Time Domain Analysis (1A)

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2^{nd} Order Systems

$$\frac{9}{s^2+9s+9}$$

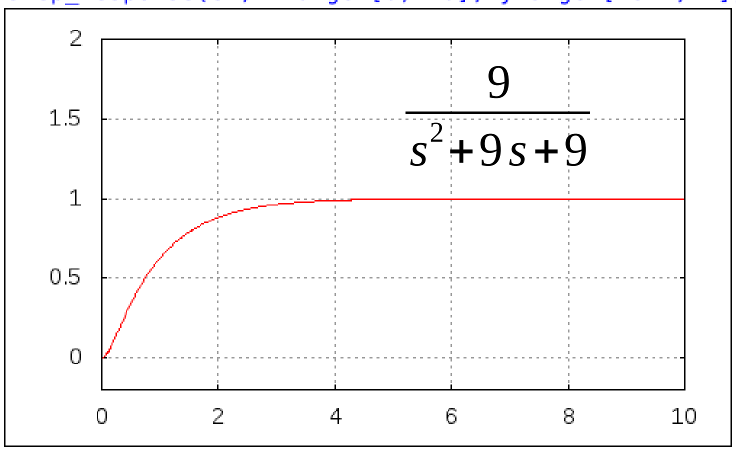
$$\frac{9}{s^2+2s+9}$$

$$\frac{9}{s^2+9}$$

$$\frac{9}{s^2+6s+9}$$

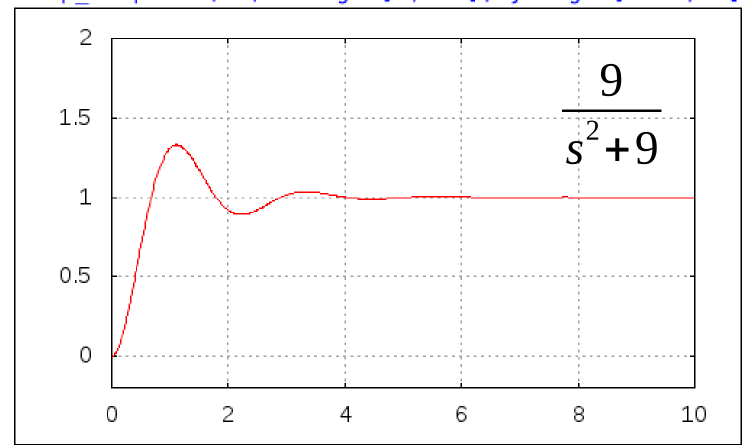
Step Responses

(%i39) `step_response(G1, xrange=[0, 10], yrange=[-0.2, 2]);`

(%t39)


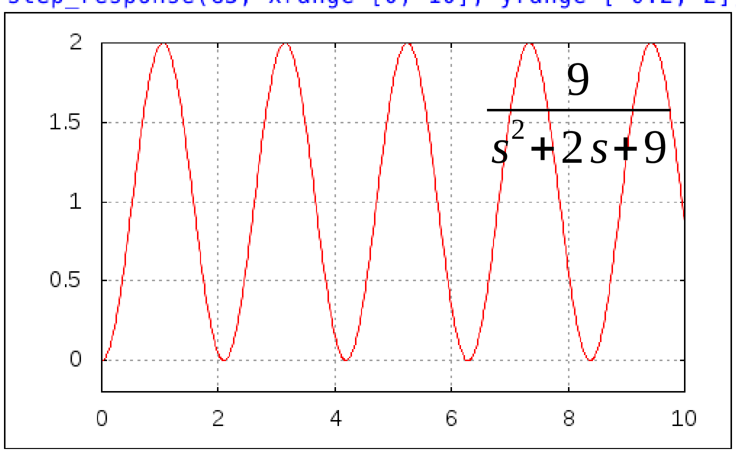
(%o39)

(%i40) `step_response(G2, xrange=[0, 10], yrange=[-0.2, 2]);`

(%t40)


