

Background – Functions (1C)

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Based on

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

Function Definition

Function Definition I.

```
square x = x * x
```

- function type is inferred → not efficient

Type Inference

Function Definition II.

```
square :: Double -> Double
```

```
square x = x * x
```

– function type declaration

Function Type Declaration

- function type declaration
- function definition

Type Declaration

the declaration of an identifier's type

the identifier name :: the type name ...

type names in Haskell always begin with a capital letter,

identifier names (including function identifiers) must always begin with a lower-case letter

<http://www.toves.org/books/hsfun/>

Function Types and Type Classes

Function Definition I.

```
square x = x * x
```

function definition

```
=
```

Function Definition II.

```
square :: Double -> Double
```

```
square x = x * x
```

function definition

- **function type declaration**

```
=
```

type class – a set of types

- **function type 1**
- **function type 2**
-
- **function type n**

Requirements

Subclasses

<http://www.toves.org/books/hsfun/>

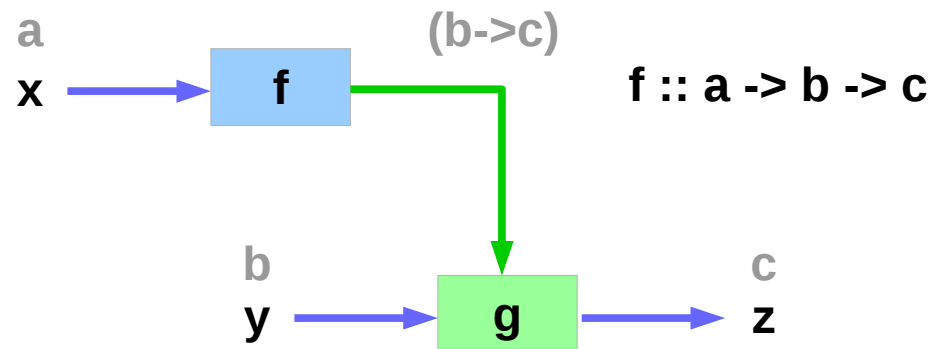
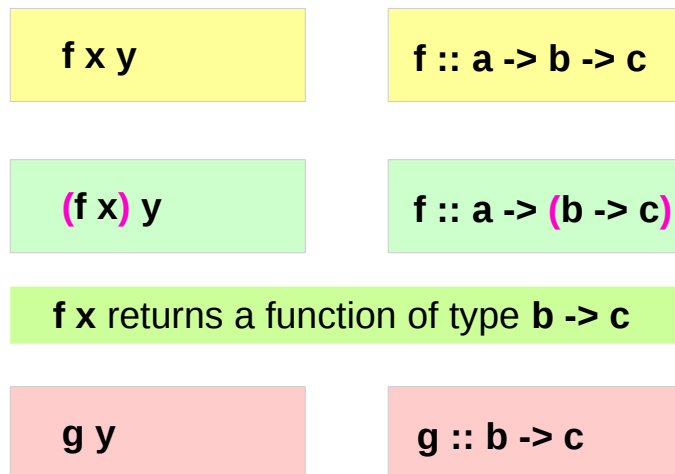
Functions : First-class Data Types

functions are **first-class data types**

Haskell **treats functions as regular data**,

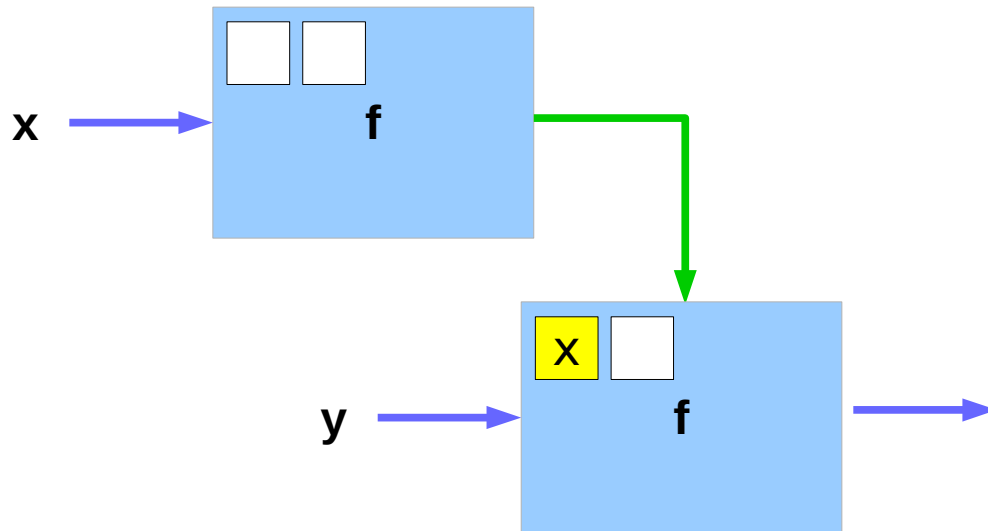
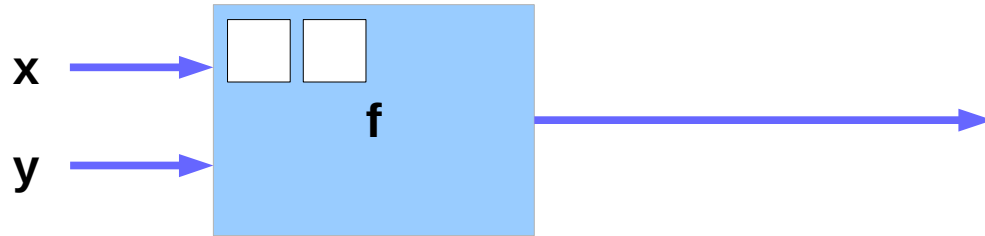
just like integers, or floating-point values, or other types.

- a function can take other functions as **parameters**
- a function takes a **parameter** and produces **another function** (curried function)



<http://www.toves.org/books/hsfun/>

Currying Examples



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Polymorphic Functions

specific types vs. arbitrary types

a **polymorphic** functions – an **abstract** type
each type variable is generally a **lower-case letter**.

