

Digital Signal Octave Codes (0B)

- Aliasing and Folding Frequencies

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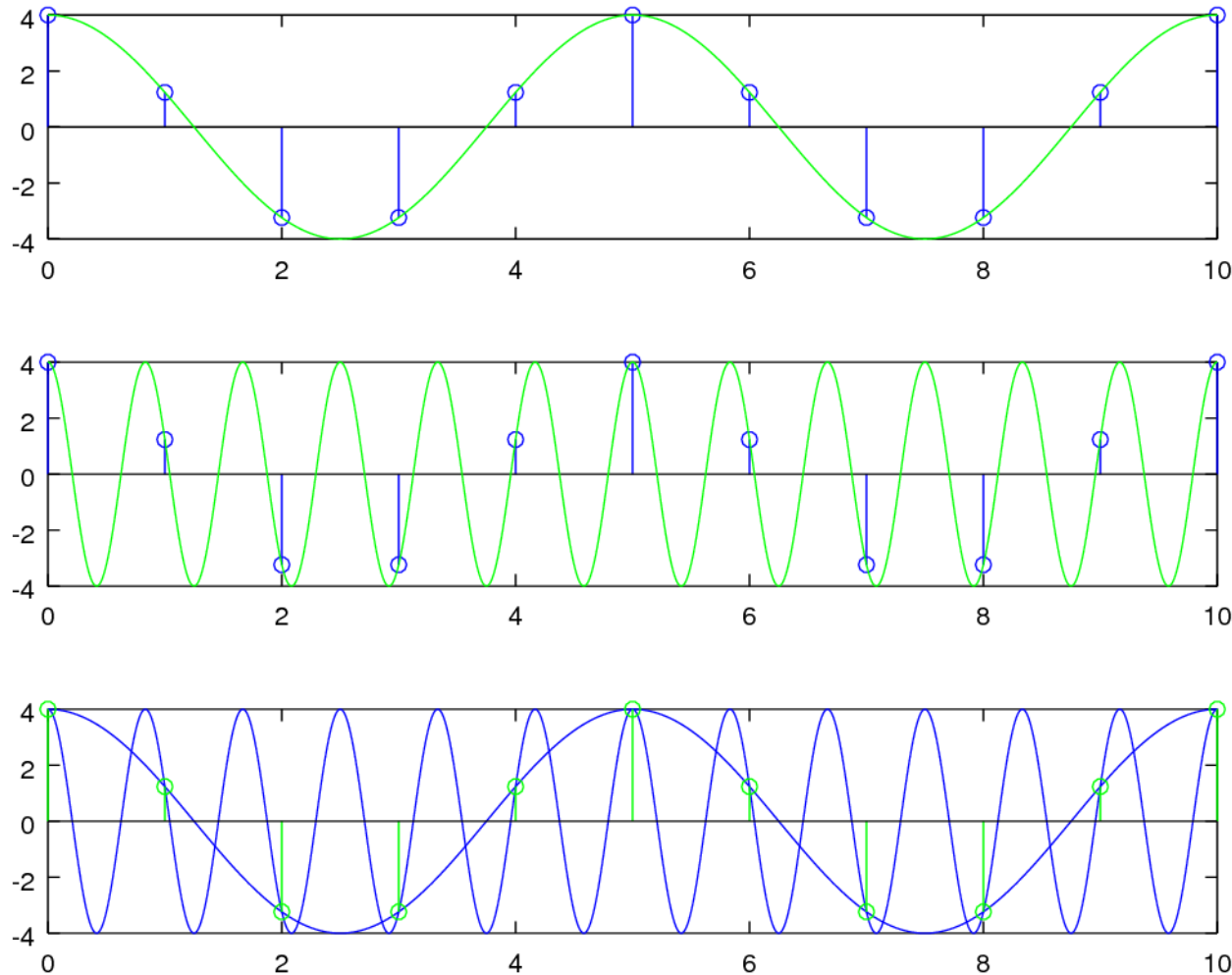
Based on

M.J. Roberts, Fundamentals of Signals and Systems

S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed

S.D. Stearns, Digital Signal Processing with Examples in MATLAB

Aliasing Condition Examples

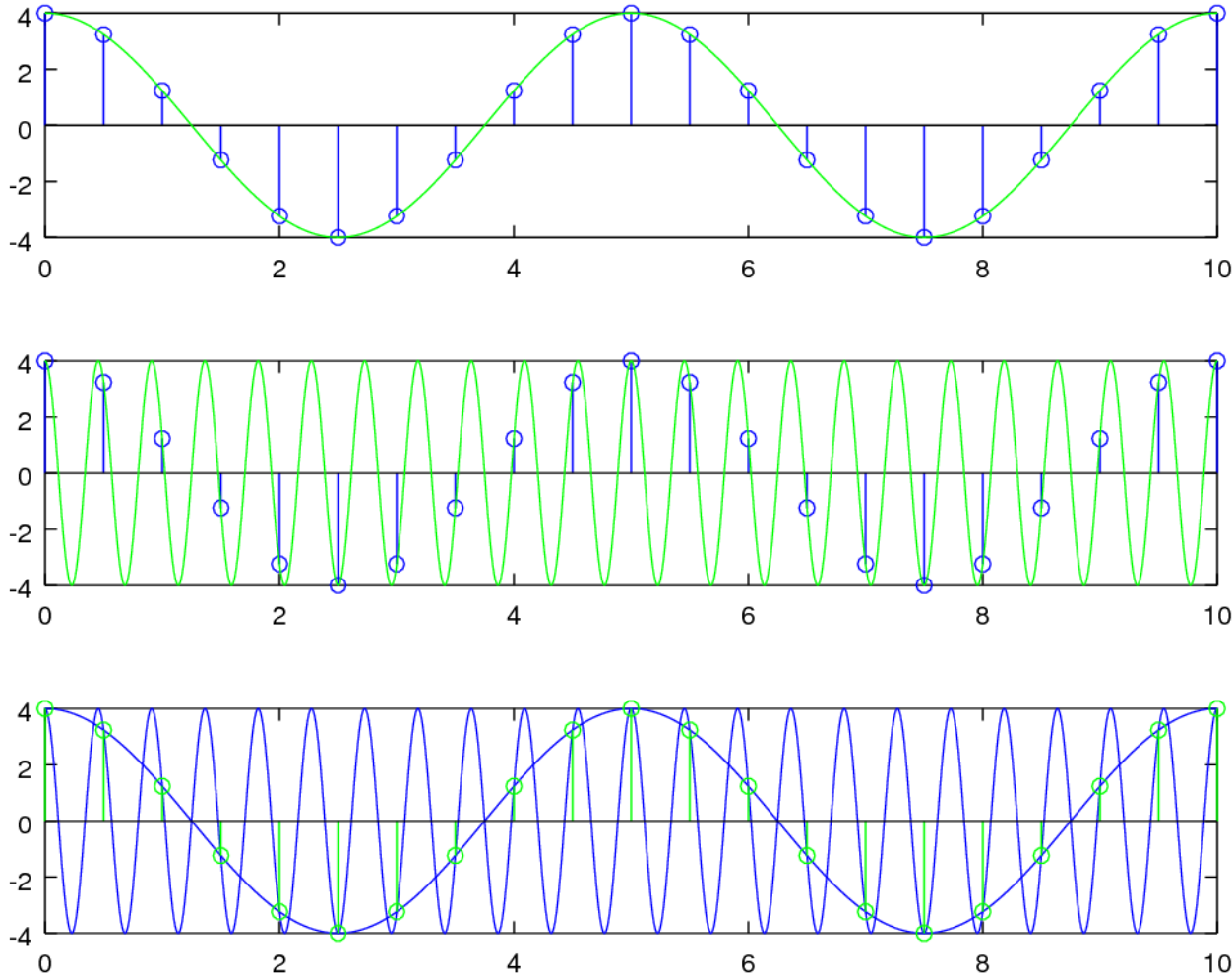


```
clf
n = [0:1:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(6/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
```

```
subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

M.J. Roberts, Fundamentals of Signals and Systems

Aliasing Condition Examples