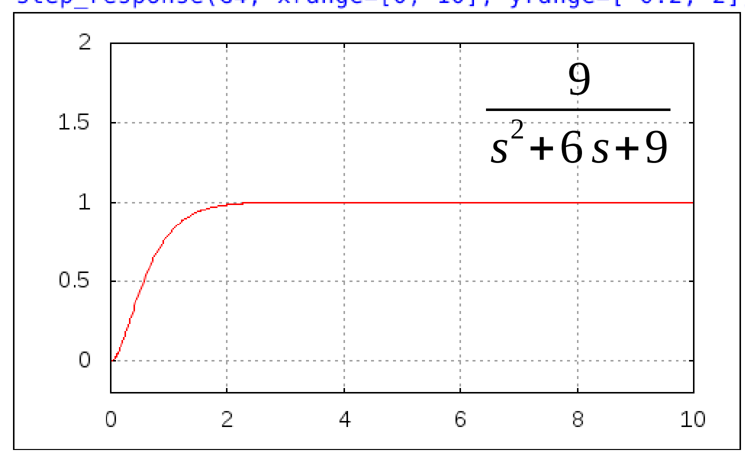
(%o40)

(%i41) `step_response(G3, xrange=[0, 10], yrange=[-0.2, 2]);`

(%t41)


(%o41)

(%i42) `step_response(G4, xrange=[0, 10], yrange=[-0.2, 2]);`

(%t42)


(%o42)

2^{nd} Order Transfer Function: Standard Form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 - \omega_n^2}$$

$$= -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}\,\omega_n$$

$$= -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\,\omega_n$$

$$s = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}\,\omega_n \qquad \zeta > 1$$

$$s = -\omega_n \qquad \zeta = 1$$

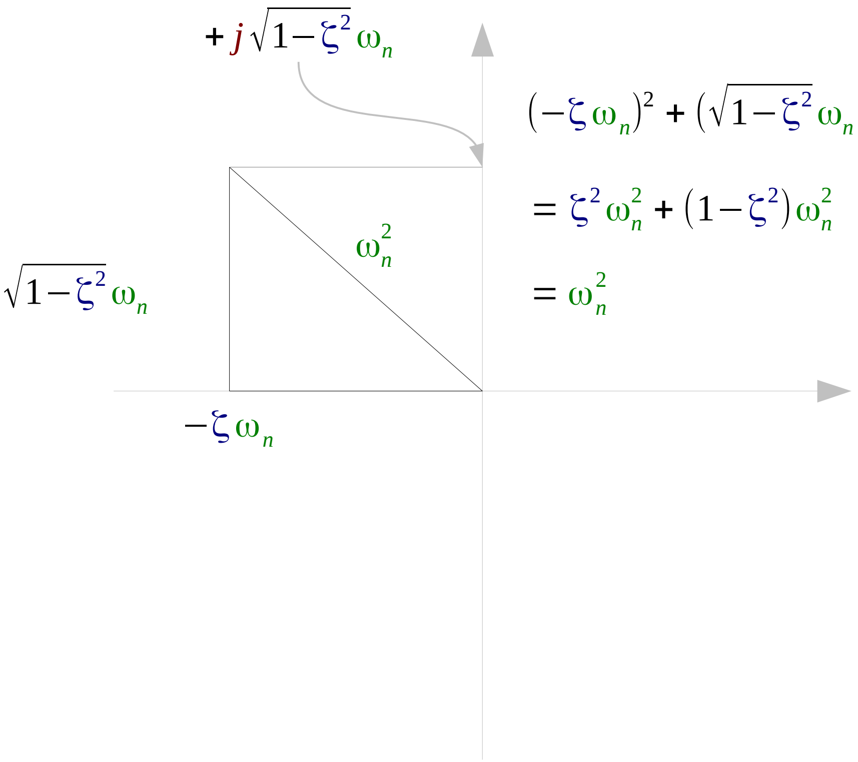
$$s = -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\,\omega_n \qquad 0 < \zeta < 1$$

$$s = \pm j\omega_n \qquad \zeta = 0$$

2^{nd} Order Transfer Function: Standard Form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$



$$(-\zeta\omega_n)^2 + \left(\sqrt{1-\zeta^2}\,\omega_n\right)^2$$

$$= \zeta^2\omega_n^2 + (1-\zeta^2)\omega_n^2$$

$$= \omega_n^2$$

$s = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}\,\omega_n$	$\zeta > 1$
$s = -\omega_n$	$\zeta = 1$
$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n$	$0 < \zeta < 1$
$s = \pm j\omega_n$	$\zeta = 0$

