

# Carry and Overflow

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- 1 "Self-service Linux: Mastering the Art of Problem Determination",

Mark Wilding

- 1 "Computer Architecture: A Programmer's Perspective", Bryant & O'Hallaron

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# Compiling 32-bit program on 64-bit gcc

- `gcc -v`
- `gcc -m32 t.c`
- `sudo apt-get install gcc-multilib`
- `sudo apt-get install g++-multilib`
- `gcc-multilib`
- `g++-multilib`
- `gcc -m32`
- `objdump -m i386`

- Carry flag and overflow flag
- Signed and unsigned computations
- Flags for an unsigned number
- Flags for a signed number
- Detecting errors in unsigned and signed arithmetic
- The verb to overflow v.s. the overflow flag

# Carry flag and overflow flag

- considering carry and overflow flags in **x86**
- do not confuse the **carry flag** with the **overflow flag** in integer arithmetic.
- the *ALU* always sets these flags appropriately when doing any integer math.
- these flags can occur on its *own*, or *both* together.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Signed and unsigned computations

- the CPU's ALU doesn't care or know whether **signed** or **unsigned** computations are performed;
- the ALU just performs integer arithmetic and sets the flags appropriately.
- It's up to the programmer to know which flag to check after the arithmetic is done.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Flags for an unsigned number

- if a word is treated as an **unsigned** number,
  - the **carry** flag must be used to check if the result is fit into  $n$ -bit or  $(n+1)$ -bit number
  - the **overflow** flag is *irrelevant* to an **unsigned** number arithmetic

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Flags for a signed number

- if a word is treated as an **signed** number,
  - the **carry** flag is *irrelevant* to an **signed** number arithmetic
  - the **overflow** flag must be used to check if the result is wrong or not

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Detecting errors in unsigned and signed arithmetic (1)

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**unsigned** integer  
arithmetic

---

**signed** integer  
arithmetic

---

**CF** Carry Flag

detects *overflows*  
extends an  $n$ -bit result  
into an  $(n+1)$ -bit result

---

**OF** Overflow Flag

detects *overflows*  
errors  
the result cannot be used

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[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Detecting errors in unsigned and signed arithmetic (2)

- **unsigned** integer arithmetic *overflow*  
is indicated by the **carry** flag
  - $P + P$  **CF=1** → carry out – the result is too large for an  $n$ -bit integer
  - $P - P$  **CF=1** → borrow in – the result is too small for an  $n$ -bit integer
- **signed** integer arithmetic *overflow*  
is indicated by the **overflow** flag
  - $P + P \rightarrow N$  **OF=1** → overflow – the result is not correct
  - $N + N \rightarrow P$  **OF=1** → overflow – the result is not correct
- $P$  (positive),  $N$  (negative)

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Detecting errors in unsigned and signed arithmetic (3)

- **unsigned** integer arithmetic *overflow* is indicated by the **carry** flag
  - the *overflowed*  $n$ -bit result can be extended into  $(n+1)$ -bit result by using the carry flag
- **signed** integer arithmetic *overflow* is indicated by the **overflow** flag
  - the *overflowed*  $n$ -bit result cannot be used

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# The verb to overflow v.s. the overflow flag (1)

- Do not confuse the English verb *to overflow* with the **overflow flag** in the ALU.
- The verb *to overflow* is used casually to indicate that some math result doesn't fit in the number of bits available;
- it could be integer math, or floating-point math, or whatever.
- The **overflow flag** is set specifically by the ALU it isn't the same as the casual English verb "to overflow"

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## The verb to overflow v.s. the overflow flag (2)

- In English, we may say "the binary/integer math overflowed the number of bits available for the result, causing the carry flag to come on".
- Note how this English usage of the verb "to overflow" is **not** the same as saying the **overflow flag** is on".
- A math result can overflow (the verb) the number of bits available without turning on the ALU **overflow flag**

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing Carry and Overflow Flags

## CF (carry flag) and OF (overflow flag) computation

| ADD (addition)                                       | SUB (subtraction)   |
|--|---|
| $CF = C_n$   | $CF = \overline{C_n}$   |
| $OF = C_n \oplus C_{n-1}$                            | $OF = C_n \oplus C_{n-1}$                                       |
| a 2's complement addition<br>$A + B = A + B + 0$     | a transformed addition<br>$A - B = A + \overline{B} + 1$        |
| $\{C_n, S_{n-1}\} = a_{n-1} + b_{n-1} + c_{n-1}$     | $\{C_n, S_{n-1}\} = a_{n-1} + \overline{b_{n-1}} + c_{n-1}$     |
| $\{C_{n-1}, S_{n-2}\} = a_{n-2} + b_{n-2} + c_{n-2}$ | $\{C_{n-1}, S_{n-2}\} = a_{n-2} + \overline{b_{n-2}} + c_{n-2}$ |

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)

- Examples of signed and unsigned integer arithmetic
- Carry flag in unsigned and signed computations
- Rules for the carry flag
- Method for computing the carry flag
- More examples of the carry flag



# TOC: Examples of signed and unsigned integer arithmetic

- Examples of interpreting **signed** and **unsigned** numbers
- Examples of **signed** and **unsigned** integer arithmetic
- 2's complements
- **Unsigned** subtraction
- **Signed** subtraction
- Interpreting the result as a **signed** or an **unsigned** integer
- Summary of **signed** and **unsigned** subtractions
- Examples of **unsigned** integer overflows
- Examples of **signed** integer overflows

# Examples of interpreting **signed** and **unsigned** numbers (1)

- interpreting 0xFFFFBDC3

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as an **unsigned** (positive) number    +0xFFFFBDC3    +4294950339<sub>10</sub>

---

as a **signed** (negative) number    -0x0000423D    -16957<sub>10</sub>

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of interpreting **signed** and **unsigned** numbers (2)

- interpreting 0xFFFFBDC3
  - as an **unsigned** (positive) number | +0xFFFFBDC3 | +4294950339<sub>10</sub> |

$$15 * 16^7 + 15 * 16^6 + 15 * 16^5 + 15 * 16^4 \\ + 11 * 16^3 + 13 * 16^2 + 12 * 16^1 + 3 * 16^0$$

- as a **signed** (negative) number | -0x0000423D | -16957<sub>10</sub> |

$$0 * 16^7 + 0 * 16^6 + 0 * 16^5 + 0 * 16^4 \\ + 4 * 16^3 + 2 * 16^2 + 3 * 16^1 + 13 * 16^0$$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of interpreting signed and unsigned numbers (3)

- the 2's complement of 0xFFFFBDC3 : 0x0000423D (= +16957<sub>10</sub>)

|            | F      | F    | F    | F    | B    | D    | C    | 3    |
|------------|--------|------|------|------|------|------|------|------|
| 0xFFFFBDC3 | 0x1111 | 1111 | 1111 | 1111 | 1011 | 1101 | 1100 | 0011 |
| 0x0000423D | 0x0000 | 0000 | 0000 | 0000 | 0100 | 0010 | 0011 | 1100 |
| 0x0000423D | 0x0000 | 0000 | 0000 | 0000 | 0100 | 0010 | 0011 | 1101 |
|            | 0      | 0    | 0    | 0    | 4    | 2    | 3    | D    |

- the 2's complement of 0x0000423D : 0xFFFFBDC3 (= -16957<sub>10</sub>)

|            | 0      | 0    | 0    | 0    | 4    | 2    | 3    | D    |
|------------|--------|------|------|------|------|------|------|------|
| 0x0000423D | 0x0000 | 0000 | 0000 | 0000 | 0100 | 0010 | 0011 | 1101 |
| 0x0000BDC2 | 0x1111 | 1111 | 1111 | 1111 | 1011 | 1101 | 1100 | 0010 |
| 0xFFFFBDC3 | 0x1111 | 1111 | 1111 | 1111 | 1011 | 1101 | 1100 | 0011 |
|            | F      | F    | F    | F    | B    | D    | C    | 3    |

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of **signed** and **unsigned** integer arithmetic

- subtracting **0x0000618D** from **0x0000195D**

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**0x0000195D - 0x0000618D**      **unsigned** subtraction

subtraction by hand

---

**0x0000195D + (-0x0000618D)**      **signed** subtraction

the *transformed addition* using  
the 2's complement of subtrahend

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## 2's complements

- the 2's complement of **0x0000618D** : 0xFFFF8E73 (= -24973<sub>10</sub>)

|            |  |   |   |   |   |   |   |   |   |                  |
|------------|--|---|---|---|---|---|---|---|---|------------------|
|            |  | F   | F | F | F | 8 | E | 7 | 3 |                  |
| 0xFFFF9E73 |  | 0x1111_1111_1111_1111_1001_1110_0111_0011 |   |   |   |   |   |   |   |                  |
| 0x0000617C |  | 0x0000_0000_0000_0000_0110_0001_1000_1100 |   |   |   |   |   |   |   | (1's complement) |
| 0x0000618D |  | 0x0000_0000_0000_0000_0110_0001_1000_1101 |   |   |   |   |   |   |   | (2's complement) |
|            |  | 0   | 0 | 0 | 0 | 6 | 1 | 8 | D |                  |

