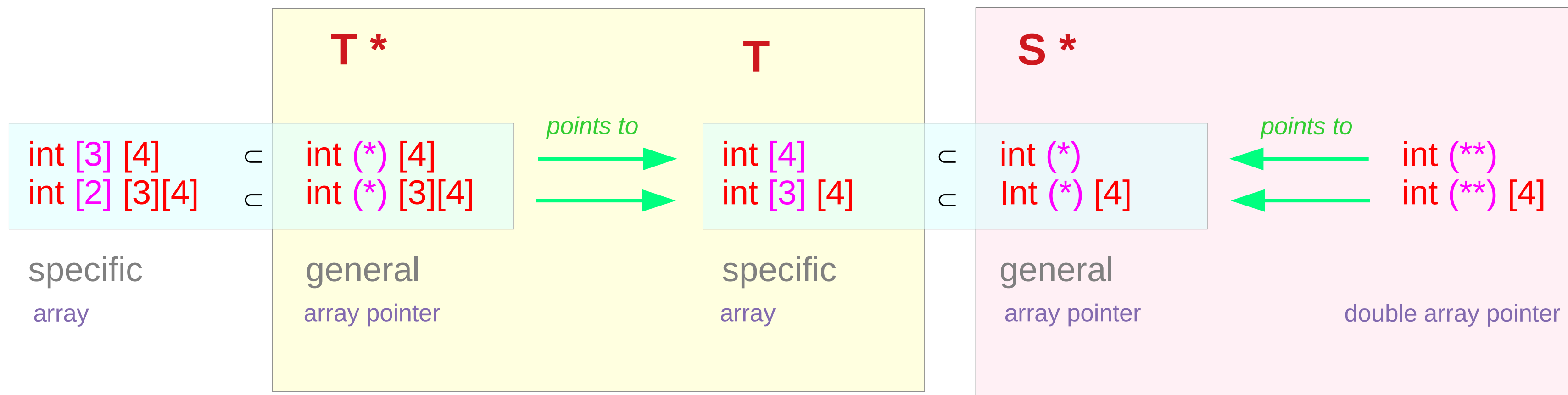
type -------- array
value -------- start address
increment size -------- size

Array pointer types v.s. array types





General array pointer types v.s. specific array types



Array pointers have augmented dimensions

```
typedef int (*T1) [4];          typedef int T2[4];
typedef int (*T1) [3][4];       typedef int T2[3][4];

      int (*) [4]                     int [4]
      int (*) [3][4]                  int [3] [4]
      general                         specific
```

```
T1  a;
T2  b;
```

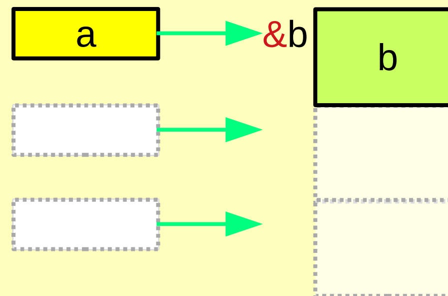
T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

```
 a  = &b;
*a  =  b;
```

a references **b**

b is the dereference of **a**

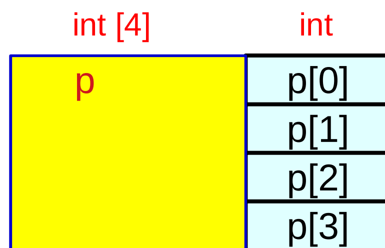
```
  (a+1) = ?
* (a+1) = ?

  (a+2) = ?
* (a+2) = ?
```



Dual types in an array of integers

int p[3];

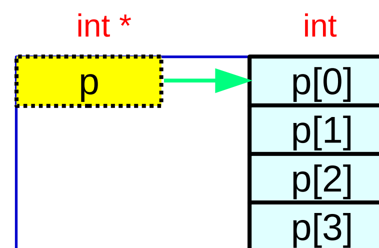
p is an abstract data (array)



p is the name of an array
p has the size of the whole array

p has an array type (abstract data)

p can also be viewed as a pointer

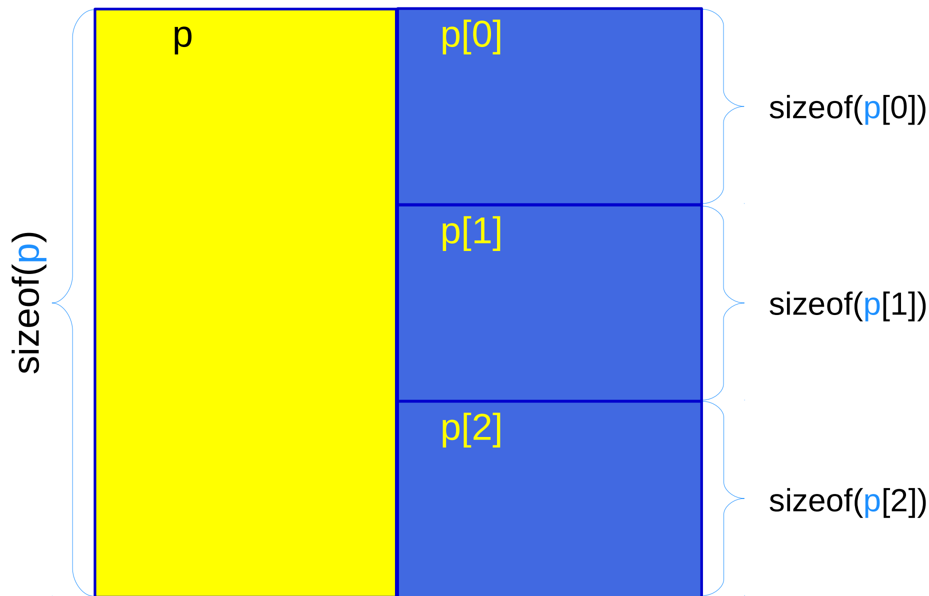


p also has pointer characteristics
p has the value of the starting address

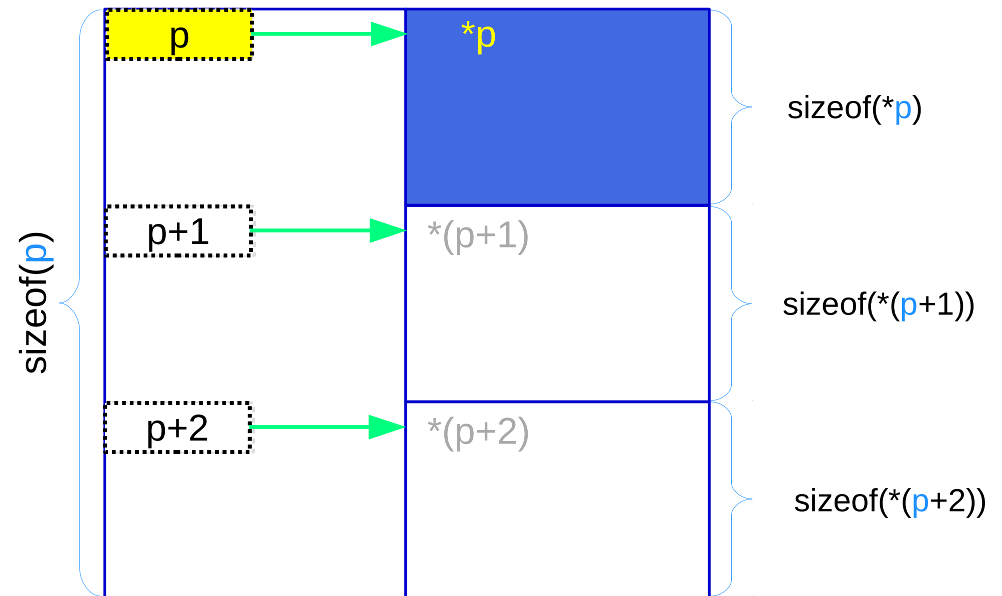
p is a virtual pointer

Dual types in an array of abstract data

Abstract data array p



Virtual pointer p

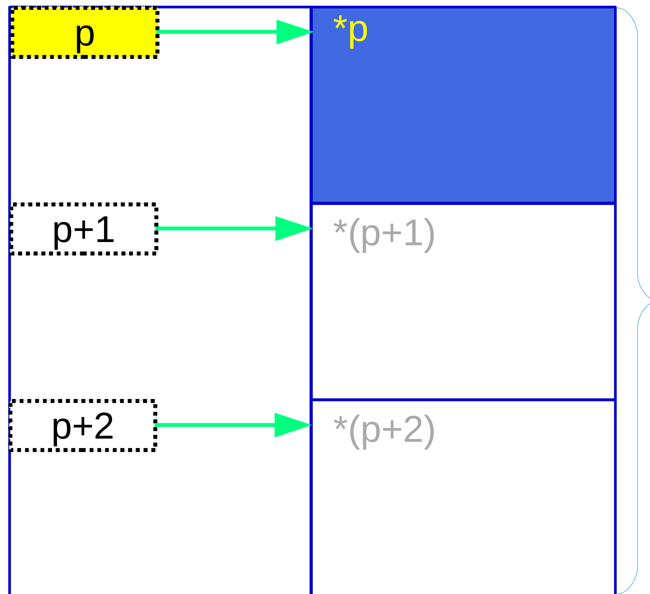


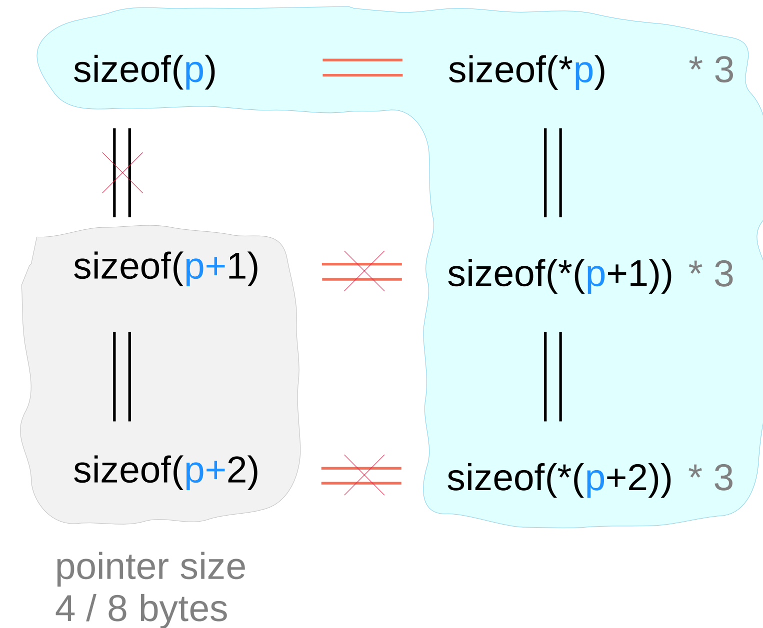
p has an array type (abstract data element)
p is the name of an array
p has the size of the whole array