Example) A translate function

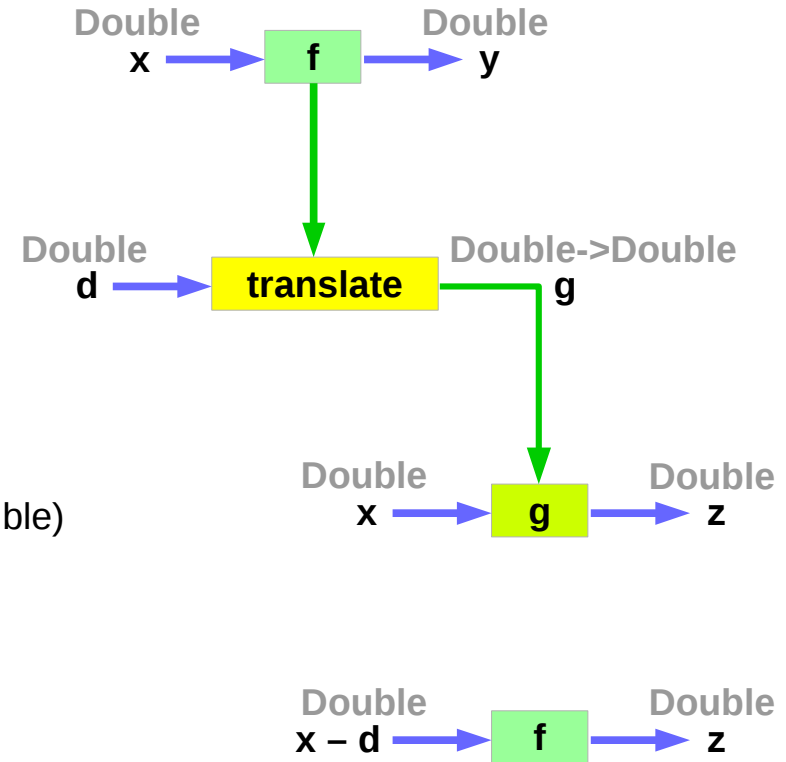
takes a function f and a distance d

returns a new function g

that is f "translated" d units to the right

<http://www.toves.org/books/hsfun/>

Polymorphic Function Examples



`translate` :: (Double -> Double) -> Double -> (Double -> Double)

`translate f d = g` where `g x = f (x - d)`

`translate` :: (Double -> `a`) -> Double -> (Double -> `a`)

<http://www.toves.org/books/hsfun/>

Currying

Currying recursively transforms
a function that takes multiple arguments
into a function that takes just a single argument and
returns another function if any arguments are still needed.

$f :: a \rightarrow b \rightarrow c$

$f\ x\ y$

$f :: a \rightarrow b \rightarrow c$

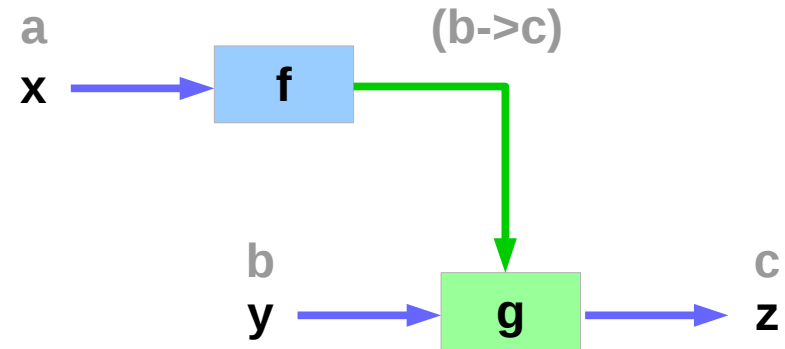
$(f\ x)\ y$

$f :: a \rightarrow (b \rightarrow c)$

$g\ y$

$g :: b \rightarrow c$

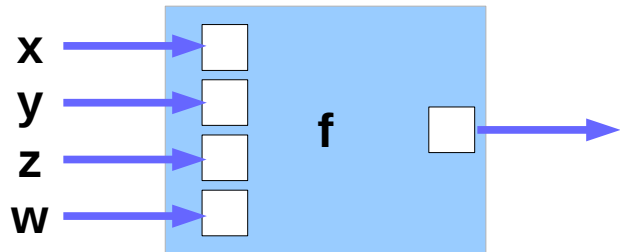
$f :: a \rightarrow b \rightarrow c$



<https://wiki.haskell.org/Currying>

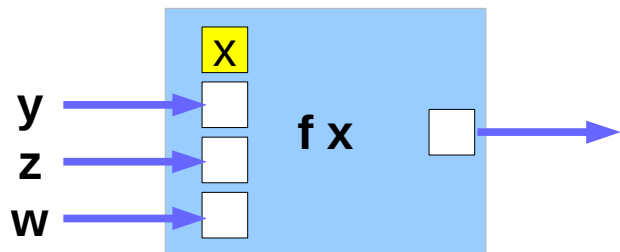
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Partially Applied Functions – f , $(f\ x)$



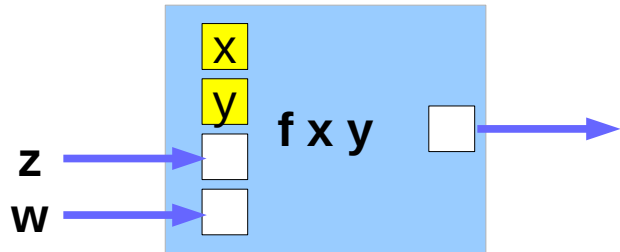
$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$
 $f\ x\ y\ z\ w = \dots$

$(f\ x)\ y\ z\ w$



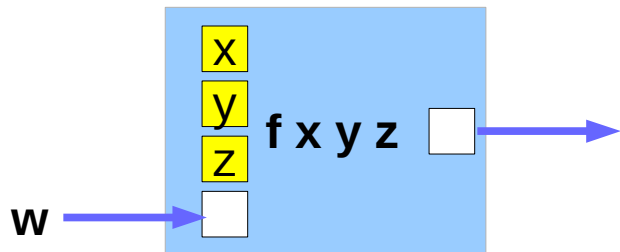
$g1 :: b \rightarrow c \rightarrow d \rightarrow e$
 $g1\ y\ z\ w = \dots$

Partially Applied Functions – (f x y), (f x y z)