```
clf  
n = [0:0.5:10];  
t = [0:1000]/100;  
y1 = 4*cos(2*pi*(1/5)*n);  
y2 = 4*cos(2*pi*(11/5)*n);  
yt1 = 4*cos(2*pi*(1/5)*t);  
yt2 = 4*cos(2*pi*(11/5)*t);
```

```
subplot(3,1,1);  
stem(n, y1); hold on;  
plot(t, yt1, 'g');  
subplot(3,1,2);  
stem(n, y2); hold on;  
plot(t, yt2, 'g');  
subplot(3,1,3);  
plot(t, yt1); hold on;  
plot(t, yt2);  
stem(n, y1, 'g');
```

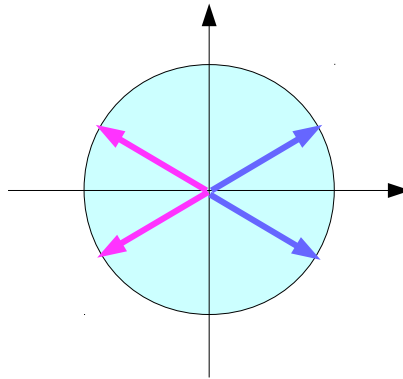
M.J. Roberts, Fundamentals of Signals and Systems

cos(2πft) & cos(2πft)

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

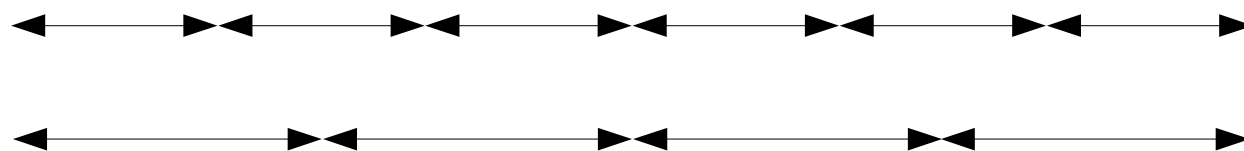
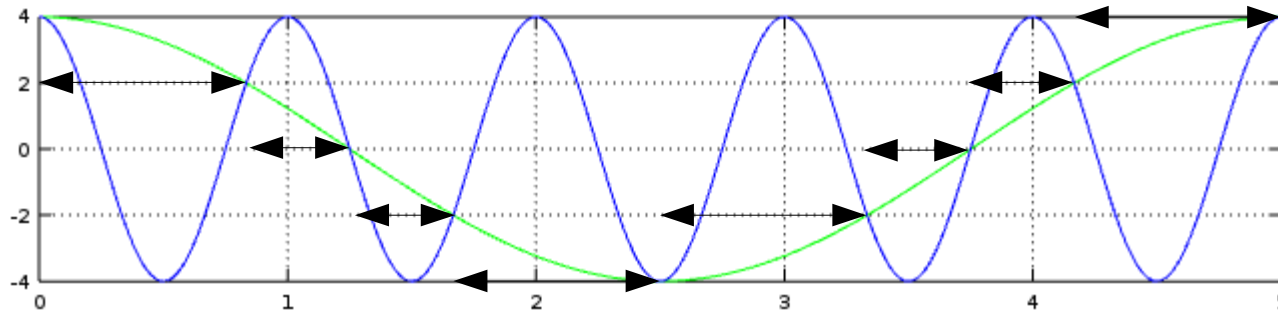
$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\omega_1 t + \omega_2 t = 2n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = n \end{cases}$$

$$\begin{cases} \frac{6}{5}t = n \\ \frac{4}{5}t = n \end{cases}$$



$$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \dots$$

$$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \dots$$

M.J. Roberts, Fundamentals of Signals and Systems

Aliasing Condition Examples

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

M.J. Roberts, Fundamentals of Signals and Systems

Aliasing Condition Examples

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

```
n1 = 0: 5/2 : 5;
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on
```

```
subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');
```

```
subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');
```

```
subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');
```

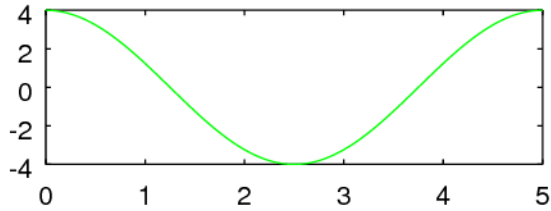
```
subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');
```

```
subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');
```

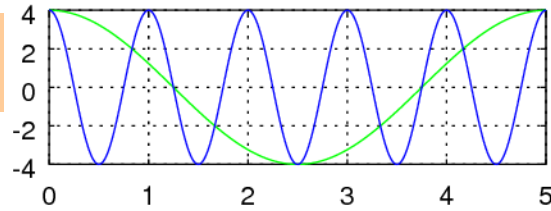
```
subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```