p also has a pointer type
p has the value of the starting address

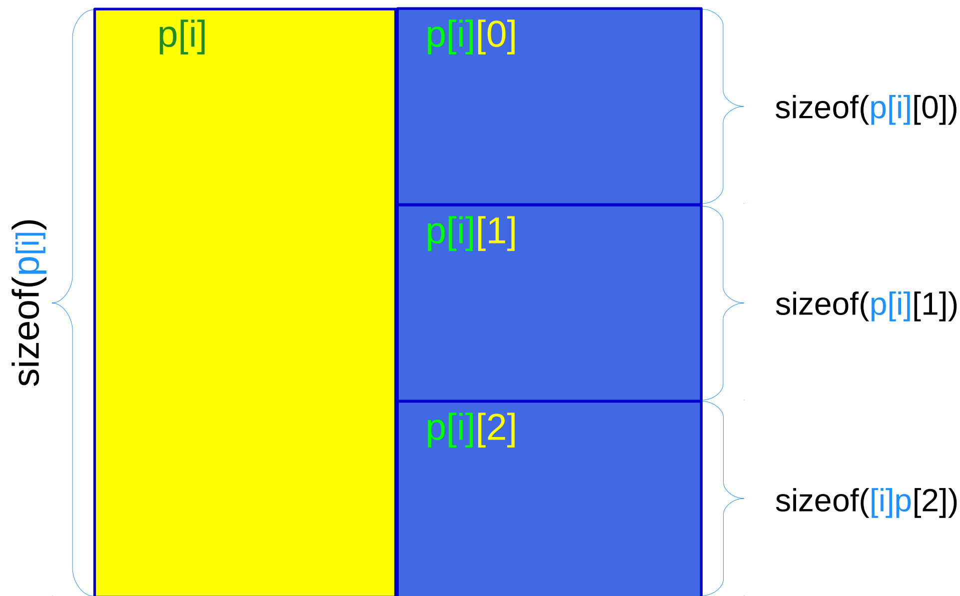
p is a virtual array pointer

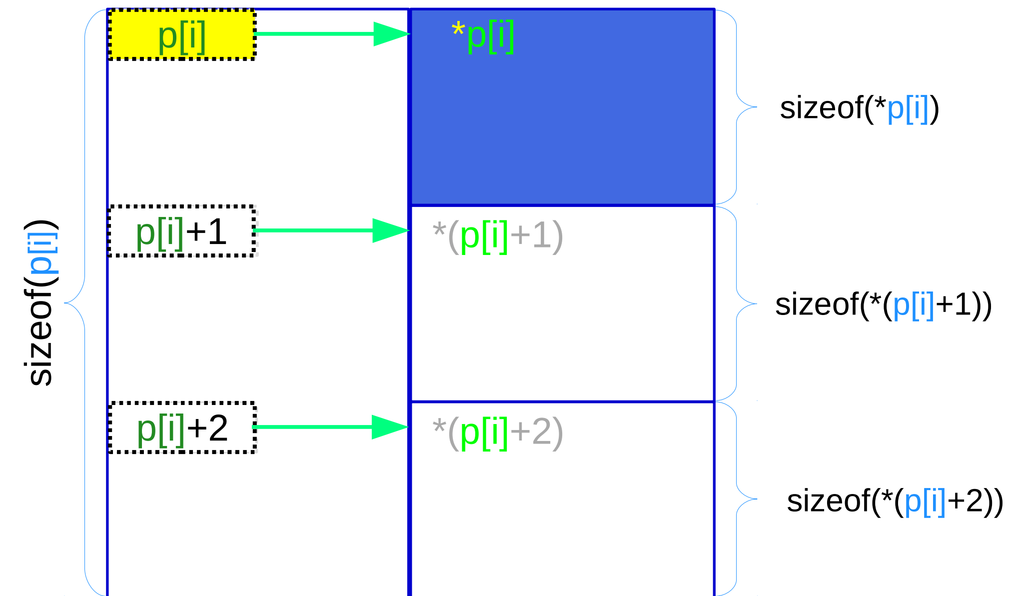
Virtual pointer to abstract data





Dual types in a multi-dimensional array





p[i] has an array type (abstract data)
p[i] is the name of an array
p[i] has the size of the whole array

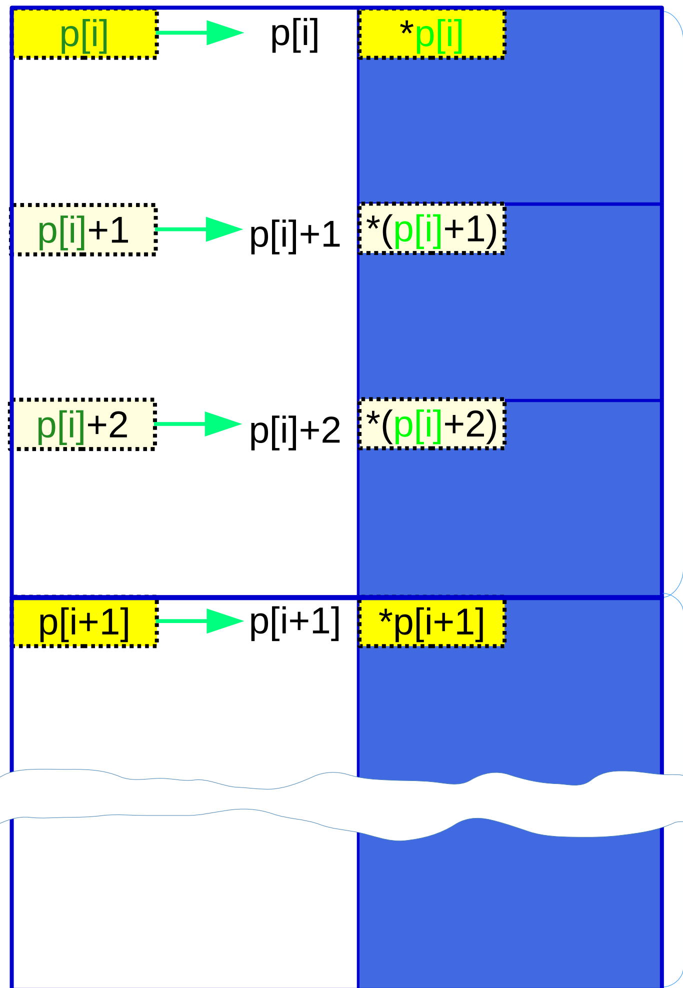
p[i] also has an array pointer type
p[i] has the value of the starting address

p[i] is a <u>virtual</u> array pointer

Virtual pointers to sub-arrays

p[i] :: T *   ***p[i], *p[i+1] :: T**

array pointer type   array type



sizeof(p[i])

= sizeof(*p[i]) * N
sizeof(p[i][0]) * N

size = sizeof(*p[i]) =
sizeof(p[i][0])

||

size = sizeof(*(p[i]+1)) =
sizeof(p[i][1])

||

size = sizeof(*(p[i]+2)) =
sizeof(p[i][2])

sizeof(p[i+1])

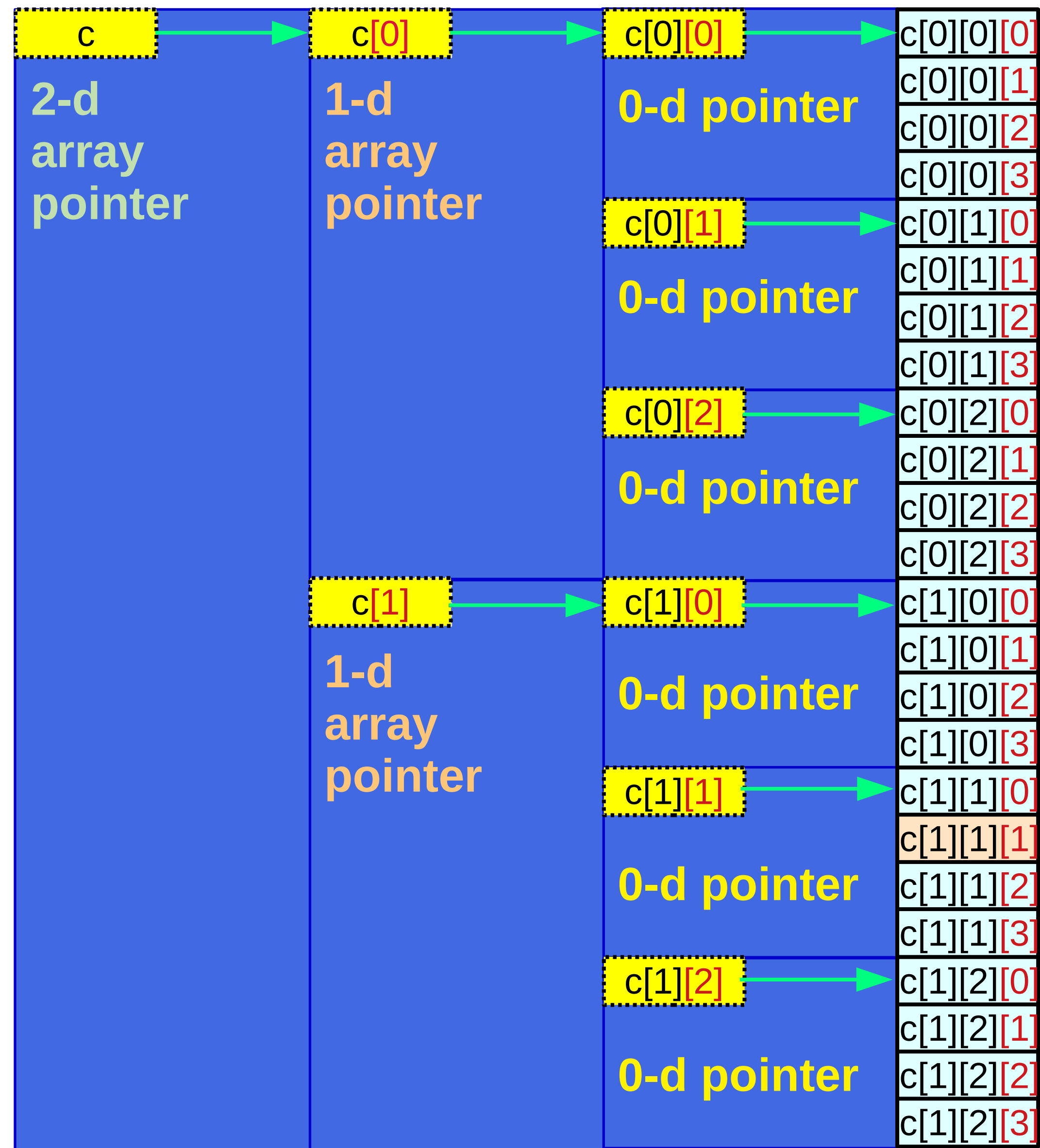
= sizeof(*p[i+1]) * N
sizeof(p[i+1][0]) * N

3-d array structure – virtual pointer representation

int c[2][3][4];

((*(c +i) +j) +k)

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order



3-d array structure – abstract data representation

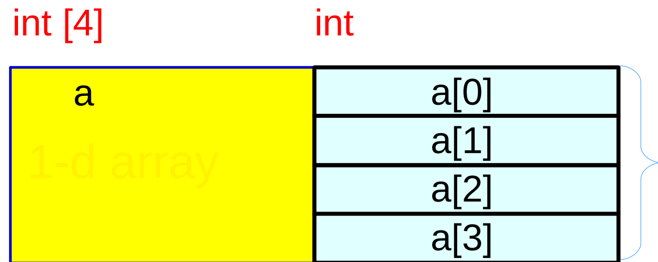
int c[2][3][4];

((c [i])[j])[k]

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

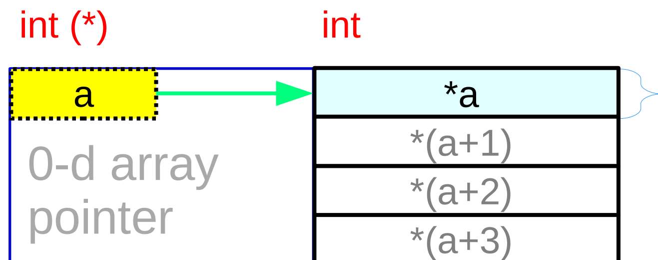
c	c[0]	c[0][0]	c[0][0][0]
3-d array name	2-d array name	1-d array name	c[0][0][1]
			c[0][0][2]
			c[0][0][3]
		c[0][1]	c[0][1][0]
		1-d array name	c[0][1][1]
			c[0][1][2]
			c[0][1][3]
		c[0][2]	c[0][2][0]
		1-d array name	c[0][2][1]
			c[0][2][2]
			c[0][2][3]
	c[1]	c[1][0]	c[1][0][0]
	2-d array name	1-d array name	c[1][0][1]
			c[1][0][2]
			c[1][0][3]
		c[1][1]	c[1][1][0]
		1-d array name	c[1][1][1]
			c[1][1][2]
			c[1][1][3]
		c[1][2]	c[1][2][0]
		1-d array name	c[1][2][1]
			c[1][2][2]
			c[1][2][3]