(f x y) z w

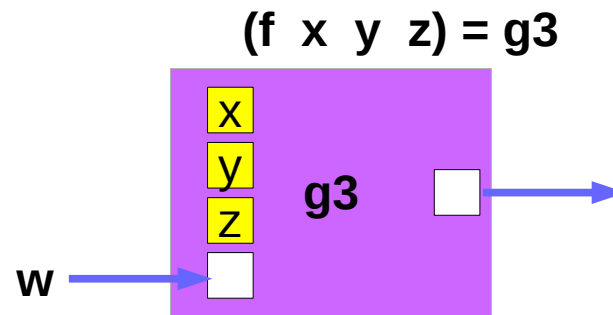
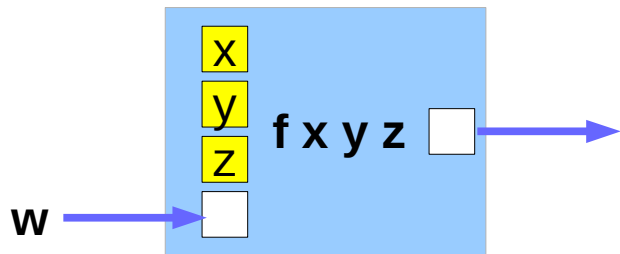
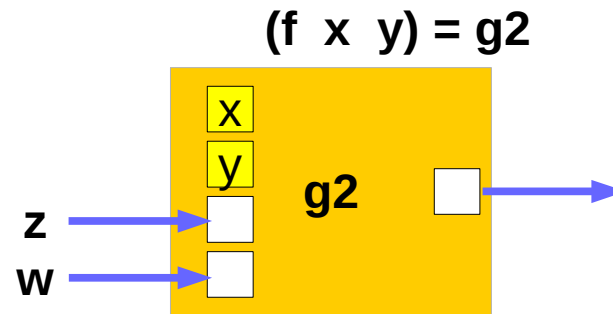
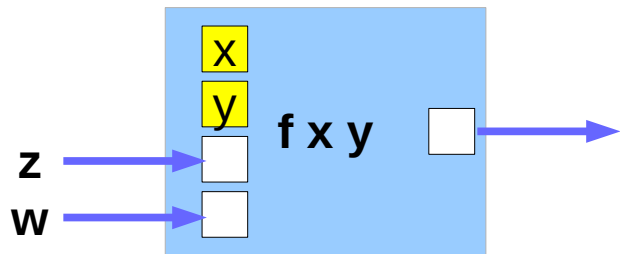
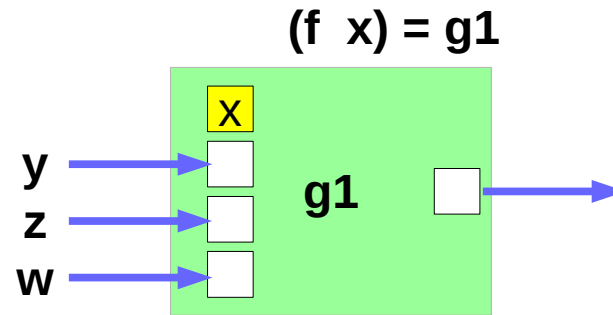
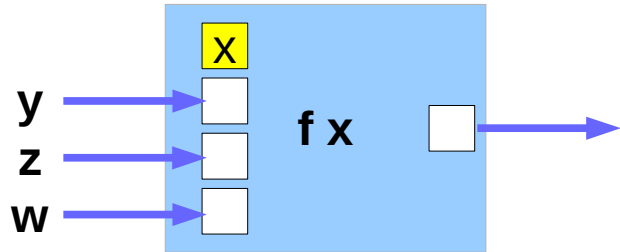
$g2 :: c \rightarrow d \rightarrow e$
 $g2 \ z \ w = \dots$