Graphs of $\cos(2\pi(n/5)t)$ & $\cos(2\pi(1/5)t)$

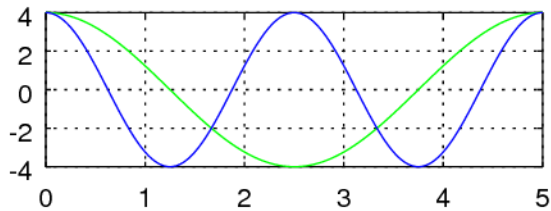
$$f = \frac{1}{5}$$



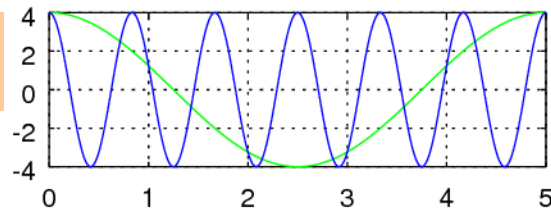
$$f = \frac{5}{5}$$



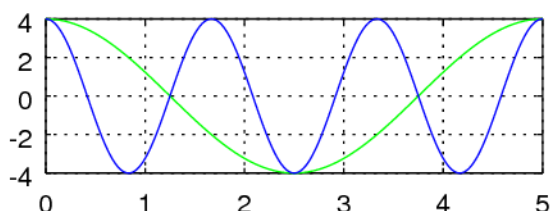
$$f = \frac{2}{5}$$



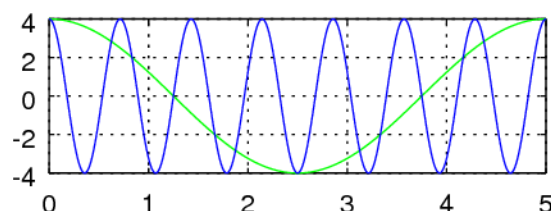
$$f = \frac{6}{5}$$



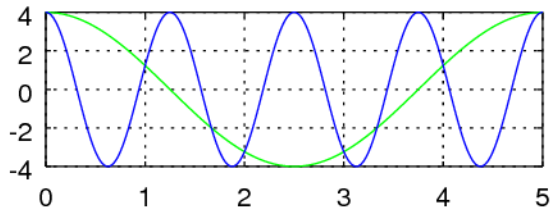
$$f = \frac{3}{5}$$



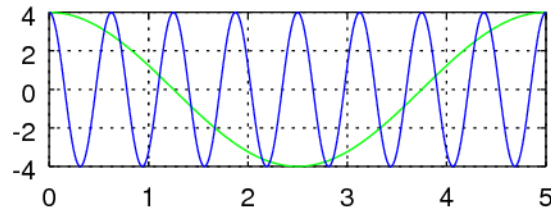
$$f = \frac{7}{5}$$



$$f = \frac{4}{5}$$



$$f = \frac{8}{5}$$

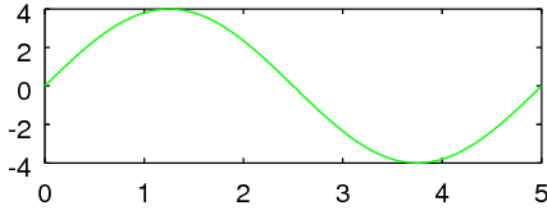


clf

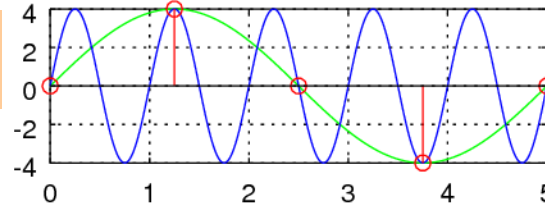
```
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

A Set of Roots of $\cos(2\pi(n/5)t) = \cos(2\pi(1/5)t)$

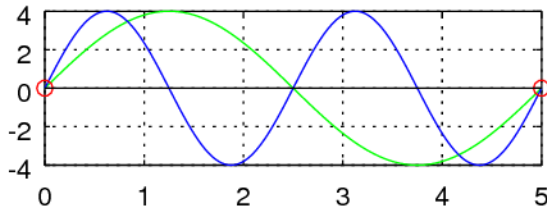
$$f = \frac{1}{5}$$



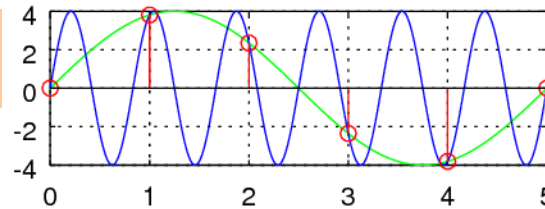
$$f = \frac{5}{5}$$



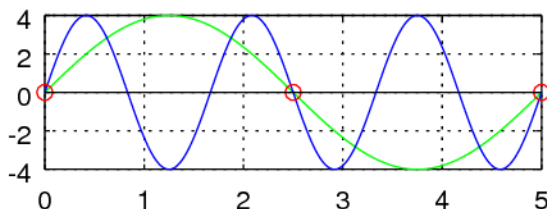
$$f = \frac{2}{5}$$



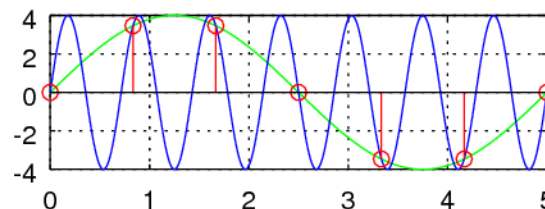
$$f = \frac{6}{5}$$



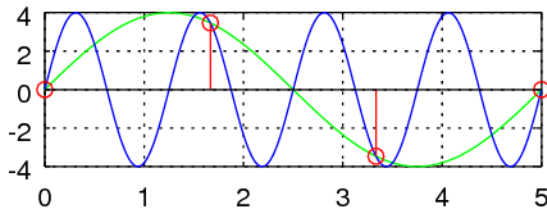
$$f = \frac{3}{5}$$



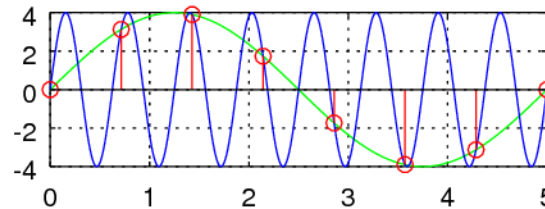
$$f = \frac{7}{5}$$



$$f = \frac{4}{5}$$



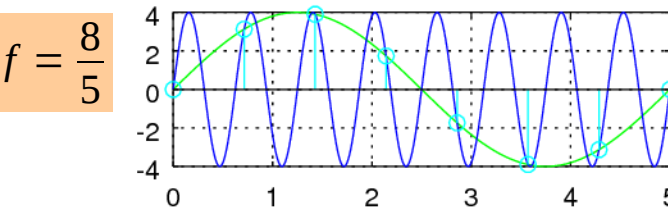
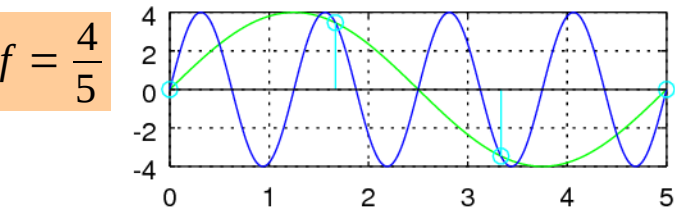
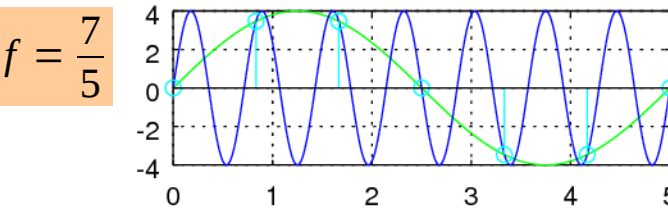
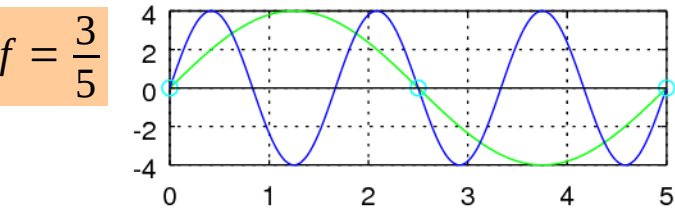