Array **a** and virtual pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

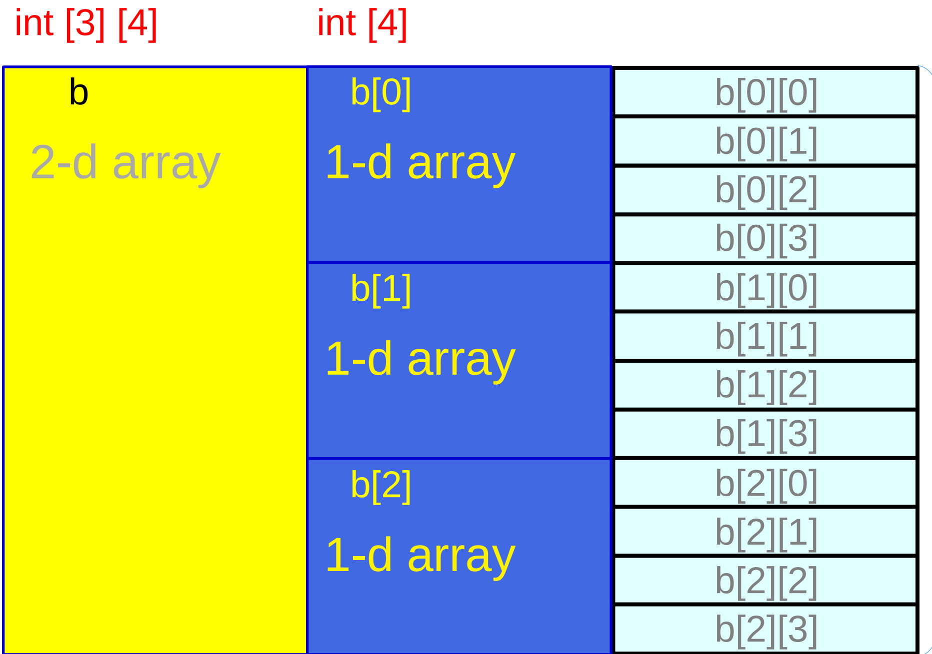
a is the name of a 1-d array
a also has a pointer type
a has the size of the array
a has the value of the starting address

a is a virtual array pointer

Array **b** and virtual pointer **b**

2-d array b specific array type

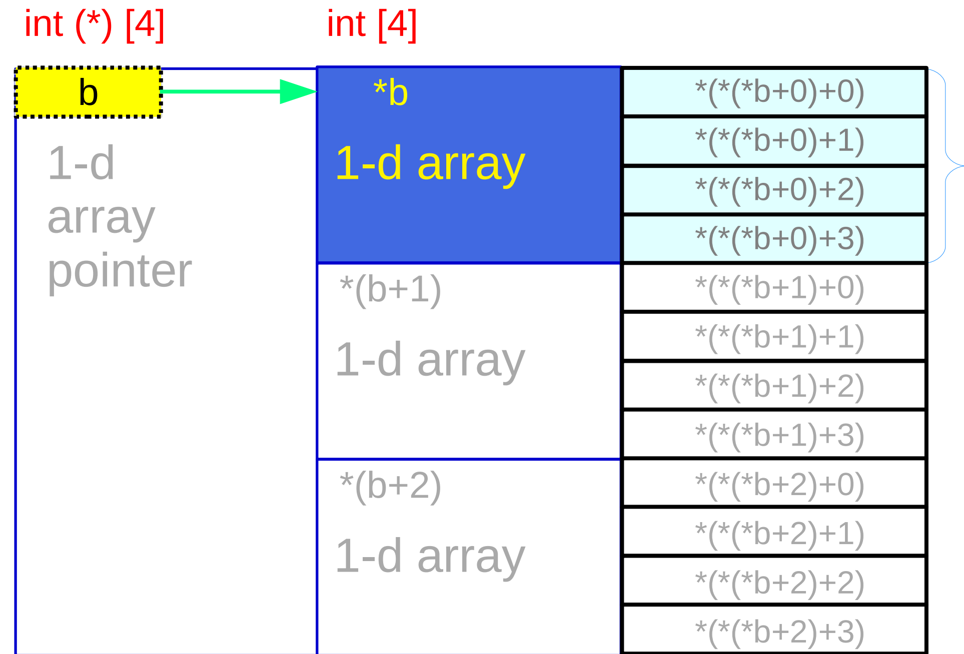
sizeof(b)



b is the name of a 2-d array
b has the size of the array

1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3



b also has a 1-d array pointer type
b has the value of the starting address

b is a virtual array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

c is the name of a 3-d array
c has the size of the array

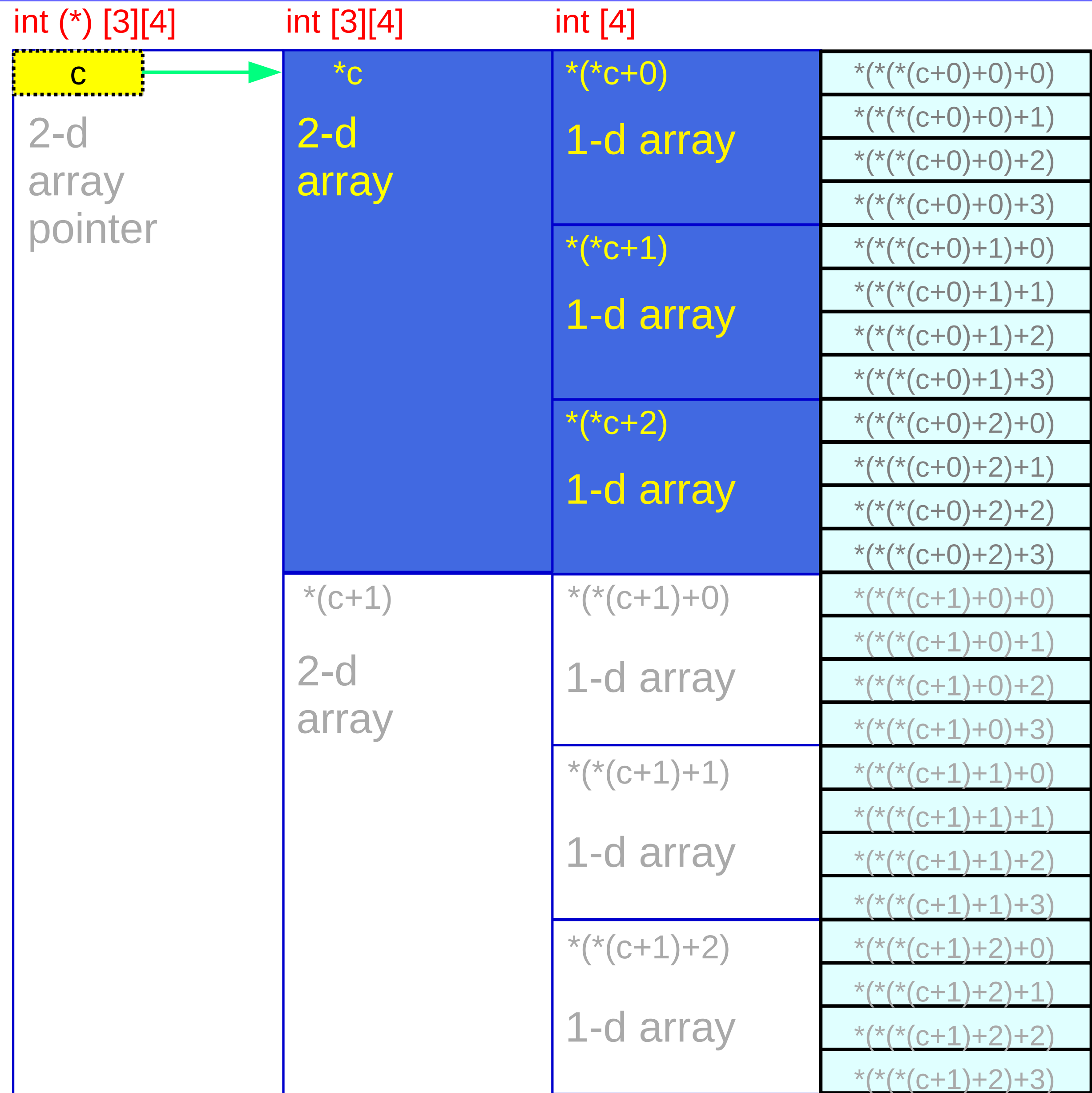
int [2][3][4]	int [3][4]	int [4]	
c 3-d array	c[0] 2-d array	c[0][0] 1-d array	c[0][0][0]
			c[0][0][1]
			c[0][0][2]
			c[0][0][3]
		c[0][1] 1-d array	c[0][1][0]
			c[0][1][1]
			c[0][1][2]
			c[0][1][3]
		c[0][2] 1-d array	c[0][2][0]
			c[0][2][1]
			c[0][2][2]
			c[0][2][3]
	c[1] 2-d array	c[1][0] 1-d array	c[1][0][0]
			c[1][0][1]
			c[1][0][2]
			c[1][0][3]
		c[1][1] 1-d array	c[1][1][0]
			c[1][1][1]
			c[1][1][2]
			c[1][1][3]
		c[1][2] 1-d array	c[1][2][0]
			c[1][2][1]
			c[1][2][2]
			c[1][2][3]

Virtual pointer **c**

2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2



c also has a 2-d array pointer type
c has the value of the starting address

c is a virtual array pointer

Finding dual types in a **3-d** array

```
int c[2][3][4];
```

The type of c

dual types

- **Int [2][3][4]** abstract data type
- **Int [ ][3][4]** relaxing the 1st dimension
- **Int (*)[3][4]** virtual pointer type - gcc displaying type

```
int c[2][3][4];
```

The type of c[i]

dual types

- **int [3][4]** abstract data type
- **int [ ][4]** relaxing the 1st dimension
- **int (*)[4]** virtual pointer type - gcc displaying type

```
int c[2][3][4];
```

The type of c[i][j]

dual types

- **int [4]** abstract data type
- **Int [ ]** relaxing the 1st dimension
- **int (*)** virtual pointer type - gcc displaying type

Types of virtual array pointers in a **3-d** array

```
int c[2][3][4];
```

	array type (name)	array pointer type
c[i][j][k]	int	int
c[i][j] [k]	int [4] [k]	int (*) [k]
c[i] [j] [k]	int [3][4] [j] [k]	int (*)[4] [j] [k]
c [i] [j] [k]	int [2][3][4] [i] [j] [k]	int (*)[3][4] [i] [j] [k]

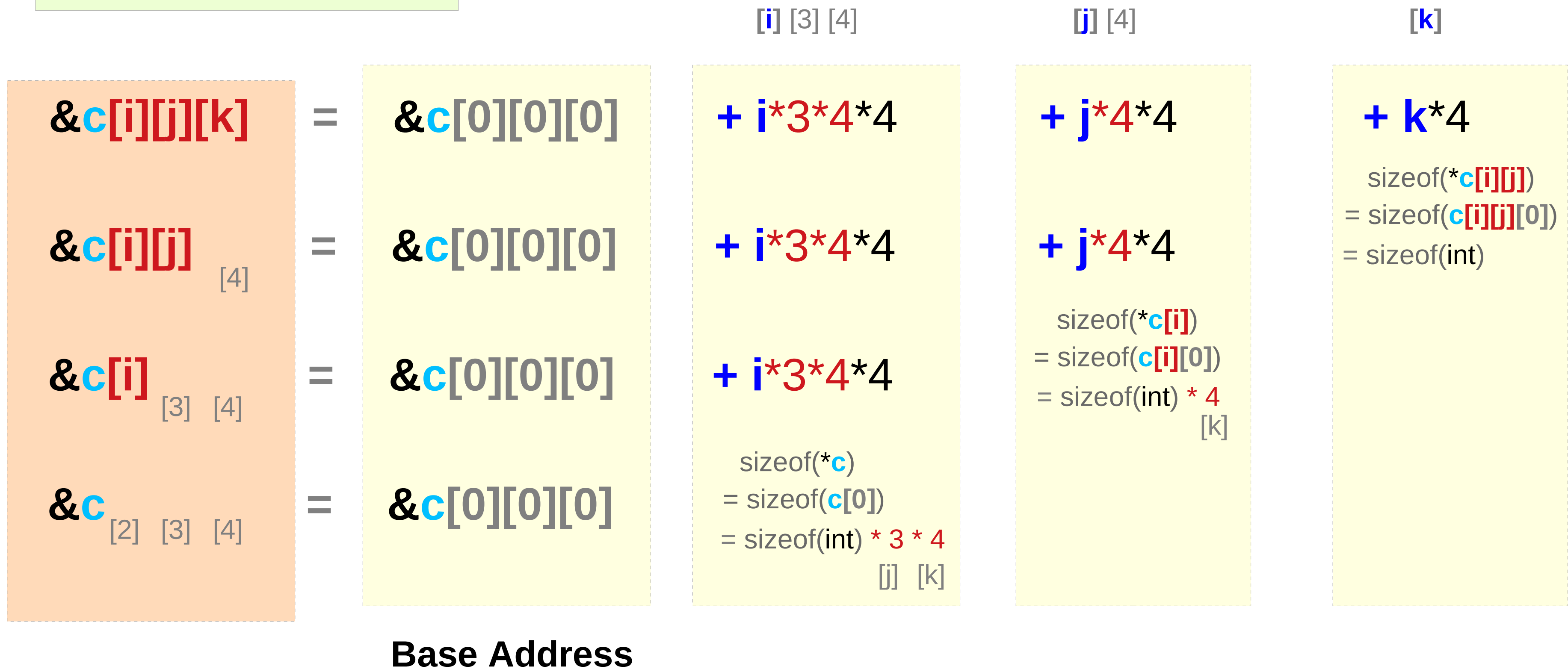
Sizes of virtual array pointers in a **3-d** array

```
int c[2][3][4];
```

