

# Power Density Spectrum - Discrete Time

Young W Lim

October 22, 2019

Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles,Jr. and B. Shi



# Bilateral z-Transform of $R_{XX}[n]$

$N$  Gaussian random variables

## Definition

$$S_{XX}(z) = \sum_{n=-\infty}^{\infty} R_{XX}[n]z^{-n}$$

# Discrete Time Fourier Transform of $R_{XX}[n]$

$N$  Gaussian random variables

## Definition

$$S_{XX}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} R_{XX}[n] e^{-jn\Omega}$$

$$R_{XX}[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{XX}(e^{j\Omega}) e^{jn\Omega} d\Omega$$

# Properties of Power Density Spectrum - DT

$N$  Gaussian random variables

- 1  $S_{XX}(e^{j\Omega}) \geq 0$
- 2  $S_{XX}(e^{-j\Omega}) = S_{XX}(e^{+j\Omega})$  for real  $X[n]$
- 3  $S_{XX}(e^{+j\Omega})$  is real
- 4  $\frac{1}{2\pi} \int_{-\pi}^{+\pi} S_{XX}(e^{j\Omega}) d\Omega = E[X^2[n]]$

# Estimating the Power Density Spectrum

$N$  Gaussian random variables

## Definition

$$\hat{R}_N[k] = \frac{1}{N} \sum_{n=0}^{N-1-|k|} X[n]X[n+|k|] \quad |k| < N$$

# DTFT, FFT

$N$  Gaussian random variables

## Definition

$$X_N(\Omega_k) = \sum_{n=0}^{N-1} X[n] e^{-j\Omega_k n} \quad k = 0, 1, \dots, N-1$$

$$\Omega_k = \frac{2\pi k}{N} \quad k = 0, 1, \dots, N-1$$