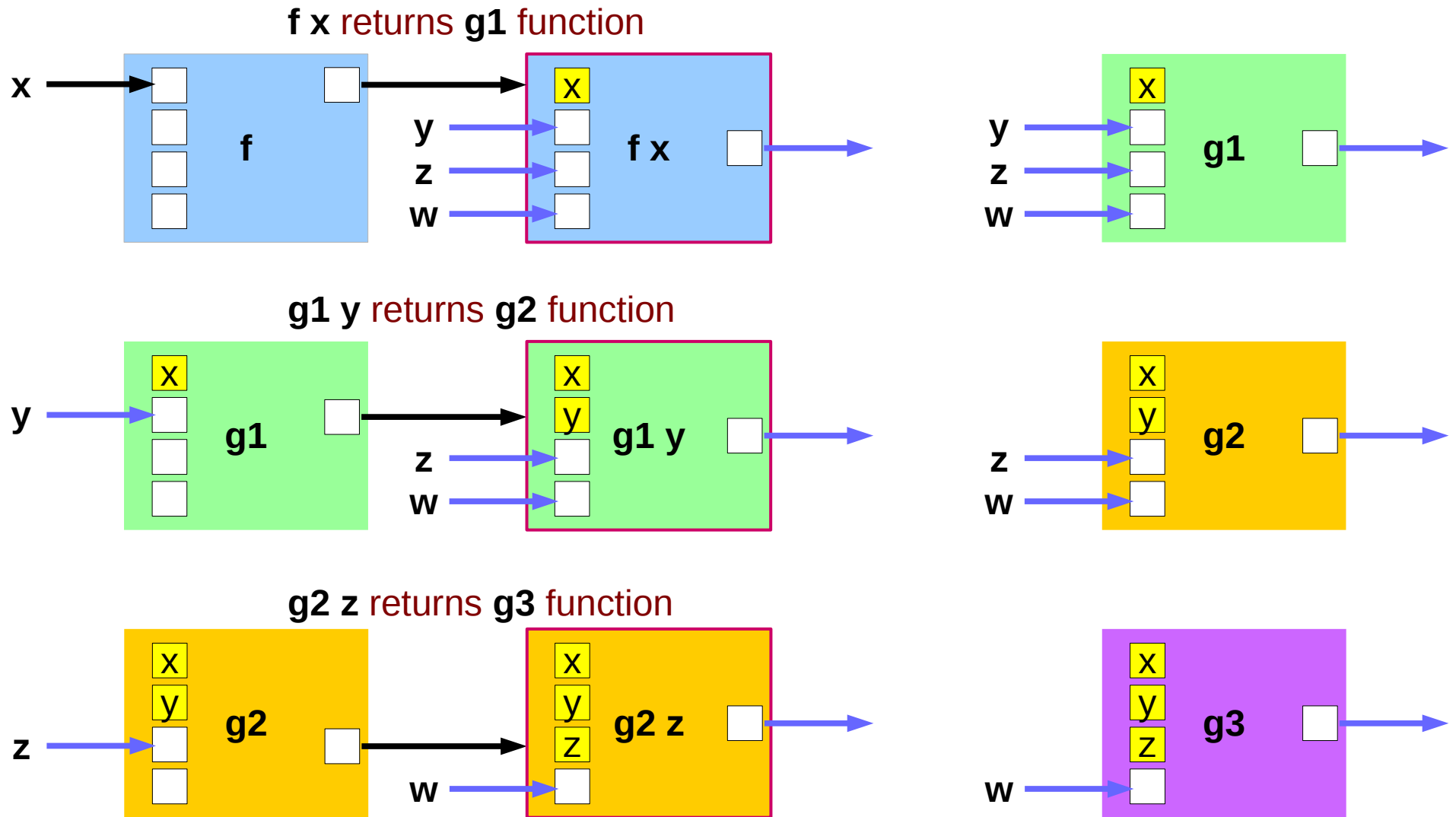
(f x y z) w

$g3 :: d \rightarrow e$
 $g3 \ w = \dots$

Partially Applied Functions – g1, g2, g3



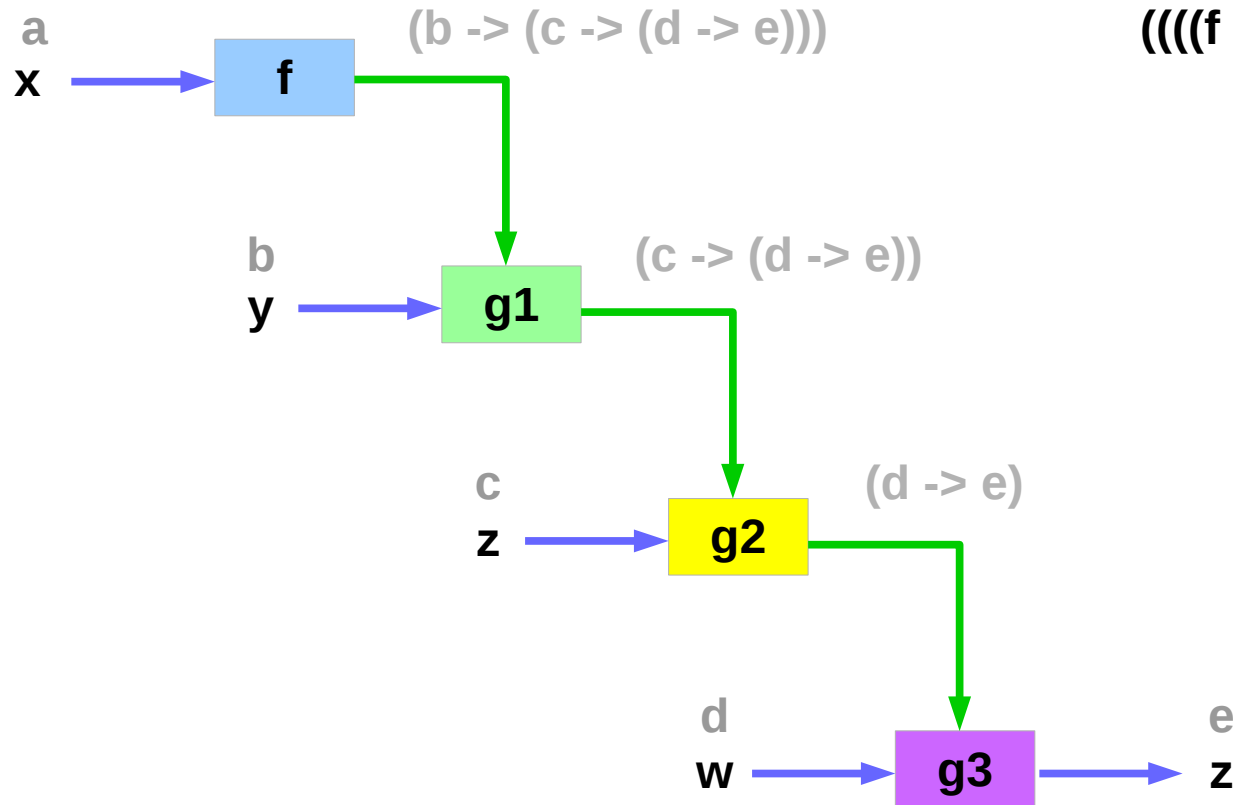
Returning Functions



Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



