

Power Density Spectrum - Discrete Time

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Bilateral z-Transform of $R_{XX}[n]$

N Gaussian random variables

Definition

$$S_{XX}(z) = \sum_{n=-\infty}^{\infty} R_{XX}[n]z^{-n}$$

Discrete Time Fourier Transform of $R_{XX}[n]$

N Gaussian random variables

Definition

$$S_{XX}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} R_{XX}[n] e^{-jn\Omega}$$

$$R_{XX}[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{XX}(e^{j\Omega}) e^{jn\Omega} d\Omega$$

Properties of Power Density Spectrum - DT

N Gaussian random variables

- 1 $S_{XX}(e^{j\Omega}) \geq 0$
- 2 $S_{XX}(e^{-j\Omega}) = S_{XX}(e^{+j\Omega})$ for real $X[n]$
- 3 $S_{XX}(e^{+j\Omega})$ is real
- 4 $\frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{XX}(e^{j\Omega}) d\Omega = E[X^2[n]]$

Estimating the Power Density Spectrum

N Gaussian random variables

Definition

$$\hat{R}_N[k] = \frac{1}{N} \sum_{n=0}^{N-1-|k|} X[n]X[n+|k|] \quad |k| < N$$

DTFT, FFT

N Gaussian random variables

Definition

$$X_N(\Omega_k) = \sum_{n=0}^{N-1} X[n] e^{-j\Omega_k n} \quad k = 0, 1, \dots, N-1$$

$$\Omega_k = \frac{2\pi k}{N} \quad k = 0, 1, \dots, N-1$$

Periodogram

N Gaussian random variables

Definition

Periodogram : the estimate of the power density spectrum

$$S_N(\Omega_k) = \frac{1}{N} |X_N(\Omega_k)|^2 \quad k = 0, 1, \dots, N-1$$

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$$\lim_{N \rightarrow \infty} E[S_N(\Omega_k)] = S_{XX}(\Omega_k) \quad k = 0, 1, \dots, N-1$$