		Element Size	
sizeof(**c[i][j][k]**)	=	sizeof(int)	
sizeof(**c[i][j]**) [k]	=	sizeof(int)	* 4 [k]
sizeof(**c[i]**) [j] [k]	=	sizeof(int)	* 3 * 4 [j] [k]
sizeof(**c**) [i] [j] [k]	=	sizeof(int)	* 2 * 3 * 4 [i] [j] [k]

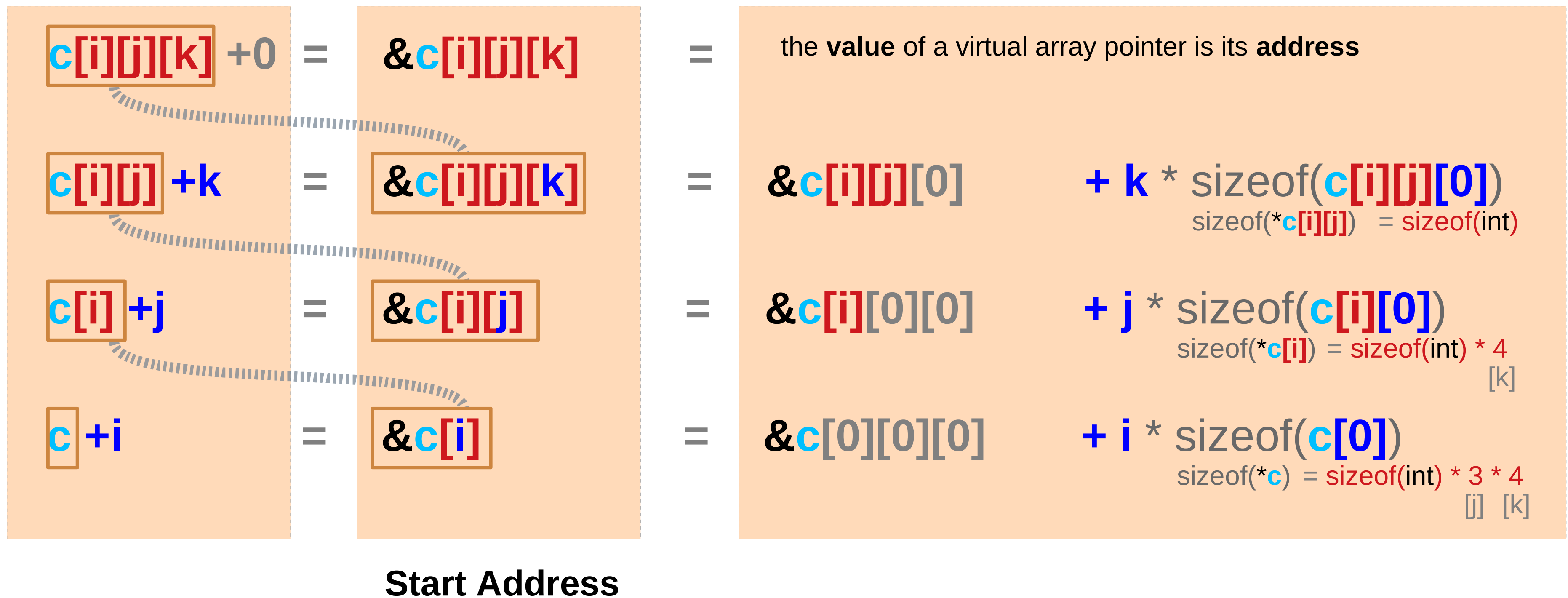
Element Size

Addresses of virtual array pointers in a **3-d** array



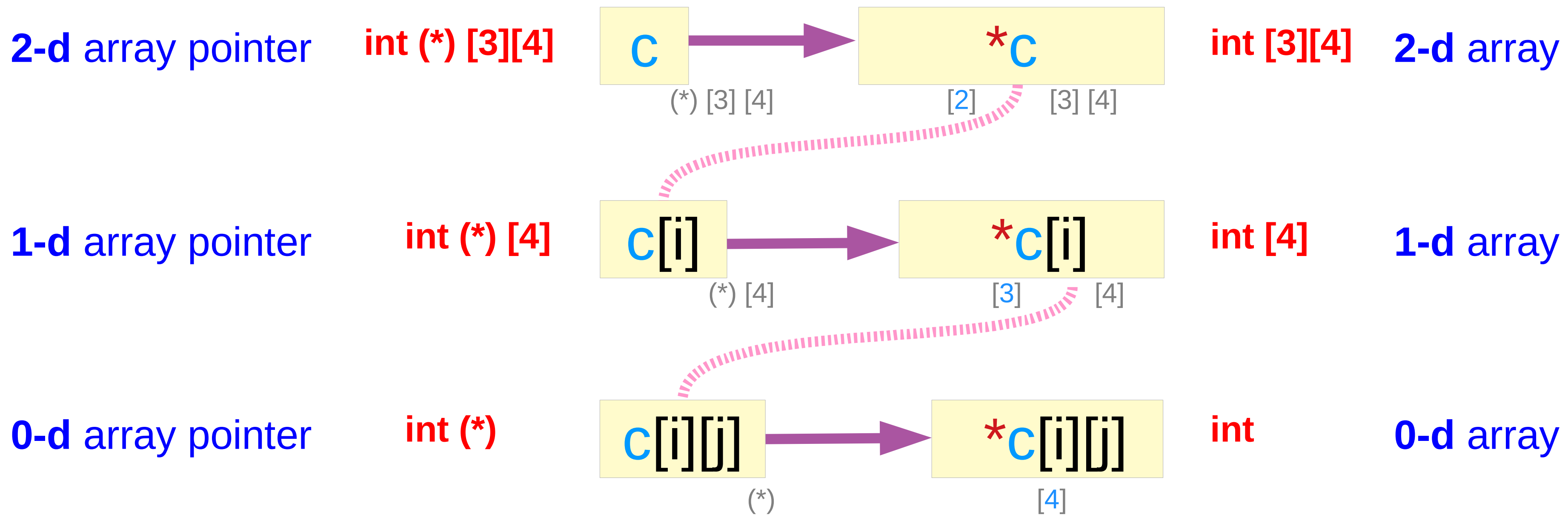
Values of virtual array pointers in a **3-d** array

int c[2][3][4];

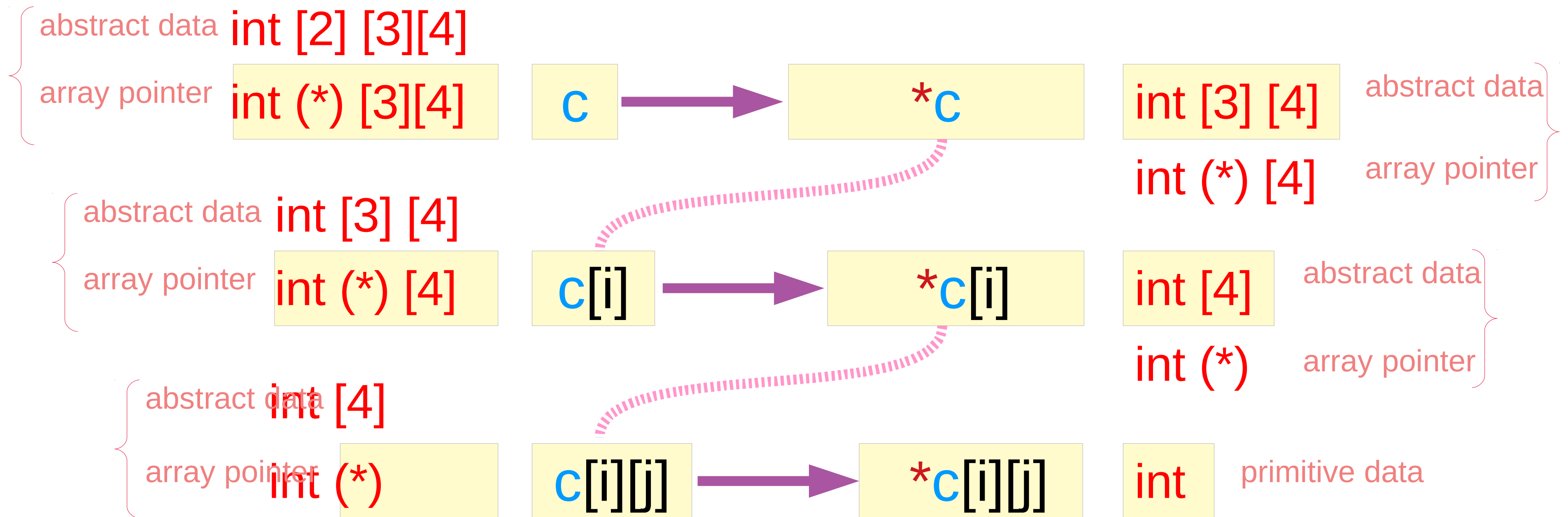


Virtual array pointers and abstract data in a **3-d** array

```
int     c [2][3][4];
```



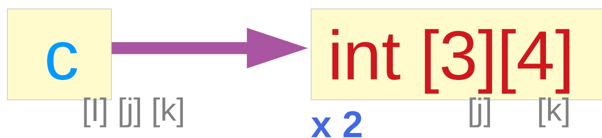
Dual types in a **3-d** array



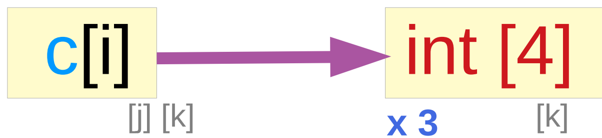
Pointed array sizes in a **3-d** array

int c [2][3][4];

the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types



sizeof(*c) = sizeof(int [3][4])



sizeof(*c[i]) = sizeof(int [4])

sizeof(*c[i][j]) = sizeof(int)

c, c[i], c[i][j] are virtual array pointers
and they are also abstract data (arrays)

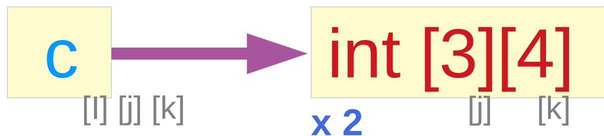
when sizes are considered,
view them as abstract data (arrays)

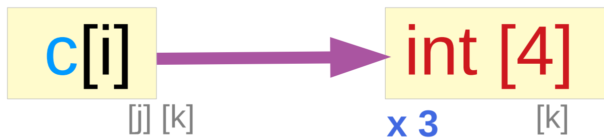
all are sizes of arrays

Virtual array pointer sizes in a **3-d** array

int c [2][3][4];

size of a virtual array pointer = size of the pointed abstract data type * the number of such data





sizeof(c) = **2** * sizeof(*c)

sizeof(c[i]) = **3** * sizeof(*c[i])

sizeof(c[i][j]) = **4** * sizeof(*c[i][j])

sizeof(Virtual Array Pointer) =
sizeof(Array of the dual type)

Virtual pointer sizes are subarray sizes

int c [2][3][4];