- the 2's complement of **0xFFFF8E73** : 0x0000618D (= +24973<sub>10</sub>)

|            |  |   |   |   |   |   |   |   |   |                  |
|------------|--|---|---|---|---|---|---|---|---|------------------|
|            |  | 0   | 0 | 0 | 0 | 6 | 1 | 8 | D |                  |
| 0x0000618D |  | 0x0000_0000_0000_0000_0110_0001_1000_1101 |   |   |   |   |   |   |   |                  |
| 0xFFFF9E72 |  | 0x1111_1111_1111_1111_1001_1110_0111_0010 |   |   |   |   |   |   |   | (1's complement) |
| 0xFFFF9E73 |  | 0x1111_1111_1111_1111_1001_1110_0111_0011 |   |   |   |   |   |   |   | (2's complement) |
|            |  | F   | F | F | F | 8 | E | 7 | 3 |                  |

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Unsigned subtraction

- **0x0000195D - 0x0000618D** : unsigned subtraction  
subtraction by hand

```

                                0  0  0  0  1  9  5  D
0x0000195D      0x0000_0000_0000_0000_0001_1001_0101_1101
- 0x0000618D    0x0000_0000_0000_0000_0110_0001_1000_1101
-----
0xFFFFB7D0     1 0x1111_1111_1111_1111_1011_0111_1101_0000 (hand subtraction)
                1      F      F      F      F      B      7      D      0
                .
                V borrow (CF=1) : unsigned integer overflow
```

- A **borrow** is indicated by the **carry** flag (CF=1)
  - whenever an **unsigned integer overflow** happened
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Signed subtraction

- $0x0000195D + (-0x0000618D)$  : signed subtraction  
the *transformed addition* using the 2's complement of subtrahend

```

          0  0  0  0  1  9  5  D
0x0000195D  0x0000_0000_0000_0001_1001_0101_1101 (+0x0000195D)
+ 0xFFFF9E73  0x1111_1111_1111_1111_1001_1110_0111_0011 (-0x0000618D)
              F  F  F  F  9  E  7  3
-----
0xFFFFB7D0  0  0x1111_1111_1111_1111_1011_0111_1101_0000 (hand addition)
0           F  F  F  F  B  7  D  0
-0x00004830  .  0x0000_0000_0000_0000_0100_1000_0011_0000 (2's complement)
.           0  0  0  0  4  8  3  0
V no carry in the transformed addition (Cn=0) --> (CF=1)
```

- signed integer overflow is indicated by the **overflow** flag (OF)
  - the **carry** flag is set by the **inverted** carry of a transformed addition

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>



# Interpreting the result as a **signed** or an **unsigned** integer

- subtracting **0x0000618D** from **0x0000195D**  
the results of **unsigned** and **signed** subtractions have  
the same bit pattern **0xFFFFB7D0**

- the 2's complement of **0xFFFFB7D0** :  $0x00004830$  ( $= +18480_{10}$ )

|            | F   | F | F | F | B | 7 | D | 0                |
|------------|---|---|---|---|---|---|---|------------------|
| 0xFFFFB7D0 | 0x1111_1111_1111_1111_1011_0111_1101_0000 |   |   |   |   |   |   |                  |
| 0x0000482F | 0x0000_0000_0000_0000_0100_1000_0010_1111 |   |   |   |   |   |   | (1's complement) |
| 0x00004830 | 0x0000_0000_0000_0000_0100_1000_0011_0000 |   |   |   |   |   |   | (2's complement) |
|            | 0   | 0 | 0 | 0 | 4 | 8 | 3 | 0                |

- the 2's complement of **0x00004830** :  $0xFFFFB7D0$  ( $= -18480_{10}$ )

|            | 0   | 0 | 0 | 0 | 4 | 8 | 3 | 0                |
|------------|---|---|---|---|---|---|---|------------------|
| 0x00004830 | 0x0000_0000_0000_0000_0100_1000_0011_0000 |   |   |   |   |   |   |                  |
| 0xFFFFB7CF | 0x1111_1111_1111_1111_1011_0111_1100_1111 |   |   |   |   |   |   | (1's complement) |
| 0xFFFFB7D0 | 0x1111_1111_1111_1111_1011_0111_1101_0000 |   |   |   |   |   |   | (2's complement) |
|            | F   | F | F | F | B | 7 | D | 0                |

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Summary of signed and unsigned subtractions (1)

- subtracting  $0x0000618D$  from  $0x0000195D$ 
  - $0x0000195D - 0x0000618D$  : unsigned integer subtraction  
hand subtraction
  - $0x0000195D + (-0x0000618D)$  : signed integer subtraction  
the *transformed addition* using the 2's complement of the subtrahend
  - the same result :  $0xFFFFB7D0$  (the same bit pattern)
    - interpreting as a unsigned integer  $4294948816_{10}$   
 $0xFFFFB7D0$  with a borrow (CF=1)
    - interpreting as a signed integer  $-18480_{10}$   
 $-0x00004830$  (meaningless CF=1)

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Summary of **signed** and **unsigned** subtractions (2)

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0xFFFFB7D0    the result of **unsigned** subtraction    4294948816<sub>10</sub>  
with CF=1    with **unsigned** integer overflow

---

-0x00004830    the result of **signed** subtraction    -18480<sub>10</sub>

---

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of **unsigned** integer overflows

- $0x0000195D - 0x0000618D$  : **unsigned** subtraction
  - there is an **unsigned** integer overflow  
so the **carry** flag will be set ( $CF=1$ ) to indicate a **borrow**
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$   
(unsigned integers can't be negative),

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Examples of signed integer overflows

- $0x0000195D + (-0x0000618D)$  : signed subtraction
  - there is no signed integer overflow  
the overflow flag won't be set ( $OF=0$ )
  - signed overflow occurs , in the transformed addition,
    - two *positive* numbers are added and the result is a *negative*, ( $P + P \rightarrow N$ ), or
    - two *negative* numbers are added and the result is a *positive*, ( $N + N \rightarrow P$ )

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# TOC Carry flag in unsigned and signed computations

- 2's complement numbers : 4-bit
- Addend and augend in a  $n$ -bit addition
- Full adder operation in each bit position
- Internal and external carry bits
- Addition and Subtraction
- Using the Carry Flag as a borrow

## 2's complement numbers : 4-bit

---

|      |      |      |      |
|------|------|------|------|
| 0111 | (+7) | 1000 | (-8) |
| 0110 | (+6) | 1001 | (-7) |
| 0101 | (+5) | 1010 | (-6) |
| 0100 | (+4) | 1011 | (-5) |
| 0011 | (+3) | 1100 | (-4) |
| 0010 | (+2) | 1101 | (-3) |
| 0001 | (+1) | 1110 | (-2) |
| 0000 | (0)  | 1111 | (-1) |

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[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Addend and augend in a $n$ -bit addition

|         |      |            |     |  |
|---------|------|------------|-----|--|
| $n$     | bits | addened    | $A$ | $\{a_{n-1}, a_{n-2}, \dots, a_1, a_0\}$      |
| $n$     | bits | augend     | $B$ | $\{b_{n-1}, b_{n-2}, \dots, b_1, b_0\}$      |
| $(n+1)$ | bits | carry bits | $C$ | $\{C_n, C_{n-1}, C_{n-2}, \dots, C_1, C_0\}$ |
| $n$     | bits | sum bits   | $S$ | $\{S_{n-1}, S_{n-2}, \dots, S_1, S_0\}$      |

external carry bits :  $C_n$  carry out,  $C_0$  carry in

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \dots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \dots & b_1 & b_0 & \\ \hline C_n & S_{n-1} & S_{n-2} & \dots & S_1 & S_0 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Full adder operation in each bit position

---

full adder operation in the  $i^{\text{th}}$  bit position

$$\{C_{i+1}, S_i\} = a_i + b_i + C_i$$

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$$\begin{array}{r} a_i \\ b_i \\ C_i \\ \hline C_{i+1} \quad S_i \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Internal and external carry bits

external carries  $C_n$  output,  $C_0$  input  
 internal carries  $\{C_{n-1}, C_{n-2}, \dots, C_2, C_1\}$  output / input  
 sum bits  $\{S_{n-1}, S_{n-2}, \dots, S_1, S_0\}$  output

$$\begin{array}{rcccccc}
 & a_{n-1} & a_{n-2} & \dots & a_1 & a_0 \\
 & b_{n-1} & b_{n-2} & \dots & b_1 & b_0 \\
 \hline
 C_n & C_{n-1} & C_{n-2} & \dots & C_1 & C_0 \\
 \hline
 & S_{n-1} & S_{n-2} & \dots & S_1 & S_0
 \end{array}$$

$$\begin{array}{rcccccc}
 & a_{n-1} & a_{n-2} & \dots & a_1 & a_0 \\
 & b_{n-1} & b_{n-2} & \dots & b_1 & b_0 \\
 & & & & & C_0 \\
 \hline
 C_n & S_{n-1} & S_{n-2} & \dots & S_1 & S_0
 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Addition and Subtraction