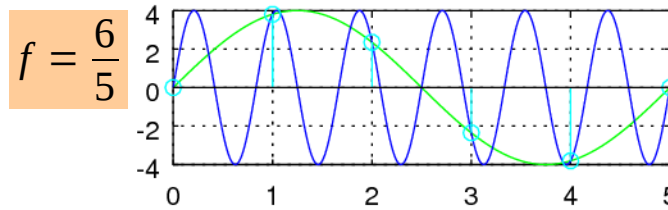
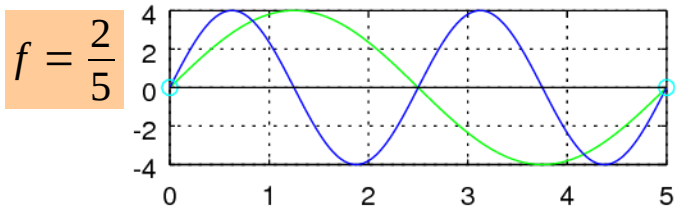
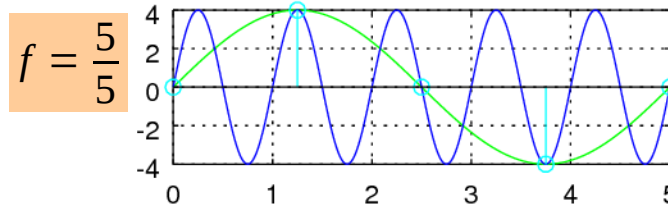
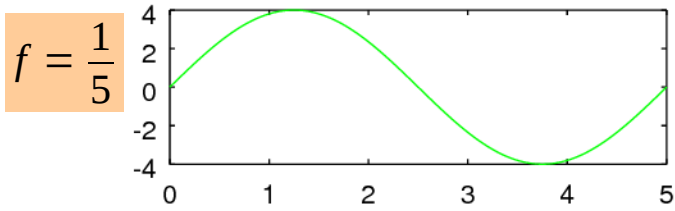
$$f = \frac{8}{5}$$



- n2 = 0: 5/3 : 5;
- n3 = 0: 5/4 : 5;
- n4 = 0: 5/5 : 5;
- n5 = 0: 5/6 : 5;
- n6 = 0: 5/7 : 5;
- n7 = 0: 5/8 : 5;
- n8 = 0: 5/9 : 5;

- y2 = 4*cos(2*pi*(2/5)*n2);
- y3 = 4*cos(2*pi*(3/5)*n3);
- y4 = 4*cos(2*pi*(4/5)*n4);
- y5 = 4*cos(2*pi*(5/5)*n5);
- y6 = 4*cos(2*pi*(6/5)*n6);
- y7 = 4*cos(2*pi*(7/5)*n7);
- y8 = 4*cos(2*pi*(8/5)*n8);

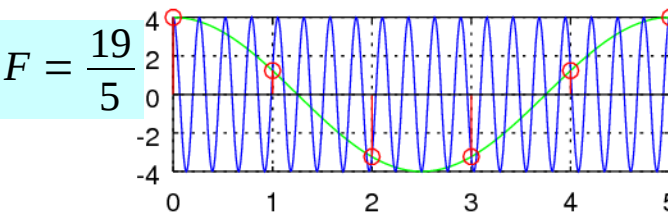
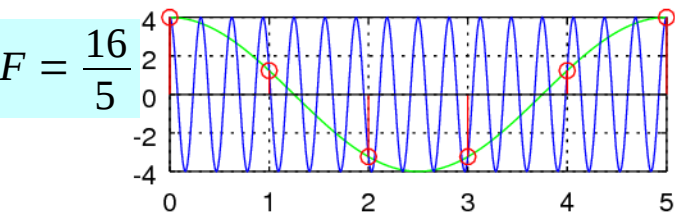
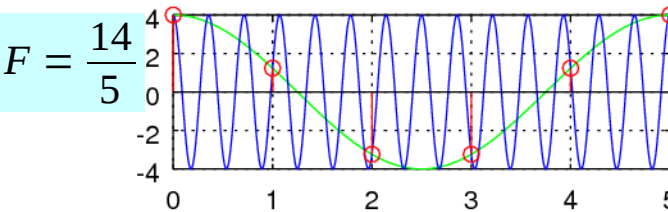
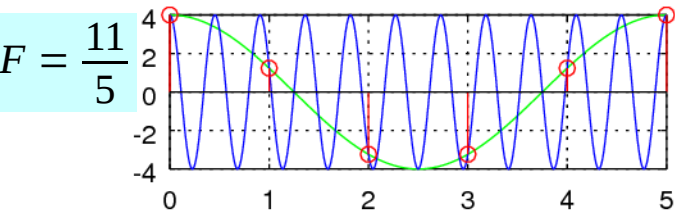
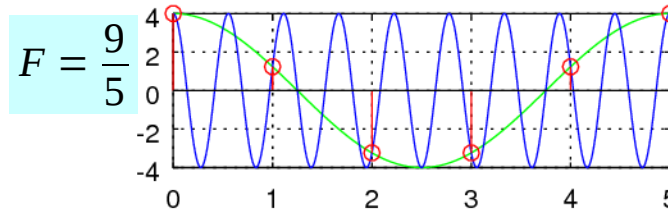
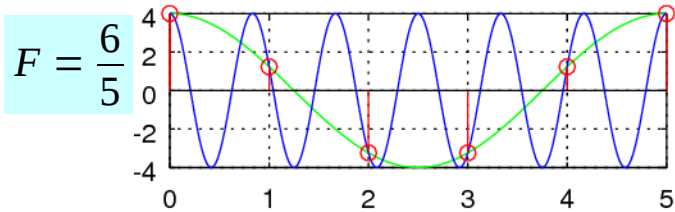
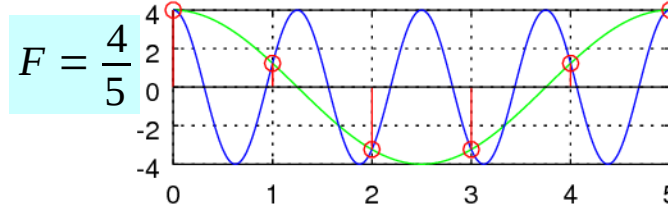
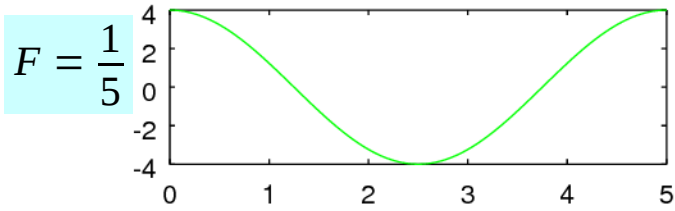
Another Set of Roots of $\cos(2\pi(n/5)t) = \cos(2\pi(1/5)t)$



$n_2 = 0: 5/1 : 5;$
 $n_3 = 0: 5/2 : 5;$
 $n_4 = 0: 5/3 : 5;$
 $n_5 = 0: 5/4 : 5;$
 $n_6 = 0: 5/5 : 5;$
 $n_7 = 0: 5/6 : 5;$
 $n_8 = 0: 5/7 : 5;$

$y_2 = 4*\cos(2*\pi*(2/5)*n_2);$
 $y_3 = 4*\cos(2*\pi*(3/5)*n_3);$
 $y_4 = 4*\cos(2*\pi*(4/5)*n_4);$
 $y_5 = 4*\cos(2*\pi*(5/5)*n_5);$
 $y_6 = 4*\cos(2*\pi*(6/5)*n_6);$
