

Trigonometry (4A)

- Trigonometric Identities
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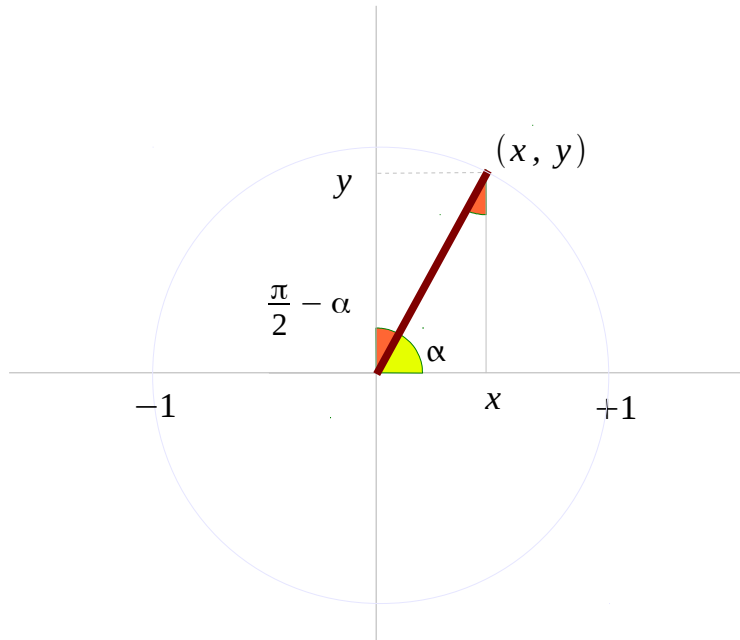
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Co-function Identities



$$\sin \alpha = y \Rightarrow \cos\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos \alpha = x \Rightarrow \sin\left(\frac{\pi}{2} - \alpha\right)$$

$$\tan \alpha = y/x \Rightarrow \cot\left(\frac{\pi}{2} - \alpha\right)$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$$

$$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$$

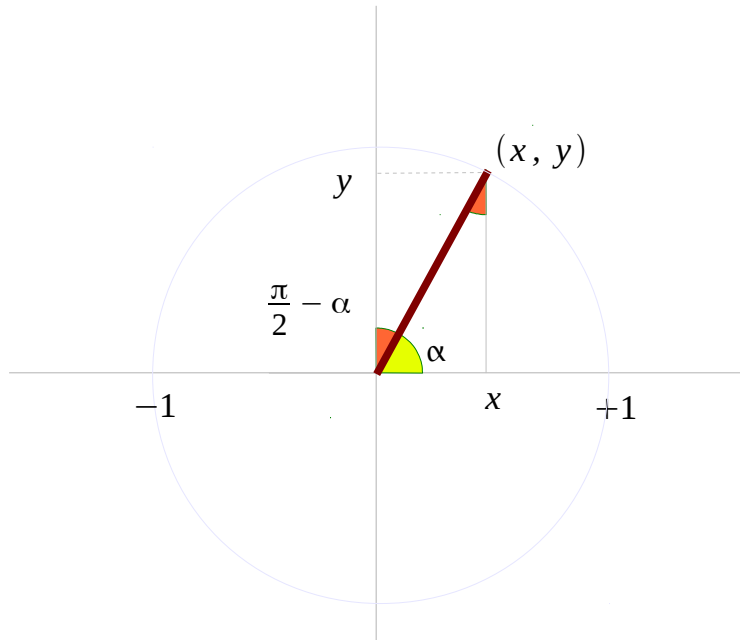
$$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$$

$$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$$

$$\csc\left(\frac{\pi}{2} - \alpha\right) = \sec \alpha$$

$$\sec\left(\frac{\pi}{2} - \alpha\right) = \csc \alpha$$

Angle Sum and Difference Identities (1)



$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(60^\circ + 30^\circ) = 1$$

$$\begin{array}{l}
 + \sin(60^\circ) = \frac{\sqrt{3}}{2} \\
 + \cos(60^\circ) = \frac{1}{2}
 \end{array}
 \quad \times \quad
 \begin{array}{l}
 \sin(30^\circ) = \frac{1}{2} \\
 \cos(30^\circ) = \frac{\sqrt{3}}{2}
 \end{array}$$

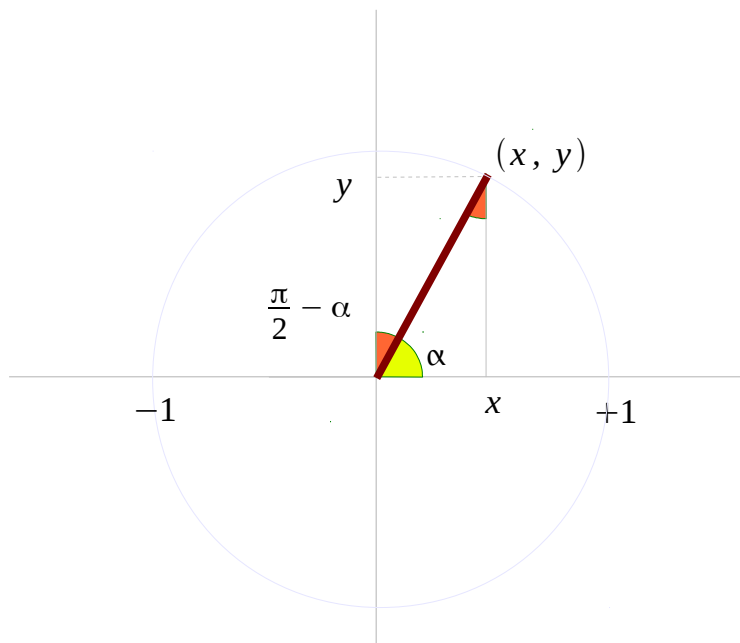
$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2} = 1$$

$$\sin(60^\circ - 30^\circ) = \frac{1}{2}$$

$$\begin{array}{l}
 + \sin(60^\circ) = \frac{\sqrt{3}}{2} \\
 - \cos(60^\circ) = \frac{1}{2}
 \end{array}
 \quad \times \quad
 \begin{array}{l}
 \sin(30^\circ) = \frac{1}{2} \\
 \cos(30^\circ) = \frac{\sqrt{3}}{2}
 \end{array}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$$