2nd Order Transfer Function: Standard Form

$$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\,\omega_n \qquad 0 < \zeta < 1$$

$\zeta = 0.1, \quad \omega_n = 200$ $\qquad s^2+4s+20\sqrt{0.99}$

$\zeta = 0.2, \quad \omega_n = 100$ $\qquad s^2+4s+10\sqrt{0.96}$

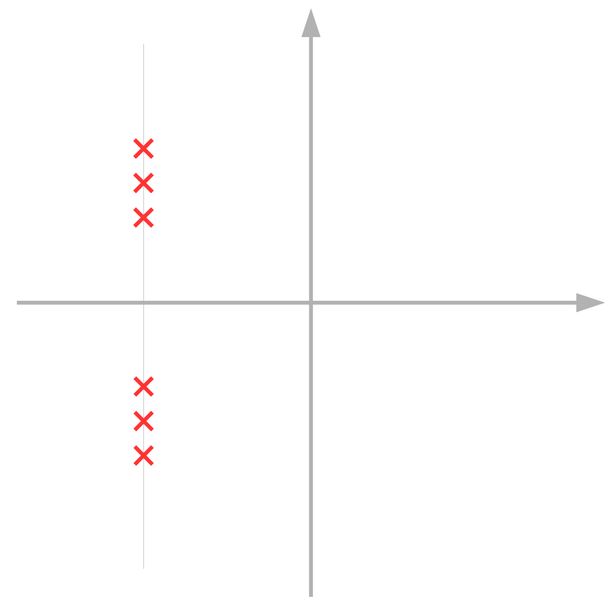
$\zeta = 0.4, \quad \omega_n = 50$ $\qquad s^2+4s+5\sqrt{0.84}$

Standard Form: varying a (1)

```
(%i78) a : 1;
       b : 2;
       'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
       ode2(%, y, t);
       ic2(%, t=0, y=0, 'diff(y, t)=0);


       a : 1;
       b : 4;
       'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
       ode2(%, y, t);
       ic2(%, t=0, y=0, 'diff(y, t)=0);


       a : 1;
       b : 6;
       'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
       ode2(%, y, t);
       ic2(%, t=0, y=0, 'diff(y, t)=0);
```

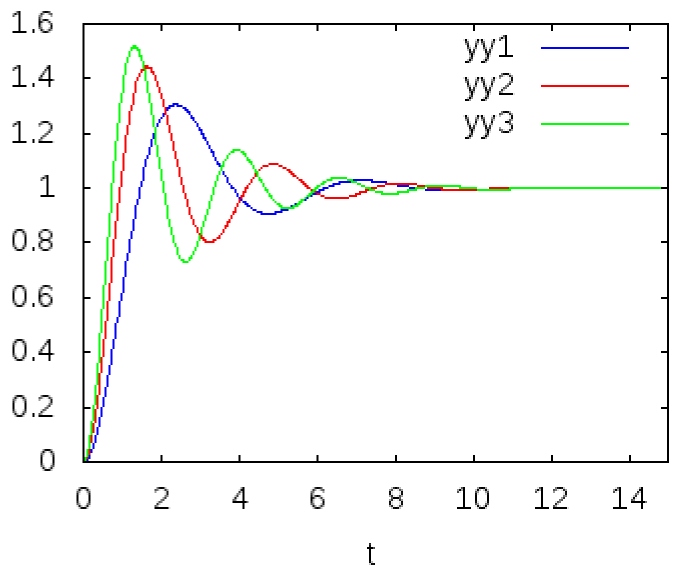


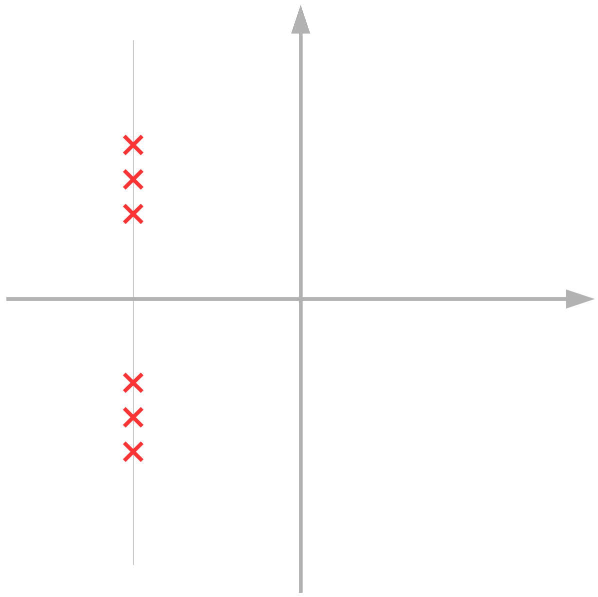
(%o97) $yy1(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{7}\,t}{2}\right)}{\sqrt{7}} - \cos\left(\frac{\sqrt{7}\,t}{2}\right) \right) + 1$

(%o98) $yy2(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{15}\,t}{2}\right)}{\sqrt{15}} - \cos\left(\frac{\sqrt{15}\,t}{2}\right) \right) + 1$

(%o99) $yy3(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}\,t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}\,t}{2}\right) \right) + 1$

Standard Form: varying a (2)





(%o97) $yy1(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{7}\,t}{2}\right)}{\sqrt{7}} - \cos\left(\frac{\sqrt{7}\,t}{2}\right) \right) + 1$

(%o98) $yy2(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{15}\,t}{2}\right)}{\sqrt{15}} - \cos\left(\frac{\sqrt{15}\,t}{2}\right) \right) + 1$

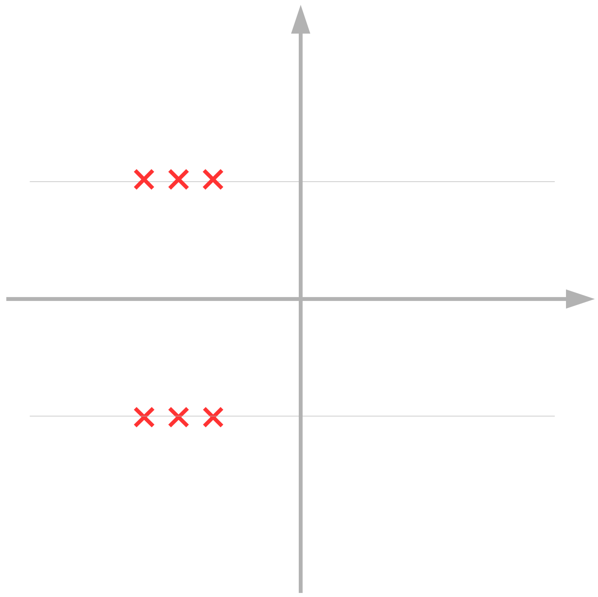
(%o99) $yy3(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}\,t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}\,t}{2}\right) \right) + 1$

Standard Form: varying b (1)

```
(%i1) a : 1;
      b : 6;
      'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);


      a : 2;
      b : 6;
      'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);


      a : 3;
      b : 6;
      'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);
```

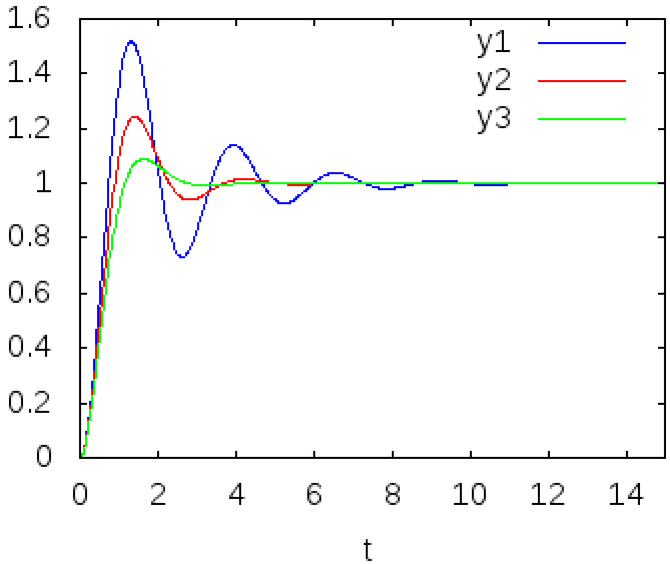


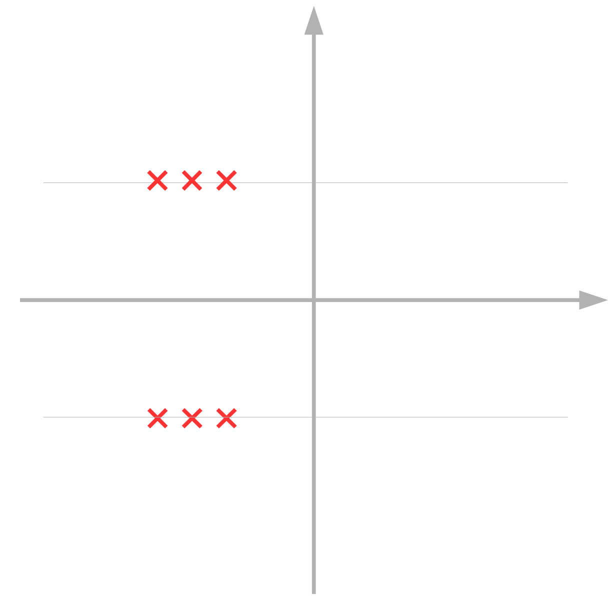
(%o16) $$y1(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}\,t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}\,t}{2}\right) \right) + 1$$

(%o17) $$y2(t) := \%e^{-t} \left(\frac{-\sin\left(\sqrt{5}\,t\right)}{\sqrt{5}} - \cos\left(\sqrt{5}\,t\right) \right) + 1$$

(%o18) $$y3(t) := \%e^{\frac{-3\,t}{2}} \left(\frac{-\sqrt{15}\sin\left(\frac{\sqrt{15}\,t}{2}\right)}{5} - \cos\left(\frac{\sqrt{15}\,t}{2}\right) \right) + 1$$

Standard Form: varying b (2)





(%o16) $y1(t) := \%e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}\, t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}\, t}{2}\right) \right) + 1$

(%o17) $y2(t) := \%e^{-t} \left(\frac{-\sin\left(\sqrt{5}\, t\right)}{\sqrt{5}} - \cos\left(\sqrt{5}\, t\right) \right) + 1$

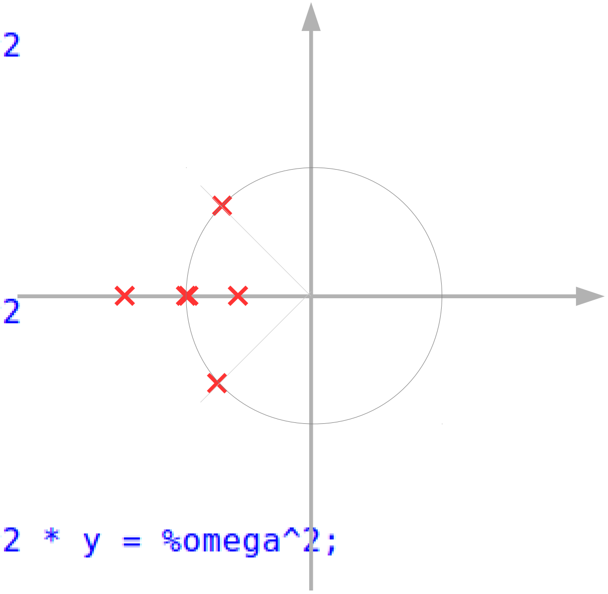
(%o18) $y3(t) := \%e^{\frac{-3\, t}{2}} \left(\frac{-\sqrt{15}\, \sin\left(\frac{\sqrt{15}\, t}{2}\right)}{5} - \cos\left(\frac{\sqrt{15}\, t}{2}\right) \right) + 1$

Standard Form: varying zeta (1)

```
(%i1) %zeta : 0.5;
      %omega : 6;
      'diff(y, t, 2)+ 2**%zeta**%omega * 'diff(y, t, 1) + %omega^2
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);


      %zeta : 1;
      %omega : 6;
      'diff(y, t, 2)+ 2**%zeta**%omega * 'diff(y, t, 1) + %omega^2
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);

      %zeta : 2;
      %omega : 6;
      'diff(y, t, 2)+ 2**%zeta**%omega * 'diff(y, t, 1) + %omega^2 * y = %omega^2;
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);
```

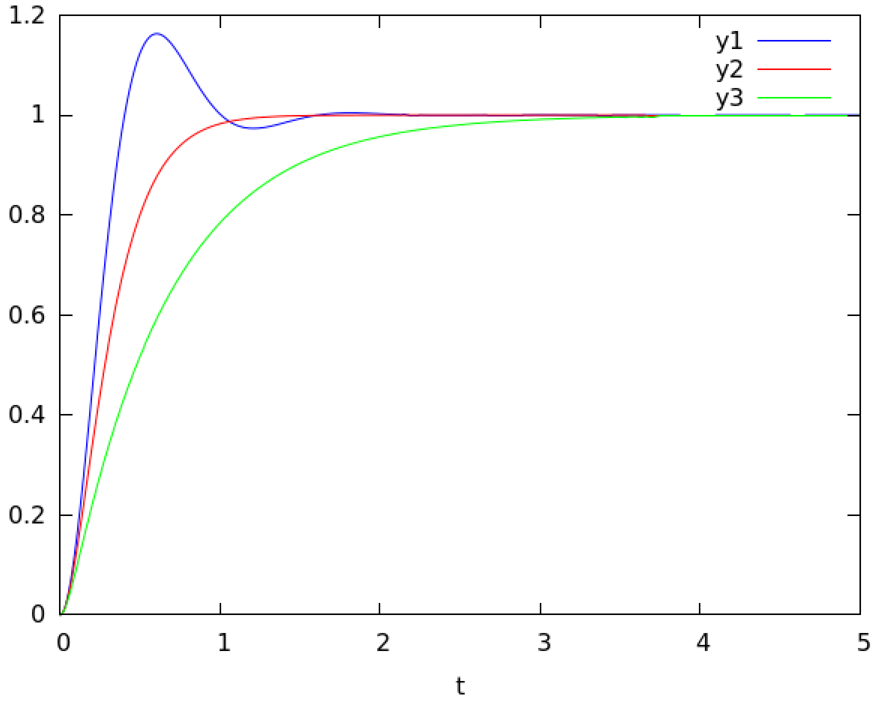


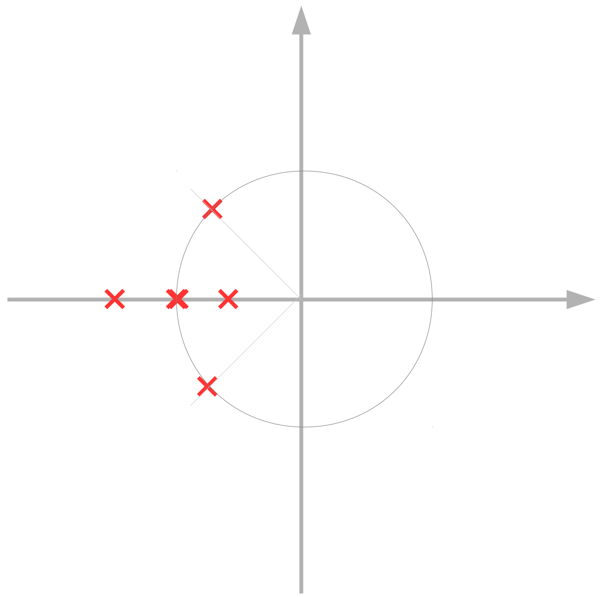
$$y1(t) := \%e^{(-3)\,t}\left(\frac{-\sin\left(3^{3/2}\,t\right)}{\sqrt{3}} - \cos\left(3^{3/2}\,t\right)\right) + 1$$

$$y2(t) := \left(\left(-6\right)t - 1\right)\%e^{(-6)\,t} + 1$$

$$y3(t) := \frac{-\left(2\sqrt{3}+3\right)\%e^{\frac{\left(4\,3^{3/2}-24\right)t}{2}}}{6} + \frac{\left(2\sqrt{3}-3\right)\%e^{\frac{\left(\left(-4\right)3^{3/2}-24\right)t}{2}}}{6} + 1$$

Standard Form: varying zeta (2)





$$y1(t) := \%e^{(-3)\,t}\left(\frac{-\sin\left(3^{3/2}\,t\right)}{\sqrt{3}} - \cos\left(3^{3/2}\,t\right)\right) + 1$$

$$y2(t) := ((-6)\,t - 1)\,\%e^{(-6)\,t} + 1$$

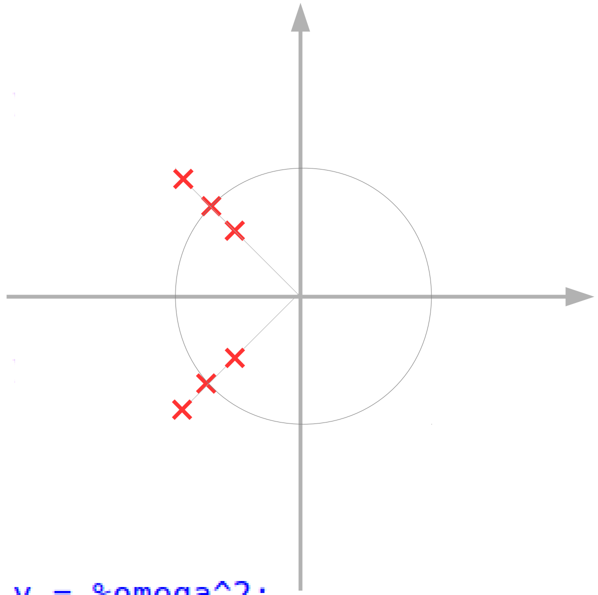
$$y3(t) := \frac{-\left(2\sqrt{3}+3\right)\%e^{\frac{\left(4\,3^{3/2}-24\right)t}{2}}}{6} + \frac{\left(2\sqrt{3}-3\right)\%e^{\frac{\left((-4)\,3^{3/2}-24\right)t}{2}}}{6} + 1$$

Standard Form: varying omega (1)

```
%zeta : 0.5;
%omega : 2;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 *
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);


%zeta : 0.5;
%omega : 4;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 *
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);

%zeta : 0.5;
%omega : 6;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 * y = %omega^2;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```

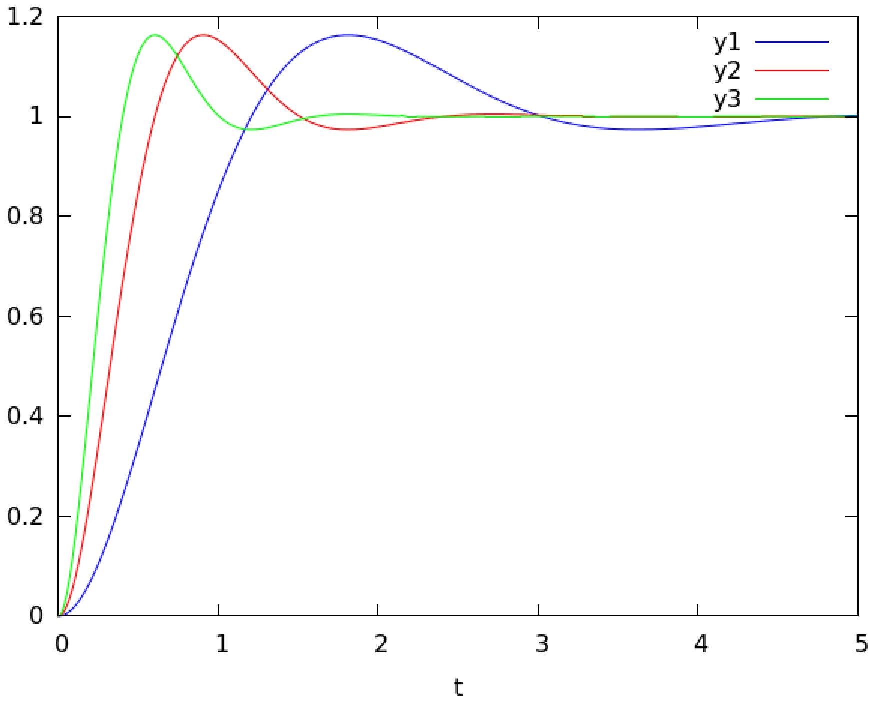


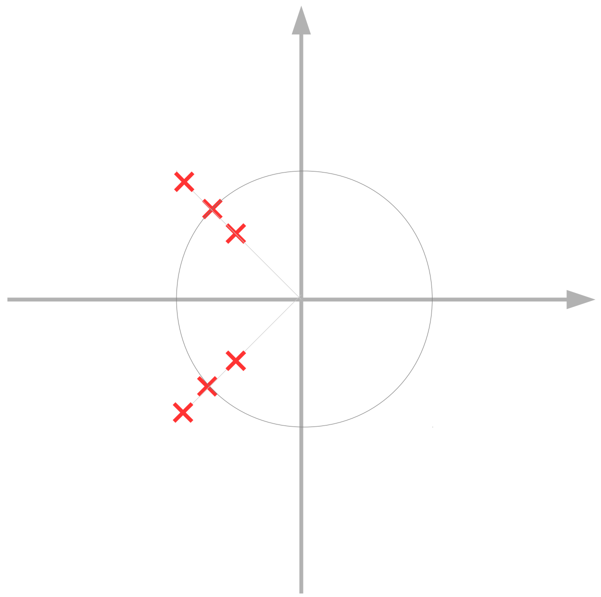
$$y1(t) := \%e^{-t}\left(\frac{-\sin(\sqrt{3}\,t)}{\sqrt{3}} - \cos(\sqrt{3}\,t)\right)+1$$

$$y2(t) := \%e^{(-2)\,t}\left(\frac{-\sin(2\sqrt{3}\,t)}{\sqrt{3}} - \cos(2\sqrt{3}\,t)\right)+1$$

$$y3(t) := \%e^{(-3)\,t}\left(\frac{-\sin(3^{3/2}\,t)}{\sqrt{3}} - \cos(3^{3/2}\,t)\right)+1$$

Standard Form: varying omega (2)





$$y1(t):=\%e^{-t}\left(\frac{-\sin\left(\sqrt{3}\,t\right)}{\sqrt{3}}-\cos\left(\sqrt{3}\,t\right)\right)+1$$

$$y2(t):=\%e^{(-2)\,t}\left(\frac{-\sin\left(2\sqrt{3}\,t\right)}{\sqrt{3}}-\cos\left(2\sqrt{3}\,t\right)\right)+1$$

$$y3(t):=\%e^{(-3)\,t}\left(\frac{-\sin\left(3^{3/2}\,t\right)}{\sqrt{3}}-\cos\left(3^{3/2}\,t\right)\right)+1$$

References

[1] http://en.wikipedia.org/
[2] M.L. Boas, "Mathematical Methods in the Physical Sciences"
[3] E. Kreyszig, "Advanced Engineering Mathematics"
[4] D. G. Zill, W. S. Wright, "Advanced Engineering Mathematics"