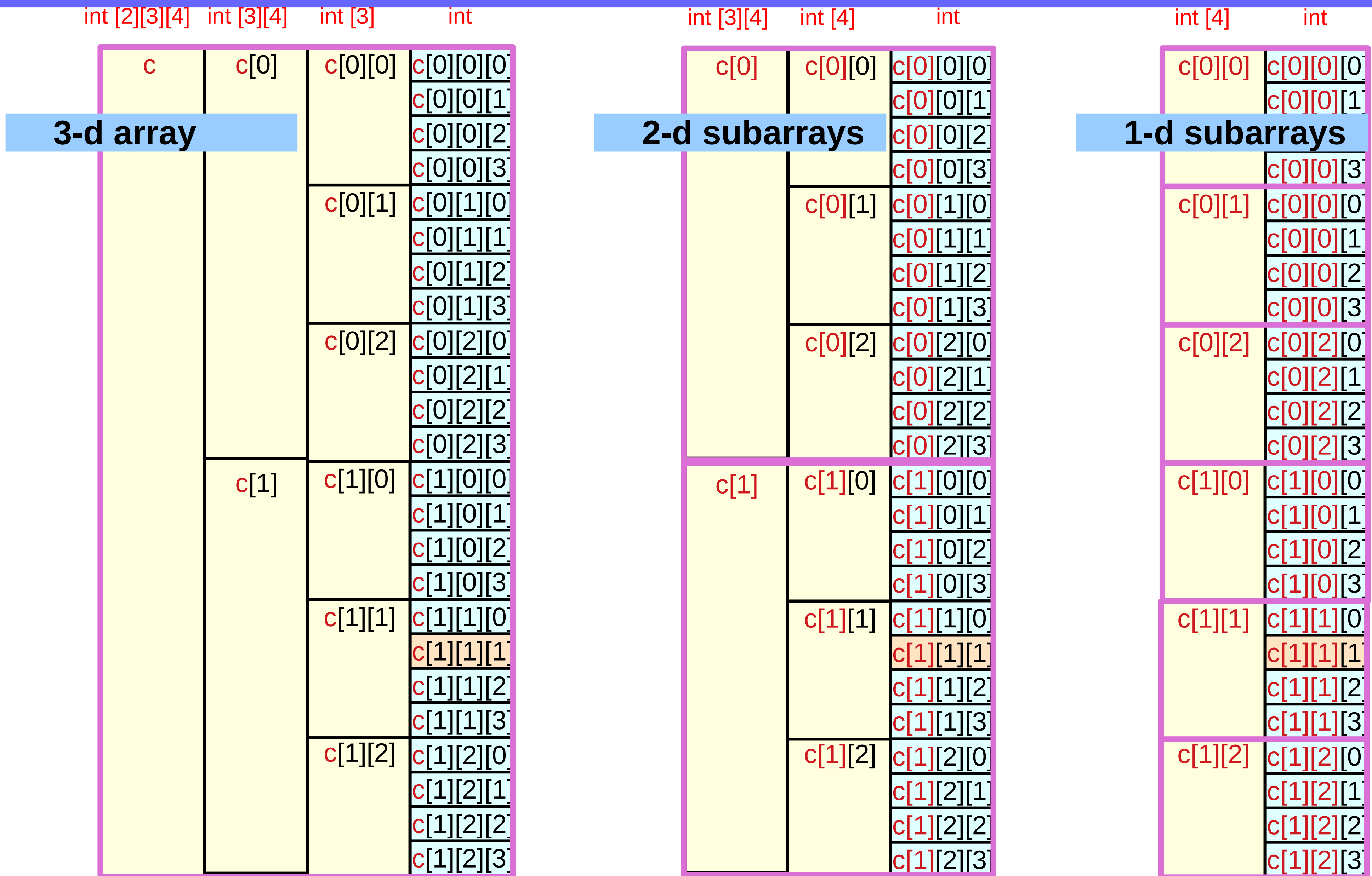
sizeof(Virtual Array Pointer) =
sizeof(Array of the dual type)

c → int [3][4] [I] [j] [k] [j] [k]	sizeof(int [2] [3][4]) = sizeof(c) = 2*3*4 * 4 sizeof(int (*) [3][4]) = pointer size = 4 or 8	
c[i] → int [4] [j] [k] [k]	sizeof(int [3] [4]) = sizeof(c[i]) = 3*4 * 4 sizeof(int (*) [4]) = pointer size = 4 or 8	
c[i][j] → int [k]	sizeof(int [4]) = sizeof(c[i][j]) = 4 * 4 sizeof(int [4]) = pointer size = 4 or 8	

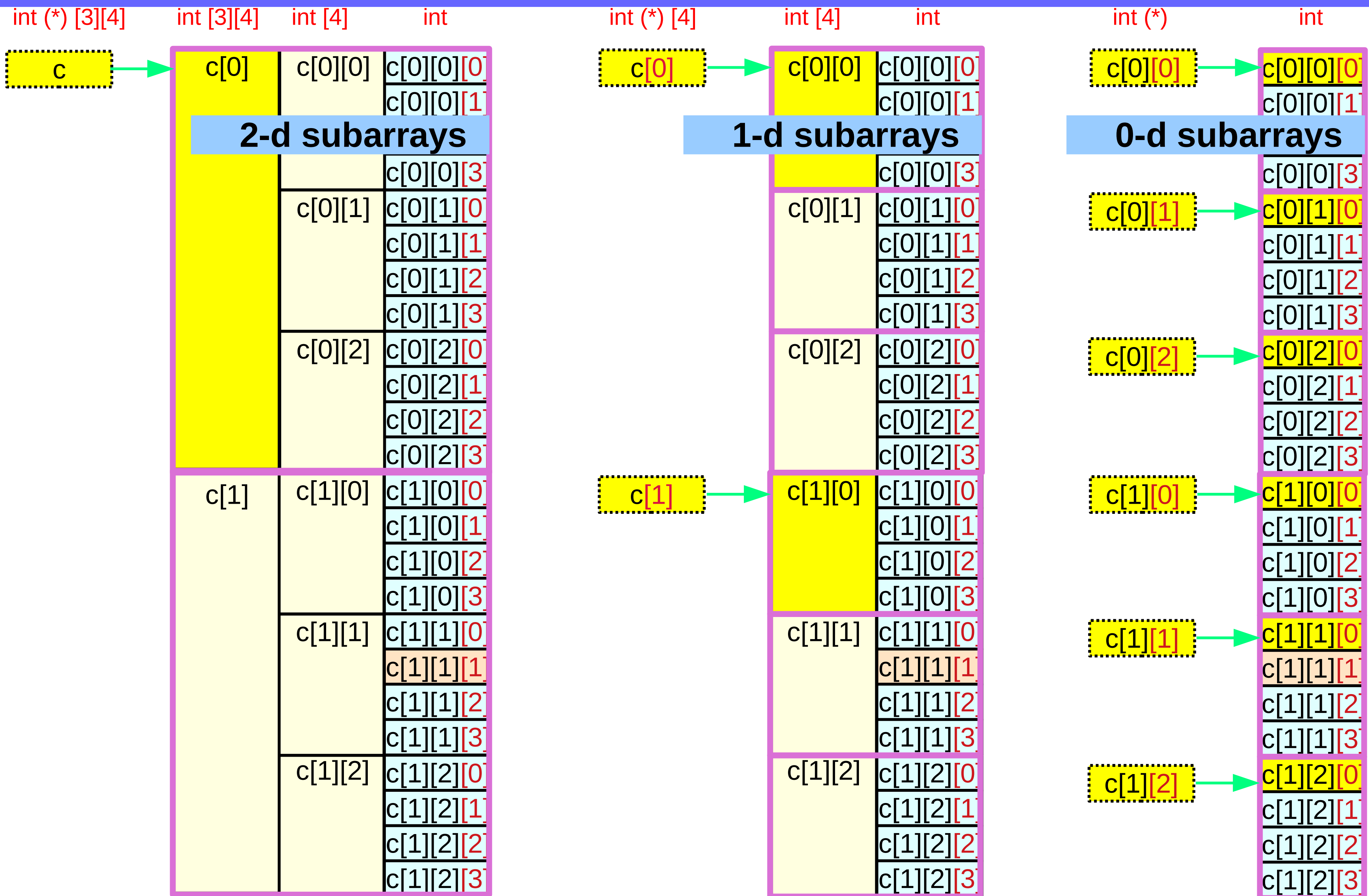
4 bytes for 32-bit machines
8 bytes for 64-bit machines

not real array pointers
virtual array pointers

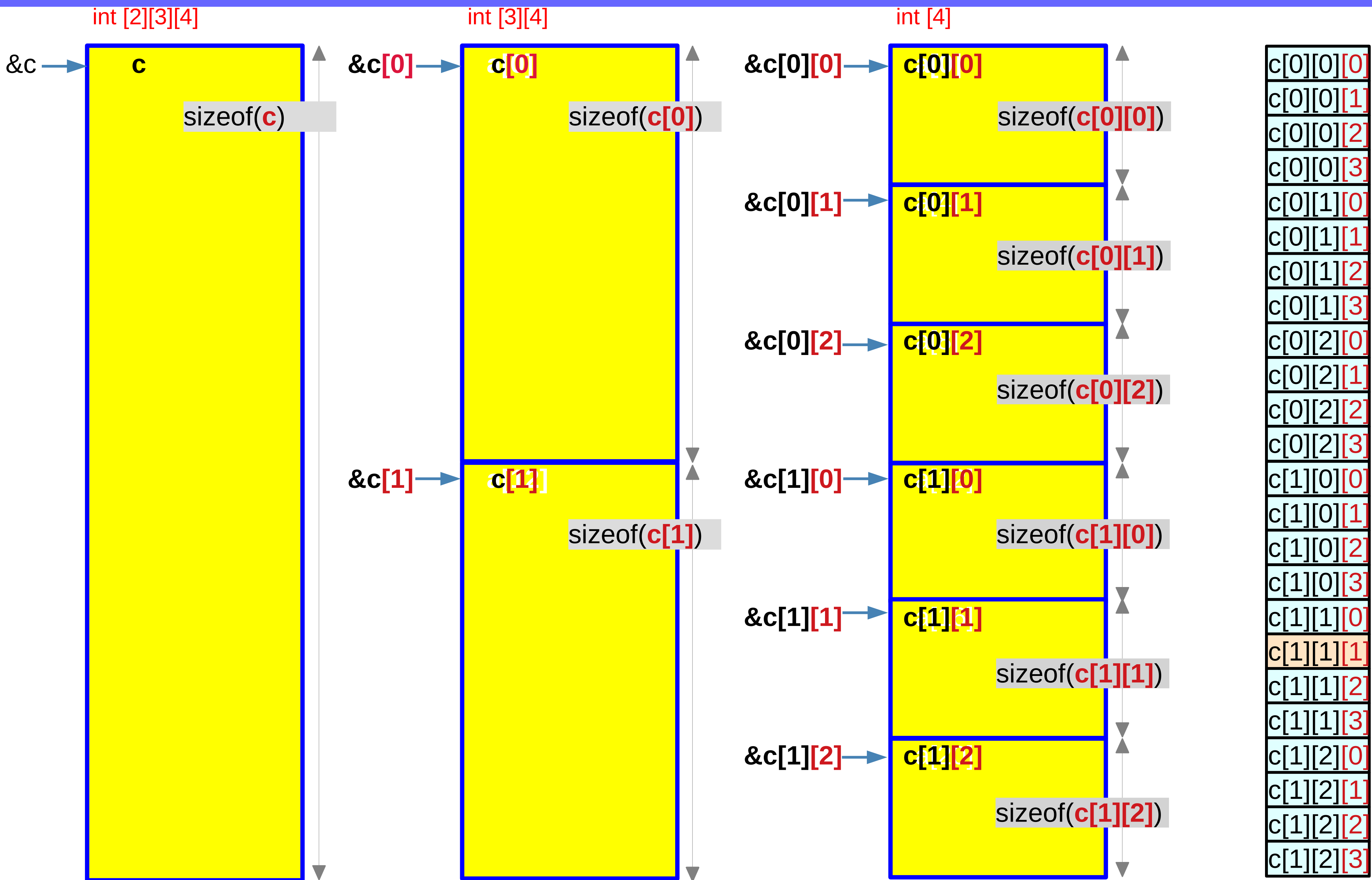
Subarrays **c**, **c[i]**, **c[i][j]** in a **3-d** array