Angle Sum and Difference Identities (2)



$$\cos(\alpha + \beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\cos(30^\circ + 60^\circ) = 0$$

$$\begin{array}{l} \color{red}{-} \sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \color{blue}{-} \sin(30^\circ) = \frac{1}{2} \\ \color{blue}{+} \cos(60^\circ) = \frac{1}{2} \quad \color{blue}{-} \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

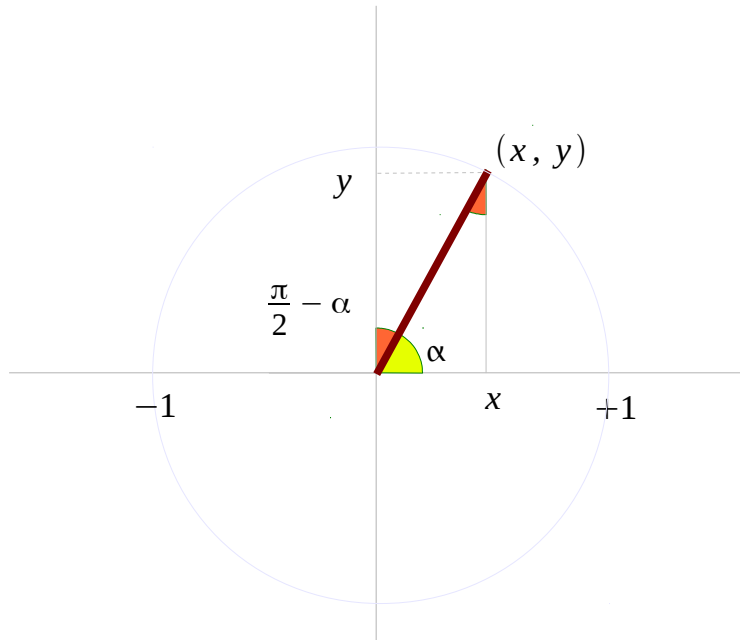
$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = 0$$

$$\cos(30^\circ - 60^\circ) = \frac{\sqrt{3}}{2}$$

$$\begin{array}{l} \color{blue}{+} \sin(60^\circ) = \frac{\sqrt{3}}{2} \quad \color{blue}{-} \sin(30^\circ) = \frac{1}{2} \\ \color{blue}{+} \cos(60^\circ) = \frac{1}{2} \quad \color{blue}{-} \cos(30^\circ) = \frac{\sqrt{3}}{2} \end{array}$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

Angle Sum and Difference Identities (3)



$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

$$\tan(30^\circ + 60^\circ) = +\infty$$

$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \text{—————} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \text{—————} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

$$\frac{\frac{1}{\sqrt{3}} + \sqrt{3}}{1 - \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = +\infty$$

$$\tan(30^\circ - 60^\circ) = -\frac{1}{\sqrt{3}}$$

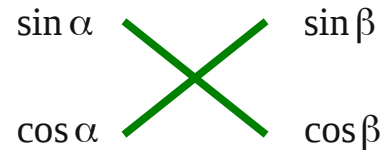
$$\tan(30^\circ) = \frac{1}{\sqrt{3}} \quad \text{—————} \quad \tan(60^\circ) = \sqrt{3}$$

$$\tan(60^\circ) = \sqrt{3} \quad \text{—————} \quad \tan(30^\circ) = \frac{1}{\sqrt{3}}$$

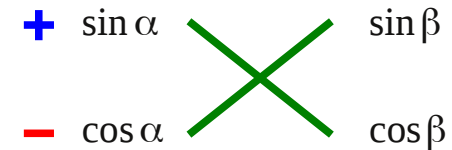
$$\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \cdot \sqrt{3}} = -\frac{1}{\sqrt{3}}$$

Angle Sum and Difference Identities (4)

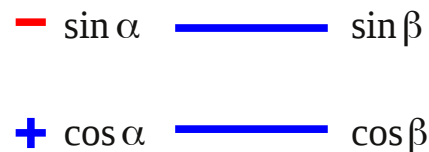
$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$



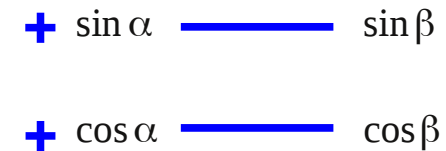
$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$



$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$



$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$



$$\frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)} = \frac{\sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta}$$

$$\frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta}{\cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta}$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

Product to Sum (1)

$$+ \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$+ \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) + \sin(\alpha - \beta) = 2 \sin \alpha \cdot \cos \beta$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$+ \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$+ \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \cos \alpha \cdot \cos \beta$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ + \cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$+ \sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$- \sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta$$

$$\cos \alpha \cdot \sin \beta = \frac{1}{2} \{ \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

$$- \cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$+ \cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$- \cos(\alpha + \beta) + \cos(\alpha - \beta) = 2 \sin \alpha \sin \beta$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \{ - \cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

Product to Sum (2)

$$\sin(\alpha \pm \beta) = \sin \alpha \cdot \cos \beta \pm \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cdot \cos \beta \mp \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} \{ + \sin(\alpha + \beta) + \sin(\alpha - \beta) \}$$

$$\cos \alpha \cdot \sin \beta = \frac{1}{2} \{ + \sin(\alpha + \beta) - \sin(\alpha - \beta) \}$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ + \cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \{ + \cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} \{ -\cos(\alpha + \beta) + \cos(\alpha - \beta) \}$$

