Row Reduction (B)

Young Won Lim
01/17/2015

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using OpenOffice and Octave.

Leading and Free Variables

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 9 \end{array}\right)$$

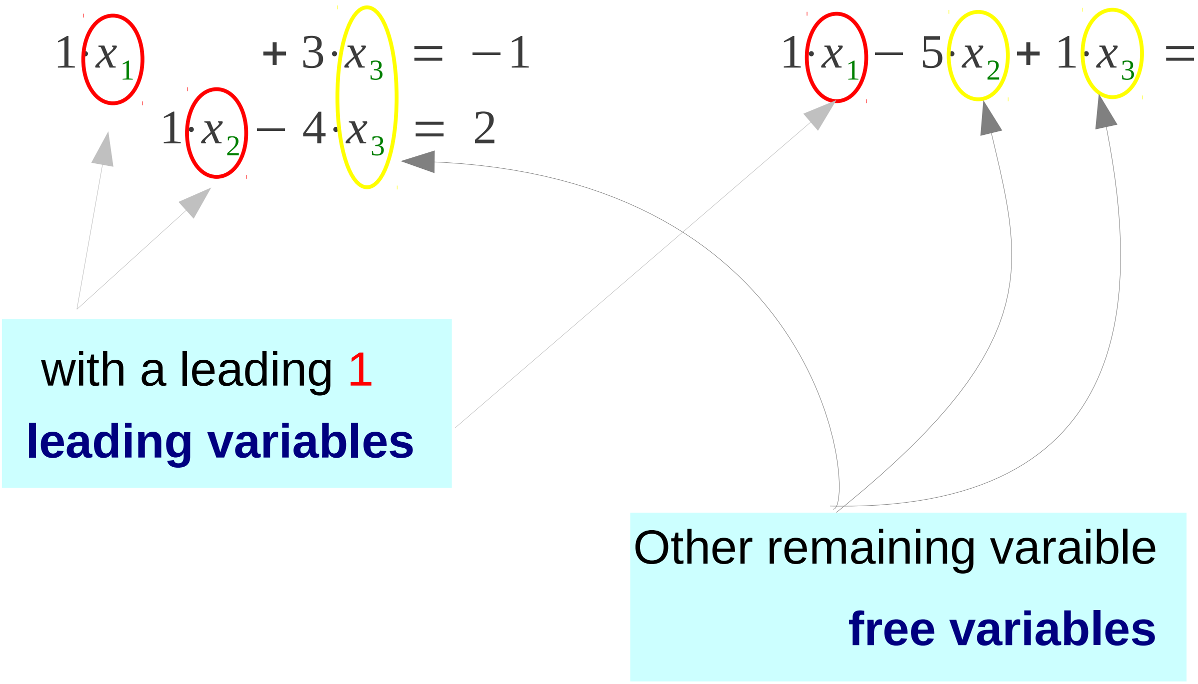
$$\begin{aligned} 1\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 &= 5 \\ 0\cdot x_1 + 1\cdot x_2 + 0\cdot x_3 &= 7 \\ 0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 &= 9 \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

$$0\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 = 1$$

$$\cancel{0 = 1}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & -5 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$



$$\begin{aligned} 1\cdot x_1 \qquad\quad + 3\cdot x_3 &= -1 \\ 1\cdot x_2 - 4\cdot x_3 &= 2 \end{aligned}$$

$$1\cdot x_1 - 5\cdot x_2 + 1\cdot x_3 = 4$$

with a leading 1

leading variables

Other remaining varaible

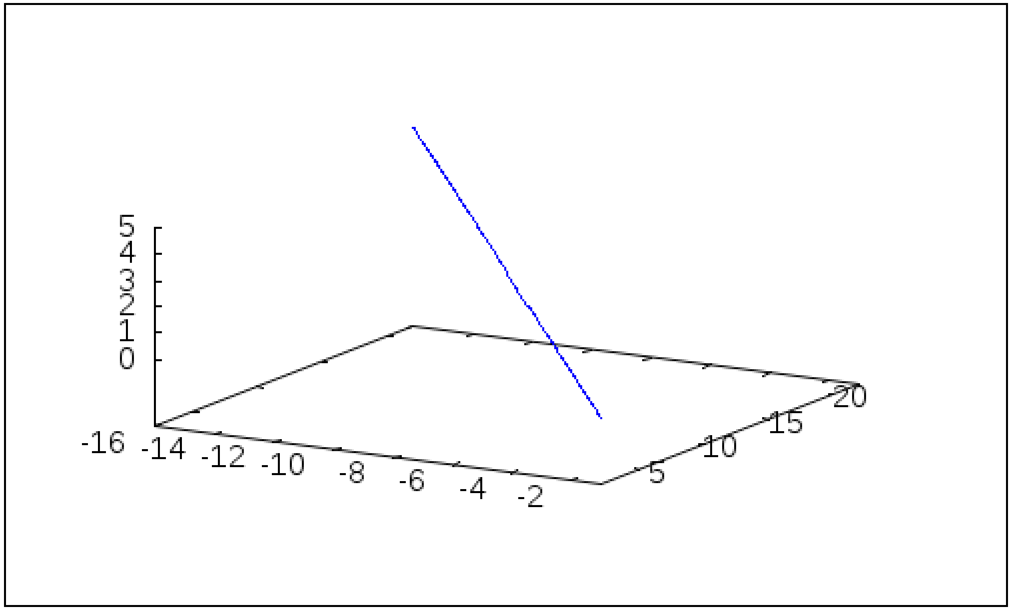
free variables

Pulse

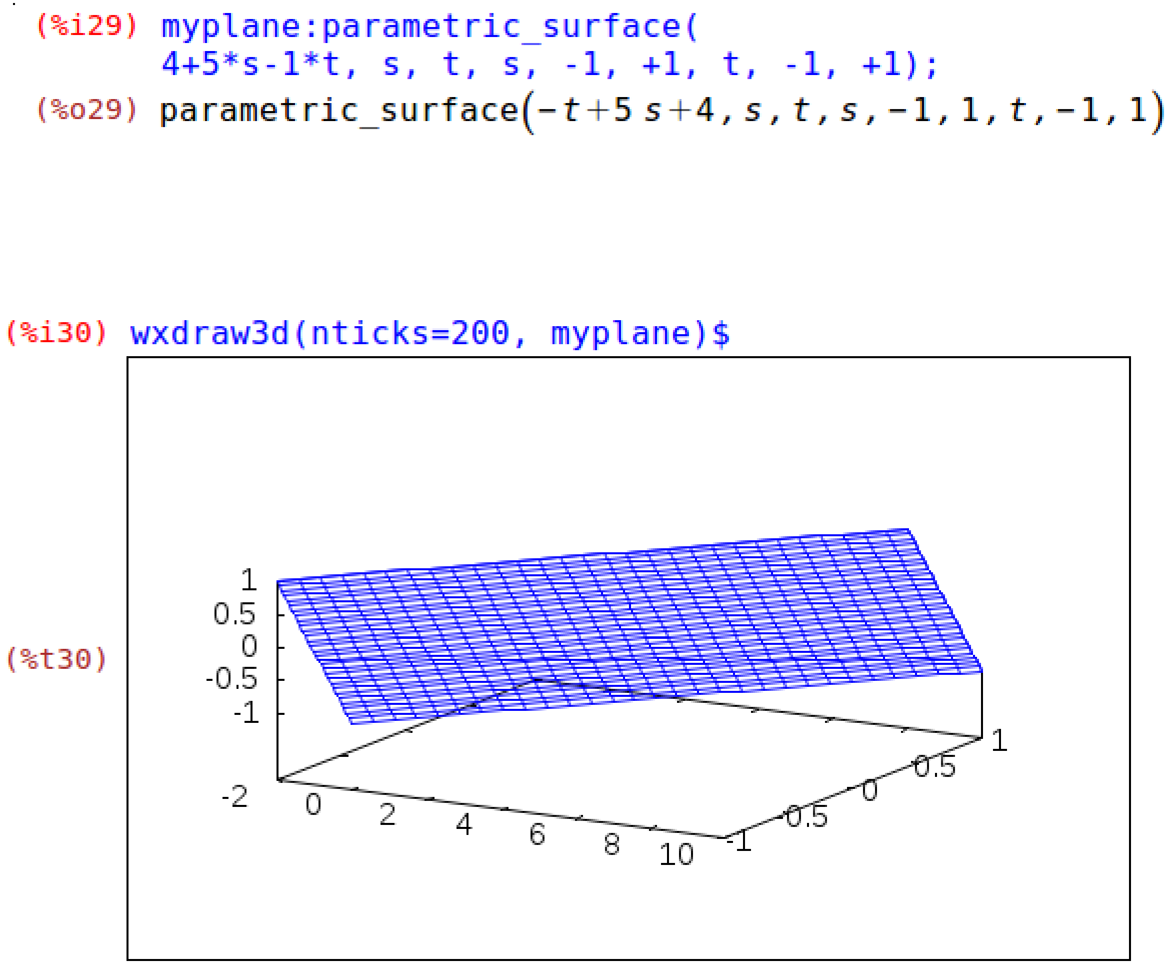
```
(%i27) myline:parametric(-1-3*t, 2+4*t, t, t, 0, 5);
(%o27) parametric(-3 t-1, 4 t+2, t, t, 0, 5)
```

```
(%i28) wxdraw3d(nticks=200, myline)$
```

(%t28)



Pulse



```
(%i29) myplane:parametric_surface(
       4+5*s-1*t, s, t, s, -1, +1, t, -1, +1);
(%o29) parametric_surface(-t+5 s+4, s, t, s, -1, 1, t, -1, 1)
```

```
(%i30) wxdraw3d(nticks=200, myplane)$
```

(%t30)

Free Variables as Parameters (1)

$$\begin{aligned} 1\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 &= 5 \\ 0\cdot x_1 + 1\cdot x_2 + 0\cdot x_3 &= 7 \\ 0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 &= 9 \end{aligned}$$

$$\begin{aligned} 1\cdot x_1 \quad\quad + 3\cdot x_3 &= -1 \\ 1\cdot x_2 - 4\cdot x_3 &= 2 \end{aligned}$$

$$1\cdot x_1 - 5\cdot x_2 + 1\cdot x_3 = 4$$

Solve for a leading variable

$$\begin{cases} x_1 = 5 \\ x_2 = 7 \\ x_3 = 9 \end{cases}$$

$$\begin{cases} x_1 = -1 - 3\cdot x_3 \\ x_2 = 2 + 4\cdot x_3 \end{cases}$$

$$x_1 = 4 + 5\cdot x_2 - 1\cdot x_3$$

Treat a free variable
as a parameter

$$x_3 = t$$

$$x_2 = s \qquad x_3 = t$$

$$\begin{cases} x_1 = 5 \\ x_2 = 7 \\ x_3 = 9 \end{cases}$$

$$\begin{cases} x_1 = -1 - 3t \\ x_2 = 2 + 4t \\ x_3 = t \end{cases}$$

$$\begin{cases} x_1 = 4 + 5\cdot s - 1\cdot t \\ x_2 = s \\ x_3 = t \end{cases}$$

Parametric Solutions (2)

$$\begin{aligned} 1\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 &= 5 \\ 0\cdot x_1 + 1\cdot x_2 + 0\cdot x_3 &= 7 \\ 0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 &= 9 \end{aligned}$$

$$\begin{cases} x_1 = 5 \\ x_2 = 7 \\ x_3 = 9 \end{cases}$$

many other forms of parametric solutions

$$\begin{aligned} 1\cdot x_1 \qquad + 3\cdot x_3 &= -1 \\ 1\cdot x_2 - 4\cdot x_3 &= 2 \end{aligned}$$

$$\begin{cases} x_1 = t \\ x_2 = \frac{1}{3}(-4t+2) \\ x_3 = \frac{1}{3}(-t-1) \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{4}(-3t+2) \\ x_2 = t \\ x_3 = \frac{1}{4}(t-2) \end{cases}$$

$$\begin{cases} x_1 = -1 - 3t \\ x_2 = 2 + 4t \\ x_3 = t \end{cases}$$

$$1\cdot x_1 - 5\cdot x_2 + 1\cdot x_3 = 4$$

$$\begin{cases} x_1 = s \\ x_2 = t \\ x_3 = -s + 5t + 4 \end{cases}$$

$$\begin{cases} x_1 = s \\ x_2 = \frac{1}{5}(s + t - 4) \\ x_3 = t \end{cases}$$

$$\begin{cases} x_1 = 4 + 5\cdot s - 1\cdot t \\ x_2 = s \\ x_3 = t \end{cases}$$

Many Solutions (3)

$$\begin{aligned} 1\cdot x_1 + 0\cdot x_2 + 0\cdot x_3 &= 5 \\ 0\cdot x_1 + 1\cdot x_2 + 0\cdot x_3 &= 7 \\ 0\cdot x_1 + 0\cdot x_2 + 1\cdot x_3 &= 9 \end{aligned}$$

$$\begin{cases} x_1 = 5 \\ x_2 = 7 \\ x_3 = 9 \end{cases}$$

$$\begin{aligned} 1\cdot x_1 \qquad + 3\cdot x_3 &= -1 \\ 1\cdot x_2 - 4\cdot x_3 &= 2 \end{aligned}$$

$$\begin{cases} x_1 = -1 - 3t \\ x_2 = 2 + 4t \\ x_3 = t \end{cases}$$

$$\begin{aligned} &(-1,\ +2,\ 0) \\ &(-4,\ +6,\ 1) \\ &(-7,\ +10,\ 2) \\ &(\underbrace{-10,\ +14,}\ 2) \end{aligned}$$

$$x_2 = -\frac{4}{3}x_1 + \frac{2}{3}$$

$$1\cdot x_1 - 5\cdot x_2 + 1\cdot x_3 = 4$$

$$\begin{cases} x_1 = 4 + 5\cdot s - 1\cdot t \\ x_2 = s \\ x_3 = t \end{cases}$$

Solutions in R^3 (4)

$$\begin{cases} x_1 = -1 - 3t \\ x_2 = 2 + 4t \\ x_3 = t \end{cases}$$

⇐ free variable ($x_3 = t$)

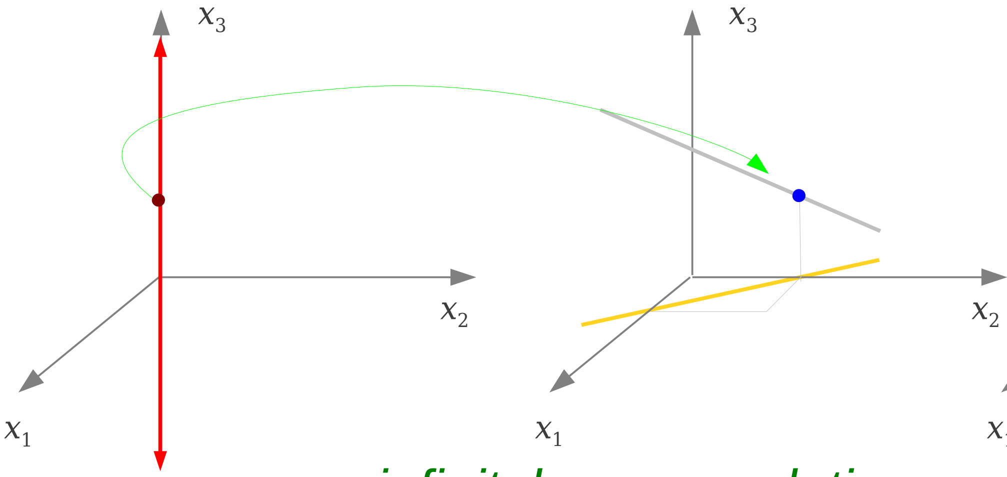
$$\begin{cases} x_1 = 4 + 5\cdot s - 1\cdot t \\ x_2 = s \\ x_3 = t \end{cases}$$

⇐ free variable ($x_2 = s$)

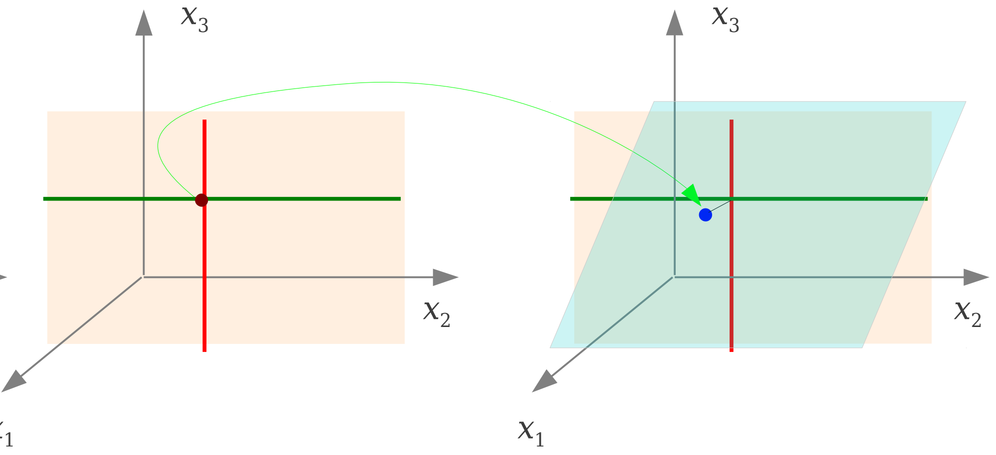
⇐ free variable ($x_3 = t$)

$$4x_1 + 3x_2 = 2$$

$$x_1 - 5x_2 + x_3 = 4$$



infinitely many solutions



infinitely many solutions

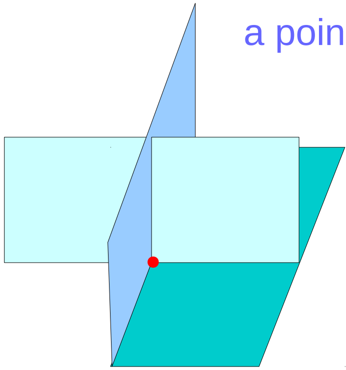
Solutions in R^3 and No of Free Variables (5)

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 9 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 3 & -1 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & -5 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

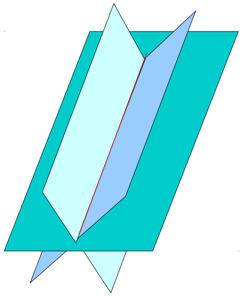
$$\begin{cases} x_1 = 5 \\ x_2 = 7 \\ x_3 = 9 \end{cases}$$

$$\begin{cases} x_1 = -1 - 3t \\ x_2 = 2 + 4t \\ x_3 = t & \Leftarrow \text{free variable} \end{cases}$$

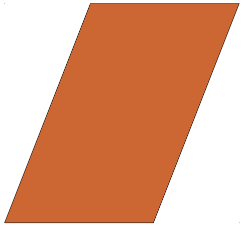
$$\begin{cases} x_1 = 4 + 5 \cdot s - 1 \cdot t \\ x_2 = s & \Leftarrow \text{free variable} \\ x_3 = t & \Leftarrow \text{free variable} \end{cases}$$



a point in R^3



a line in R^3



a plane in R^3

Pulse

References

[1] http://en.wikipedia.org/
[2] Anton & Busby, “Contemporary Linear Algebra”
[3] Anton & Rorres, “Elementary Linear Algebra”