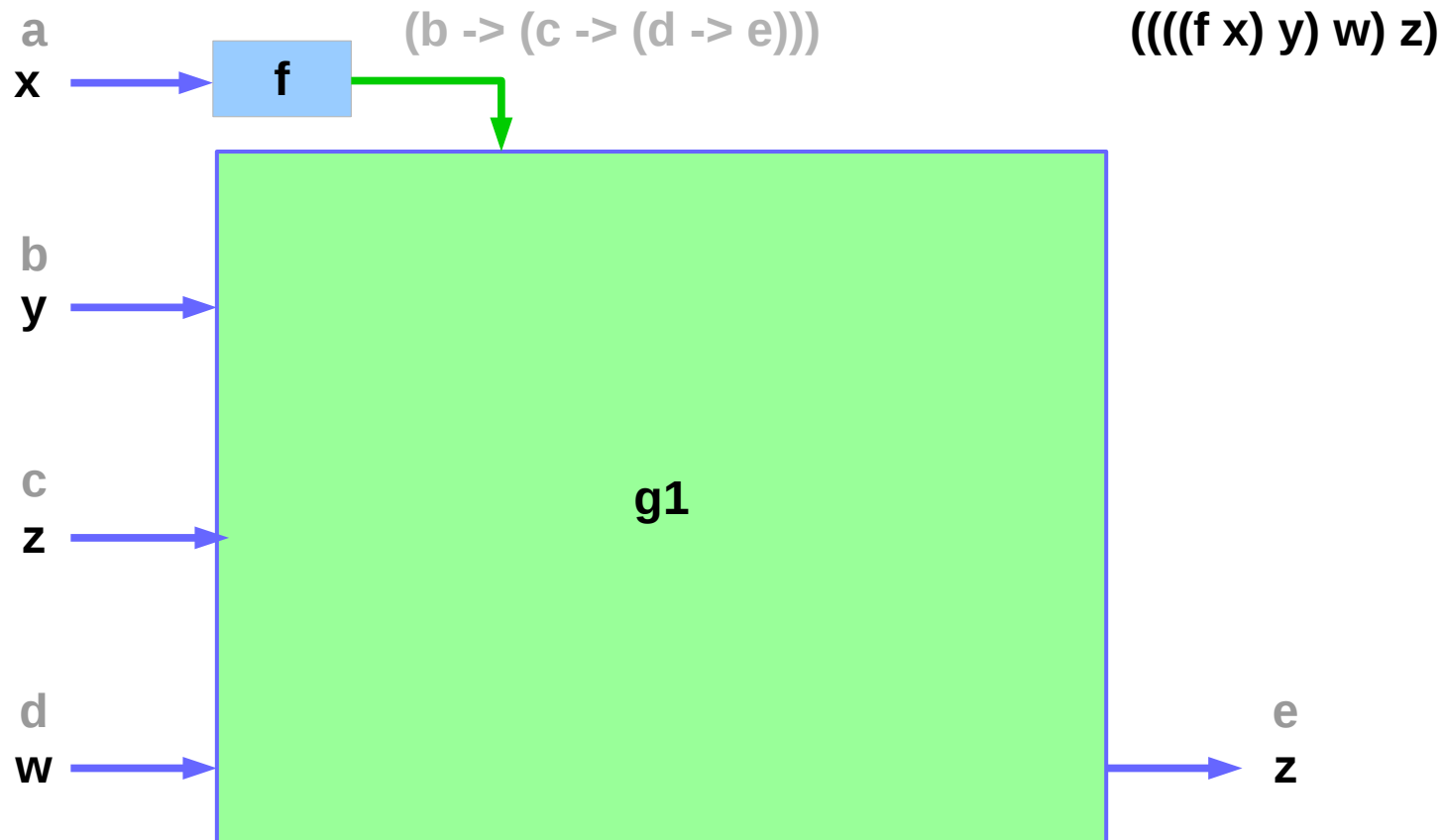
$(((((f\ x)\ y)\ z)\ w))$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



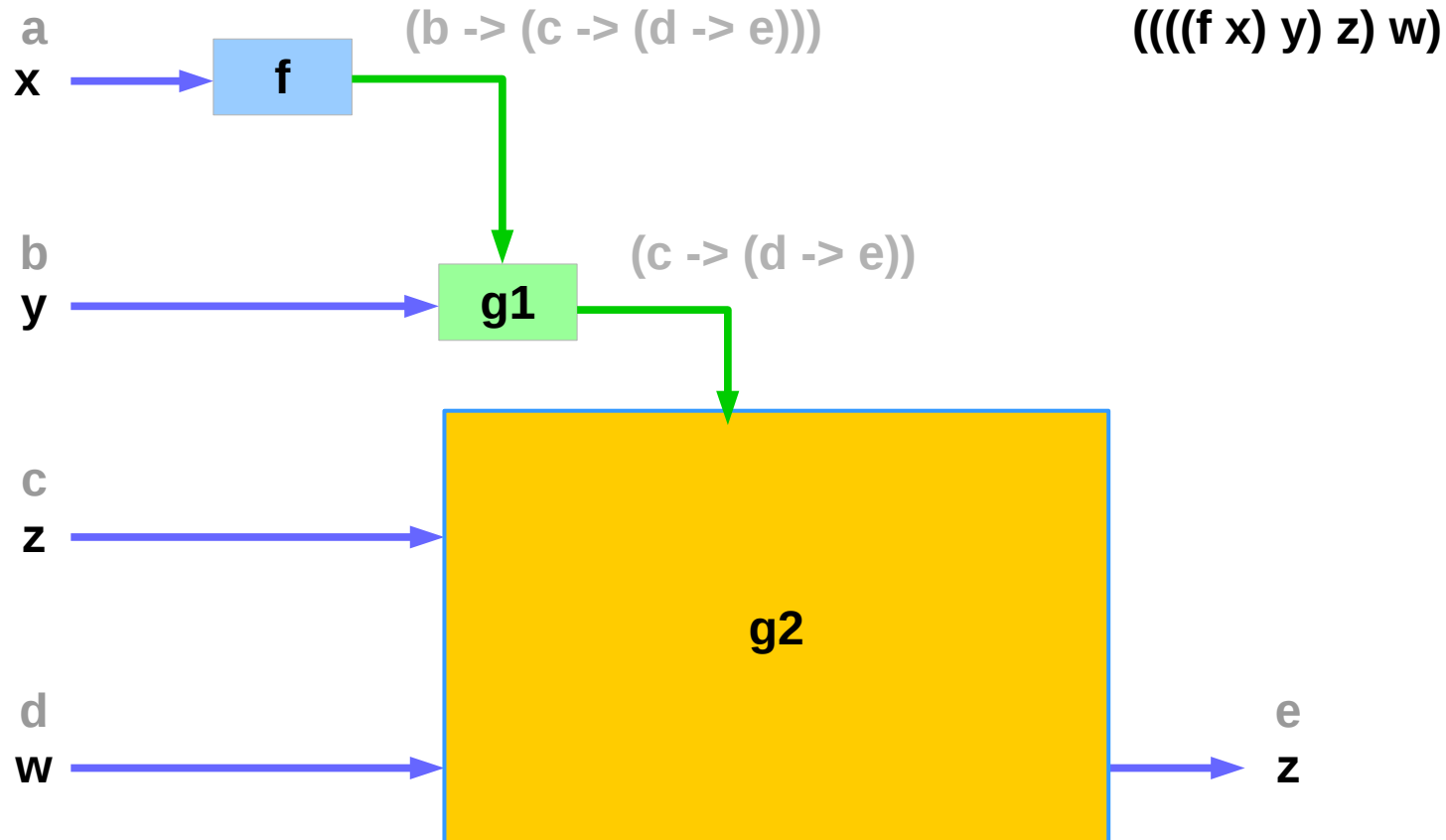
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Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$