Angle sum and difference identities

Sine	$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$ ^{[8][9]}
Cosine	$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$ ^{[9][10]}
Tangent	$\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$ ^{[9][11]}
Arcsine	$\arcsin \alpha \pm \arcsin \beta = \arcsin \left(\alpha \sqrt{1 - \beta^2} \pm \beta \sqrt{1 - \alpha^2} \right)$ ^[12]
Arccosine	$\arccos \alpha \pm \arccos \beta = \arccos \left(\alpha \beta \mp \sqrt{(1 - \alpha^2)(1 - \beta^2)} \right)$ ^[13]
Arctangent	$\arctan \alpha \pm \arctan \beta = \arctan \left(\frac{\alpha \pm \beta}{1 \mp \alpha \beta} \right)$ ^[14]

<http://en.wikipedia.org/wiki/Derivative>

Double Angle Formula

Double-angle formulae ^{[18][19]}			
$\sin 2\theta = 2 \sin \theta \cos \theta$ $= \frac{2 \tan \theta}{1 + \tan^2 \theta}$	$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= 2 \cos^2 \theta - 1$ $= 1 - 2 \sin^2 \theta$ $= \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$	$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$	$\cot 2\theta = \frac{\cot^2 \theta - 1}{2 \cot \theta}$

<http://en.wikipedia.org/wiki/Derivative>

Triple-angle formulae

Triple-angle formulae ^{[16][20]}			
$\begin{aligned}\sin 3\theta &= -\sin^3 \theta + 3 \cos^2 \theta \sin \theta \\ &= -4 \sin^3 \theta + 3 \sin \theta\end{aligned}$	$\begin{aligned}\cos 3\theta &= \cos^3 \theta - 3 \sin^2 \theta \cos \theta \\ &= 4 \cos^3 \theta - 3 \cos \theta\end{aligned}$	$\tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$	$\cot 3\theta = \frac{3 \cot \theta - \cot^3 \theta}{1 - 3 \cot^2 \theta}$

<http://en.wikipedia.org/wiki/Derivative>

Half-angle formulae

Half-angle formulae ^{[21][22]}			
$\sin \frac{\theta}{2} = \operatorname{sgn}\left(2\pi - \theta + 4\pi \left\lfloor \frac{\theta}{4\pi} \right\rfloor\right) \sqrt{\frac{1 - \cos \theta}{2}}$ <p>(or $\sin^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{2}$)</p>	$\cos \frac{\theta}{2} = \operatorname{sgn}\left(\pi + \theta + 4\pi \left\lfloor \frac{\pi - \theta}{4\pi} \right\rfloor\right) \sqrt{\frac{1 + \cos \theta}{2}}$ <p>(or $\cos^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}$)</p>	$\begin{aligned} \tan \frac{\theta}{2} &= \csc \theta - \cot \theta \\ &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} \\ &= \frac{\sin \theta}{1 + \cos \theta} \\ &= \frac{1 - \cos \theta}{\sin \theta} \end{aligned}$ $\tan \frac{\eta + \theta}{2} = \frac{\sin \eta + \sin \theta}{\cos \eta + \cos \theta}$ $\tan \left(\frac{\theta}{2} + \frac{\pi}{4}\right) = \sec \theta + \tan \theta$ $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \frac{1 - \tan(\theta/2)}{1 + \tan(\theta/2)}$ $\tan \frac{1}{2}\theta = \frac{\tan \theta}{1 + \sqrt{1 + \tan^2 \theta}}$ <p>for $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$</p>	$\begin{aligned} \cot \frac{\theta}{2} &= \csc \theta + \cot \theta \\ &= \pm \sqrt{\frac{1 + \cos \theta}{1 - \cos \theta}} \\ &= \frac{\sin \theta}{1 - \cos \theta} \\ &= \frac{1 + \cos \theta}{\sin \theta} \end{aligned}$

<http://en.wikipedia.org/wiki/Derivative>

Power-reduction formula

Sine	Cosine	Other
$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$	$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$	$\sin^2 \theta \cos^2 \theta = \frac{1 - \cos 4\theta}{8}$
$\sin^3 \theta = \frac{3 \sin \theta - \sin 3\theta}{4}$	$\cos^3 \theta = \frac{3 \cos \theta + \cos 3\theta}{4}$	$\sin^3 \theta \cos^3 \theta = \frac{3 \sin 2\theta - \sin 6\theta}{32}$
$\sin^4 \theta = \frac{3 - 4 \cos 2\theta + \cos 4\theta}{8}$	$\cos^4 \theta = \frac{3 + 4 \cos 2\theta + \cos 4\theta}{8}$	$\sin^4 \theta \cos^4 \theta = \frac{3 - 4 \cos 4\theta + \cos 8\theta}{128}$
$\sin^5 \theta = \frac{10 \sin \theta - 5 \sin 3\theta + \sin 5\theta}{16}$	$\cos^5 \theta = \frac{10 \cos \theta + 5 \cos 3\theta + \cos 5\theta}{16}$	$\sin^5 \theta \cos^5 \theta = \frac{10 \sin 2\theta - 5 \sin 6\theta + \sin 10\theta}{512}$

<http://en.wikipedia.org/wiki/Derivative>

Product-to-sum

Product-to-sum ^[24]
$\cos \theta \cos \varphi = \frac{\cos(\theta - \varphi) + \cos(\theta + \varphi)}{2}$
$\sin \theta \sin \varphi = \frac{\cos(\theta - \varphi) - \cos(\theta + \varphi)}{2}$
$\sin \theta \cos \varphi = \frac{\sin(\theta + \varphi) + \sin(\theta - \varphi)}{2}$
$\cos \theta \sin \varphi = \frac{\sin(\theta + \varphi) - \sin(\theta - \varphi)}{2}$
$\tan \theta \tan \varphi = \frac{\cos(\theta - \varphi) - \cos(\theta + \varphi)}{\cos(\theta - \varphi) + \cos(\theta + \varphi)}$
$\prod_{k=1}^n \cos \theta_k = \frac{1}{2^n} \sum_{e \in S} \cos(e_1 \theta_1 + \cdots + e_n \theta_n)$ where $S = \{1, -1\}^n$