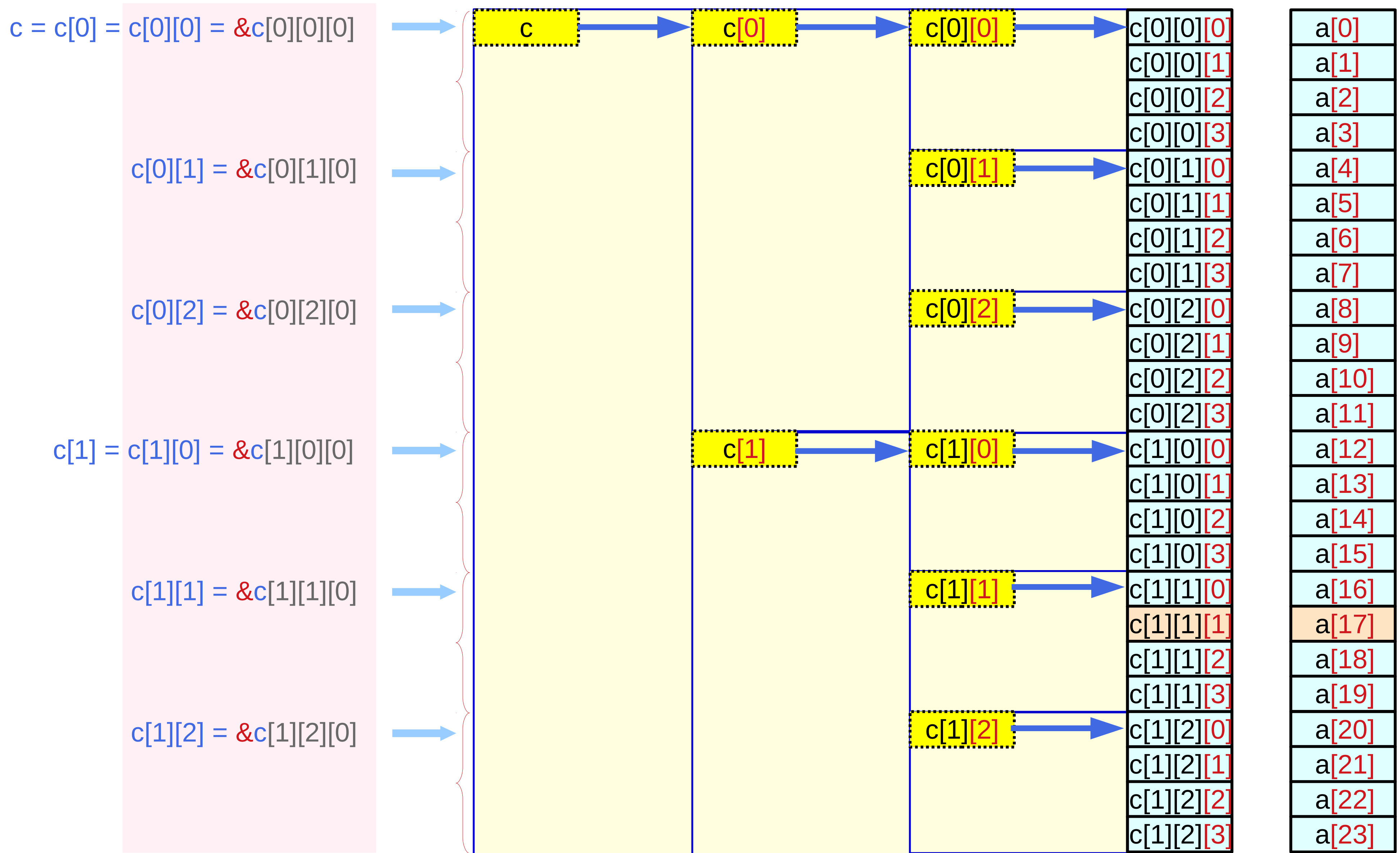
Virtual array pointers **c**, **c[i]**, **c[i][j]** in a **3-d** array



Abstract data **c**, **c[i]**, **c[i][j]** – start addresses and sizes



Virtual array pointer **c**, **c[i]**, **c[i][j]** – values (addresses)



Virtual array pointer **c**, **c[i]**, **c[i][j]** – values and types

c = c[0] = c[0][0] = &c[0][0][0]

means

value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])

type(c) ≠ type(c[0]) ≠ type(c[0][0]) = type(&c[0][0][0])

int (*) [3][4] int (*) [4] int * int *

c[0][1] = &c[0][1][0]

means

value(c[0][1]) = value(&c[0][1][0])

type(c[0][1]) = type(&c[0][1][0])

int * int *

c[0][2] = &c[0][2][0]

means

value(c[0][2]) = value(&c[0][2][0])

type(c[0][2]) = type(&c[0][2][0])

int * int *

c[1] = c[1][0] = &c[1][0][0]

means

value(c[1]) = value(c[1][0]) = value(&c[1][0][0])

type(c[1]) ≠ type(c[1][0]) = type(&c[1][0][0])

int (*) [4] int * int *

c[1][1] = &c[1][1][0]

means

value(c[1][1]) = value(&c[1][1][0])

type(c[1][1]) = type(&c[1][1][0])

int * int *

c[1][2] = &c[1][2][0]

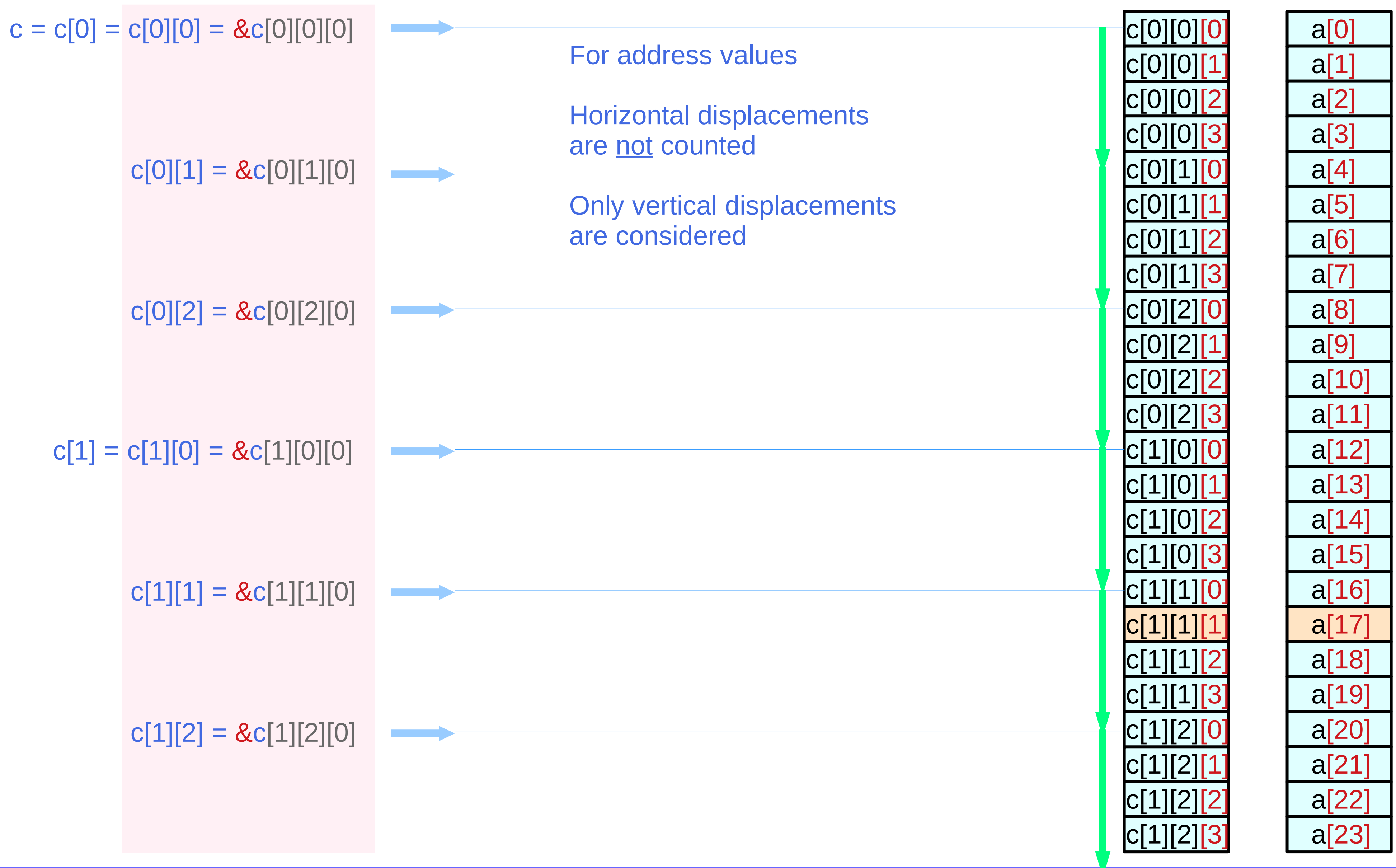
means

value(c[1][2]) = value(&c[1][2][0])

type(c[1][2]) = type(&c[1][2][0])

int * int *

Virtual array pointer **c**, **c[i]**, **c[i][j]** – vertical displacement



Virtual array pointers – types, sizes, and values

int c[2][3][4];	c[i][j]	c[i][j][0]	
type	int [4] int (*)	int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * 4	= sizeof(int) * 4
value (address)	c[i][j] =	&c[i][j][0]	

int c[2][3][4];	c[i]	c[i][0]	
type	int [3][4] int (*)[4]	int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =	sizeof(c[i][0]) * 3	= sizeof(int) * 4 * 3
value (address)	c[i] =	&c[i][0][0]	

int c[2][3][4];	c	c[0]	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =	sizeof(c[0]) * 2	= sizeof(int) * 4 * 3 * 2
value (address)	c =	&c[0][0][0]	

Summary of virtual array pointers in a **3-d** array

c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
int (*) [3][4] 2-d array pointer c int [2] [3][4] 3-d array name c	int (*) [4] 1-d array pointers c[i] Int [3] [4] 2-d array names c[i]	int (*) 0-d array pointers c[i][j] int [4] 1-d array names c[i][j]
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 3 * 4	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

Dual type constraints in a multi-dimensional array

Virtual array pointers to subarrays in a 3-d array

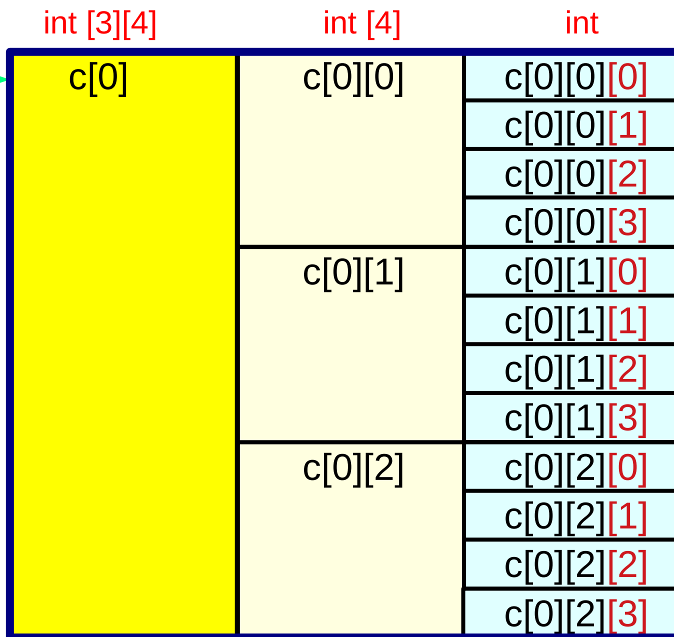
virtual 2-d array pointer

sizeof(**c**) =
sizeof(**c[0]**) * **2**

int (*) [3][4]

c

int c [2][3][4];



the first 2-d subarray

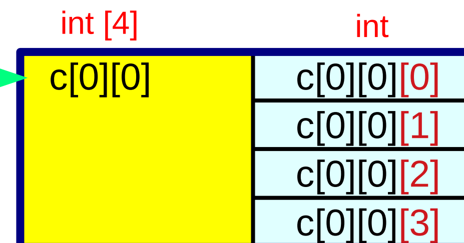
sizeof(**c[0]**) =
sizeof(**int [3][4]**)

virtual 1-d array pointer

sizeof(**c[0]**) =
sizeof(**c[0][0]**) * **3**

int (*) [4]

c[0]



the first 1-d subarray

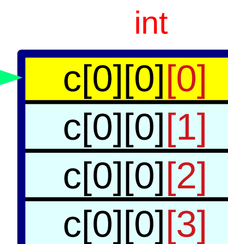
sizeof(**c[0][0]**) =
sizeof(**int [4]**)

virtual 0-d array pointer

sizeof(**c[0][0]**) =
sizeof(**c[0][0][0]**) * **4**

int (*)

c[0][0]



the first 0-d subarray

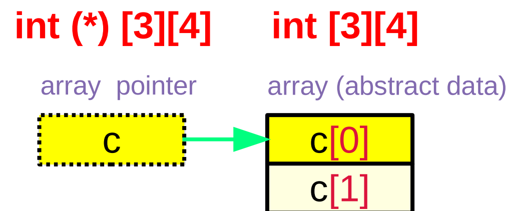
sizeof(**c[0][0][0]**) =
sizeof(**int**)

Virtual array pointer **c**, **c[0]**, **c[0][0]** – types and sizes

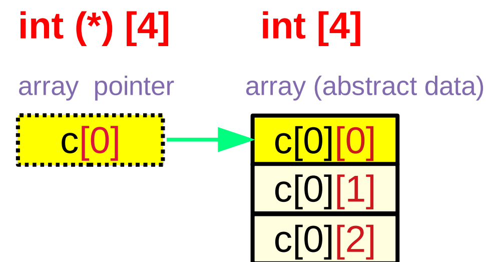
int c [2][3][4];