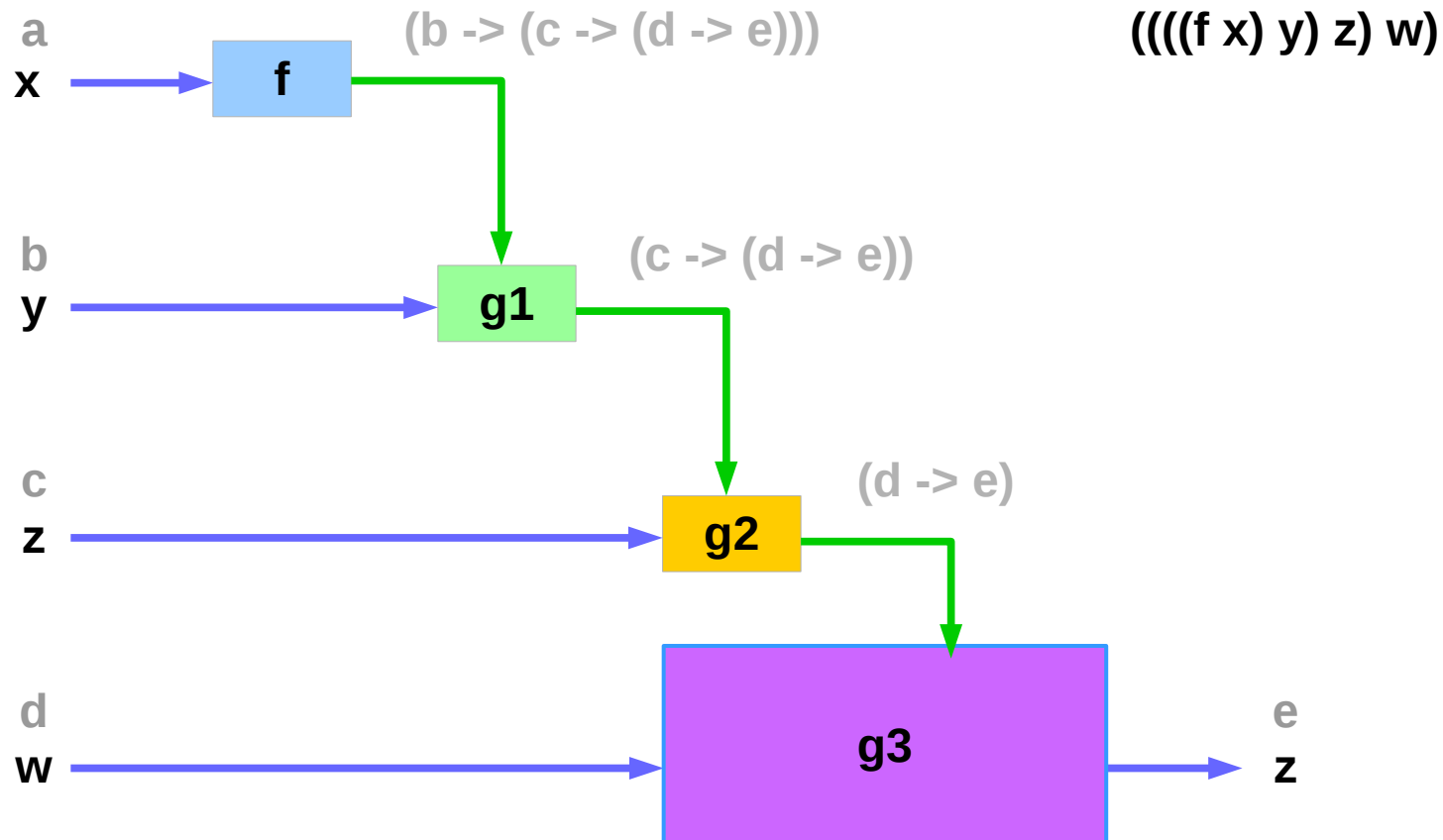


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Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



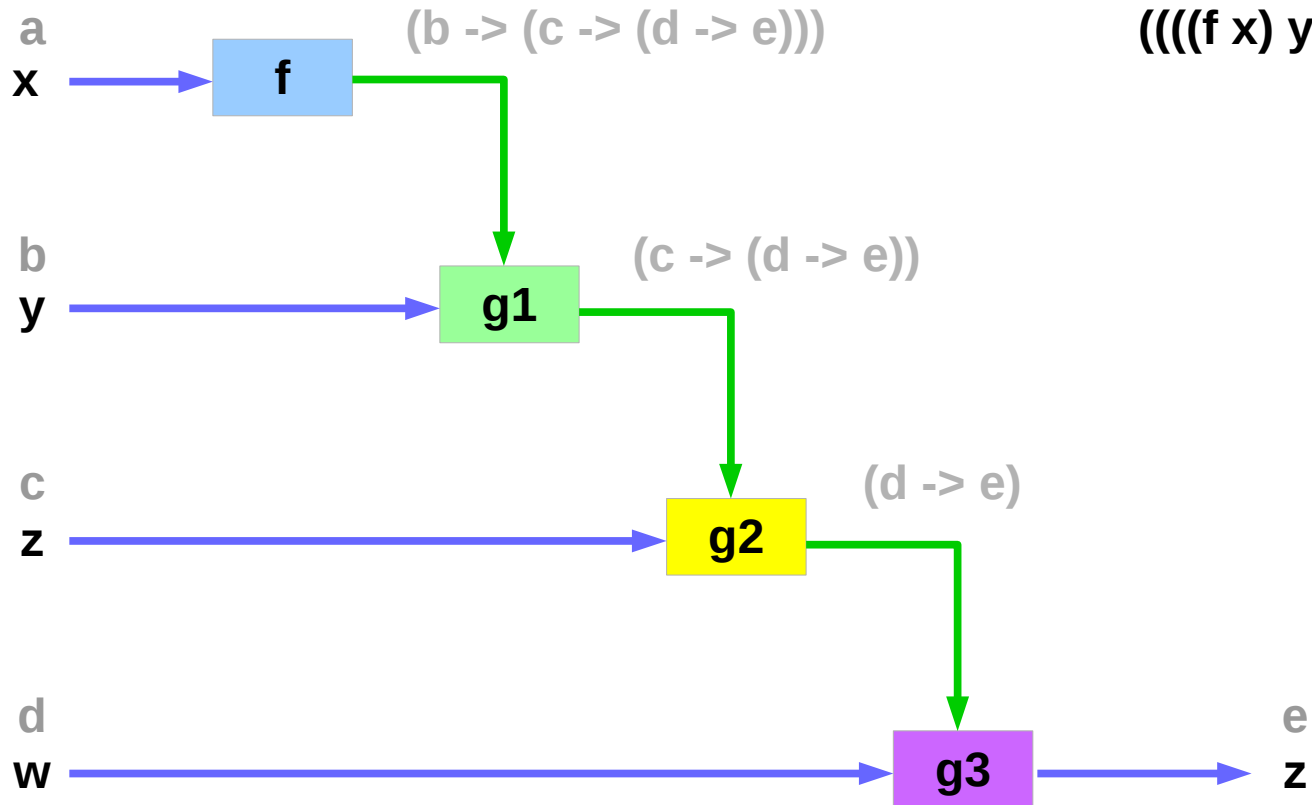
$((((f\ x)\ y)\ z)\ w)$

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Currying Examples

$f :: a \rightarrow b \rightarrow c \rightarrow d \rightarrow e$

$f :: a \rightarrow (b \rightarrow (c \rightarrow (d \rightarrow e)))$



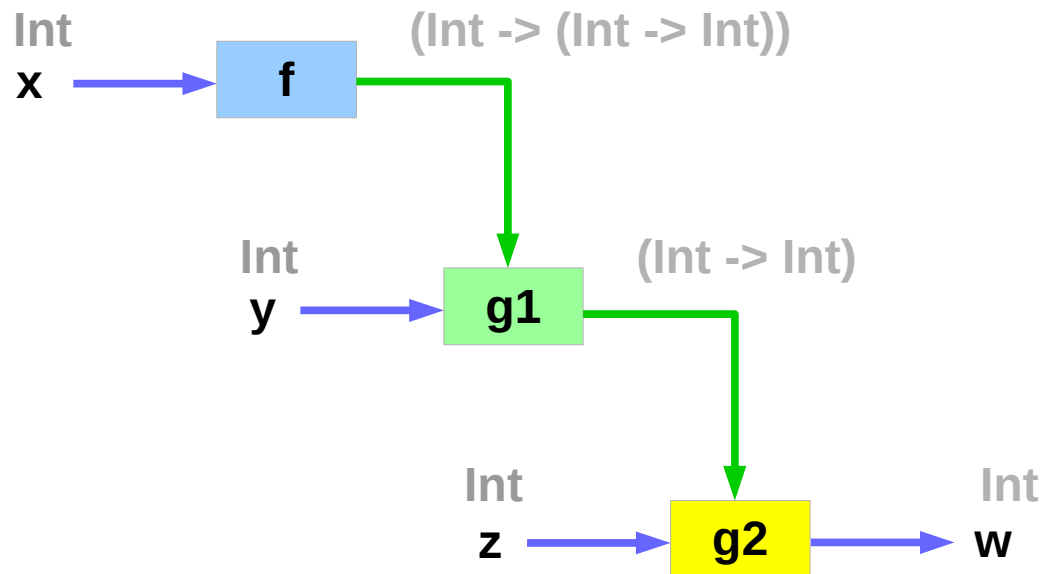
$(((((f\ x)\ y)\ z)\ w))$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

`mult :: Int -> Int -> Int -> Int`

`f :: a -> (b -> (c -> (d -> e)))`



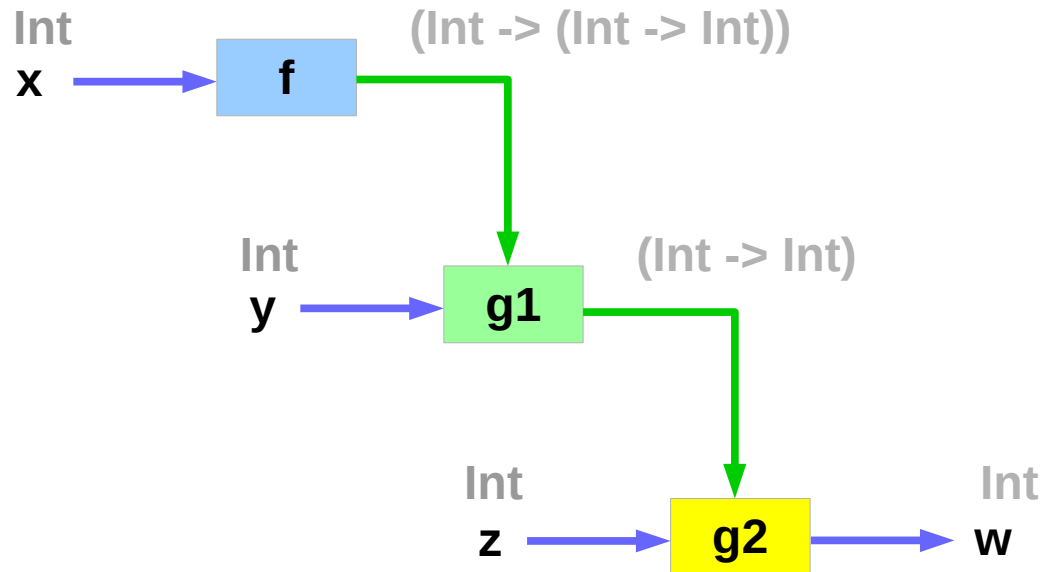
`(((mult x) y) z)`

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Currying Examples

`mult :: Int -> Int -> Int -> Int`

`f :: Int -> (Int -> (Int -> Int))`



`f x :: Int -> (Int -> Int)`

`g1 :: Int -> (Int -> Int)`

`f x y :: Int -> Int`

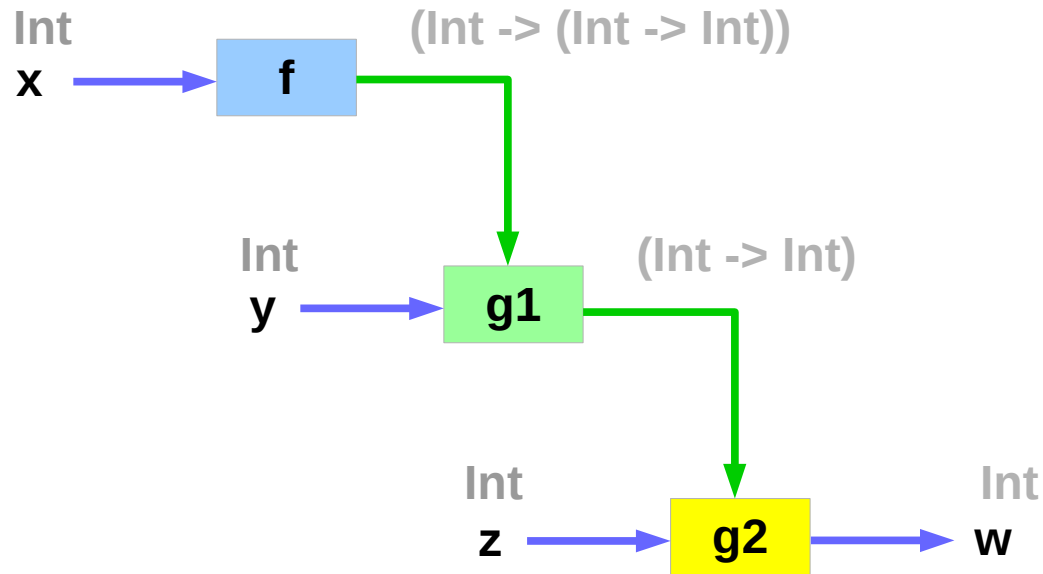
`g2 :: Int -> Int`

`f x y z :: Int`

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Currying Examples

`mult :: Int -> Int -> Int -> Int`



`mult x y z`

`mult a1 y z`

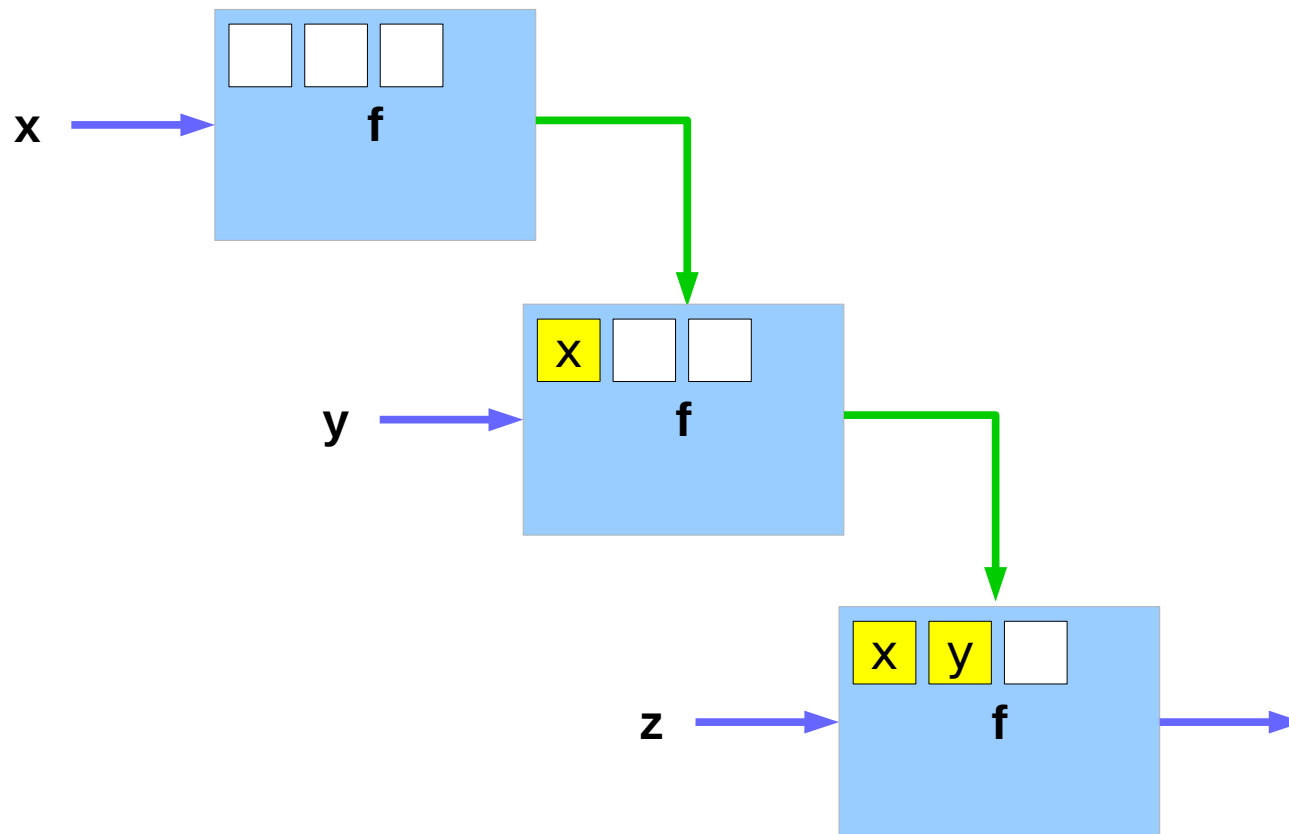
`mult a1 a2 z`

`mult a1 a2 a3`

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Currying Examples

`mult :: Int -> Int -> Int -> Int`



`mult` `x` `y` `z`

`mult` `a1` `y` `z`

`mult` `a1` `a2` `z`

`mult` `a1` `a2` `a3`

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Curry & Uncurry

$f :: a \rightarrow b \rightarrow c$ the curried form of $g :: (a, b) \rightarrow c$

$f = \text{curry } g$
 $g = \text{uncurry } f$

$f \ x \ y = g \ (x,y)$

the curried form is usually more convenient because it allows **partial application**.

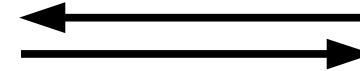
all functions are considered **curried**

all functions take **just one argument**

the curried form

$f :: a \rightarrow b \rightarrow c$

currying



$g :: (a, b) \rightarrow c$

uncurrying

$f \ x \ y$

$g \ (x,y)$

<https://wiki.haskell.org/Currying>

References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>