 $y_7 = 4*\cos(2*\pi*(7/5)*n_7);$
 $y_8 = 4*\cos(2*\pi*(8/5)*n_8);$

Aliasing and Folding



Plotting of Aliasing & Folding Frequencies

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
yt3 = 4*cos(2*pi*(11/5)*t);
yt4 = 4*cos(2*pi*(16/5)*t);
yt5 = 4*cos(2*pi*(4/5)*t);
yt6 = 4*cos(2*pi*(9/5)*t);
yt7 = 4*cos(2*pi*(14/5)*t);
yt8 = 4*cos(2*pi*(19/5)*t);

n1 = 0: 5/5 : 5;
n2 = 0: 5/5 : 5;
n3 = 0: 5/5 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/5 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/5 : 5;
n8 = 0: 5/5 : 5;

y2 = 4*cos(2*pi*(6/5)*n2);
y3 = 4*cos(2*pi*(11/5)*n2);
y4 = 4*cos(2*pi*(16/5)*n2);
y5 = 4*cos(2*pi*(4/5)*n5);
y6 = 4*cos(2*pi*(9/5)*n5);
y7 = 4*cos(2*pi*(14/5)*n5);
y8 = 4*cos(2*pi*(19/5)*n5);

subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n2, y3, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n2, y4, 'r');

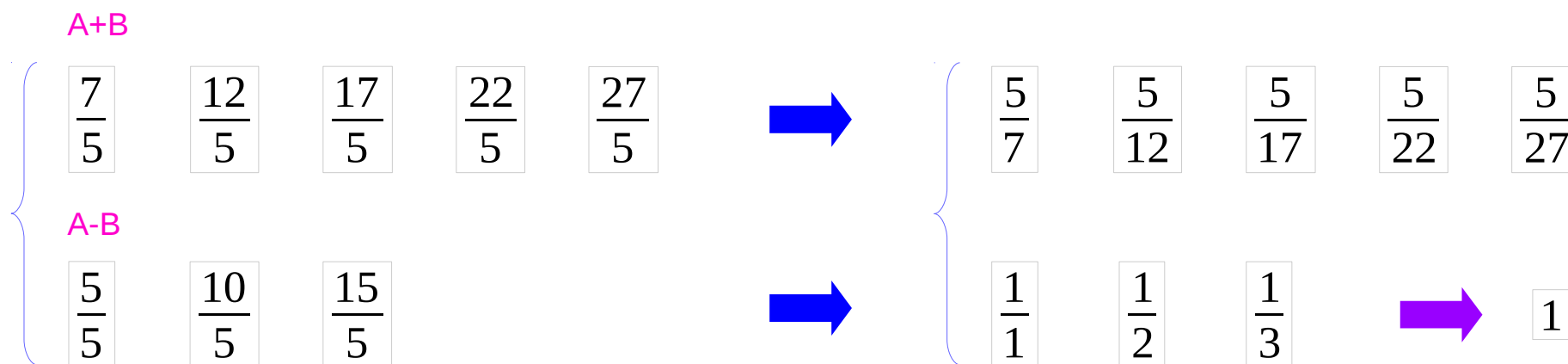
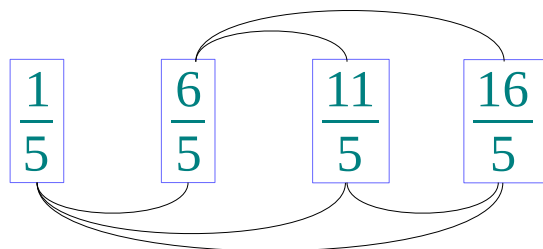
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n5, y6, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n5, y7, 'r');

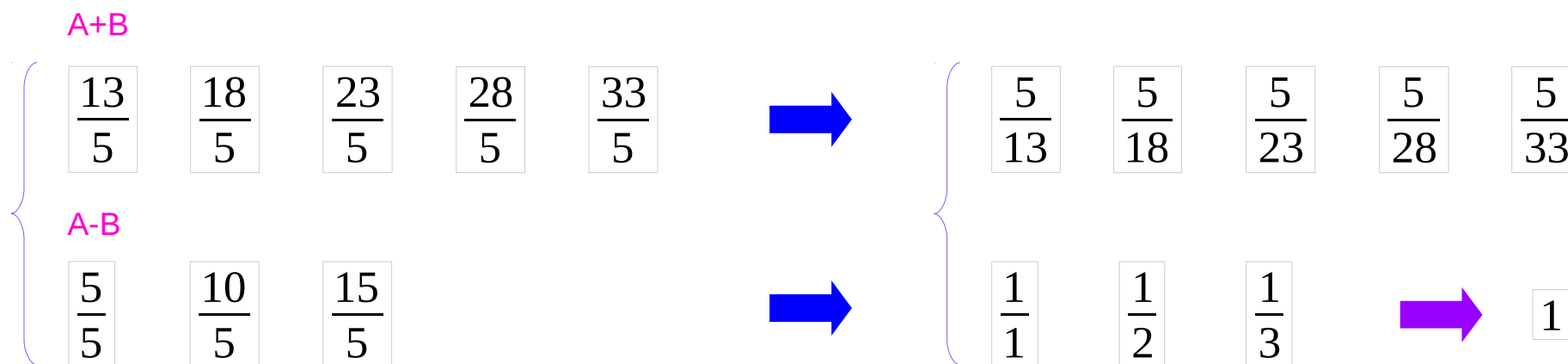
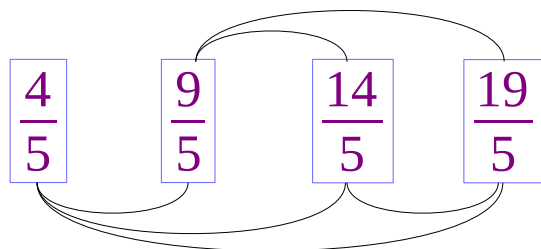
subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n5, y8, 'r');
```