Types – array pointers

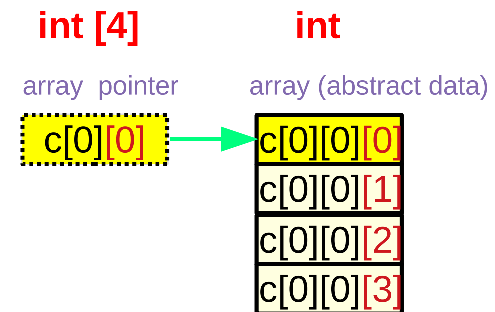
int (*) [3][4] **int [3][4]**



int (*) [4] **int [4]**



int [4] **int**



Sizes – abstract data

sizeof(**c**)
sizeof(**int [2][3][4]**)
sizeof(**int**) * **2** * **3** * **4**

sizeof(**c[0]**)
sizeof(**int [3][4]**)
sizeof(**int**) * **3** * **4**

sizeof(**c[0][0]**)
sizeof(**int [4]**)
sizeof(**int**) * **4**

sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

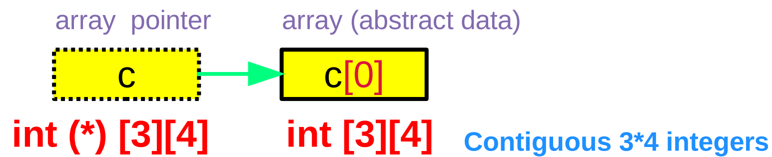
sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

Contiguous subarrays **c[i]**, **c[i][j]**, **c[i][j][k]**

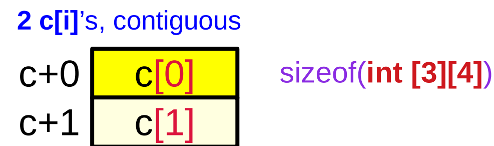
int c [2][3][4];

Contiguous 2*3*4 integers

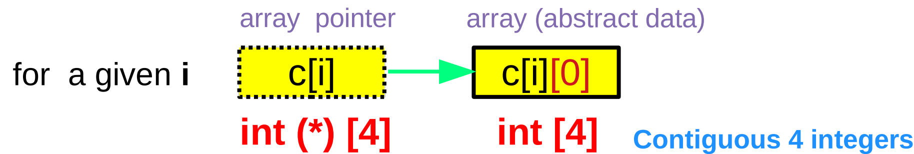


Contiguous 3*4 integers

2 c[i]'s, contiguous

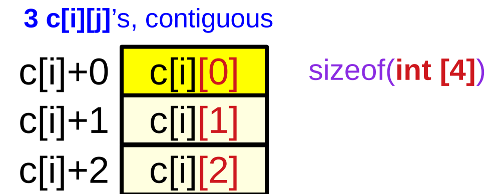


sizeof(**int [3][4]**)

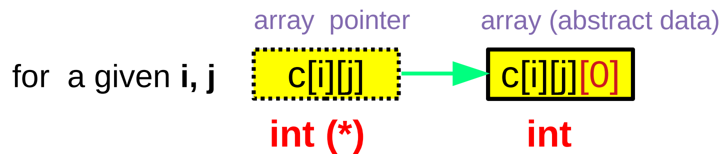


Contiguous 4 integers

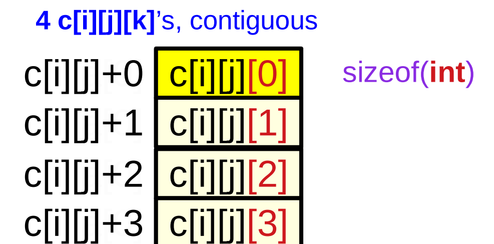
3 c[i][j]'s, contiguous



sizeof(**int [4]**)



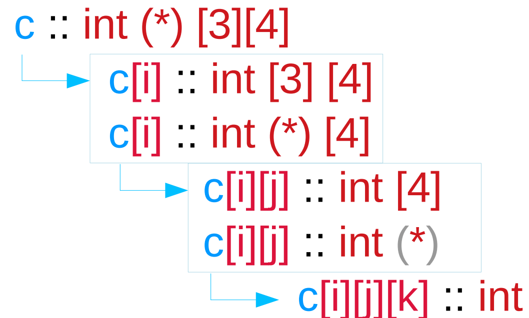
4 c[i][j][k]'s, contiguous



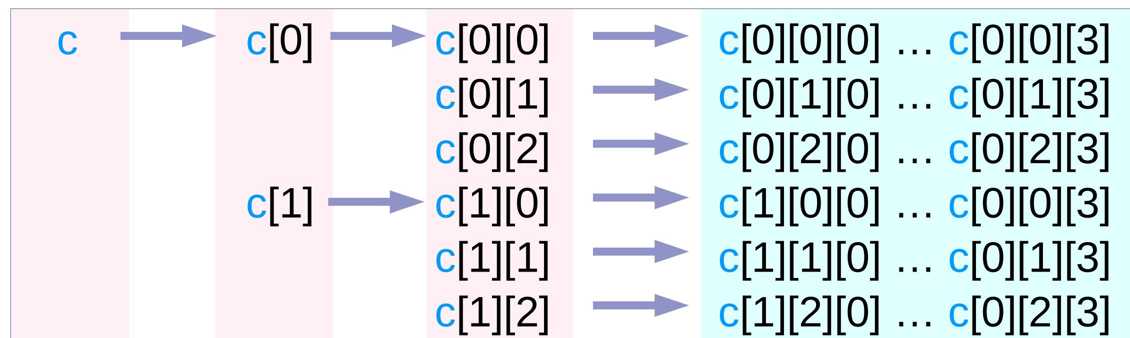
sizeof(**int**)

Dual types of **c**, **c[i]**, **c[i][j]**

int c [2][3][4];



2-d array pointers
2-d arrays
1-d array pointers
1-d arrays
0-d array pointers
0-d arrays (integers)

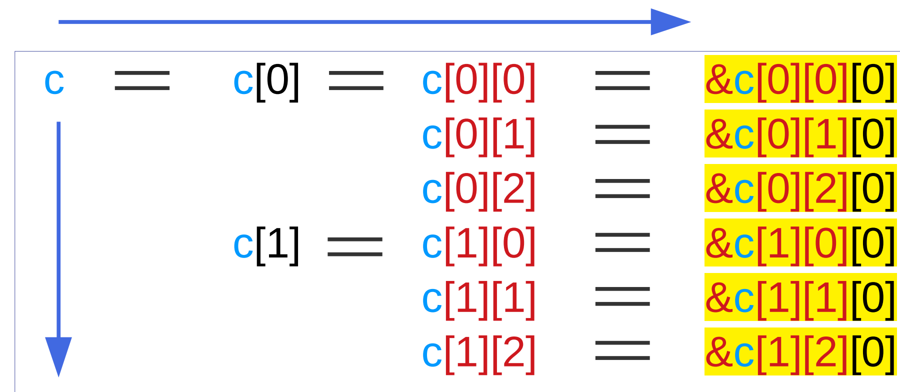


int [2] [3][4]	int [3] [4]	int [4]	int	…	int
int (*) [3][4]	int (*) [4]	int (*)	int	…	int

Values of virtual array pointers **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

virtual array pointers have address values
in each row in the following figure
have the same address value



Horizontal displacements are not counted
only **vertical displacements** are considered
for address values

virtual assignments

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Finding values of virtual array pointers **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

append [0] to the right

c	+[0] =	c[0]	+[0] =	c[0][0]	+[0] =	&c[0][0][0]
				c[0][1]	+[0] =	&c[0][1][0]
				c[0][2]	+[0] =	&c[0][2][0]
		c[1]	+[0] =	c[1][0]	+[0] =	&c[1][0][0]
				c[1][1]	+[0] =	&c[1][1][0]
				c[1][2]	+[0] =	&c[1][2][0]
int (*) [3][4]		int (*) [4]		int [4]		int

c[0][0][0] **:** leading elements of **c**

c[i][0][0] **:** leading elements of **c[i]**

c[i][j][0] **:** leading elements of **c[i][j]**

&c[0][0][0]

&c[0][0][0]
&c[1][0][0]

&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]

Finding sub-arrays with the address &c[i][j][0]

```
int   c [2][3][4];
```

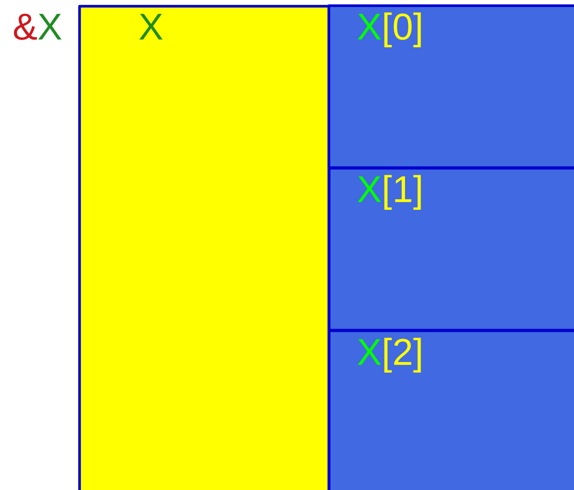
delete [0] from the right

&c[0][0][0]	-[0] ==	c[0][0]	-[0] ==	c[0]	-[0] ==	c
&c[0][1][0]	-[0] ==	c[0][1]				
&c[0][2][0]	-[0] ==	c[0][2]				
&c[1][0][0]	-[0] ==	c[1][0]	-[0] ==	c[1]		
&c[1][1][0]	-[0] ==	c[1][1]				
&c[1][2][0]	-[0] ==	c[1][2]				
int		int [4]		int (*) [4]		int (*) [3][4]

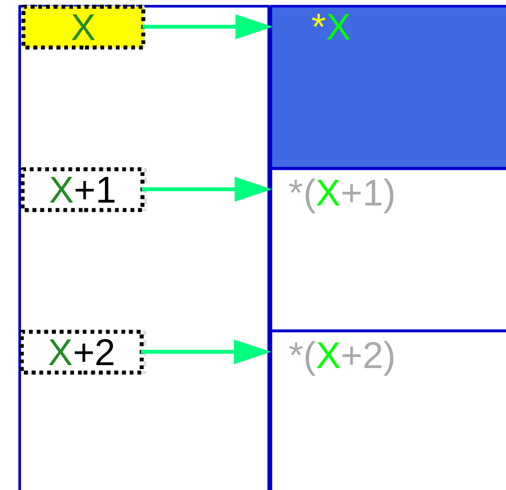
c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

Dual types in a **3-d** array

Abstract data (array) X



Virtual array pointer X



array (abstract data) | array (abstract data)

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

&c[i][j]	=	&c[i][j][0]
&c[i]	=	&c[i][0]
&c	=	&c[0]

array pointer | array (abstract data)

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

c[i][j]	=	&c[i][j][0]
c[i]	=	&c[i][0]
c	=	&c[0]

Dual type constraints

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

Virtual array pointer

array pointer — array (abstract data)

c[i][j]	points to	c[i][j][0]
c[i]	points to	c[i][0]
c	points to	c[0]

address value

X

&c[i][j][0] = &c[i][j]
&c[i][0] = &c[i]
&c[0] = &c

Abstract data (array)

array (abstract data) — array (abstract data)

c[i][j]	starts from	&c[i][j][0]
c[i]	starts from	&c[i][0]
c	starts from	&c[0]

&X X

c[i][j] = &c[i][j]
c[i] = &c[i]
c = &c

array (abstract data) — Address of an array pointer

c[i][j]	pointer value	=	pointer address	&c[i][j]
c[i]	pointer value	=	pointer address	&c[i]
c	pointer value	=	pointer address	&c

X — &X

c[0] = c[0][0] relation

```
int    c [2][3][4];
```

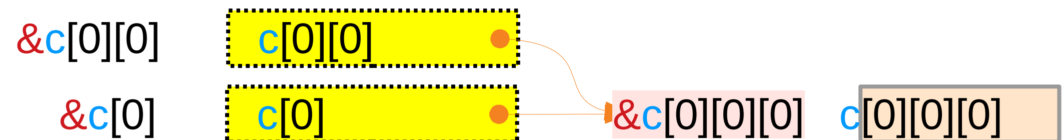
c ═ c[0] ═ c[0][0] ═ &c[0][0][0]

value(c[0]) = &c[0][0][0]
value(c[0][0]) = &c[0][0][0]

type(c[0]) = int (*)[4]
type(c[0][0]) = int [4]

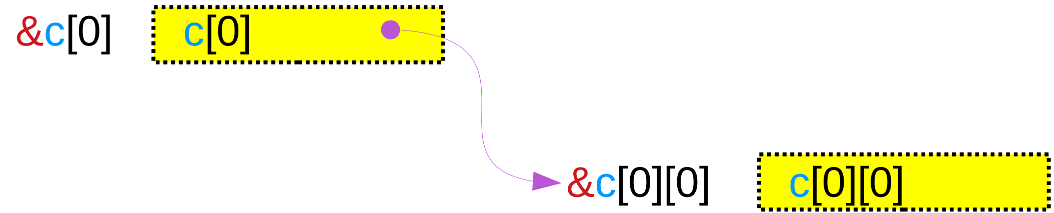
c[0] = c[0][0] *means*
value(c[0]) = value(c[0][0])