- addition

$$\{C_n, S\} = A + B = A + B + 0$$

$$\begin{array}{rcccccc} & a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \\ & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \\ \hline & C_{n-1} & C_{n-2} & \cdots & C_1 & 0 \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

- subtraction - transformed addition

$$\{C_n, S\} = A - B = A + \overline{B} + 1$$

$$\begin{array}{rcccccc} & a_{n-1} & a_{n-2} & \cdots & a_1 & a_0 \\ & b_{n-1} & b_{n-2} & \cdots & b_1 & b_0 \\ \hline & C_{n-1} & C_{n-2} & \cdots & C_1 & 1 \\ \hline C_n & S_{n-1} & S_{n-2} & \cdots & S_1 & S_0 \end{array}$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Using the Carry Flag as a borrow (1)

- a **borrow** (CF=1) occurs in the **subtraction**  $A - B$  when  $b$  is larger than  $a$  ( $A < B$ ) as unsigned numbers
- Computer hardware can detect a **borrow** (CF=1) in **subtraction** by looking at whether a carry out (Cn) occurred in the transformed addition

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Using the Carry flag as a borrow (2)

- a **borrow** ( $CF=1$ ) occurs in the **subtraction**  $A - B$  ( $A < B$ ) as unsigned numbers
- a carry out ( $C_n$ ) in the transformed addition
  - If there is no **carry** ( $C_n=0$ ) then there is a **borrow** ( $CF=1$ )
  - If there is a **carry** ( $C_n=1$ ) then there is no **borrow** ( $CF=0$ )
  - **$CF = !C_n$**

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Using the Carry Flag as a borrow (3)

- the same *addition* and *subtraction* instructions are used for both **unsigned** and **signed** integer arithmetic.
  - no special *addition* and *subtraction* instructions for **unsigned** and **signed** integer arithmetic
- the only difference is
  - which flags you *test* afterwards and
  - how you *interpret* the result

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# TOC Rules for the carry flag

- 2's complement numbers : 4-bit
- The 1st rule for setting the carry flag
- The 2nd rule for setting the carry flag
- Cases for clearing the carry flag
- Computing CF in unsigned additions and subtractions

## 2's complement numbers : 4-bit

---

|      |      |      |      |
|------|------|------|------|
| 0111 | (+7) | 1000 | (-8) |
| 0110 | (+6) | 1001 | (-7) |
| 0101 | (+5) | 1010 | (-6) |
| 0100 | (+4) | 1011 | (-5) |
| 0011 | (+3) | 1100 | (-4) |
| 0010 | (+2) | 1101 | (-3) |
| 0001 | (+1) | 1110 | (-2) |
| 0000 | (0)  | 1111 | (-1) |

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[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# The 1st rule for setting the carry flag

- 1 **CF = 1** : carry in unsigned addition
  - the carry flag is set if the addition of two unsigned numbers causes a carry out of the most significant bits added.
  - unsigned integer overflow in unsigned addition
  - *hand addition rule*

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# The 2nd rule for setting the carry flag

- ② **CF = 1 : borrow in unsigned subtraction**
  - the **carry flag** is also set if the **subtraction** of two **unsigned** numbers requires a **borrow** into the most significant bits subtracted.
  - **unsigned integer overflow in unsigned subtraction**
  - *hand subtraction rule*

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (1)

- Otherwise, the **carry flag** is turned off (zero).
  - all three interpretations have the same CF=1, the same S=0000

| unsigned addition |  | signed addition                          |  | signed subtraction |
|-------------------|--|--|--|--------------------|
| 0111 ( 7)         |  | 0111 (+7)                                |  | 0111 (+7)          |
| +1001 +( 9)       |  | +1001 +(-7)                              |  | -0111 -(+7)        |
| -----             |  | -----                                    |  | -----              |
| 10000 (16)        |  | 10000 ( 0)                               |  | 10000 ( 0)         |
|                   |  |  |  |                    |
| CF=1              |  | Cn=1 -> CF=1                             |  | Cn=1 -> CF=1       |
|                   |  |  |  |                    |
| CF means 16       |  | CF meaningless                           |  | CF meaningless     |
| S = 0000          |  | S = 0000                                 |  | S = 0000           |
|                   |  |  |  |                    |
| * think hand      |  | * think Cn of the corresponding addition |  |                    |
| addition          |  | CF <- Cn                                 |  |                    |

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the carry flag (2)

- Otherwise, the **carry flag** is turned off (zero).
  - all three interpretations have the same CF=0, the same S=1111

| unsigned addition |  | signed addition                          |  | signed subtraction |
|-------------------|--|--|--|--------------------|
| 0111 ( 7)         |  | 0111 (+7)                                |  | 0111 (+7)          |
| +1001 +( 9)       |  | +1001 +(-7)                              |  | -0111 -(+7)        |
| -----             |  | -----                                    |  | -----              |
| 10000 (16)        |  | 10000 ( 0)                               |  | 10000 ( 0)         |
|                   |  |  |  |                    |
| CF=1              |  | Cn=1 -> CF=1                             |  | Cn=1 -> CF=1       |
|                   |  |  |  |                    |
| CF means 16       |  | CF meaningless                           |  | CF meaningless     |
| S = 0000          |  | S = 0000                                 |  | S = 0000           |
|                   |  |  |  |                    |
| * think hand      |  | * think Cn of the corresponding addition |  |                    |
| addition          |  | CF <- Cn                                 |  |                    |

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing CF in unsigned additions and subtractions

- Computing CF in an **unsigned** addition
  - do the **signed** addition
  - $C_n$  is the carry out
  - $CF \leftarrow C_n$
- Computing CF in an **unsigned** subtraction
  - do the transformed **signed** addition
  - do the **signed** addition
  - $C_n$  is the carry out
  - $CF \leftarrow !C_n$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: Method for computing the carry flag

- Carry flag computation

# Carry flag computation (1)

| ADD (addition)  | SUB (subtraction)  |
|---|--|
| $CF = C_n$  | $CF = \overline{C_n}$  |
| normal carry of<br>a 2's complement addition          | <b>inverted</b> carry of<br>a transformed addition               |
| $A + B = A + B + 0$                                   | $A - B = A + \overline{B} + 1$                                   |
| $\{C_n, S_{n-1}\}$<br>$= a_{n-1} + b_{n-1} + c_{n-1}$ | $\{C_n, S_{n-1}\}$<br>$= a_{n-1} + \overline{b_{n-1}} + c_{n-1}$ |

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)

## Carry flag computation (2)

- In **unsigned** arithmetic,
  - the **carry flag** is used to detect *overflow*
  - the **carry flag** is used to extend *n-bit* result into *(n+1)-bit* result
  - for **addition**, the **carry flag** is a **carry out**
  - for **subtraction**, the **carry flag** is a **borrow in**
- In **signed** arithmetic,
  - the **carry flag** is useless
  - the **carry flag** neither detects overflow nor extends n-bit result

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Carry flag computation (3)

- In **unsigned** arithmetic,

---

|             |                                      |                               |
|-------------|--------------------------------------|-------------------------------|
| Addition    | <b>CF = 1</b> means <b>carry out</b> | when <b>C<sub>n</sub> = 1</b> |
| Subtraction | <b>CF = 1</b> means <b>borrow in</b> | when <b>C<sub>n</sub> = 0</b> |

---

- **CF** - Carry Flag in x86
- **C<sub>n</sub>** - the normal carry out
  - the carry out of a 2's complement addition for **ADD**
  - the carry out of a *transformed* addition for **SUB**
- In **signed** arithmetic,
  - the **carry** flag is useless

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: More examples of the carry flag

- Summary I
- Summary II
- Cases for setting the carry flag
- Cases for clearing the carry flag

# Summary I

| unsigned add/sub |       |       | signed addition |       |     | signed subtraction |       |     | CF | OF |
|------------------|-------|-------|-----------------|-------|-----|--------------------|-------|-----|----|----|
| 1101             | (13)  |       | 1101            | (-3)  |     | 1101               | (-3)  |     |    |    |
| +1110            | +(14) | ADD   | +1110           | +(-2) | ADD | -0010              | -(-2) |     |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 11011            | (11)  | (+16) | 11011           | (-5)  |     | 11011              | (-5)  |     | 1  | 0  |
|                  |       |       |                 |       |     |                    |       |     |    |    |
| 0011             | ( 3)  |       | 0011            | (+3)  |     | 0011               | (+3)  |     |    |    |
| -1110            | -(14) | SUB   | +0010           | +(+2) |     | -1110              | -(-2) | SUB |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 10101            | ( 5)  | (-16) | 00101           | (+5)  |     | 00101              | (+5)  |     | 1  | 0  |
|                  |       |       |                 |       |     |                    |       |     |    |    |
| 0011             | ( 3)  |       | 0011            | (+3)  |     | 0011               | (+3)  |     |    |    |
| +0010            | +( 2) | ADD   | +0010           | +(+2) | ADD | -1110              | -(-2) |     |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 00101            | ( 5)  | (+ 0) | 00101           | (+5)  |     | 00101              | (+5)  |     | 0  | 0  |
|                  |       |       |                 |       |     |                    |       |     |    |    |
| 1101             | (13)  |       | 1101            | (-3)  |     | 1101               | (-3)  |     |    |    |
| -0010            | -( 2) | SUB   | +1110           | +(-2) |     | -0010              | -(-2) | SUB |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 11011            | (11)  | (-16) | 11011           | (-5)  |     | 11011              | (-5)  |     | 0  | 0  |

# Summary II

| unsigned add/sub |            |     | signed addition |       |     | signed subtraction |       |     | CF | OF |
|------------------|------------|-----|-----------------|-------|-----|--------------------|-------|-----|----|----|
| 1011             | (11)       |     | 1011            | (-5)  |     | 1011               | (-5)  |     |    |    |
| +1100            | +(12)      | ADD | +1100           | +(-4) | ADD | -0100              | -(+4) |     |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 10111            | ( 7) (+16) |     | 10111           | (+7)  |     | 10111              | (+7)  |     | 1  | 1  |
| 0101             | ( 5)       |     | 0101            | (+5)  |     | 0101               | (+5)  |     |    |    |
| -1100            | -(12)      | SUB | +0100           | +(+4) |     | -1100              | -(-4) | SUB |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 11001            | ( 9) (-16) |     | 01001           | (-7)  |     | 01001              | (-7)  |     | 1  | 1  |
| 0101             | ( 5)       |     | 0101            | (+5)  |     | 0101               | (+5)  |     |    |    |
| +0100            | +( 4)      | ADD | +0100           | +(+4) | ADD | -1100              | -(-4) |     |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 01001            | ( 9) (+ 0) |     | 01001           | (-7)  |     | 01001              | (-7)  |     | 0  | 1  |
| 1011             | (11)       |     | 1011            | (-5)  |     | 1011               | (-5)  |     |    |    |
| -0100            | -( 4)      | SUB | +1100           | +(-4) |     | -0100              | -(+4) | SUB |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 00111            | ( 7) ( 0)  |     | 10111           | (+7)  |     | 10111              | (+7)  |     | 0  | 1  |

# Cases for setting the carry flag (1) CF=1, OF=0

- unsigned integer overflow (CF=1 means +16)

| * unsigned addition |  | * signed addition                        |  | signed subtraction |
|---------------------|--|--|--|--------------------|
| 1101 (13)           |  | 1101 (-3)                                |  | 1101 (-3)          |
| +1110 +(14) ADD     |  | +1110 +(-2) ADD                          |  | -0010 -(+2)        |
| -----               |  | -----                                    |  | -----              |
| 11011 (11) (+16)    |  | 11011 (-5)                               |  | 11011 (-5)         |
|                     |  |  |  |                    |
| CF=1                |  | Cn=1 -> CF=1                             |  | Cn=1 -> CF=1       |
|                     |  |  |  |                    |
| CF means 16         |  | CF meaningless                           |  | CF meaningless     |
| S = 0000            |  | S = 0000                                 |  | S = 0000           |
|                     |  |  |  |                    |
| * think hand        |  | * think Cn of the corresponding addition |  |                    |
| addition            |  | CF <- Cn (for unsigned addition)         |  |                    |

\* CF=1, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (2) CF=1, OF=0

- unsigned integer overflow (CF=1 means -16)

| * unsigned subtraction   |  | signed addition                        |  | * signed subtraction                 |
|--------------------------|--|--|--|--------------------------------------|
| 0011 (3)                 |  | 0011 (+3)                              |  | 0011 (+3)                            |
| -1110 -(14) SUB          |  | +0010 +(2)                             |  | -1110 -(-2) SUB                      |
| -----                    |  | -----                                  |  | -----                                |
| 10101 (5) (-16)          |  | 00101 (+5)                             |  | 00101 (+5)                           |
|                          |  |  |  |                                      |
| CF=1                     |  | Cn=0 -> CF=1                           |  | Cn=0 -> CF=1                         |
|                          |  |  |  |                                      |
| CF means -16             |  | CF meaningless                         |  | CF meaningless                       |
| S = 0101                 |  | S = 0101                               |  | S = 0101                             |
| -----                    |  | -----                                  |  | -----                                |
| * think hand subtraction |  | * think Cn of the transformed addition |  | CF <- !Cn (for unsigned subtraction) |
| -----                    |  | -----                                  |  | -----                                |

\* CF=1, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (3) CF=1, OF=1

- unsigned integer overflow (CF=1 means +16)

| * unsigned addition |  | * signed addition                        |  | signed subtraction |
|---------------------|--|--|--|--------------------|
| 1011 (11)           |  | 1011 (-5)                                |  | 1011 (-5)          |
| +1100 +(12) ADD     |  | +1100 +(-4) ADD                          |  | -0100 -(+4)        |
| -----               |  | -----                                    |  | -----              |
| 10111 ( 7) (+16)    |  | 10111 (+7)                               |  | 10111 (+7)         |
|                     |  |  |  |                    |
| CF=1                |  | Cn=1 -> CF=1                             |  | Cn=1 -> CF=1       |
|                     |  |  |  |                    |
| CF means +16        |  | CF meaningless                           |  | CF meaningless     |
| S = 0111            |  | S = 0111                                 |  | S = 0111           |
|                     |  |  |  |                    |
| * think hand        |  | * think Cn of the corresponding addition |  |                    |
| addition            |  | CF <- Cn (for unsigned addition)         |  |                    |

\* CF=1, S=0111, OF=1 for all three interpretations

# Cases for setting the carry flag (4) CF=1, OF=1

- unsigned integer overflow (CF=1 means -16)

| * unsigned subtraction   |  | signed addition                        |  | * signed subtraction                 |
|--------------------------|--|--|--|--------------------------------------|
| 0101 ( 5)                |  | 0101 (+5)                              |  | 0101 (+5)                            |
| -1100 -(12) SUB          |  | +0100 +(4)                             |  | -1100 -(-4) SUB                      |
| -----                    |  | -----                                  |  | -----                                |
| 11001 ( 9) (-16)         |  | 01001 (-7)                             |  | 01001 (-7)                           |
| CF=1                     |  | Cn=0 -> CF=1                           |  | Cn=0 -> CF=1                         |
| CF means -16             |  | CF meaningless                         |  | CF meaningless                       |
| S = 1001                 |  | S = 1001                               |  | S = 1001                             |
| * think hand subtraction |  | * think Cn of the transformed addition |  | CF <- !Cn (for unsigned subtraction) |

\* CF=1, S=1001, OF=1 for all three interpretations



# Cases for clearing the carry flag (1) CF=0, OF=0

- no unsigned integer overflow (CF=0)

| * unsigned addition |  | * signed addition                        | signed subtraction |
|---------------------|--|--|--------------------|
| 0011 ( 3)           |  | 0011 (+3)                                | 0011 (+3)          |
| +0010 +( 2) ADD     |  | +0010 +( +2) ADD                         | -1110 -(-2)        |
| -----               |  | -----                                    | -----              |
| 00101 ( 5) (+ 0)    |  | 00101 (+5)                               | 00101 (+5)         |
|                     |  |  |                    |
| CF=0                |  | Cn=0 -> CF=0                             | Cn=0 -> CF=0       |
|                     |  |  |                    |
| CF means 0          |  | CF meaningless                           | CF meaningless     |
| S = 0101            |  | S = 0101                                 | S = 0101           |
|                     |  |  |                    |
| * think hand        |  | * think Cn of the corresponding addition |                    |
| addition            |  | CF <- Cn (for unsigned addition)         |                    |

\* CF=0, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (2) CF=0, OF=0

- no unsigned integer overflow (CF=0)

| * unsigned addition      |  | * signed addition                        |  | signed subtraction |
|--------------------------|--|--|--|--------------------|
| 1101 (13)                |  | 1101 (-3)                                |  | 1101 (-3)          |
| -0010 -( 2) SUB          |  | +1110 +(-2)                              |  | -0010 -(+2) SUB    |
| -----                    |  | -----                                    |  | -----              |
| 11011 (11) (-16)         |  | 11011 (-5)                               |  | 11011 (-5)         |
|                          |  |  |  |                    |
| CF=0                     |  | Cn=0 -> CF=0                             |  | Cn=0 -> CF=0       |
|                          |  |  |  |                    |
| CF means 0               |  | CF meaningless                           |  | CF meaningless     |
| S = 1011                 |  | S = 1011                                 |  | S = 1011           |
|                          |  |  |  |                    |
| * think hand subtraction |  | * think Cn of the corresponding addition |  |                    |
|                          |  | CF <- Cn (for unsigned addition)         |  |                    |

\* CF=0, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (3) CF=0, OF=1

- no unsigned integer overflow (CF=0)

| * unsigned addition |  | * signed addition                        |  | signed subtraction |
|---------------------|--|--|--|--------------------|
| 0101 ( 5)           |  | 0101 (+5)                                |  | 0101 (+5)          |
| +0100 +( 4) ADD     |  | +0100 +( +4) ADD                         |  | -1100 -(-4)        |
| -----               |  | -----                                    |  | -----              |
| 01001 ( 9) (+ 0)    |  | 01001 (-7)                               |  | 01001 (-7)         |
|                     |  |  |  |                    |
| CF=0                |  | Cn=0 -> CF=0                             |  | Cn=0 -> CF=0       |
|                     |  |  |  |                    |
| CF means +0         |  | CF meaningless                           |  | CF meaningless     |
| S = 1001            |  | S = 1001                                 |  | S = 1001           |
|                     |  |  |  |                    |
| * think hand        |  | * think Cn of the corresponding addition |  |                    |
| addition            |  | CF <- Cn (for unsigned addition)         |  |                    |

\* CF=0, S=1001, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (4) CF=0, OF=1

- no unsigned integer overflow (CF=0)

| * unsigned subtraction   |  | signed addition                        |  | * signed subtraction |
|--------------------------|--|--|--|----------------------|
| 1011 (11)                |  | 1011 (-5)                              |  | 1011 (-5)            |
| -0100 -(4) SUB           |  | +1100 +(-4)                            |  | -0100 -(+4) SUB      |
| -----                    |  | -----                                  |  | -----                |
| 00111 (7) (0)            |  | 10111 (+7)                             |  | 10111 (+7)           |
|                          |  |  |  |                      |
| CF=0                     |  | Cn=1 -> CF=0                           |  | Cn=1 -> CF=0         |
|                          |  |  |  |                      |
| CF means 0               |  | CF meaningless                         |  | CF meaningless       |
| S = 0111                 |  | S = 0111                               |  | S = 0111             |
|                          |  |  |  |                      |
| * think hand subtraction |  | * think Cn of the transformed addition |  |                      |
|                          |  | CF <- !Cn (for unsigned subtraction)   |  |                      |

\* CF=0, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: Overflow flag

- Overflow flag in unsigned and signed computations
- Rules for the overflow flag
- Method 1 for computing the overflow flag
- Method 2 for computing the overflow flag
- More examples of the overflow flag

- Overflow flag

# Overflow flag (1)

- only need to look at the **sign bits** (leftmost) of the three numbers

$$\begin{array}{rclcl} \text{augend} & + & \text{addend} & = & \text{sum} \\ \text{minuend} & - & \text{subrahend} & = & \text{difference} \end{array}$$

to decide if the **overflow** flag is turned on or off.

- overflow** flag is based on **signed** arithmetic

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Overflow flag (2)

- in **signed** arithmetic,
  - watch the **overflow** flag to detect errors
  - **overflow** flag on means the result is wrong
  - errors can be detected by examining the sign of the result, in the 2's complement arithmetic
- in **unsigned** arithmetic,
  - the **overflow** flag tells you nothing interesting

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Overflow flag (3)

- when two positive numbers are added
  - if the result is a negative, ( $P + P \rightarrow N$ ), then overflow
  - if the result is a positive, ( $P + P \rightarrow P$ ), then no overflow
- when two negative numbers are added
  - the result is a positive, ( $N + N \rightarrow P$ ), then overflow
  - the result is a negative, ( $N + N \rightarrow N$ ), then no overflow

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow flag (4)

- adding negative and positive numbers cannot be wrong, because the sum is between the addends.
  - mixed-sign addition never turns on the **overflow** flag.
  - opposite signed numbers are added, then no **overflow**
  - both of the addends lies in the allowable range of numbers, and their sum is between the addends, therefore the sum lies also in the allowable range
- $(P + N \rightarrow P)$  no overflow
- $(P + N \rightarrow N)$  no overflow
- $(N + P \rightarrow P)$  no overflow
- $(N + P \rightarrow N)$  no overflow

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Rules for the overflow flag

- the 1st rule for setting OF
- the 2nd rule for setting OF
- cases for clearing OF (1 ~ 6)

# Overflow flag setting and clearing conditions

ADD conditions      SUB conditions

|      |                       |                       |                          |
|------|-----------------------|-----------------------|--------------------------|
| OF=1 | $P + P \rightarrow N$ | $P - N \rightarrow N$ | $C_n \oplus C_{n-1} = 1$ |
| OF=1 | $N + N \rightarrow P$ | $N - P \rightarrow P$ | $C_n \oplus C_{n-1} = 1$ |
| OF=0 | $P + P \rightarrow P$ | $P - N \rightarrow P$ | $C_n \oplus C_{n-1} = 0$ |
| OF=0 | $N + N \rightarrow N$ | $N - P \rightarrow N$ | $C_n \oplus C_{n-1} = 0$ |
| OF=0 | $P + N \rightarrow P$ | $P - N \rightarrow P$ | $C_n \oplus C_{n-1} = 0$ |
| OF=0 | $P + N \rightarrow N$ | $P - P \rightarrow N$ | $C_n \oplus C_{n-1} = 0$ |
| OF=0 | $N + P \rightarrow P$ | $N - N \rightarrow P$ | $C_n \oplus C_{n-1} = 0$ |
| OF=0 | $N + P \rightarrow N$ | $N - P \rightarrow N$ | $C_n \oplus C_{n-1} = 0$ |

$$+P = -(-P) = -N$$

$$+N = -(-N) = -P$$

# The 1st rule for setting the overflow flag

- 1 If the **sum** of two **signed** numbers with the sign bits off (0, 0) yields a result number with the sign bit on (1) the **overflow flag** is turned on ( $OF = 1 : P + P \rightarrow N$ )

signed addition

```
0100 carries
 0100 (+4)
+0100 (+4)
-----
01000 (-8)
```

signed subtraction

```
0100 (+4)
-1100 -(-4)
-----
01000 (-8)
```

unsigned addition

```
0100 ( 4)
+0100 +( 4)
-----
01000 ( 8)
```

$$\bullet OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 1 = 1$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## The 2nd rule for setting the overflow flag

- 2 If the **sum** of two numbers with the sign bits on (1, 1) yields a result number with the sign bit off (0) the **overflow flag** is turned on. ( $OF = 1 : N + N \rightarrow P$ )

signed addition

```
1001 carries
 1001 (-7)
+1001 +(-7)
-----
10010 ( 2)
```

signed subtraction

```
1001 (-7)
-0111 -(+7)
-----
10010 ( 2)
```

unsigned addition

```
1001 ( 9)
+1001 +( 9)
-----
10010 (18)
```

$$\bullet OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 0 = 1$$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (1)

- **overflow flag** is turned off. ( $OF = 0 : P + P \rightarrow P$ )

signed addition

```
0011  carries
0011  (+3)
+0011  +(+3)
-----
00110  (+6)
```

signed subtraction

```
0011  (+3)
-1101  -(-3)
-----
00110  (+6)
```

unsigned addition

```
0011  ( 3)
+0011  +( 3)
-----
00110  ( 6)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

## Cases for clearing the overflow flag (2)

- **overflow flag** is turned off. ( $OF = 0 : N + N \rightarrow N$ )

signed addition

```
1101 carries
 1101 (-3)
+1101 +(-3)
-----
11010 (-6)
```

signed subtraction

```
1101 (-3)
-0011 -(+3)
-----
11010 (-6)
```

unsigned addition

```
1101 (13)
+1101 +(13)
-----
11010 (26)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Cases for clearing the overflow flag (3)

- **overflow flag** is turned off. ( $OF = 0 : P + N \rightarrow P$ )

signed addition

```
1100 carries
 0100 (+4)
+1101 +(-3)
-----
10001 (+1)
```

signed subtraction

```
0100 (+4)
-0011 -(+3)
-----
10001 (+1)
```

unsigned addition

```
0100 ( 4)
+1101 +(13)
-----
10001 (17)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (4)

- **overflow flag** is turned off. ( $OF = 0 : P + N \rightarrow N$ )

signed addition

```
0000 carries
0011 (+3)
+1100 +(-4)
-----
01111 (-1)
```

signed subtraction

```
0011 (+3)
-0100 -(+4)
-----
01111 (-1)
```

unsigned addition

```
0011 ( 3)
+1100 +(12)
-----
01111 (15)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (5)

- **overflow flag** is turned off. ( $OF = 0 : N + P \rightarrow P$ )

signed addition

```
1100 carries
 1101 (-3)
+0100 (+4)
-----
10001 (+1)
```

signed subtraction

```
0011 (-3)
-1100 -(-4)
-----
10001 (+1)
```

unsigned addition

```
1101 (13)
+0100 +( 4)
-----
10001 (17)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 1 \oplus 1 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the overflow flag (6)

- **overflow flag** is turned off. ( $OF = 0 : N + P \rightarrow N$ )

signed addition

```
0000 carries
1100 (-4)
+0011 +(+3)
-----
01111 (-1)
```

signed subtraction

```
0100 (-4)
-1101 -(-3)
-----
01111 (-1)
```

unsigned addition

```
1100 (12)
+0011 +( 3)
-----
01111 (15)
```

- $OF = C_n \oplus C_{n-1} = C_4 \oplus C_3 = 0 \oplus 0 = 0$

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Method 1 for computing the overflow flag

- Adding two numbers with the same sign
- Overflow conditions for additions and subtractions
- Overflow condition for an addition
- Overflow conditions for a subtraction
- Overflow in signed computations

# Adding two numbers with the same sign

- **overflow** can only happen when adding two numbers of the same sign results in a different sign ( $P + P \rightarrow N, \quad N + N \rightarrow P$ )

- $n$ -bit **signed** binary arithmetic  $A + B = C$

$$A = (a_{n-1}, \dots, a_1, a_0)$$

$$B = (b_{n-1}, \dots, b_1, b_0)$$

$$C = (c_{n-1}, \dots, c_1, c_0)$$

- to detect overflow
  - only the **sign** bits are considered
  - **msb** (most significant bit)  $a_{n-1}, b_{n-1}, c_{n-1}$
  - the other bits are ignored

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow conditions for additions and subtractions

- with two operands ( $A$  and  $B$ ) and one result ( $C$ ), three sign bits ( $a_{n-1}, b_{n-1}, c_{n-1}$ ) are considered  
→  $2^3 = 8$  possible combinations
- only two cases result in **overflow** for an addition
  - 0 0 1     ( $p + p \rightarrow n$ )
  - 1 1 0     ( $n + n \rightarrow p$ )
- only two cases are considered as **overflow** for an subtraction
  - 0 1 1     ( $p - n \rightarrow n$ )
  - 1 0 0     ( $n - p \rightarrow p$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow condition for an addition

- Overflow in an addition ( $A + B$ )

|      | $a_{n-1}$ | $b_{n-1}$ | $c_{n-1}$ |                       |
|------|-----------|-----------|-----------|-----------------------|
|      | 0         | 0         | 0         | $p + p \rightarrow p$ |
| OVER | 0         | 0         | 1         | $p + p \rightarrow n$ |
|      | 0         | 1         | 0         | $p + n \rightarrow p$ |
|      | 0         | 1         | 1         | $p + n \rightarrow n$ |
|      | 1         | 0         | 0         | $n + p \rightarrow p$ |
|      | 1         | 0         | 1         | $n + p \rightarrow n$ |
| OVER | 1         | 1         | 0         | $n + n \rightarrow p$ |
|      | 1         | 1         | 1         | $n + n \rightarrow n$ |

- adding two positives should be positive
- adding two negatives should be negative

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Overflow conditions for a subtraction

- Overflow in a subtraction ( $A - B$ )

|      | $a_{n-1}$ | $b_{n-1}$ | $c_{n-1}$ |                       |
|------|-----------|-----------|-----------|-----------------------|
|      | 0         | 0         | 0         | $p - p \rightarrow p$ |
|      | 0         | 0         | 1         | $p - p \rightarrow n$ |
|      | 0         | 1         | 0         | $p - n \rightarrow p$ |
| OVER | 0         | 1         | 1         | $p - n \rightarrow n$ |
| OVER | 1         | 0         | 0         | $n - p \rightarrow p$ |
|      | 1         | 0         | 1         | $n - p \rightarrow n$ |
|      | 1         | 1         | 0         | $n - n \rightarrow p$ |
|      | 1         | 1         | 1         | $n - n \rightarrow n$ |

- subtracting a negative is the same as adding a positive
- subtracting a positive is the same as adding a negative

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow in signed computations

- ALU might contain a small logic that sets the **overflow** flag to "1" if and only if any one of the above four **OV conditions** is met.
- in **signed** computations, adding two numbers of the same sign must produce a result of the same sign, otherwise overflow happened.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC Method 2 for computing the overflow flag

- Carry into and carry out of the sign bit
- Overflow in 2's complement arithmetic
- Overflow flag =  $C_n \oplus C_{n-1}$
- Examples of 4-bit signed additions
- $C_n$  and  $C_{n-1}$  in a  $n$ -bit addition
- Overflow flag computation
- Examples of computing overflow flag
- Hexadecimal carry, octal carry, decimal carry
- No carry into the sign bit

# Carry into and carry out of the sign bit

- When adding two  $n$ -bit binary values, consider
  - the **carry** *coming into* the most significant bit (msb)  
 $C_{n-1}$  : **carry** into the **sign** bit
  - the **carry** *going out of* the most significant bit (msb)  
 $C_n$  : **carry** out of the **sign** bit  
this is the **carry** flag (**CF**) in the processor

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Overflow in 2's complement arithmetic

- **overflow** in 2's complement happens (**OF=1**) when
  - there is a **carry into** the **sign** bit ( $C_{n-1} = 1$ )  
but no carry out of the **sign** bit ( $C_n = 0$ )
  - there is no carry into the **sign** bit ( $C_{n-1} = 0$ )  
but a **carry out** of the **sign** bit ( $C_n = 1$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

$$\text{Overflow flag} = C_n \oplus C_{n-1}$$

- the **overflow** flag is the **XOR** ( $C_n \oplus C_{n-1}$ ) of
  - of the **carry coming into** the **sign** bit ( $C_{n-1}$ )
  - with the **carry going out of** the **sign** bit ( $C_n$ )
- **overflow** happens when the **carry in** ( $C_{n-1}$ ) does not equal to the **carry out** ( $C_n$ )

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Examples of 4-bit signed additions (1)

- 4-bit 2's complement addition examples

```
0000
 0100 (+4) (pos sign 0)
+1000 (-8) (neg sign 1)
=====
01100 (-4) (neg sign 1)
```

```
C4 carry out 0 (1+0+0)
C3 carry in 0 (0+1+0)
0 XOR 0 = NO OVERFLOW
```

```
1100
 1100 (-4) (neg sign 1)
+0100 (+4) (pos sign 0)
=====
10000 ( 0) (pos sign 0)
```

```
C4 carry out 1 (1+0+1)
C3 carry in 1 (1+1+0)
1 XOR 1 = NO OVERFLOW
```

```
0100
 0100 (+4) (pos sign 0)
+0100 (+4) (pos sign 0)
=====
01000 (-8) (neg sign 1)
```

```
C4 carry out 0 (0+0+1)
C3 carry in 1 (1+1+0)
0 XOR 1 = OVERFLOW!
```

```
1000
 1100 (-4) (neg sign 1)
+1000 (-8) (neg sign 1)
=====
10100 (+4) (pos sign 0)
```

```
C4 carry out 1 (1+1+0)
C3 carry in 0 (1+0+0)
1 XOR 0 = OVERFLOW!
```

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Examples of 4-bit signed additions (2)

- same sign addition → possible overflow

|                          |                          |                          |                          |
|--------------------------|--------------------------|--------------------------|--------------------------|
| -----<br>+ +, -<br>----- | -----<br>- -, +<br>----- | -----<br>+ +, +<br>----- | -----<br>- -, -<br>----- |
| +5<br>+5                 | -5<br>-5                 | +5<br>+1                 | -5<br>-1                 |
| -----<br>-6(OF)          | -----<br>+6(OF)          | -----<br>+6              | -----<br>-6              |

|                      |                      |                      |                      |
|----------------------|----------------------|----------------------|----------------------|
| 0101<br>0101<br>0101 | 1011<br>1011<br>1011 | 0001<br>0101<br>0001 | 1111<br>1011<br>1111 |
| -----<br>01010       | -----<br>10110       | -----<br>00110       | -----<br>11010       |
| -----<br>C4 = 0      | -----<br>C4 = 1      | -----<br>C4 = 0      | -----<br>C4 = 1      |
| -----<br>C3 = 1      | -----<br>C3 = 0      | -----<br>C3 = 0      | -----<br>C3 = 1      |
| -----<br>OF = 1      | -----<br>OF = 1      | -----<br>OF = 0      | -----<br>OF = 0      |

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)



# Examples of 4-bit signed additions (3)

- mixed sign addition → no overflow

|  |  |  |  |
|--|--|--|--|
| -----<br>+ -, +<br>-----<br>+5<br>-1<br>-----<br>+4                                    | -----<br>+ -, -<br>-----<br>+5<br>-6<br>-----<br>-1                                    | -----<br>- +, +<br>-----<br>-5<br>+6<br>-----<br>+1                                    | -----<br>- +, -<br>-----<br>-5<br>+1<br>-----<br>-4                                    |
| 1111<br>0101<br>1111<br>-----<br>10100<br>-----<br>C4 = 1<br>C3 = 1<br>-----<br>OF = 0 | 0000<br>0101<br>1010<br>-----<br>01111<br>-----<br>C4 = 0<br>C3 = 0<br>-----<br>OF = 0 | 1110<br>1011<br>0110<br>-----<br>10001<br>-----<br>C4 = 1<br>C3 = 1<br>-----<br>OF = 0 | 0011<br>1011<br>0001<br>-----<br>01100<br>-----<br>C4 = 0<br>C3 = 0<br>-----<br>OF = 0 |

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# $C_n$ and $C_{n-1}$ in a $n$ -bit addition

## $(n-1)^{th}$ bit – MSB

- adding operations at the  $(n-1)$  bit position
- $\{C_n, S_{n-1}\} = a_{n-1} + b_{n-1} + c_{n-1}$

$$\begin{array}{r} \text{msb} \\ a_{n-1} \\ b_{n-1} \\ \hline C_{n-1} \\ \hline C_n \quad S_{n-1} \end{array}$$

- $C_n$  : carry coming out of the msb

## $(n-2)^{th}$ bit

- adding operations at the  $(n-2)$  bit position
- $\{C_{n-1}, S_{n-2}\} = a_{n-2} + b_{n-2} + c_{n-2}$

$$\begin{array}{r} \text{msb} \\ a_{n-2} \\ b_{n-2} \\ \hline C_{n-2} \\ \hline C_{n-1} \quad S_{n-2} \end{array}$$

- $C_{n-1}$  : carry coming into the msb

# Overflow flag computation

---

**ADD** (addition)

---

$$OF = C_n \oplus C_{n-1}$$

a 2's complement addition

$$A + B = A + B + \mathbf{0} \quad (C_0 = 0)$$

$$\begin{aligned} &\{C_n, S_{n-1}\} \\ &= a_{n-1} + b_{n-1} + c_{n-1} \end{aligned}$$

$$\begin{aligned} &\{C_{n-1}, S_{n-2}\} \\ &= a_{n-2} + b_{n-2} + c_{n-2} \end{aligned}$$

---

**SUB** (subtraction)

---

$$OF = C_n \oplus C_{n-1}$$

the transformed addition

$$A - B = A + \overline{B} + \mathbf{1} \quad (C_0 = 1)$$

$$\begin{aligned} &\{C_n, S_{n-1}\} \\ &= a_{n-1} + \overline{b_{n-1}} + c_{n-1} \end{aligned}$$

$$\begin{aligned} &\{C_{n-1}, S_{n-2}\} \\ &= a_{n-2} + \overline{b_{n-2}} + c_{n-2} \end{aligned}$$

---

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Hexadecimal carry, octal carry, decimal carry

- Note that this XOR method only works with the **binary** carry that goes into the sign **bit**.
- not works with **hexadecimal carry**  
**decimal carry**, **octal carry**
  - the carry doesn't go into the sign **bit**
  - can't XOR that non-binary carry with the outgoing carry.

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# No carry into the sign bit

- Hexadecimal addition example  
(showing that XOR doesn't work for hex carry):

```
8Ah
+8Ah
====
114h
```

- The hexadecimal carry of 1 resulting from A+A does not affect the sign bit.
- If you do the math in binary, you'll see that there is **no** carry **into** the sign bit; but, there is carry out of the sign bit. Therefore, the above example sets OVERFLOW on. (The example adds two negative numbers and gets a positive number.)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Summary I

| unsigned add/sub |       |       | signed addition |       |     | signed subtraction |       |     | CF | OF |
|------------------|-------|-------|-----------------|-------|-----|--------------------|-------|-----|----|----|
| 1101             | (13)  |       | 1101            | (-3)  |     | 1101               | (-3)  |     |    |    |
| +1110            | +(14) | ADD   | +1110           | +(-2) | ADD | -0010              | -(-2) |     |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 11011            | (11)  | (+16) | 11011           | (-5)  |     | 11011              | (-5)  |     | 1  | 0  |
|                  |       |       |                 |       |     |                    |       |     |    |    |
| 0011             | ( 3)  |       | 0011            | (+3)  |     | 0011               | (+3)  |     |    |    |
| -1110            | -(14) | SUB   | +0010           | +(+2) |     | -1110              | -(-2) | SUB |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 10101            | ( 5)  | (-16) | 00101           | (+5)  |     | 00101              | (+5)  |     | 1  | 0  |
|                  |       |       |                 |       |     |                    |       |     |    |    |
| 0011             | ( 3)  |       | 0011            | (+3)  |     | 0011               | (+3)  |     |    |    |
| +0010            | +( 2) | ADD   | +0010           | +(+2) | ADD | -1110              | -(-2) |     |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 00101            | ( 5)  | (+ 0) | 00101           | (+5)  |     | 00101              | (+5)  |     | 0  | 0  |
|                  |       |       |                 |       |     |                    |       |     |    |    |
| 1101             | (13)  |       | 1101            | (-3)  |     | 1101               | (-3)  |     |    |    |
| -0010            | -( 2) | SUB   | +1110           | +(-2) |     | -0010              | -(-2) | SUB |    |    |
| -----            | ----- |       | -----           | ----- |     | -----              | ----- |     |    |    |
| 11011            | (11)  | (-16) | 11011           | (-5)  |     | 11011              | (-5)  |     | 0  | 0  |

# Summary II

| unsigned add/sub |            |     | signed addition |       |     | signed subtraction |       |     | CF | OF |
|------------------|------------|-----|-----------------|-------|-----|--------------------|-------|-----|----|----|
| 1011             | (11)       |     | 1011            | (-5)  |     | 1011               | (-5)  |     |    |    |
| +1100            | +(12)      | ADD | +1100           | +(-4) | ADD | -0100              | -(+4) |     |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 10111            | ( 7) (+16) |     | 10111           | (+7)  |     | 10111              | (+7)  |     | 1  | 1  |
| 0101             | ( 5)       |     | 0101            | (+5)  |     | 0101               | (+5)  |     |    |    |
| -1100            | -(12)      | SUB | +0100           | +(+4) |     | -1100              | -(-4) | SUB |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 11001            | ( 9) (-16) |     | 01001           | (-7)  |     | 01001              | (-7)  |     | 1  | 1  |
| 0101             | ( 5)       |     | 0101            | (+5)  |     | 0101               | (+5)  |     |    |    |
| +0100            | +( 4)      | ADD | +0100           | +(+4) | ADD | -1100              | -(-4) |     |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 01001            | ( 9) (+ 0) |     | 01001           | (-7)  |     | 01001              | (-7)  |     | 0  | 1  |
| 1011             | (11)       |     | 1011            | (-5)  |     | 1011               | (-5)  |     |    |    |
| -0100            | -( 4)      | SUB | +1100           | +(-4) |     | -0100              | -(+4) | SUB |    |    |
| -----            | -----      |     | -----           | ----- |     | -----              | ----- |     |    |    |
| 00111            | ( 7) ( 0)  |     | 10111           | (+7)  |     | 10111              | (+7)  |     | 0  | 1  |

# Cases for setting the overflow flag (1) CF=1, OF=1

- signed integer overflow (OF=1 means incorrect S)

| * unsigned addition |  | * signed addition               |  | signed subtraction |
|---------------------|--|---------------------------------|--|--------------------|
| 1011 (11)           |  | 1000                            |  |                    |
| +1100 +(12) ADD     |  | 1011 (-5)                       |  | 1011 (-5)          |
| -----               |  | +1100 +(-4) ADD                 |  | -0100 -(+4)        |
| 10111 ( 7) (+16)    |  | -----                           |  | -----              |
|                     |  | 10111 (+7)                      |  | 10111 (+7)         |
| OF=1                |  | n + n -> p (OF=1)               |  | n - p -> p (OF=1)  |
| OF meaningless      |  | -> incorrect S                  |  | -> incorrect S     |
| S = 0111            |  | S = 0111                        |  | S = 0111           |
| * think hand        |  | * OF <- C4 XOR C3 = 1 XOR 0 = 1 |  |                    |
| addition            |  | of signed addition              |  |                    |

\* CF=1, S=0111, OF=1 for all three interpretations



# Cases for setting the overflow flag (2) CF=1, OF=1

- signed integer overflow (OF=1 means incorrect S)

| * unsigned subtraction   |  | signed addition                 |  | * signed subtraction |
|--------------------------|--|---------------------------------|--|----------------------|
| 0101 (5)                 |  | 0100                            |  | 0101 (+5)            |
| -1100 -(12) SUB          |  | +0100 +(4)                      |  | -1100 -(-4) SUB      |
| -----                    |  | -----                           |  | -----                |
| 11001 (9) (-16)          |  | 01001 (-7)                      |  | 01001 (-7)           |
| OF=1                     |  | p + p -> n (OF=1)               |  | p - n -> n (OF=1)    |
| OF meaningless           |  | -> incorrect S                  |  | -> incorrect S       |
| S = 1001                 |  | S = 1001                        |  | S = 1001             |
| -----                    |  |                                 |  |                      |
| * think hand subtraction |  | * OF <- C4 XOR C3 = 0 XOR 1 = 1 |  | of signed addition   |
| -----                    |  |                                 |  |                      |

\* CF=1, S=1001, OF=1 for all three interpretations

# Cases for setting the overflow flag (3) CF=0, OF=1

- signed integer overflow (OF=1 means incorrect S)

| * unsigned addition |  | * signed addition               | signed subtraction |
|---------------------|--|---------------------------------|--------------------|
| 0101 ( 5)           |  | 0100                            |                    |
| +0100 +( 4) ADD     |  | 0101 (+5)                       | 0101 (+5)          |
| -----               |  | +0100 +(4) ADD                  | -1100 -(-4)        |
| 01001 ( 9) (+ 0)    |  | -----                           | -----              |
|                     |  | 01001 (-7)                      | 01001 (-7)         |
| OF=1                |  | p + p -> n (OF=1)               | p - n -> n (OF=1)  |
| OF meaningless      |  | -> incorrect S                  | -> incorrect S     |
| S = 1001            |  | S = 1001                        | S = 1001           |
| * think hand        |  | * OF <- C4 XOR C3 = 0 XOR 1 = 1 |                    |
| addition            |  | of signed addition              |                    |

\* CF=0, S=1001, OF=1 for all three interpretations

# Cases for setting the overflow flag (4) CF=0, OF=1

- signed integer overflow (OF=1 means incorrect S)

| * unsigned subtraction   |  | signed addition                 |  | * signed subtraction |
|--------------------------|--|---------------------------------|--|----------------------|
| 1011 (11)                |  | 1000                            |  | 1011 (-5)            |
| -0100 -(4) SUB           |  | +1100 +(-4)                     |  | -0100 -(+4) SUB      |
| -----                    |  | -----                           |  | -----                |
| 00111 (7) (0)            |  | 10111 (+7)                      |  | 10111 (+7)           |
| OF=1                     |  | n + n -> p (OF=1)               |  | n - p -> p (OF=1)    |
| OF meaningless           |  | -> incorrect S                  |  | -> incorrect S       |
| S = 0111                 |  | S = 0111                        |  | S = 0111             |
| * think hand subtraction |  | * OF <- C4 XOR C3 = 1 XOR 0 = 1 |  | of signed addition   |

\* CF=0, S=0111, OF=1 for all three interpretations

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# Cases for clearing the overflow flag (1) CF=1, OF=0

- no signed integer overflow (CF=0 means correct S)

| * unsigned addition |  | * signed addition               |  | signed subtraction |
|---------------------|--|---------------------------------|--|--------------------|
| 1101 (13)           |  | 1100                            |  |                    |
| +1110 +(14) ADD     |  | 1101 (-3)                       |  | 1101 (-3)          |
| -----               |  | +1110 +(-2) ADD                 |  | -0010 -(+2)        |
| -----               |  | -----                           |  | -----              |
| 11011 (11) (+16)    |  | 11011 (-5)                      |  | 11011 (-5)         |
|                     |  |                                 |  |                    |
| OF=0                |  | n + n -> n (OF=0)               |  | n - p -> n (OF=0)  |
|                     |  |                                 |  |                    |
| OF meaningless      |  | -> correct S                    |  | -> correct S       |
| S = 0000            |  | S = 0000                        |  | S = 0000           |
|                     |  |                                 |  |                    |
| * think hand        |  | * OF <- C4 XOR C3 = 1 XOR 1 = 0 |  |                    |
| addition            |  | of signed addition              |  |                    |

\* CF=1, S=1011, OF=0 for all three interpretations

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## Cases for clearing the overflow flag (2) CF=1, OF=0

- no signed integer overflow (CF=0 means correct S)

| * unsigned subtraction   |  | signed addition                 |  | * signed subtraction |
|--------------------------|--|---------------------------------|--|----------------------|
| 0011 (3)                 |  | 0010                            |  | 0011 (+3)            |
| -1110 -(14) SUB          |  | +0010 +(2)                      |  | -1110 -(-2) SUB      |
| -----                    |  | -----                           |  | -----                |
| 10101 (5) (-16)          |  | 00101 (+5)                      |  | 00101 (+5)           |
| CF=1                     |  | p + p -> p (OF=0)               |  | p - n -> p (OF=0)    |
| OF meaningless           |  | -> correct S                    |  | -> correct S         |
| S = 0101                 |  | S = 0101                        |  | S = 0101             |
| * think hand subtraction |  | * OF <- C4 XOR C3 = 0 XOR 0 = 0 |  | of signed addition   |

\* CF=1, S=0101, OF=0 for all three interpretations

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# Cases for clearing the overflow flag (3) CF=0, OF=0

- no signed integer overflow (CF=0 means correct S)

| * unsigned addition |  | * signed addition               |  | signed subtraction |
|---------------------|--|---------------------------------|--|--------------------|
| 0011 (3)            |  | 0010                            |  |                    |
| +0010 +(2) ADD      |  | 0011 (+3)                       |  | 0011 (+3)          |
| -----               |  | +0010 +(2) ADD                  |  | -1110 -(-2)        |
| 00101 (5) (+0)      |  | -----                           |  | -----              |
|                     |  | 00101 (+5)                      |  | 00101 (+5)         |
|                     |  |                                 |  |                    |
| OF=0                |  | p + p -> p (OF=0)               |  | p - n -> p (OF=0)  |
|                     |  |                                 |  |                    |
| OF meaningless      |  | -> correct S                    |  | -> correct S       |
| S = 0101            |  | S = 0101                        |  | S = 0101           |
|                     |  |                                 |  |                    |
| * think hand        |  | * OF <- C4 XOR C3 = 0 XOR 0 = 0 |  |                    |
| addition            |  | of signed addition              |  |                    |

\* CF=0, S=0101, OF=0 for all three interpretations

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# Cases for clearing the overflow flag (4) CF=0, OF=0

- no signed integer overflow (CF=0 means correct S)

| * unsigned addition |  | * signed addition               |  | signed subtraction |
|---------------------|--|---------------------------------|--|--------------------|
| 1101 (13)           |  | 1100                            |  |                    |
| -0010 -(2) SUB      |  | 1101 (-3)                       |  | 1101 (-3)          |
| -----               |  | +1110 +(-2)                     |  | -0010 -(+2) SUB    |
| -----               |  | -----                           |  | -----              |
| 11011 (11) (-16)    |  | 11011 (-5)                      |  | 11011 (-5)         |
|                     |  |                                 |  |                    |
| OF=0                |  | n + n -> n (OF=0)               |  | n - p -> n (OF=0)  |
|                     |  |                                 |  |                    |
| OF meaningless      |  | -> correct S                    |  | -> correct S       |
| S = 1011            |  | S = 1011                        |  | S = 1011           |
|                     |  |                                 |  |                    |
| * think hand        |  | * OF <- C4 XOR C3 = 1 XOR 1 = 0 |  |                    |
| subtraction         |  | of signed addition              |  |                    |

\* CF=0, S=1011, OF=0 for all three interpretations

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# 4-bit addition table (1)

|              |              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|              | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) |
| 0000<br>(0)  | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) |
| 0001<br>(+1) | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) |
| 0010<br>(+2) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) |
| 0011<br>(+3) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) |
| 0100<br>(+4) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) |
| 0101<br>(+5) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) |
| 0110<br>(+6) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) |
| 0111<br>(+7) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) |

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# 4-bit addition table (3)

|              |              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|              | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) |
| 0000<br>(0)  | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) |
| 0001<br>(+1) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  |
| 0010<br>(+2) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) |
| 0011<br>(+3) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) |
| 0100<br>(+4) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) |
| 0101<br>(+5) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) |
| 0110<br>(+6) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) |
| 0111<br>(+7) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) |

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# 4-bit addition table (2)

|              |              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|              | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) |
| 1000<br>(-8) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) |
| 1001<br>(-7) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  |
| 1010<br>(-6) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) |
| 1011<br>(-5) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) |
| 1100<br>(-4) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) |
| 1101<br>(-3) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) |
| 1110<br>(-2) | 1110<br>(-2) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) |
| 1111<br>(-1) | 1111<br>(-1) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) |

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# 4-bit addition table (4)

|              |              |              |              |              |              |              |              |              |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
|              | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) | 1111<br>(-1) |
| 1000<br>(-8) | 0000<br>(0)  | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) |
| 1001<br>(-7) | 0001<br>(+1) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) |
| 1010<br>(-6) | 0010<br>(+2) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) |
| 1011<br>(-5) | 0011<br>(+3) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) |
| 1100<br>(-4) | 0100<br>(+4) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) |
| 1101<br>(-3) | 0101<br>(+5) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) |
| 1110<br>(-2) | 0110<br>(+6) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) |
| 1111<br>(-1) | 0111<br>(+7) | 1000<br>(-8) | 1001<br>(-7) | 1010<br>(-6) | 1011<br>(-5) | 1100<br>(-4) | 1101<br>(-3) | 1110<br>(-2) |

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