Power Density Spectrum - Discrete Time

Young W Lim

October 22, 2019

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Bilateral z-Transform of $R_{XX}[n]$

N Gaussian random variables

Definition

$$S_{XX}(z) = \sum_{n=-\infty}^{\infty} R_{XX}[n] z^{-n}$$

Definition

$$S_{XX}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} R_{XX}[n] e^{-jn\Omega}$$

$$R_{XX}[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{XX}(e^{j\Omega}) e^{jn\Omega} d\Omega$$

1. $S_{XX}(e^{j\Omega}) \geq 0$
2. $S_{XX}(e^{-j\Omega}) = S_{XX}(e^{+j\Omega})$ for real $X[n]$
3. $S_{XX}(e^{+j\Omega})$ is real
4. $\frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{XX}(e^{j\Omega}) d\Omega = E[X^2[n]]$

Definition

$$\hat{R}_N[k] = \frac{1}{N} \sum_{n=0}^{N-1-|k|} X[n]X[n+|k|] \qquad |k| < N$$

Definition

$$X_N(\Omega_k) = \sum_{n=0}^{N-1} X[n]e^{-j\Omega_k n} \qquad k = 0, 1, ..., N-1$$

$$\Omega_k = \frac{2\pi k}{N} \qquad k = 0, 1, ..., N-1$$