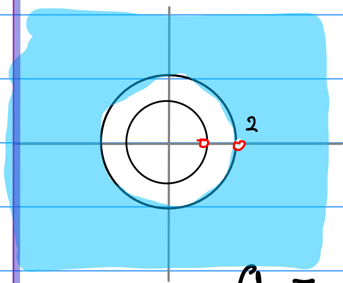
Laurent Series and z-Transform Examples case 4.B

20171012



$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1-\left(\frac{1}{2}\right)^{n+1} & (n<0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n - \sum_{n=-1}^{-\infty} 2^{-n-1} z^n$$

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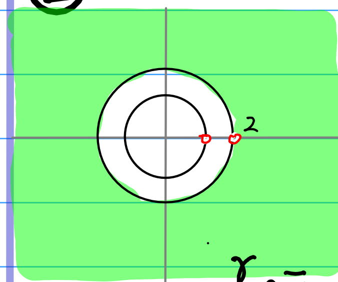
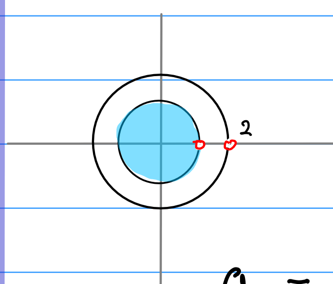


$$x_n = \begin{cases} 1-2^{n-1} & (n>0) \\ 0 & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} z^{-n}$$



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1}-1 & (n \geq 0) \\ 0 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n - \sum_{n=0}^{\infty} 1 \cdot z^n$$

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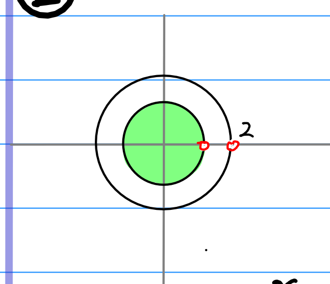
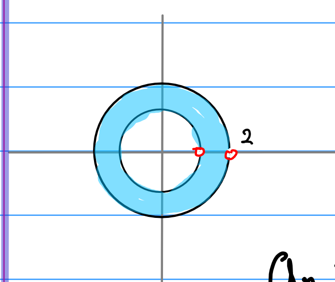


$$x_n = \begin{cases} 0 & (n>0) \\ 2^{n-1}-1 & (n \leq 0) \end{cases}$$

$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} - \sum_{n=0}^{-\infty} 1 \cdot z^{-n}$$



$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n<0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} 1 \cdot z^n + \sum_{n=0}^{\infty} 2^{-n-1} \cdot z^n$$

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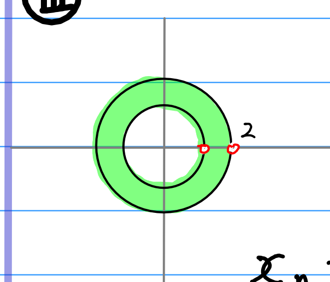
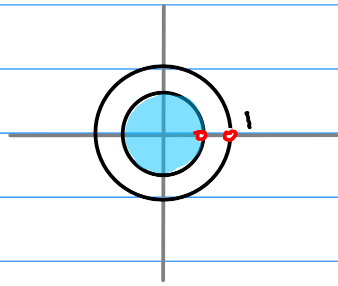
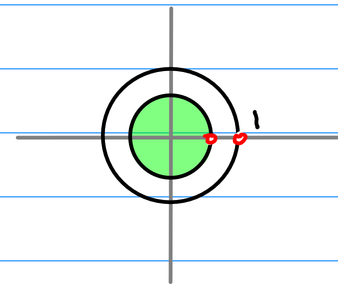


$$x_n = \begin{cases} 1 & (n>0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

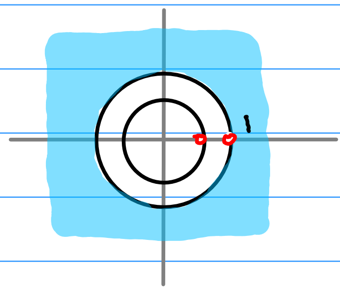
$$X(z) = \sum_{n=0}^{-\infty} 2^{n-1} \cdot z^{-n} + \sum_{n=1}^{\infty} 1 \cdot z^{-n}$$

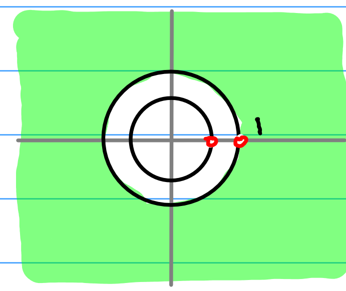
4.B $X(z) = \dfrac{-0.5\,z^2}{(z-1)(z-0.5)} = f(z) = \dfrac{-0.5\,z^2}{(z-1)(z-0.5)}$



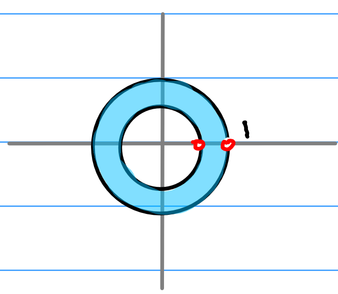


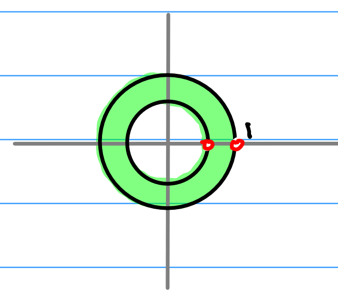
$$\sum_{n=1}^{\infty} [1 - 2^{n-1}]\, z^{n} \xleftarrow{-n} \sum_{n=-1}^{\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$





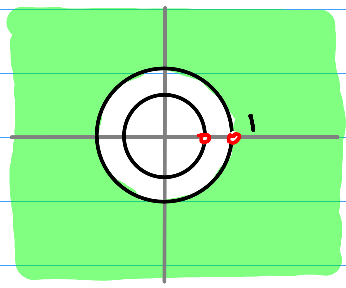
$$\sum_{n=-1}^{-\infty} [2^{n-1} - 1]\, z^{n} \xleftarrow{-n} \sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$





$$+\sum_{n=1}^{\infty} z^{n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{n} \xleftarrow{-n} +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5\,z}{z-0.5}$$

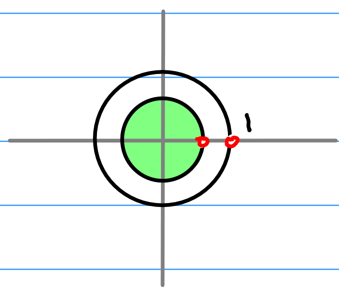


$$-\frac{(1)}{1-(\frac{1}{z})} + \frac{(\frac{1}{2})}{1-(\frac{1}{2z})}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= -\sum_{n=0}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= \sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$

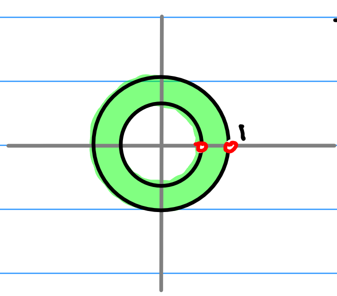


$$+\frac{(\frac{z}{1})}{1-(\frac{z}{1})} - \frac{(\frac{z}{1})}{1-(\frac{2z}{1})}$$

$$= +\sum_{n=0}^{\infty}(z)(z)^n - \sum_{n=0}^{\infty}(z)(2z)^n$$

$$= \sum_{n=0}^{\infty}\left[1 - 2^n\right] z^{n+1}$$

$$= \sum_{n=-1}^{-\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$



$$\frac{-z}{z-1} + \frac{0.5\,z}{z-0.5} \qquad -\frac{(1)}{1-(\frac{1}{z})} - \frac{(\frac{z}{1})}{1-(\frac{2z}{1})}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n$$

$$= -\sum_{n=0}^{\infty} z^{-n} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= -\sum_{n=0}^{\infty} z^{-n} - \sum_{n=-1}^{\infty} 2^{-n-1} z^{-n}$$