<http://en.wikipedia.org/wiki/Derivative>

Sum-to-product

Sum-to-product ^[25]
$\sin \theta \pm \sin \varphi = 2 \sin \left(\frac{\theta \pm \varphi}{2} \right) \cos \left(\frac{\theta \mp \varphi}{2} \right)$
$\cos \theta + \cos \varphi = 2 \cos \left(\frac{\theta + \varphi}{2} \right) \cos \left(\frac{\theta - \varphi}{2} \right)$
$\cos \theta - \cos \varphi = -2 \sin \left(\frac{\theta + \varphi}{2} \right) \sin \left(\frac{\theta - \varphi}{2} \right)$

<http://en.wikipedia.org/wiki/Derivative>

Euler's Formula

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

$$e^{i(A+B)} = \cos(A+B) + i\sin(A+B)$$

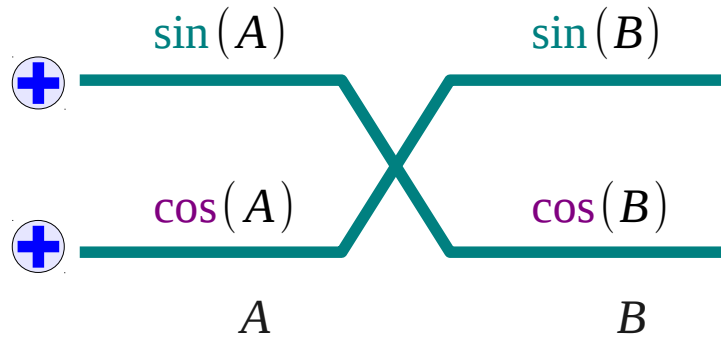
$$\begin{aligned} e^{iA} e^{iB} &= (\cos(A) + i\sin(A))(\cos(B) + i\sin(B)) \\ &= [\cos(A)\cos(B) - \sin(A)\sin(B)] \\ &\quad + i[\cos(A)\sin(B) + \sin(A)\cos(B)] \end{aligned}$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

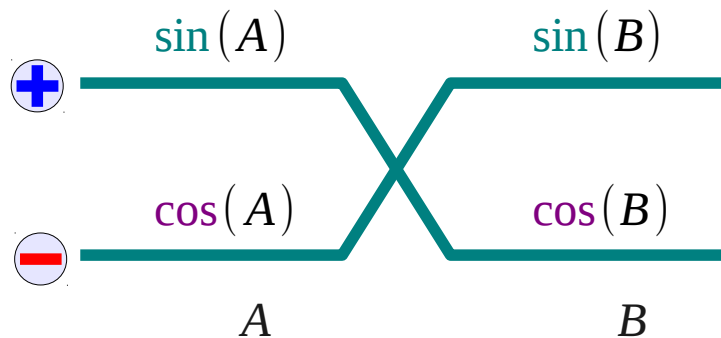
Sin(angle sum and difference)

$$\sin(A+B)$$



$$\sin(A)\cos(B) - \cos(A)\sin(B)$$

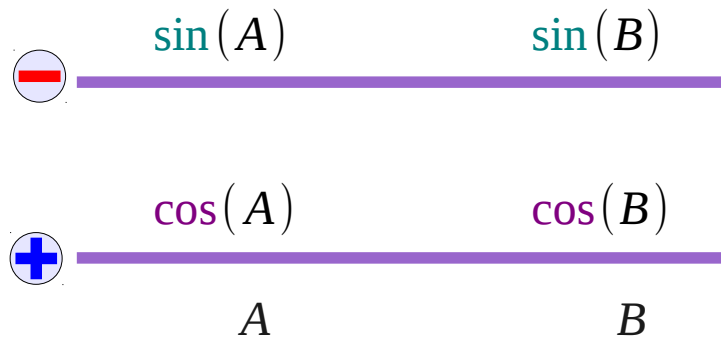
$$\sin(A-B)$$



$$\sin(A)\cos(B) - \cos(A)\sin(B)$$

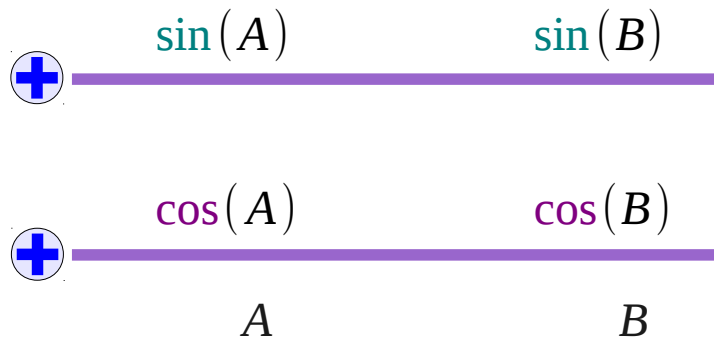
Cos(angle sum and difference)

$$\cos(A+B)$$



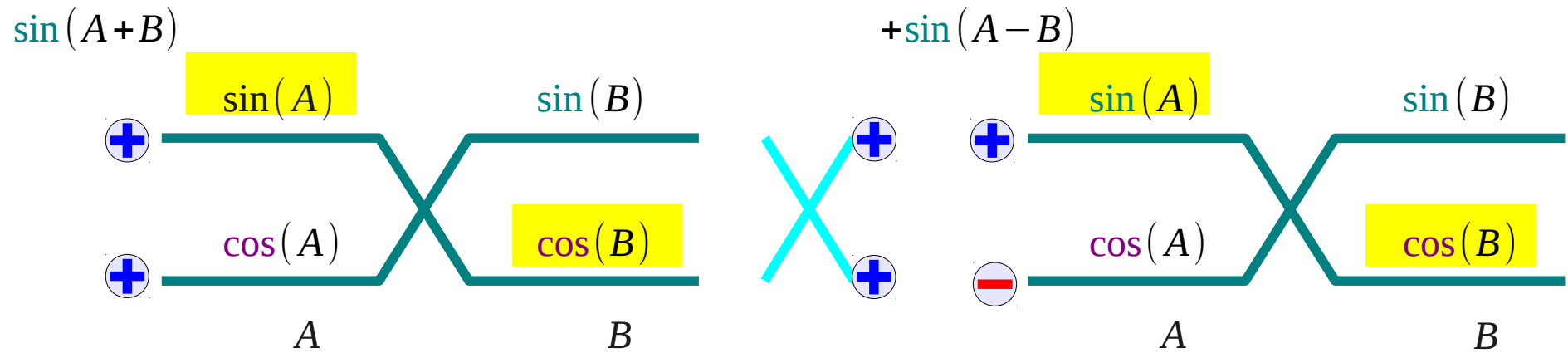
$$\cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\cos(A-B)$$



$$\cos(A)\cos(B) + \sin(A)\sin(B)$$

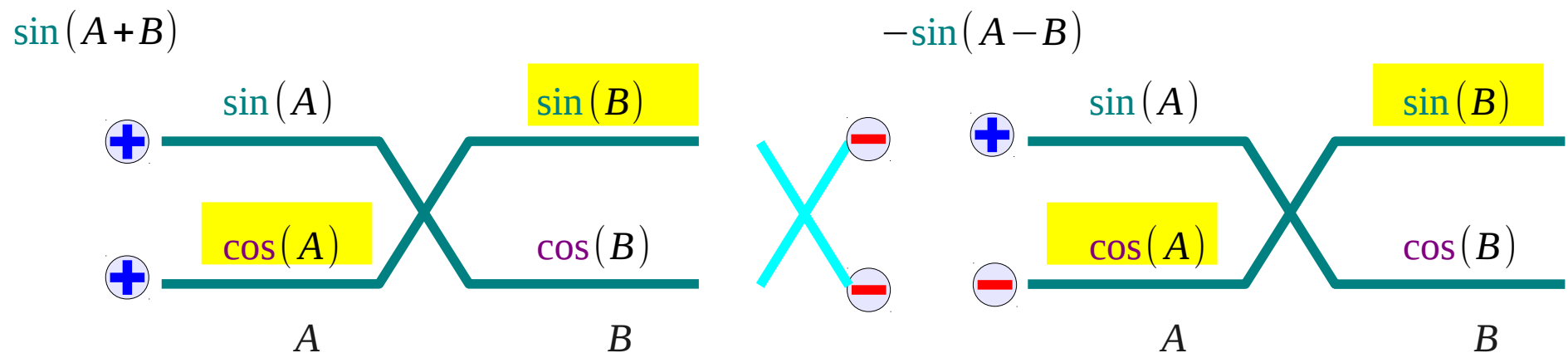
Product to Sum : sin cos



$$\sin(A+B) + \sin(A-B) \Leftarrow 2\sin(A)\cos(B)$$

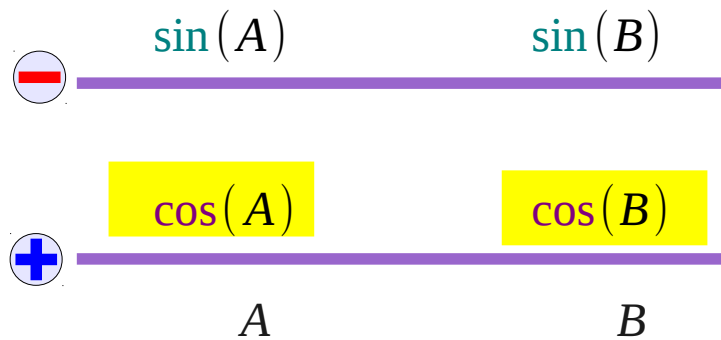
Product to Sum : cos sin

$$\sin(A+B) - \sin(A-B) \Leftrightarrow 2\cos(A)\sin(B)$$

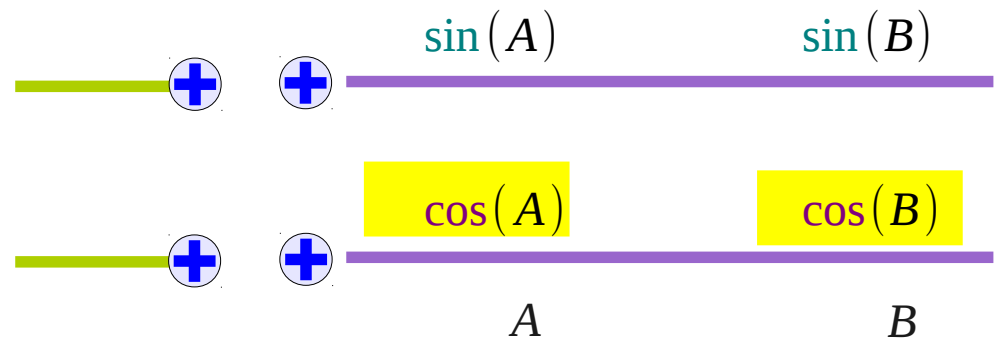


Product to Sum : cos cos

$\cos(A+B)$



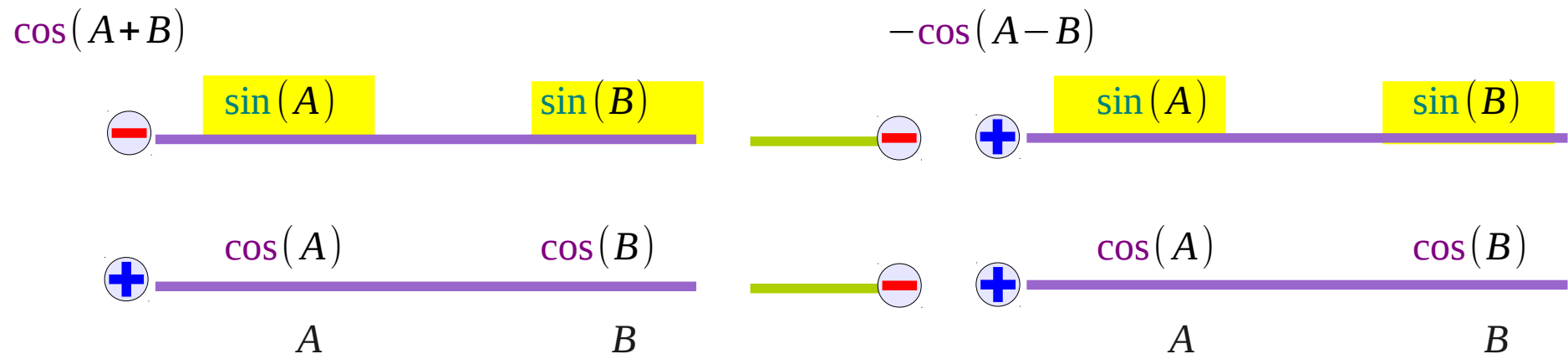
$+\cos(A-B)$



$$\cos(A+B) + \cos(A-B) \Leftarrow 2\cos(A)\cos(B)$$

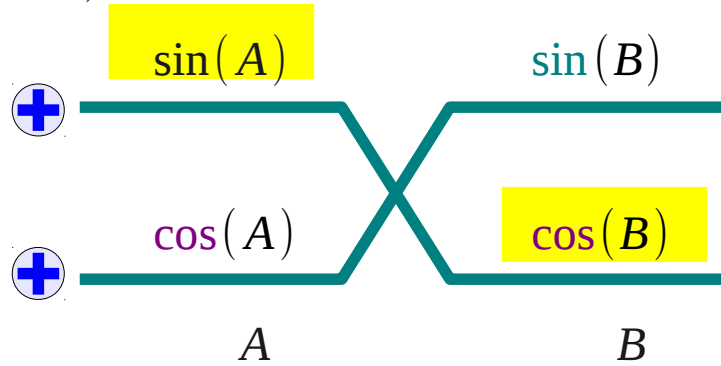
Product to Sum : sin sin

$$\begin{aligned}\cos(A+B) - \cos(A-B) &\Leftarrow -2\sin(A)\sin(B) \\ -\cos(A+B) + \cos(A-B) &\Leftarrow +2\sin(A)\sin(B)\end{aligned}$$

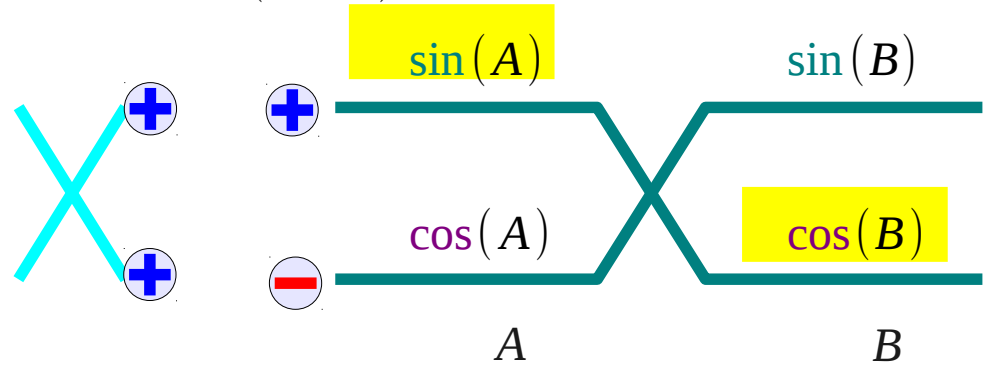


Product to Sum

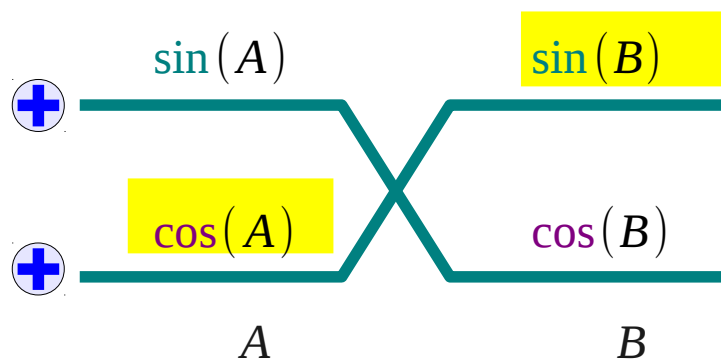
$$\sin(A+B)$$



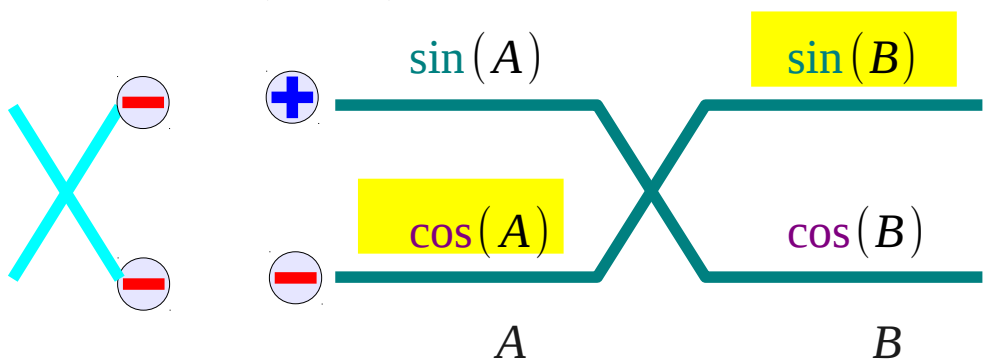
$$+\sin(A-B)$$



$$\sin(A+B)$$

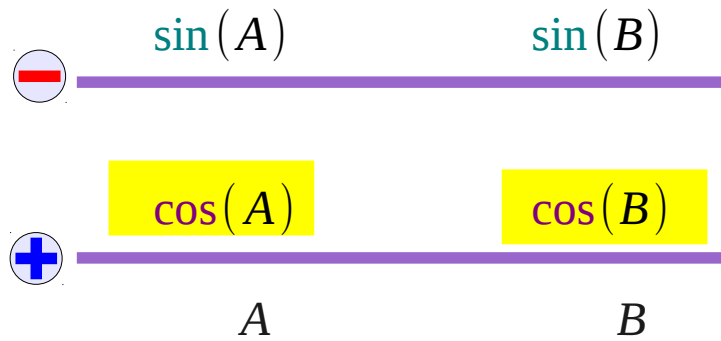


$$-\sin(A-B)$$

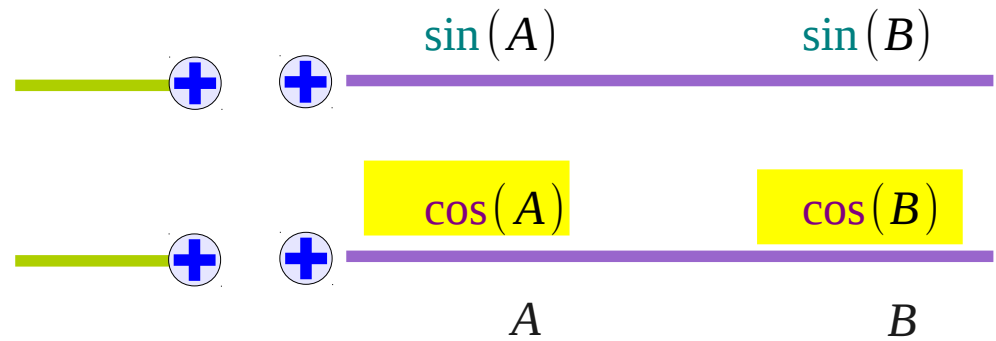


Product to Sum

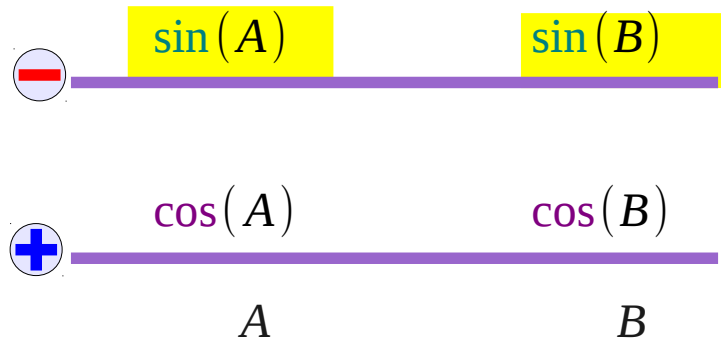
$$\cos(A+B)$$



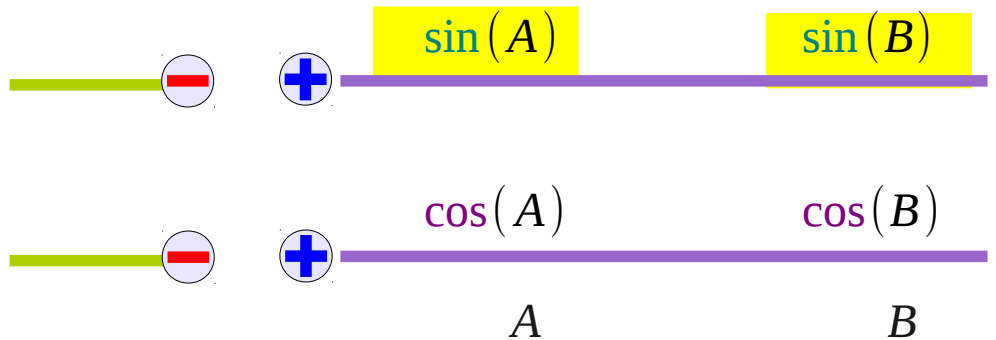
$$+\cos(A-B)$$



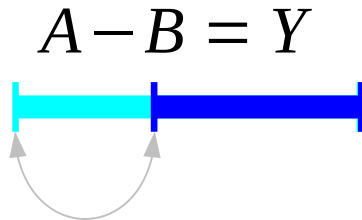
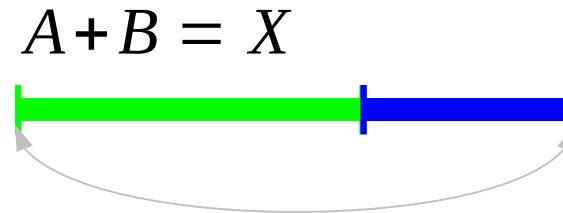
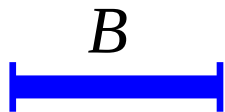
$$\cos(A+B)$$



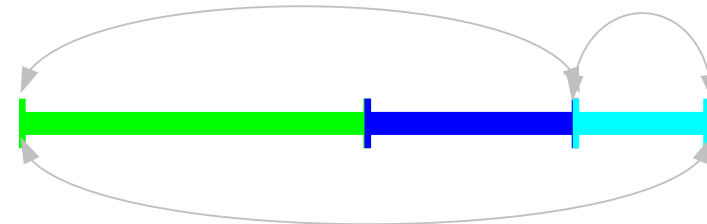
$$-\cos(A-B)$$



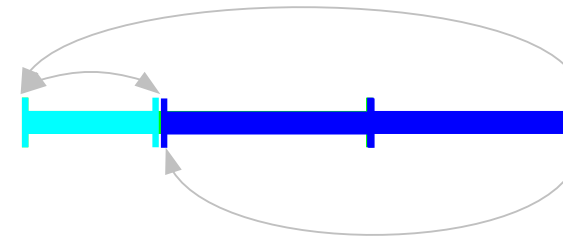
Sum and Difference



$$X + Y = A + B + A - B = 2A$$

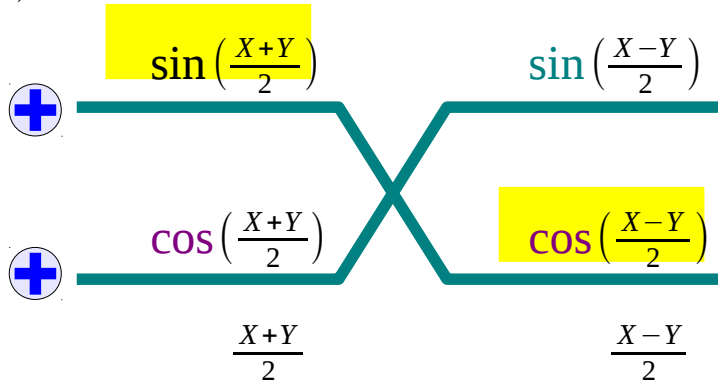


$$X - Y = A + B - A + B = 2B$$

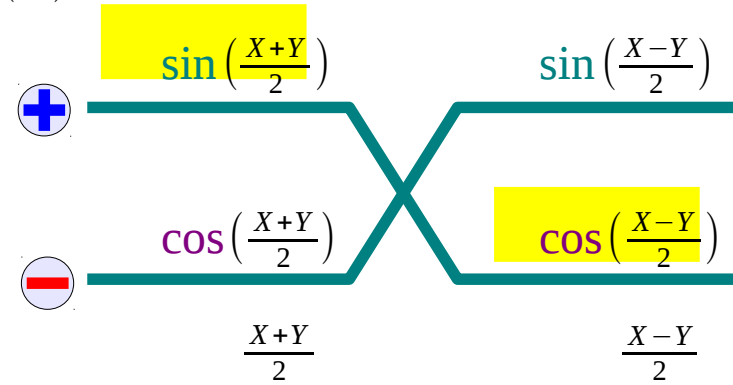


Product to Sum

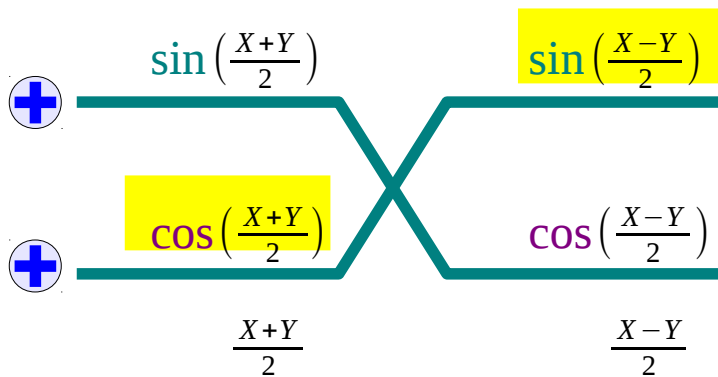
$\sin(X)$



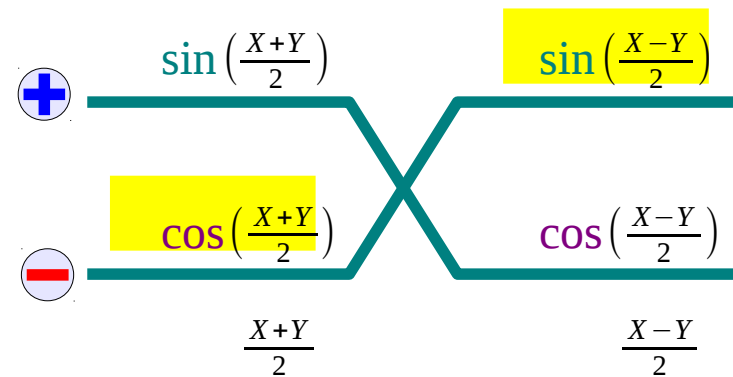
$+\sin(Y)$



$\sin(X)$



$-\sin(Y)$



Product to Sum

$\cos(X)$

$$\begin{aligned} & \ominus \sin\left(\frac{X+Y}{2}\right) \sin\left(\frac{X-Y}{2}\right) \\ & \oplus \cos\left(\frac{X+Y}{2}\right) \cos\left(\frac{X-Y}{2}\right) \end{aligned}$$

$+\cos(Y)$

$$\begin{aligned} & \oplus \sin\left(\frac{X+Y}{2}\right) \sin\left(\frac{X-Y}{2}\right) \\ & \oplus \cos\left(\frac{X+Y}{2}\right) \cos\left(\frac{X-Y}{2}\right) \end{aligned}$$

$\cos(X)$

$$\begin{aligned} & \ominus \sin\left(\frac{X+Y}{2}\right) \sin\left(\frac{X-Y}{2}\right) \\ & \oplus \cos\left(\frac{X+Y}{2}\right) \cos\left(\frac{X-Y}{2}\right) \end{aligned}$$

$-\cos(Y)$

$$\begin{aligned} & \ominus \sin\left(\frac{X+Y}{2}\right) \sin\left(\frac{X-Y}{2}\right) \\ & \oplus \cos\left(\frac{X+Y}{2}\right) \cos\left(\frac{X-Y}{2}\right) \end{aligned}$$

Product-to-Sum & Sum-to-Product

SUM

PRODUCT

$$\sin(A+B) + \sin(A-B) \Leftarrow 2\sin(A)\cos(B)$$

$$\sin(A+B) - \sin(A-B) \Leftarrow 2\cos(A)\sin(B)$$

$$\cos(A+B) + \cos(A-B) \Leftarrow 2\cos(A)\cos(B)$$

$$-\cos(A+B) + \cos(A-B) \Leftarrow 2\sin(A)\sin(B)$$

SUM

PRODUCT

$$\sin(X) + \sin(Y) \Rightarrow 2\sin\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$$

$$\sin(X) - \sin(Y) \Rightarrow 2\cos\left(\frac{X+Y}{2}\right)\sin\left(\frac{X-Y}{2}\right)$$

$$\cos(X) + \cos(Y) \Rightarrow 2\cos\left(\frac{X+Y}{2}\right)\cos\left(\frac{X-Y}{2}\right)$$

$$-\cos(X) + \cos(Y) \Rightarrow 2\sin\left(\frac{X+Y}{2}\right)\sin\left(\frac{X-Y}{2}\right)$$

References

- [1] <http://en.wikipedia.org/>
- [2] <http://planetmath.org/>
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- [5] 홍성대, "기본/실력 수학의 정석," 성지출판