Aliasing Frequencies (1)



M.J. Roberts, Fundamentals of Signals and Systems

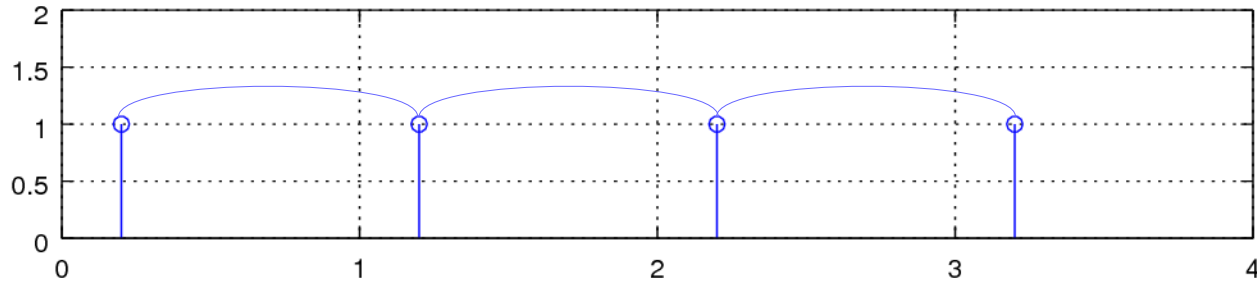
Aliasing Frequencies (2)



M.J. Roberts, Fundamentals of Signals and Systems

Aliasing and Folding Frequencies

Aliasing frequencies

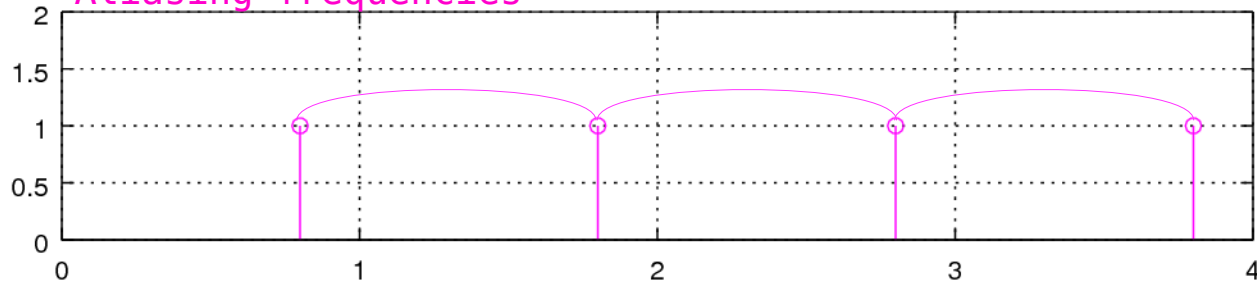


```
n1 = [1/5, 6/5, 11/5, 16/5];  
n2 = [4/5, 9/5, 14/5, 19/5];
```

```
y1 = [1, 1, 1, 1];  
y2 = [1, 1, 1, 1];
```

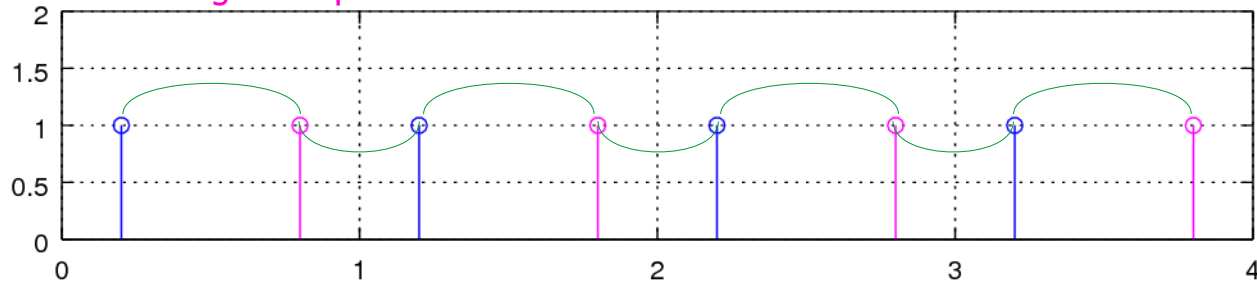
```
subplot(3, 1, 1)  
stem(n1, y1, 'b'); grid on;  
axis([0, 4, 0, 2]);
```

Aliasing frequencies



```
subplot(3, 1, 2)  
stem(n2, y2, 'm'); grid on;  
axis([0, 4, 0, 2]);
```