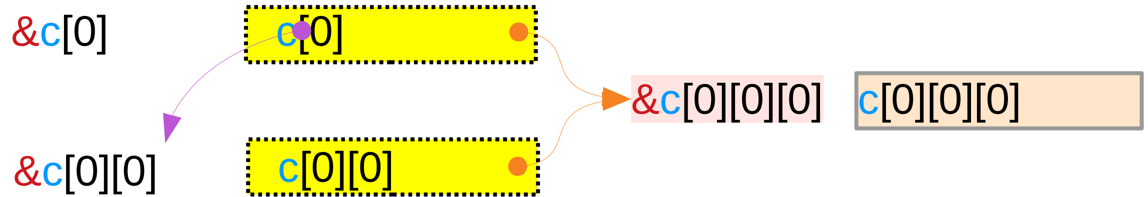
c[0] = c[0][0] *does* ***not*** *mean*
type(c[0]) = type(c[0][0])



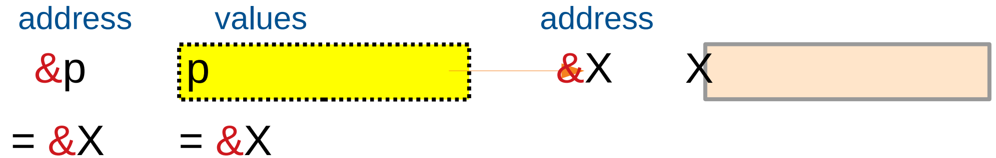
Addresses and Values of **c[0]** and **c[0][0]**

```
int    c [2][3][4];
```





A virtual pointer's address and value are the same



c[i] and c[i][0] point to the same c[i][0][0]

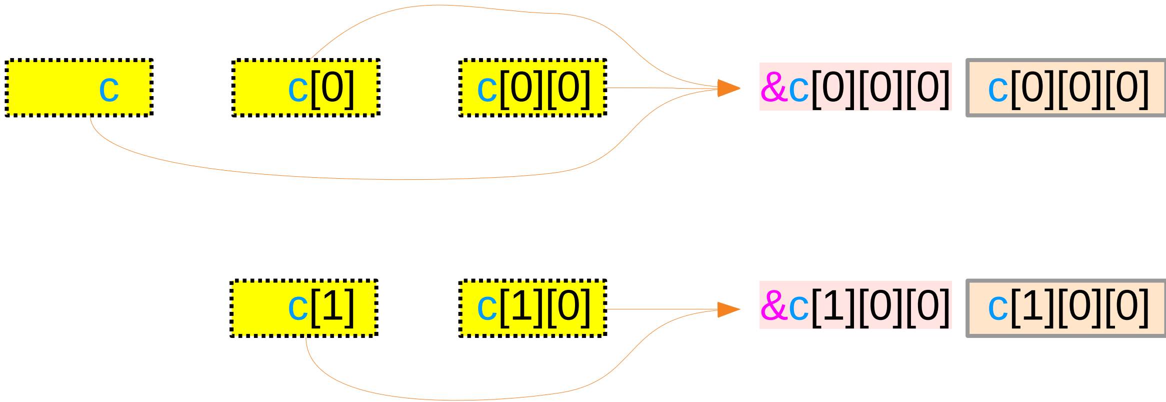
int c [2][3][4];

c = c[0] = c[0][0] = &c[0][0][0] ← value

int(*)[3][4] int(*)[4] int(*) int ← type

c[1] = c[1][0] = &c[1][0][0] ← value

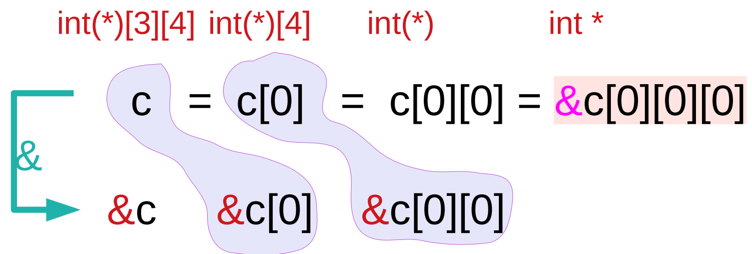
int(*)[4] int(*) int ← type



These virtual pointers have different types
but the same value (address)

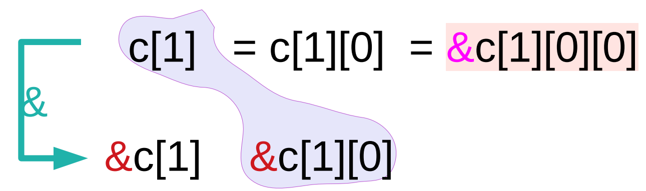
&c[i][0] and &c[i][0][0] – equivalence relations

int c [2][3][4];



equivalences

c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]



equivalences

c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]

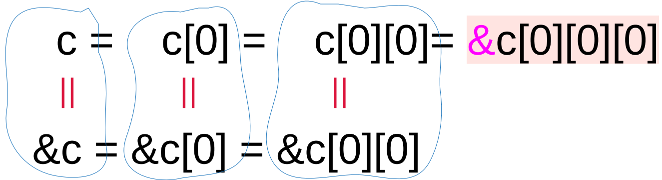
Horizontal displacements are not counted
only **vertical displacements** are considered
for **address values**

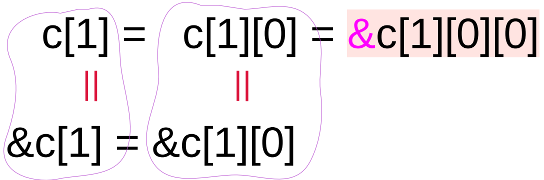
equivalences

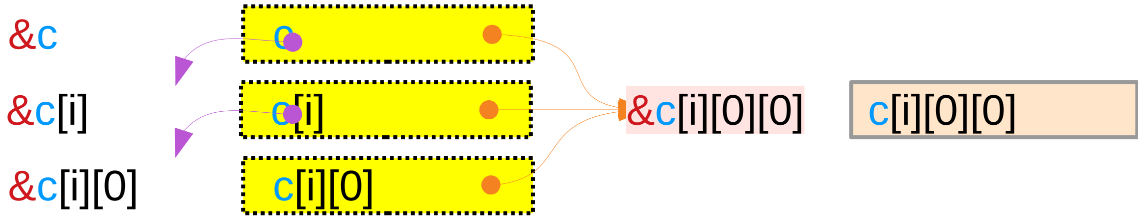
c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]

c[i] = &c[i] and c[i][0] = &c[i][0]

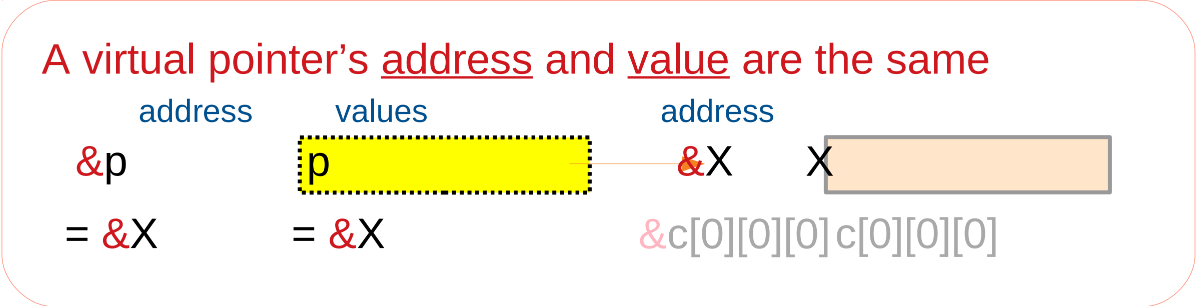
```
int    c [2][3][4];
```

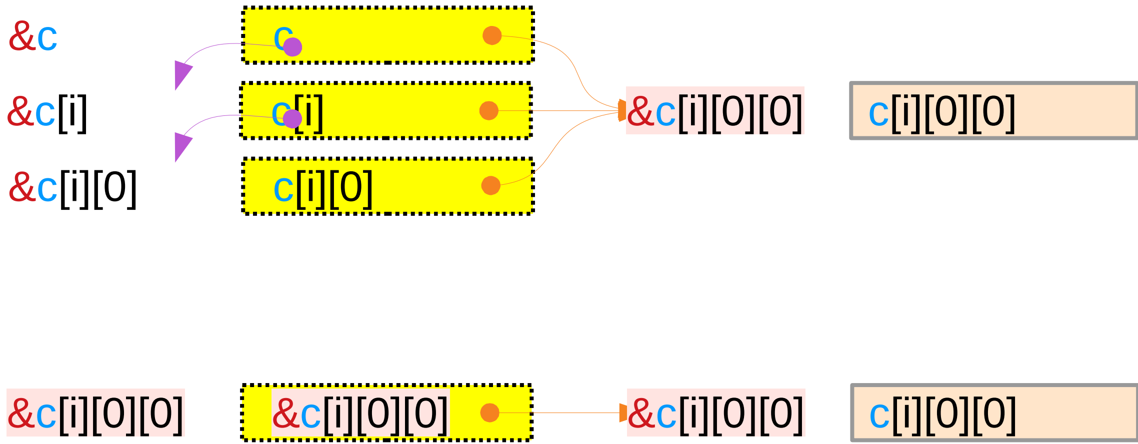




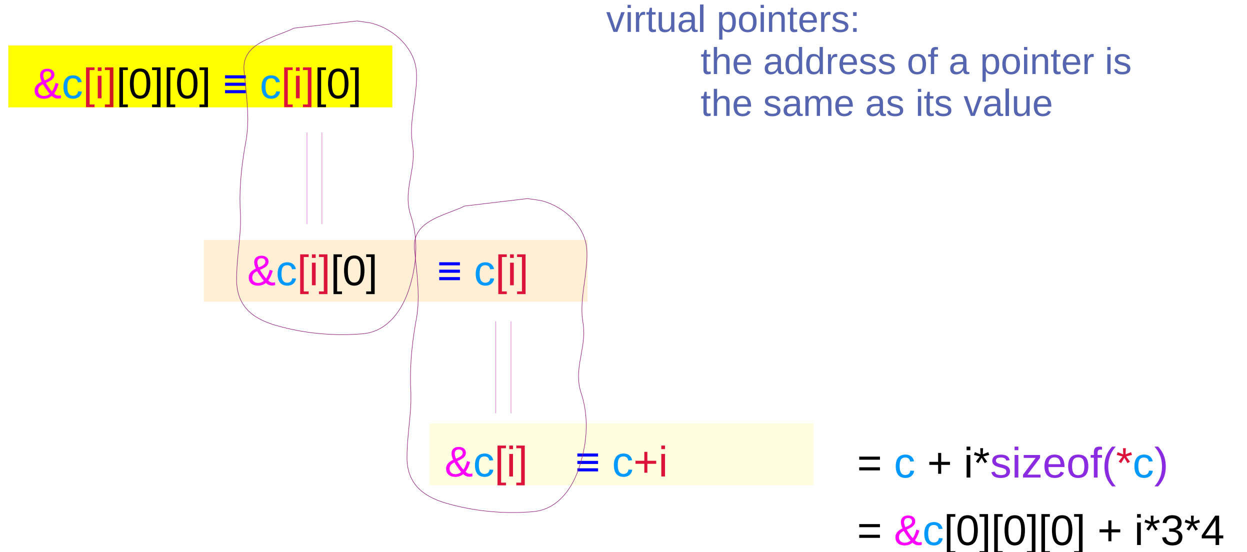


```
int    c [2][3][4];
```





Array Pointers to **c[i][0][0]**



delete [0] from the right

&c[0][0][0] $\overset{-[0]}{\equiv}$ c[0][0] $\overset{-[0]}{\equiv}$ c[0] $\overset{-[0]}{\equiv}$ c

&c[1][0][0] $\overset{-[0]}{\equiv}$ c[1][0] $\overset{-[0]}{\equiv}$ c[1]

Array Pointers to **c[i][j][0]**

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

&c[0][0][0]	-[0] =	c[0][0]	-[0] =	c[0]	-[0] =	c
&c[0][1][0]	-[0] =	c[0][1]				
&c[0][2][0]	-[0] =	c[0][2]				
&c[1][0][0]	-[0] =	c[1][0]	-[0] =	c[1]		
&c[1][1][0]	-[0] =	c[1][1]				
&c[1][2][0]	-[0] =	c[1][2]				

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun