

Time Domain Analysis (1A)

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2nd Order Systems

$$\frac{9}{s^2+9s+9}$$

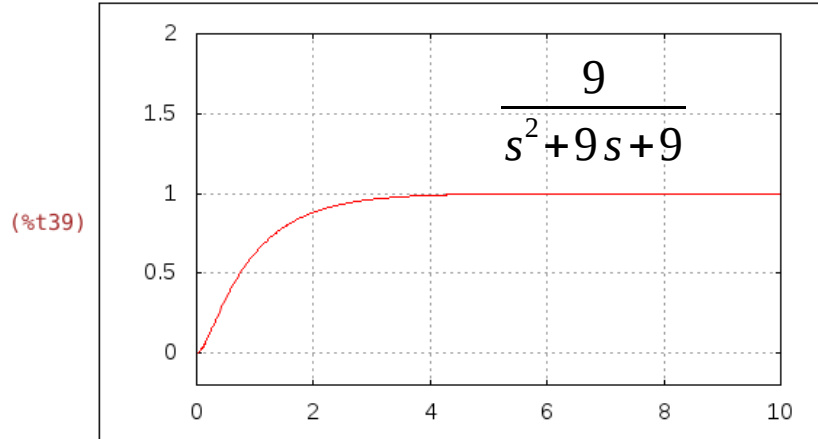
$$\frac{9}{s^2+2s+9}$$

$$\frac{9}{s^2+9}$$

$$\frac{9}{s^2+6s+9}$$

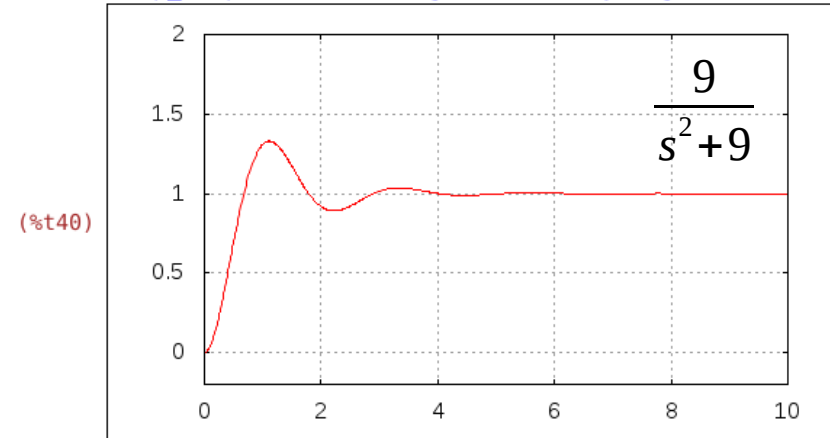
Step Responses

```
(%i39) step_response(G1, xrange=[0, 10], yrange=[-0.2, 2]);
```



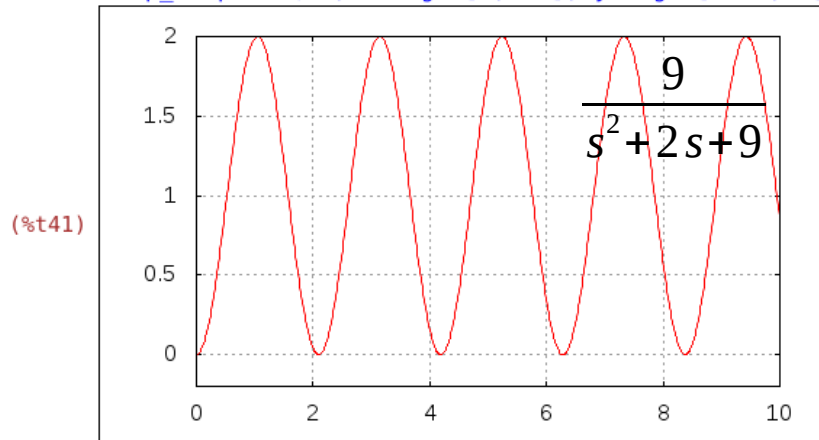
(%o39)

```
(%i40) step_response(G2, xrange=[0, 10], yrange=[-0.2, 2]);
```



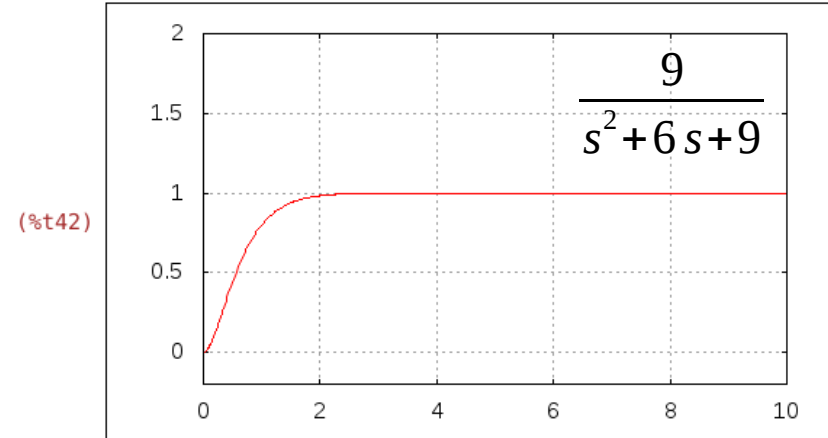
(%o40)

```
(%i41) step_response(G3, xrange=[0, 10], yrange=[-0.2, 2]);
```



(%o41)

```
(%i42) step_response(G4, xrange=[0, 10], yrange=[-0.2, 2]);
```



(%o42)

2nd Order Transfer Function: Standard Form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 - \omega_n^2}$$

$$= -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}\omega_n$$

$$= -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\omega_n$$

$$s = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}\omega_n \quad \zeta > 1$$

$$s = -\omega_n \quad \zeta = 1$$

$$s = -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\omega_n \quad 0 < \zeta < 1$$

$$s = \pm j\omega_n \quad \zeta = 0$$

2nd Order Transfer Function: Standard Form

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

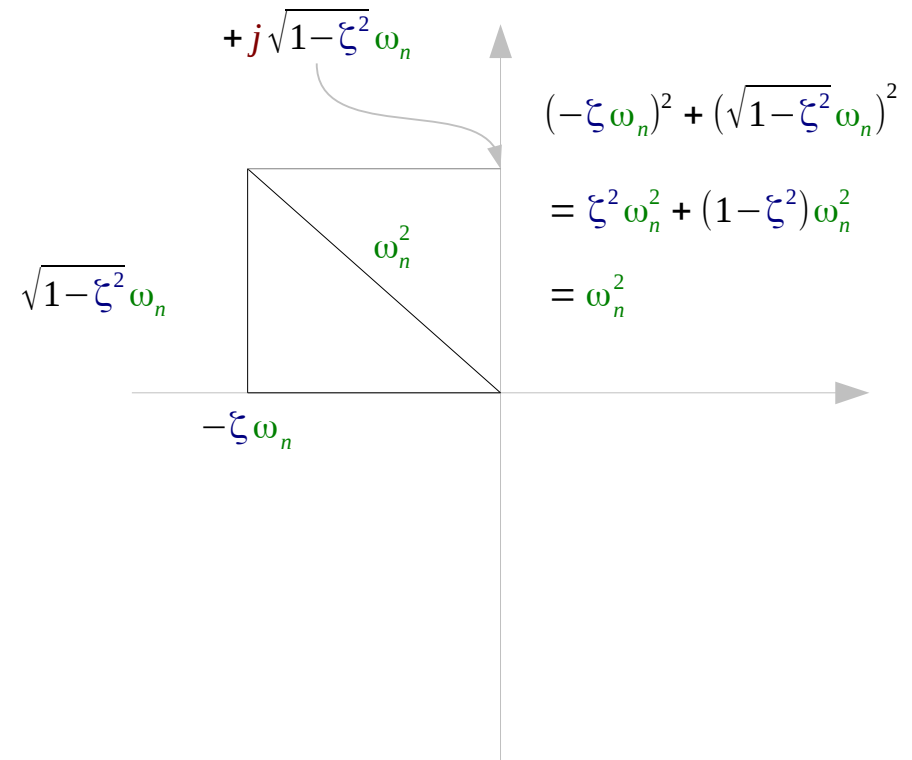
$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$s = -\zeta\omega_n \pm \sqrt{\zeta^2 - 1}\omega_n \quad \zeta > 1$$

$$s = -\omega_n \quad \zeta = 1$$

$$s = -\zeta\omega_n \pm j\sqrt{1 - \zeta^2}\omega_n \quad 0 < \zeta < 1$$

$$s = \pm j\omega_n \quad \zeta = 0$$



2nd Order Transfer Function: Standard Form

$$s = -\zeta\omega_n \pm j\sqrt{1-\zeta^2}\omega_n \quad 0 < \zeta < 1$$

$$\zeta = 0.1, \quad \omega_n = 200 \quad s^2 + 4s + 20\sqrt{0.99}$$

$$\zeta = 0.2, \quad \omega_n = 100 \quad s^2 + 4s + 10\sqrt{0.96}$$

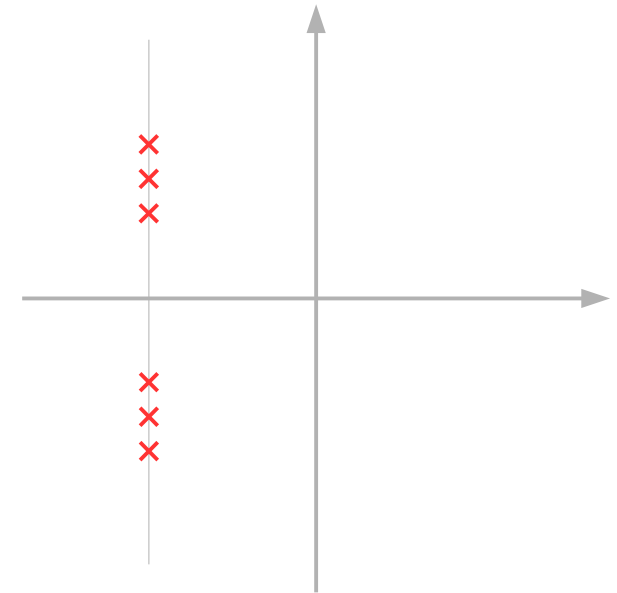
$$\zeta = 0.4, \quad \omega_n = 50 \quad s^2 + 4s + 5\sqrt{0.84}$$

Standard Form: varying a (1)

```
(%i78) a : 1;
      b : 2;
      'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);
```

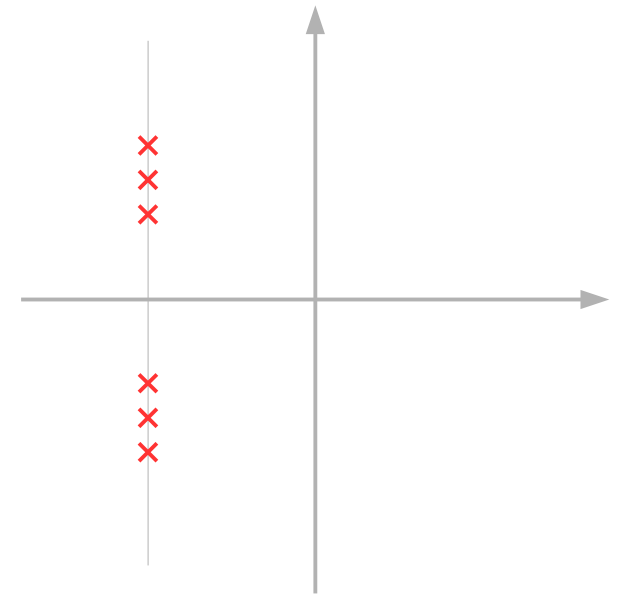
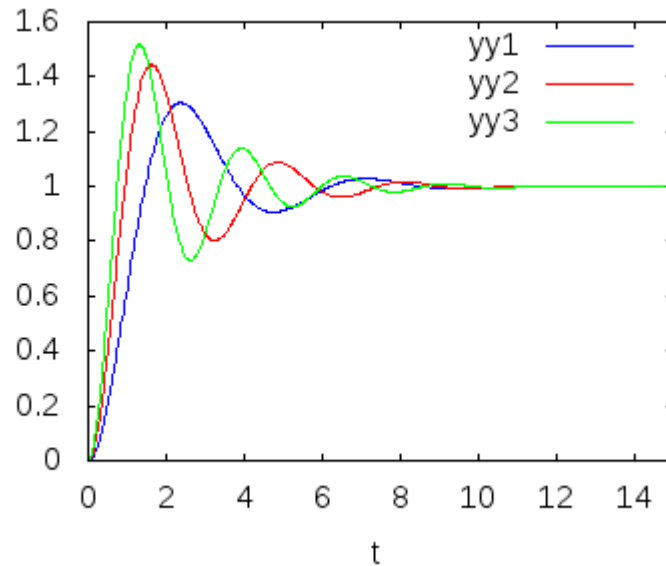
```
a : 1;
b : 4;
'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```

```
a : 1;
b : 6;
'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```



$$\begin{aligned}
 (\%097) \quad yy1(t) &:= e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{7}t}{2}\right)}{\sqrt{7}} - \cos\left(\frac{\sqrt{7}t}{2}\right) \right) + 1 \\
 (\%098) \quad yy2(t) &:= e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{15}t}{2}\right)}{\sqrt{15}} - \cos\left(\frac{\sqrt{15}t}{2}\right) \right) + 1 \\
 (\%099) \quad yy3(t) &:= e^{\frac{-t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}t}{2}\right) \right) + 1
 \end{aligned}$$

Standard Form: varying a (2)



$$(\%097) \text{ yy1}(t) := e^{-\frac{t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{7}t}{2}\right)}{\sqrt{7}} - \cos\left(\frac{\sqrt{7}t}{2}\right) \right) + 1$$

$$(\%098) \text{ yy2}(t) := e^{-\frac{t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{15}t}{2}\right)}{\sqrt{15}} - \cos\left(\frac{\sqrt{15}t}{2}\right) \right) + 1$$

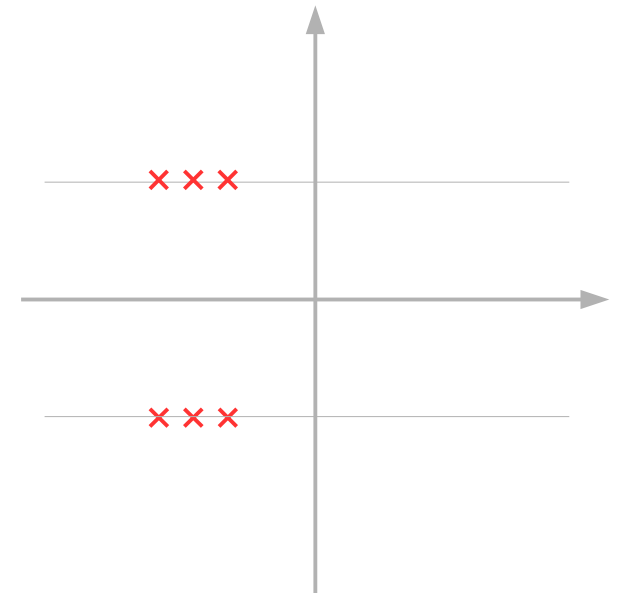
$$(\%099) \text{ yy3}(t) := e^{-\frac{t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}t}{2}\right) \right) + 1$$

Standard Form: varying b (1)

```
(%i1) a : 1;
      b : 6;
      'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);
```

```
a : 2;
b : 6;
'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```

```
a : 3;
b : 6;
'diff(y, t, 2) + a* 'diff(y, t, 1) + b * y = b;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```

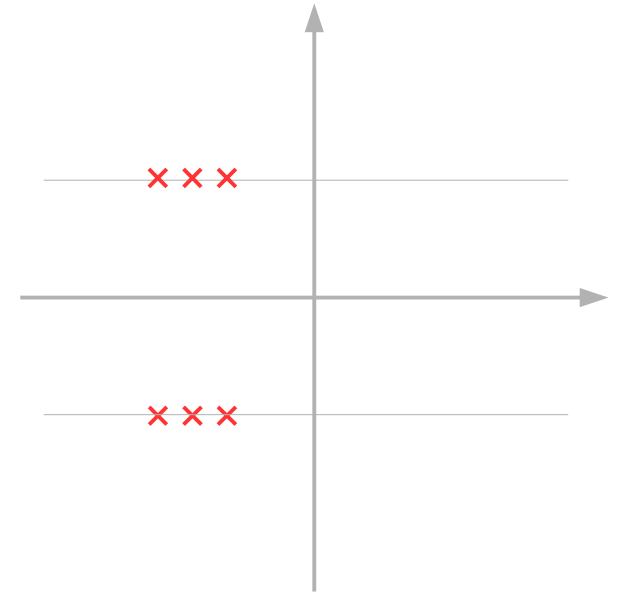
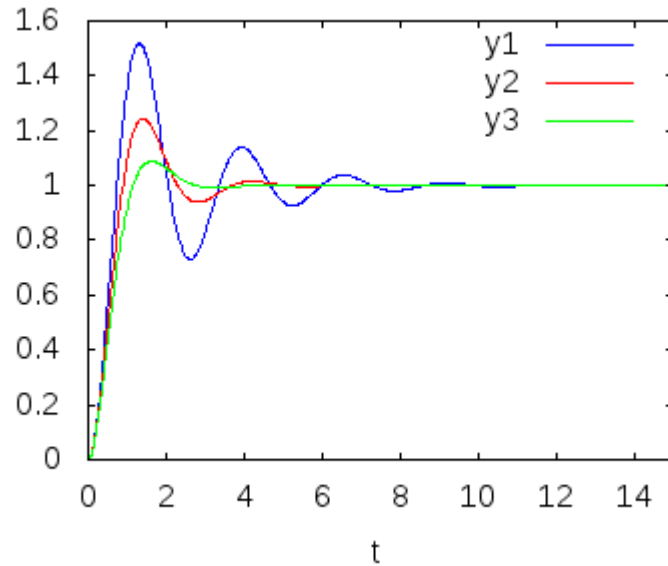


$$(\%o16) \quad y_1(t) := e^{-\frac{t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}t}{2}\right) \right) + 1$$

$$(\%o17) \quad y_2(t) := e^{-t} \left(\frac{-\sin(\sqrt{5}t)}{\sqrt{5}} - \cos(\sqrt{5}t) \right) + 1$$

$$(\%o18) \quad y_3(t) := e^{-\frac{3t}{2}} \left(\frac{-\sqrt{15} \sin\left(\frac{\sqrt{15}t}{2}\right)}{5} - \cos\left(\frac{\sqrt{15}t}{2}\right) \right) + 1$$

Standard Form: varying b (2)



$$(\%016) \quad y1(t) := \%e^{-\frac{t}{2}} \left(\frac{-\sin\left(\frac{\sqrt{23}t}{2}\right)}{\sqrt{23}} - \cos\left(\frac{\sqrt{23}t}{2}\right) \right) + 1$$

$$(\%017) \quad y2(t) := \%e^{-t} \left(\frac{-\sin(\sqrt{5}t)}{\sqrt{5}} - \cos(\sqrt{5}t) \right) + 1$$

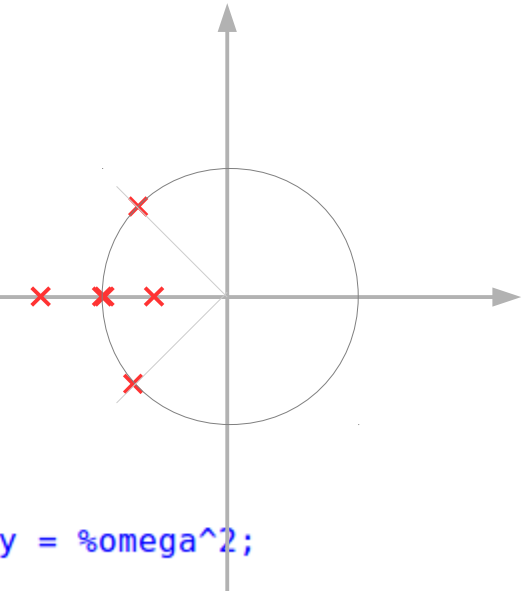
$$(\%018) \quad y3(t) := \%e^{-\frac{3t}{2}} \left(\frac{-\sqrt{15} \sin\left(\frac{\sqrt{15}t}{2}\right)}{5} - \cos\left(\frac{\sqrt{15}t}{2}\right) \right) + 1$$

Standard Form: varying zeta (1)

```
(%i1) %zeta : 0.5;
      %omega : 6;
      'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2
      ode2(%, y, t);
      ic2(%, t=0, y=0, 'diff(y, t)=0);
```

```
%zeta : 1;
%omega : 6;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```

```
%zeta : 2;
%omega : 6;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 * y = %omega^2;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```

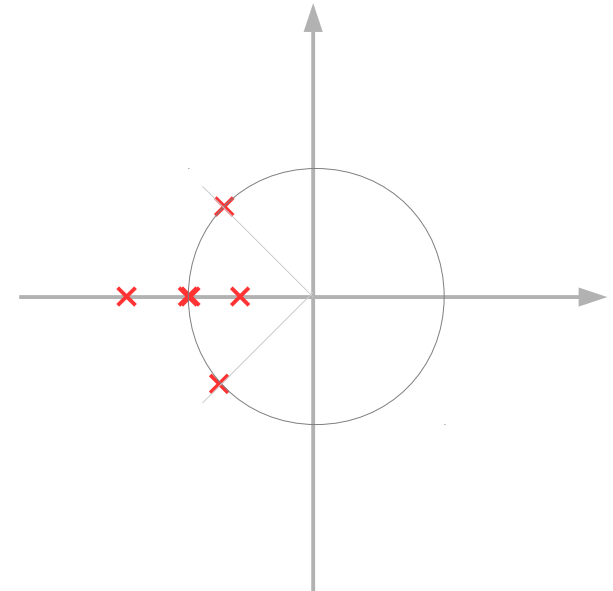
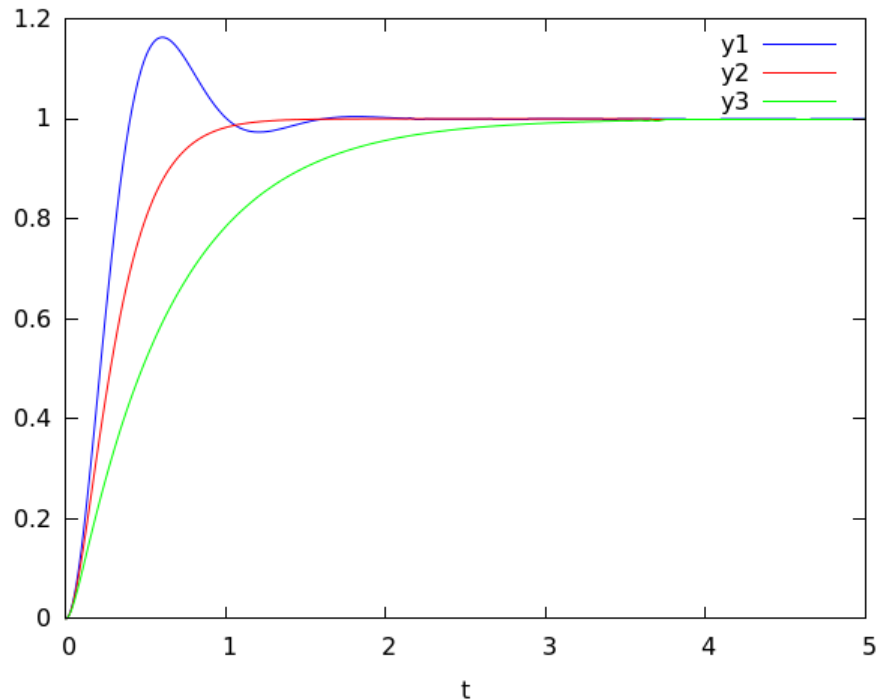


$$y_1(t) := e^{(-3)t} \left(\frac{-\sin(3^{3/2}t)}{\sqrt{3}} - \cos(3^{3/2}t) \right) + 1$$

$$y_2(t) := ((-6)t - 1) e^{(-6)t} + 1$$

$$y_3(t) := \frac{-(2\sqrt{3}+3) e^{\frac{(4 \cdot 3^{3/2} - 24)t}{2}}}{6} + \frac{(2\sqrt{3}-3) e^{\frac{((-4) \cdot 3^{3/2} - 24)t}{2}}}{6} + 1$$

Standard Form: varying zeta (2)



$$y_1(t) := e^{(-3)t} \left(\frac{-\sin(3^{3/2}t)}{\sqrt{3}} - \cos(3^{3/2}t) \right) + 1$$

$$y_2(t) := ((-6)t - 1) e^{(-6)t} + 1$$

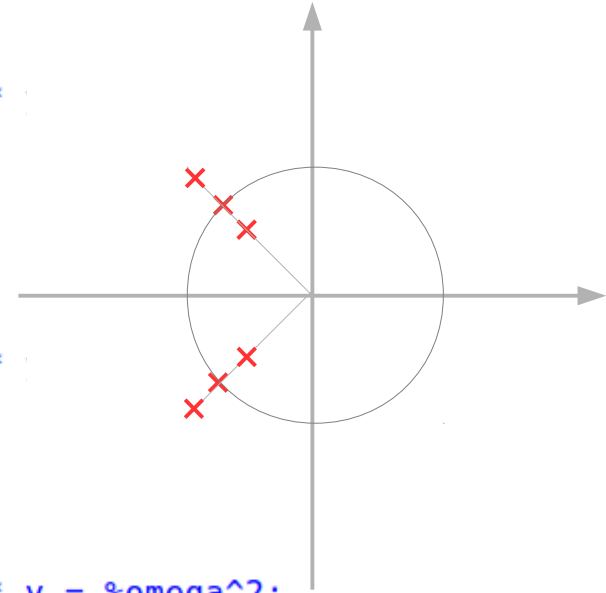
$$y_3(t) := \frac{-(2\sqrt{3}+3) e^{\frac{(4 \cdot 3^{3/2} - 24)t}{2}}}{6} + \frac{(2\sqrt{3}-3) e^{\frac{((-4) \cdot 3^{3/2} - 24)t}{2}}}{6} + 1$$

Standard Form: varying omega (1)

```
%zeta : 0.5;
%omega : 2;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 *
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);

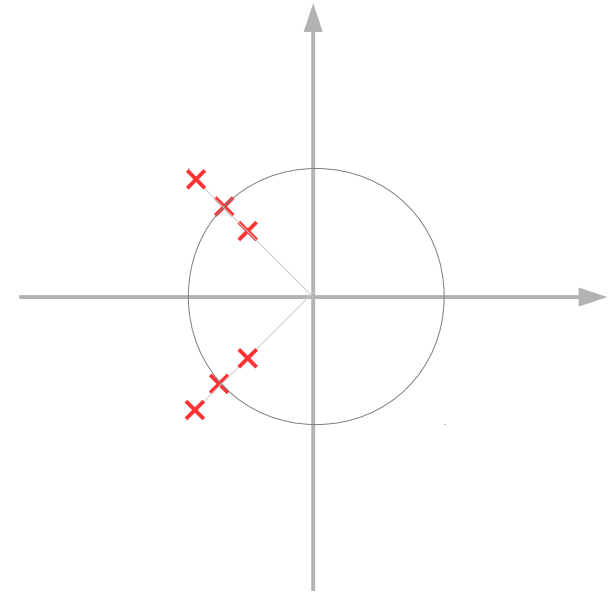
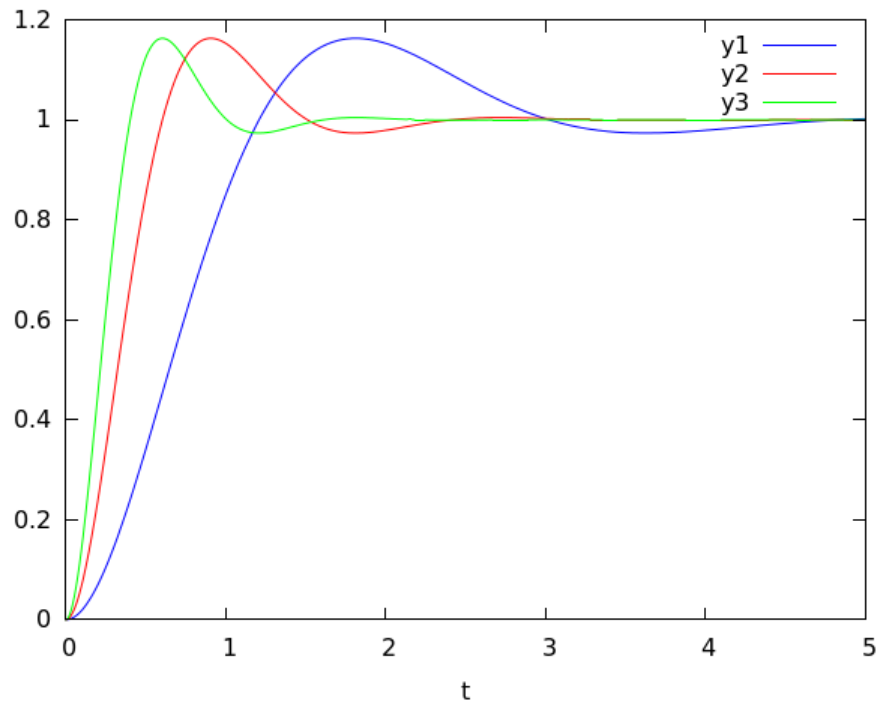
%zeta : 0.5;
%omega : 4;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 *
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);

%zeta : 0.5;
%omega : 6;
'diff(y, t, 2)+ 2*%zeta*%omega * 'diff(y, t, 1) + %omega^2 * y = %omega^2;
ode2(%, y, t);
ic2(%, t=0, y=0, 'diff(y, t)=0);
```



$$y_1(t) := e^{-t} \left(\frac{-\sin(\sqrt{3} t)}{\sqrt{3}} - \cos(\sqrt{3} t) \right) + 1$$
$$y_2(t) := e^{(-2)t} \left(\frac{-\sin(2\sqrt{3} t)}{\sqrt{3}} - \cos(2\sqrt{3} t) \right) + 1$$
$$y_3(t) := e^{(-3)t} \left(\frac{-\sin(3^{3/2} t)}{\sqrt{3}} - \cos(3^{3/2} t) \right) + 1$$

Standard Form: varying ω (2)



$$y_1(t) := e^{-t} \left(\frac{-\sin(\sqrt{3} t)}{\sqrt{3}} - \cos(\sqrt{3} t) \right) + 1$$
$$y_2(t) := e^{(-2)t} \left(\frac{-\sin(2\sqrt{3} t)}{\sqrt{3}} - \cos(2\sqrt{3} t) \right) + 1$$
$$y_3(t) := e^{(-3)t} \left(\frac{-\sin(3^{3/2} t)}{\sqrt{3}} - \cos(3^{3/2} t) \right) + 1$$

References

- [1] <http://en.wikipedia.org/>
- [2] M.L. Boas, "Mathematical Methods in the Physical Sciences"
- [3] E. Kreyszig, "Advanced Engineering Mathematics"
- [4] D. G. Zill, W. S. Wright, "Advanced Engineering Mathematics"