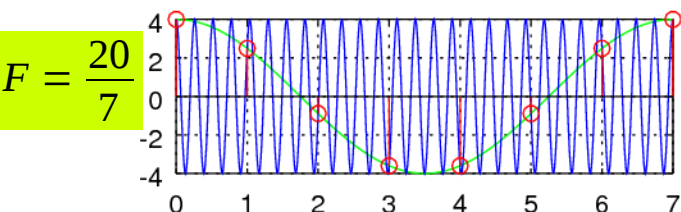
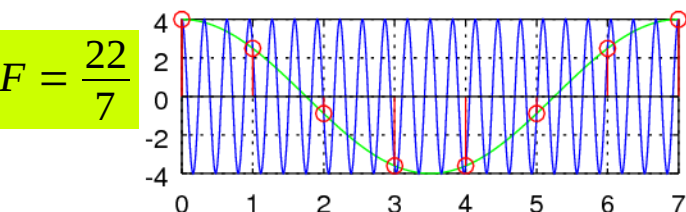
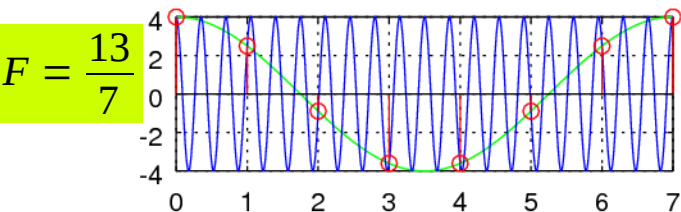
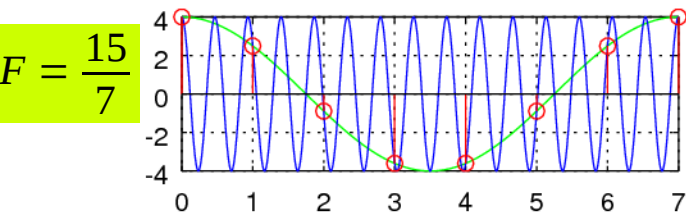
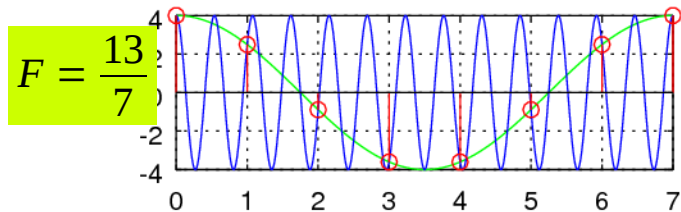
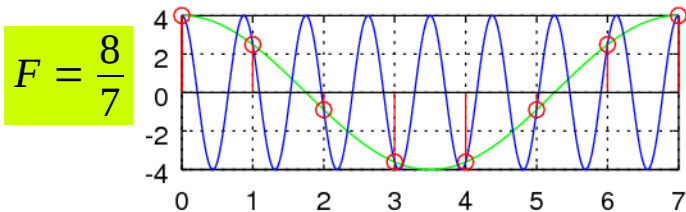
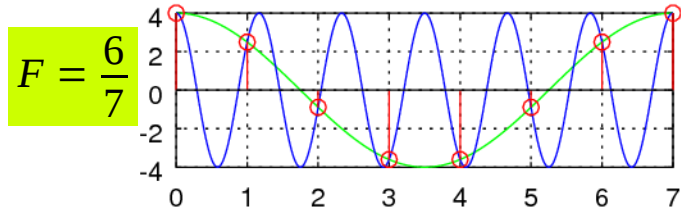
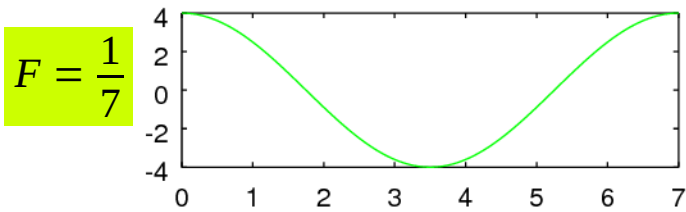
Folding frequencies



```
subplot(3, 1, 3)  
stem(n1, y1, 'b'); hold on;  
stem(n2, y2, 'm'); grid on;  
axis([0, 4, 0, 2]);
```

J.H. McClellan, et al., Signal Processing First

Graphs of $\cos(2\pi(n/7)t)$ & $\cos(2\pi(1/7)t)$

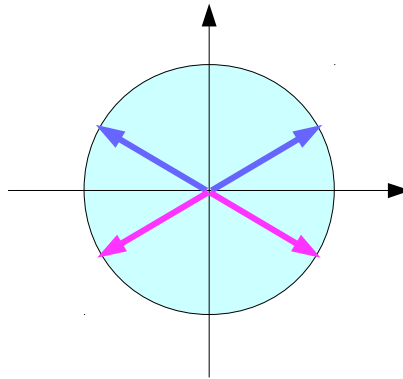


$\sin(2\pi ft)$ & $\sin(2\pi ft)$

$$\sin(\omega_1 t) = \sin(\omega_2 t)$$

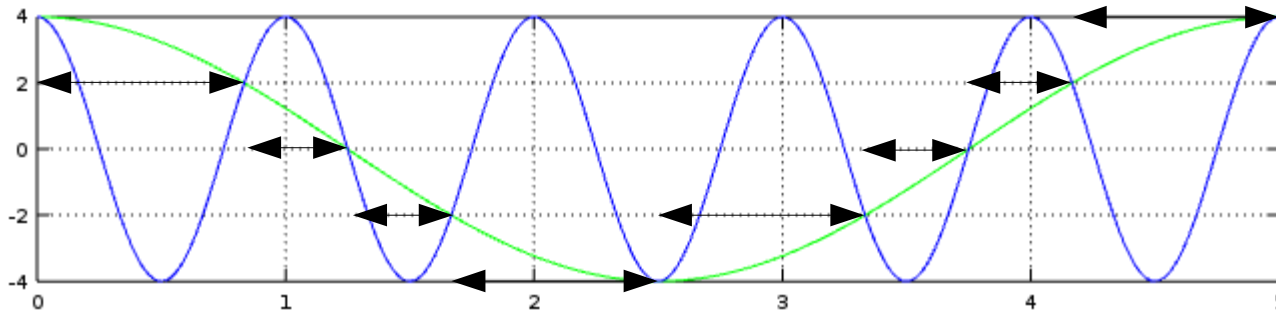
$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\omega_1 t + \omega_2 t = n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = \frac{n}{2} \end{cases}$$

$$\begin{cases} \frac{6}{5}t = n \\ \frac{4}{5}t = \frac{n}{2} \end{cases}$$



$$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \dots$$



$$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \dots$$

M.J. Roberts, Fundamentals of Signals and Systems

Aliasing Condition Examples

$\frac{2}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = \frac{n}{2}$	$\frac{1}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = 5$
$\frac{3}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = \frac{n}{2}$	$\frac{2}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = \frac{n}{2}$	$\frac{3}{5}t = n$	$T_s = \frac{5}{10}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = \frac{n}{2}$	$\frac{4}{5}t = n$	$T_s = \frac{5}{12}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = \frac{n}{2}$	$\frac{5}{5}t = n$	$T_s = \frac{5}{14}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = \frac{n}{2}$	$\frac{6}{5}t = n$	$T_s = \frac{5}{16}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = \frac{n}{2}$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = \frac{n}{2}$	$\frac{7}{5}t = n$	$T_s = \frac{5}{18}$	$T_s = \frac{5}{7}$

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Aliasing Condition Examples

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

```
n1 = 0: 5/2 : 5;
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on
```

```
subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');
```

```
subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');
```

```
subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

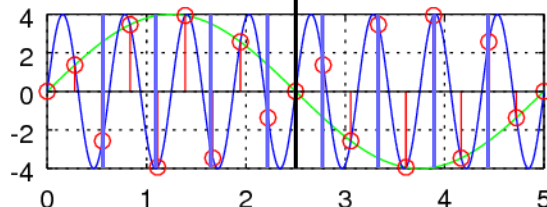
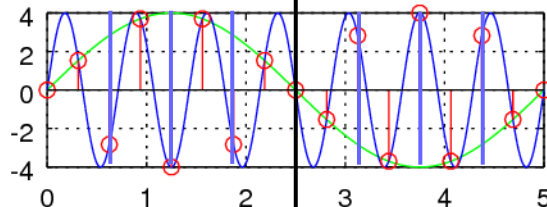
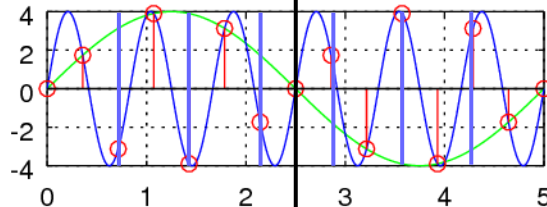
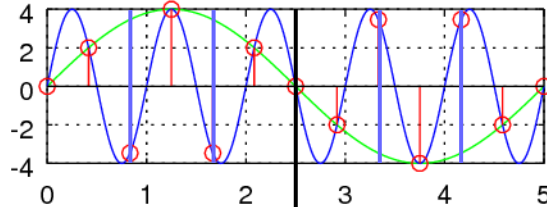
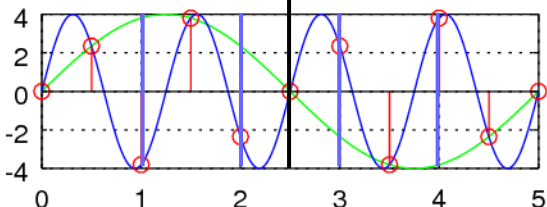
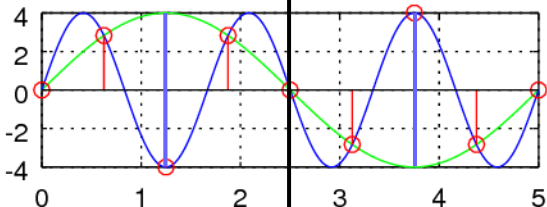
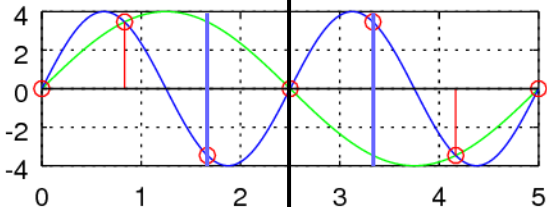
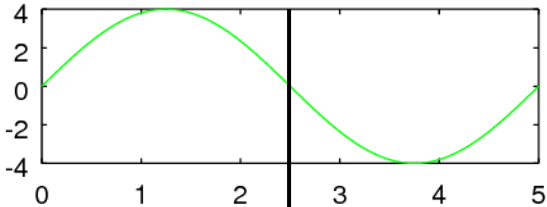
```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');
```

```
subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');
```

```
subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');
```

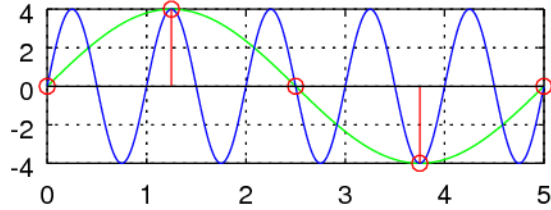
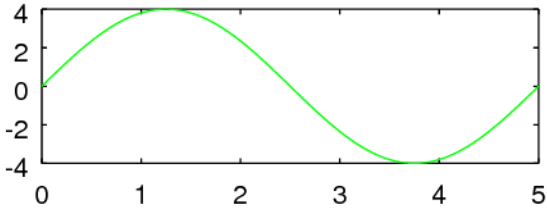
```
subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

Aliasing Condition Examples

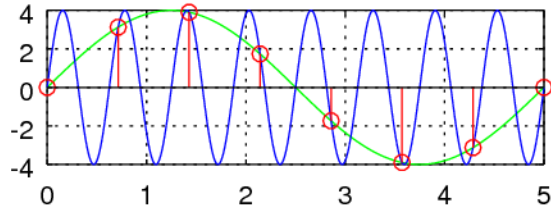
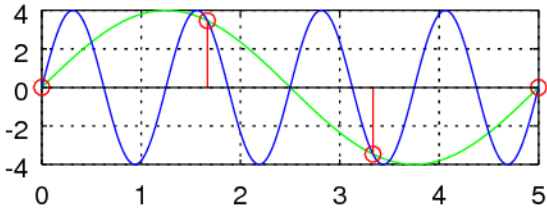
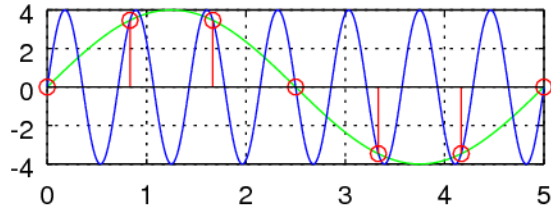
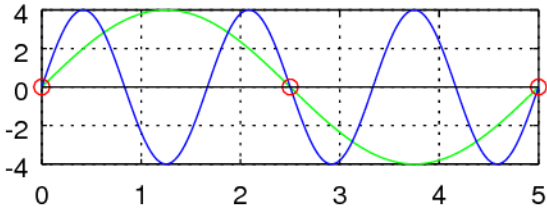
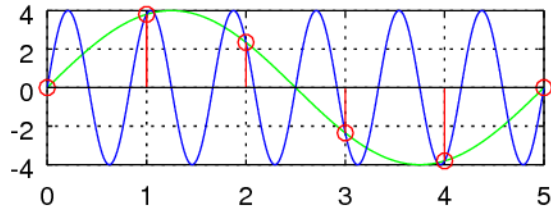
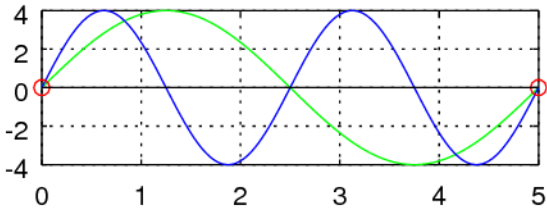


- n2 = 0: (1/2)*5/3 : 5;
- n3 = 0: (1/2)*5/4 : 5;
- n4 = 0: (1/2)*5/5 : 5;
- n5 = 0: (1/2)*5/6 : 5;
- n6 = 0: (1/2)*5/7 : 5;
- n7 = 0: (1/2)*5/8 : 5;
- n8 = 0: (1/2)*5/9 : 5;

Aliasing Condition Examples



$n_2 = 0 : 5/1 : 5;$
 $n_3 = 0 : 5/2 : 5;$
 $n_4 = 0 : 5/3 : 5;$
 $n_5 = 0 : 5/4 : 5;$
 $n_6 = 0 : 5/5 : 5;$
 $n_7 = 0 : 5/6 : 5;$
 $n_8 = 0 : 5/7 : 5;$



M.J. Roberts, Fundamentals of Signals and Systems

References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann