

Capacitor in an AC circuit

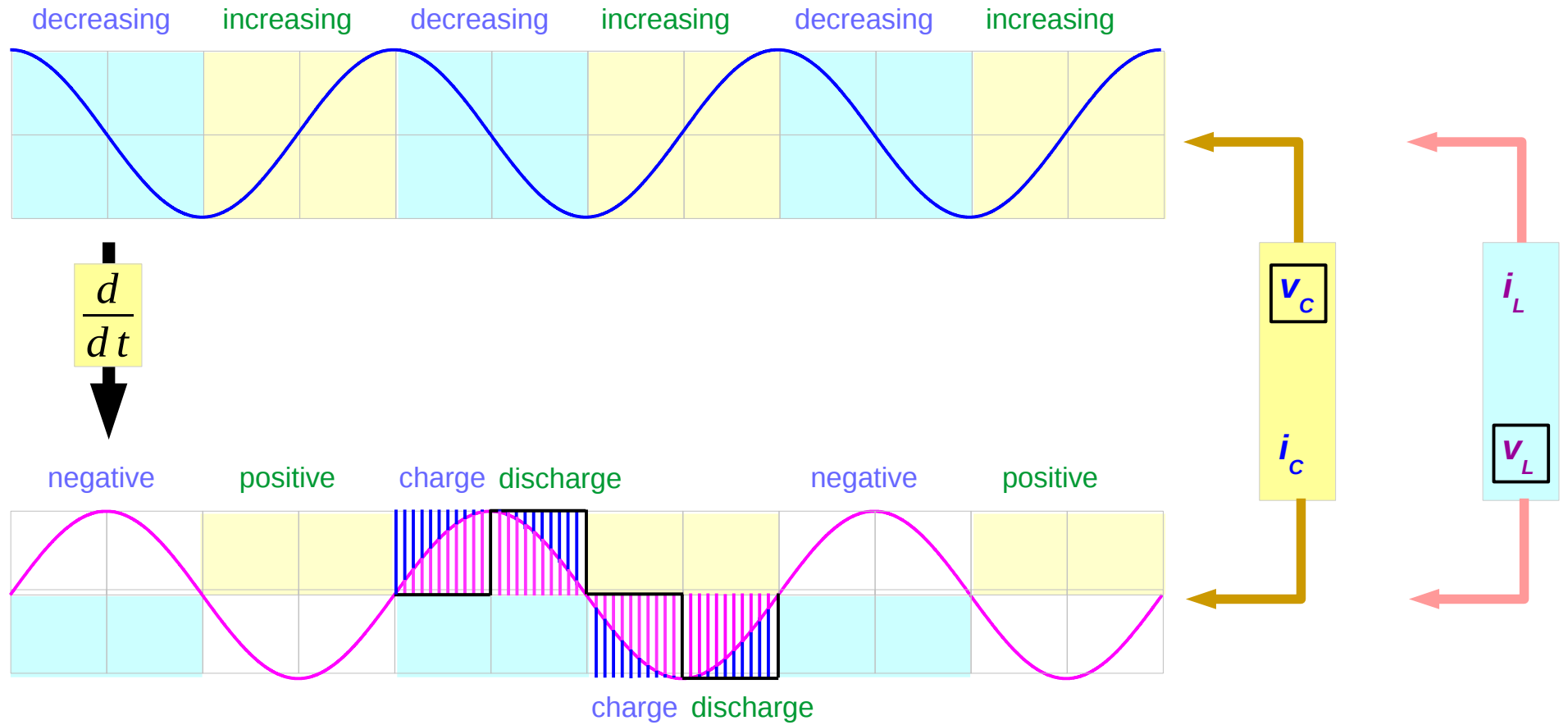
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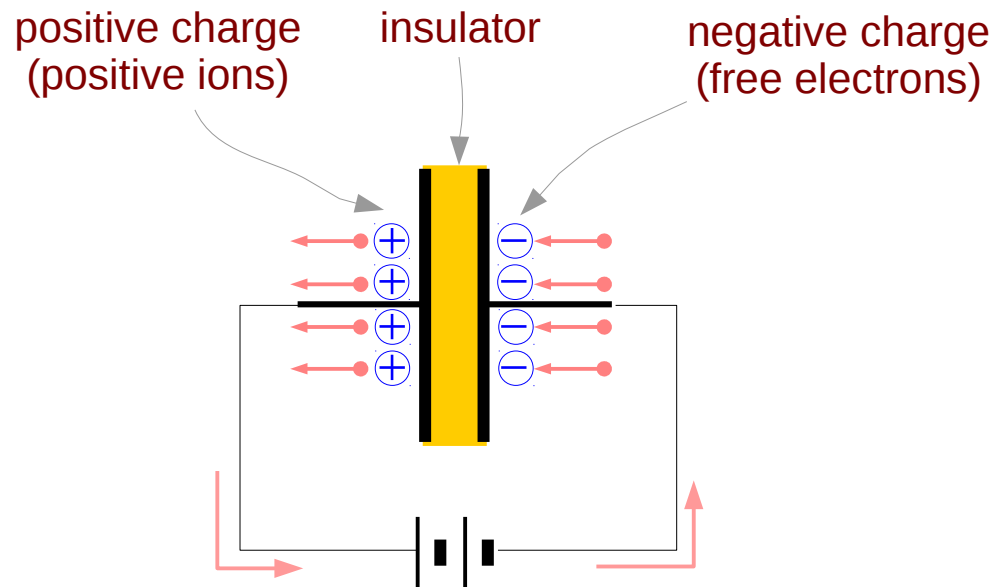
Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Everchanging signal pairs



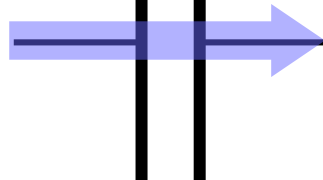
Capacitor Current



No actual electrons movement across insulator materials



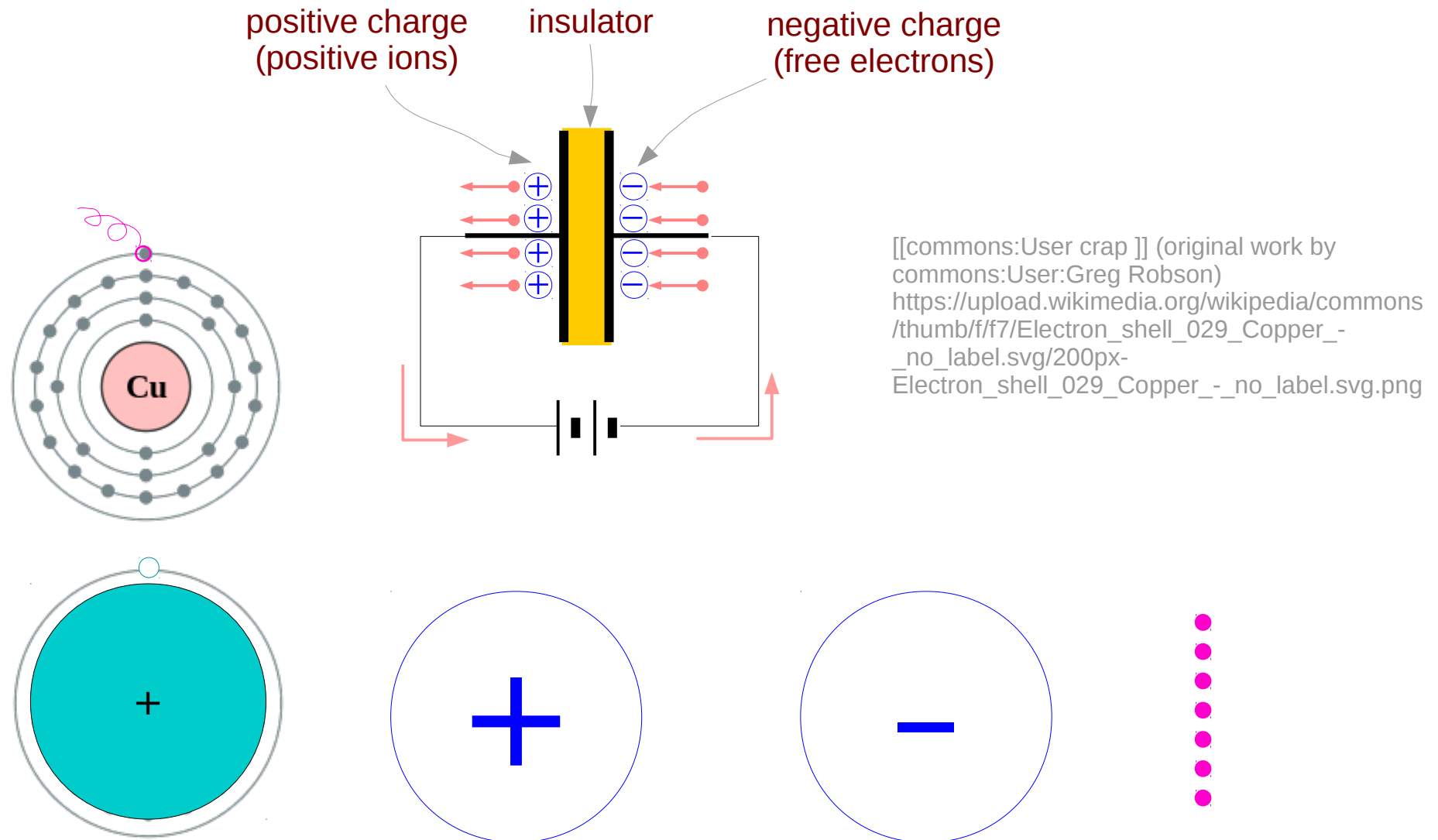
But, think as



Displacement Current

flows through the capacitor

Positive ions and free electrons

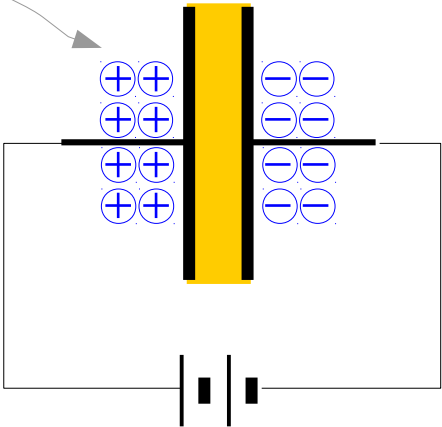


Three States

positive charge
(positive ions)

Positively Charged State

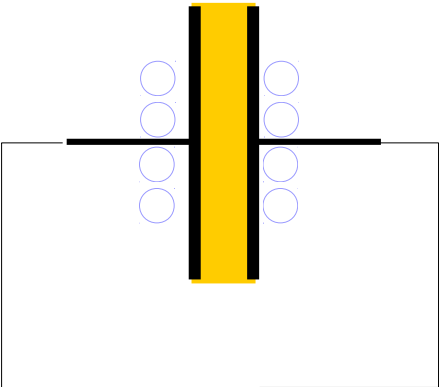
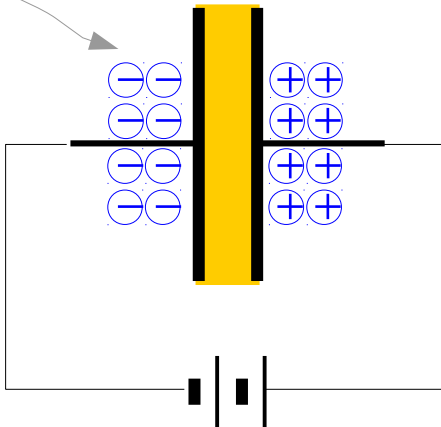
fully charged → no current



negative charge
(free electrons)

Negatively Charged State

fully charged → no current

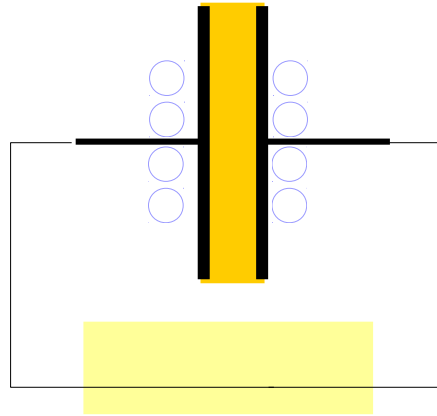


Fully Discharged State

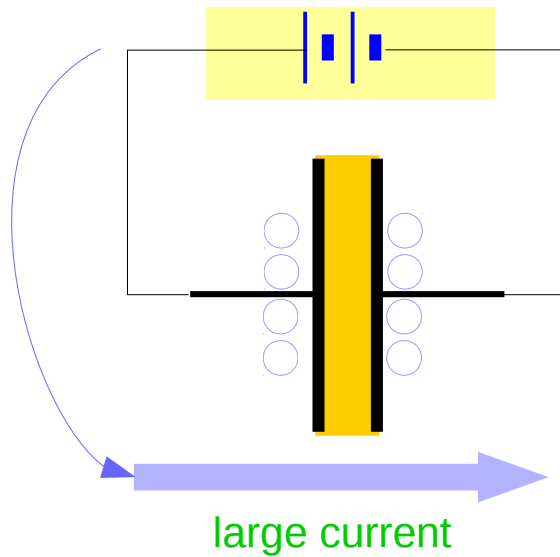
possible large current

Currents in the Fully Discharged State

Initially no current



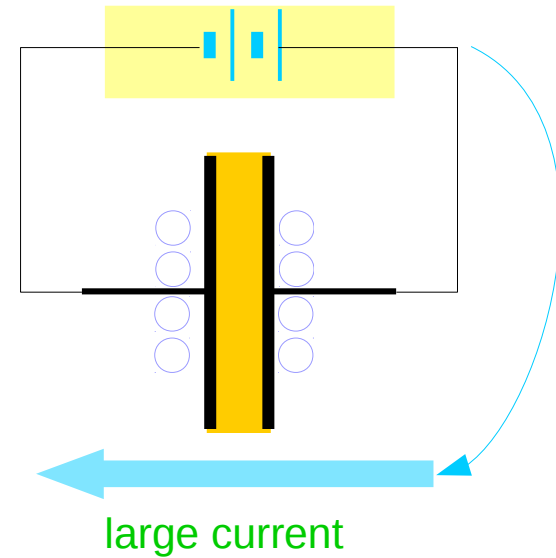
Fully Discharged State



Fully Discharged State

This state can flow large current in either direction

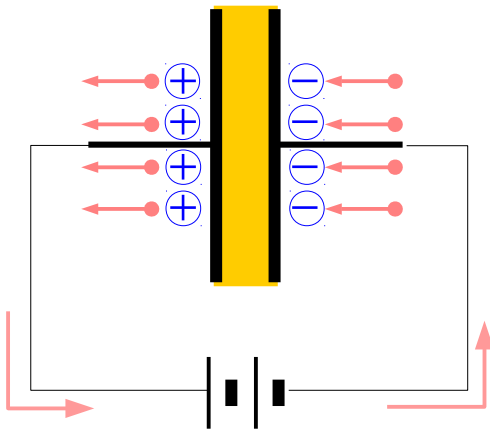
Fully Discharged State



Inter-State Current Flowing

Under Positively Charging

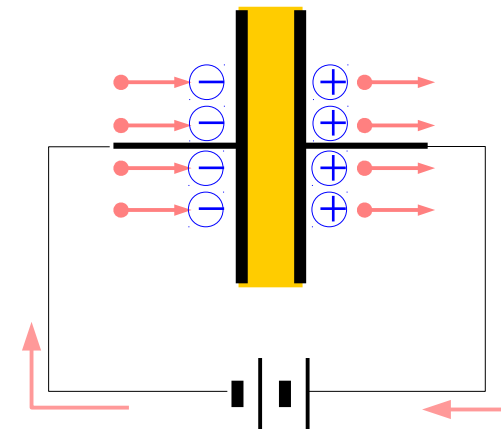
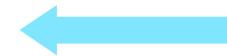
(+) current flow direction



electron flow direction

Under Negatively Charging

(-) current flow direction

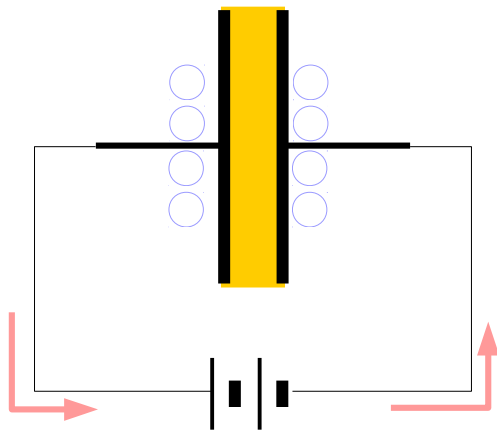


electron flow direction

Inter-State Current Flowing

Fully Discharged State

(+) current flow direction

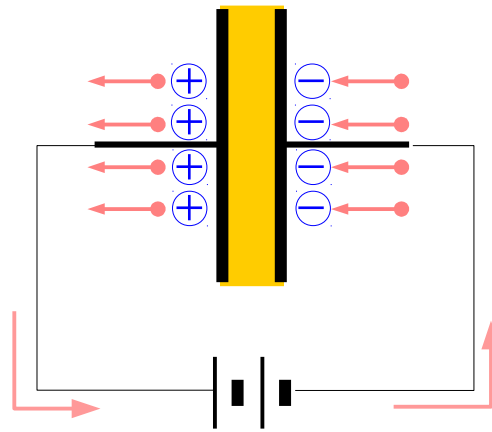


electron flow direction

Initial large current

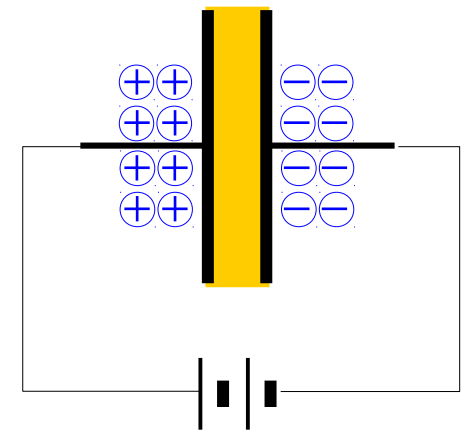
Under Positively Charging

(+) current flow direction



electron flow direction

Positively Charged State



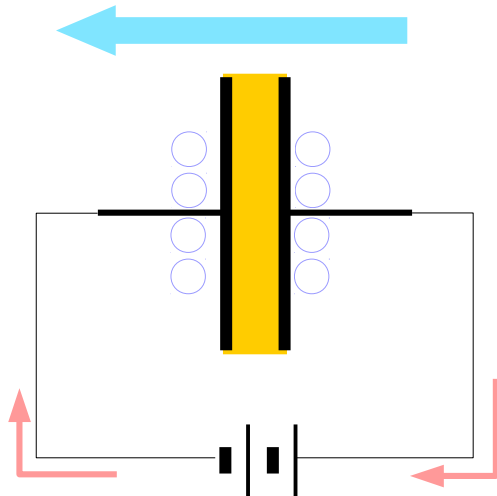
Crowded →
No more space

no current

Inter-State Current Flowing

Fully Discharged State

(-) current flow direction

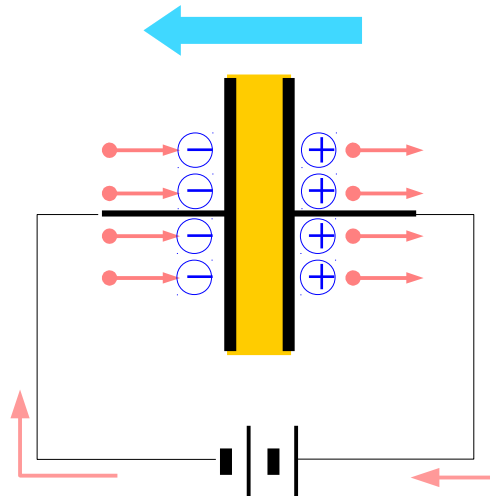


electron flow direction

Initial large current

Under Negatively Charging

(-) current flow direction

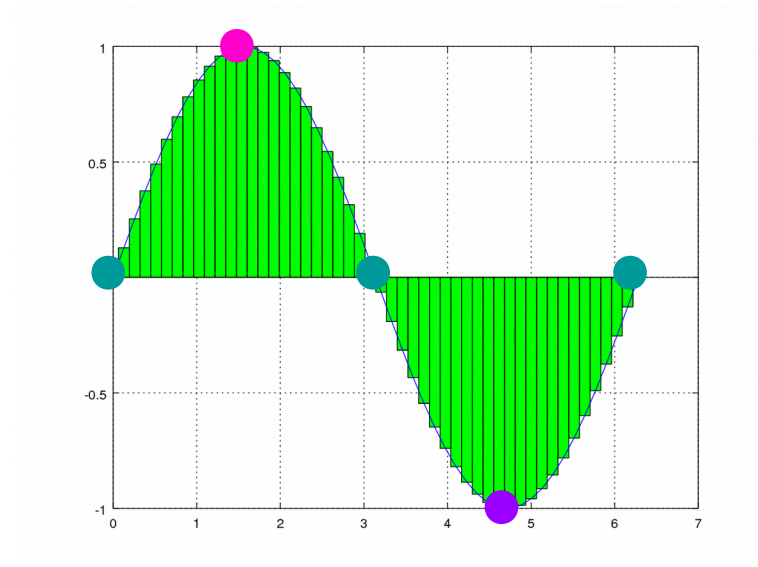
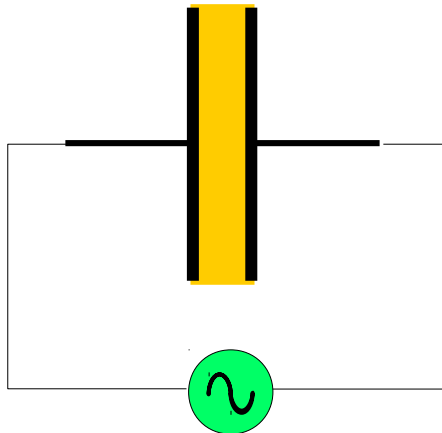


electron flow direction

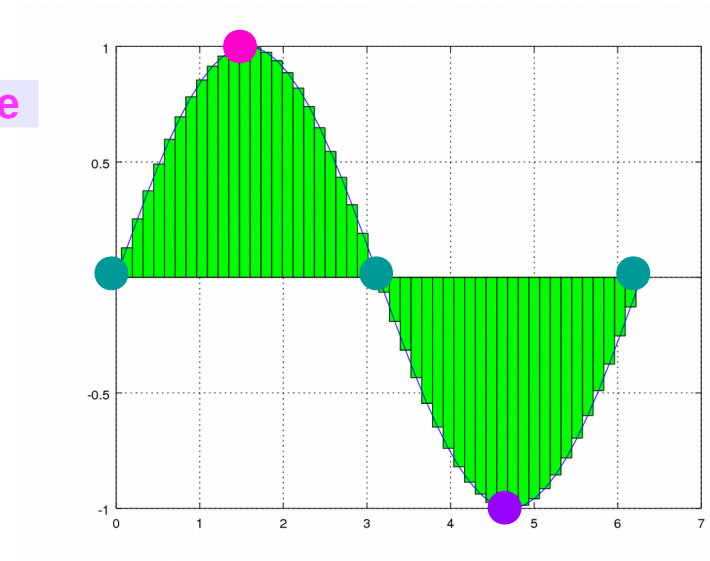
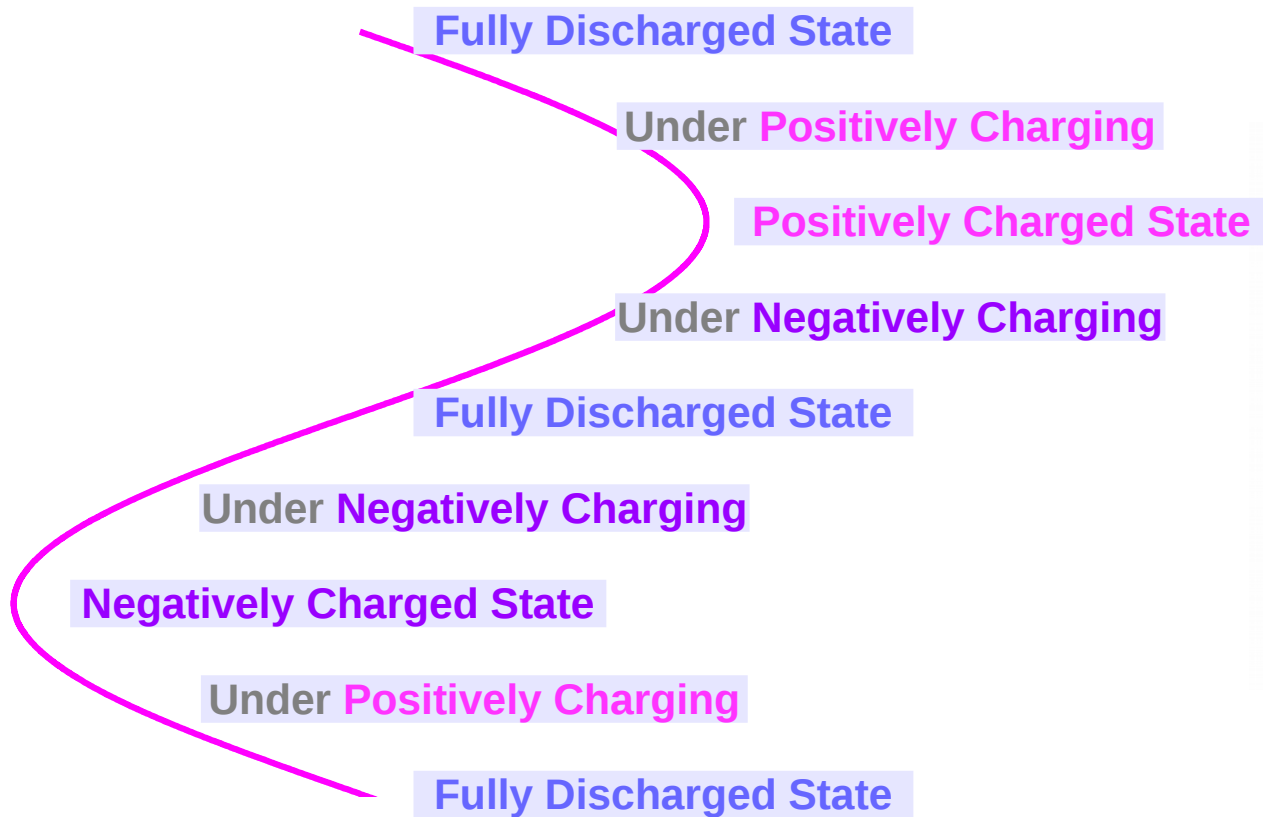
Crowded →
No more space

no current

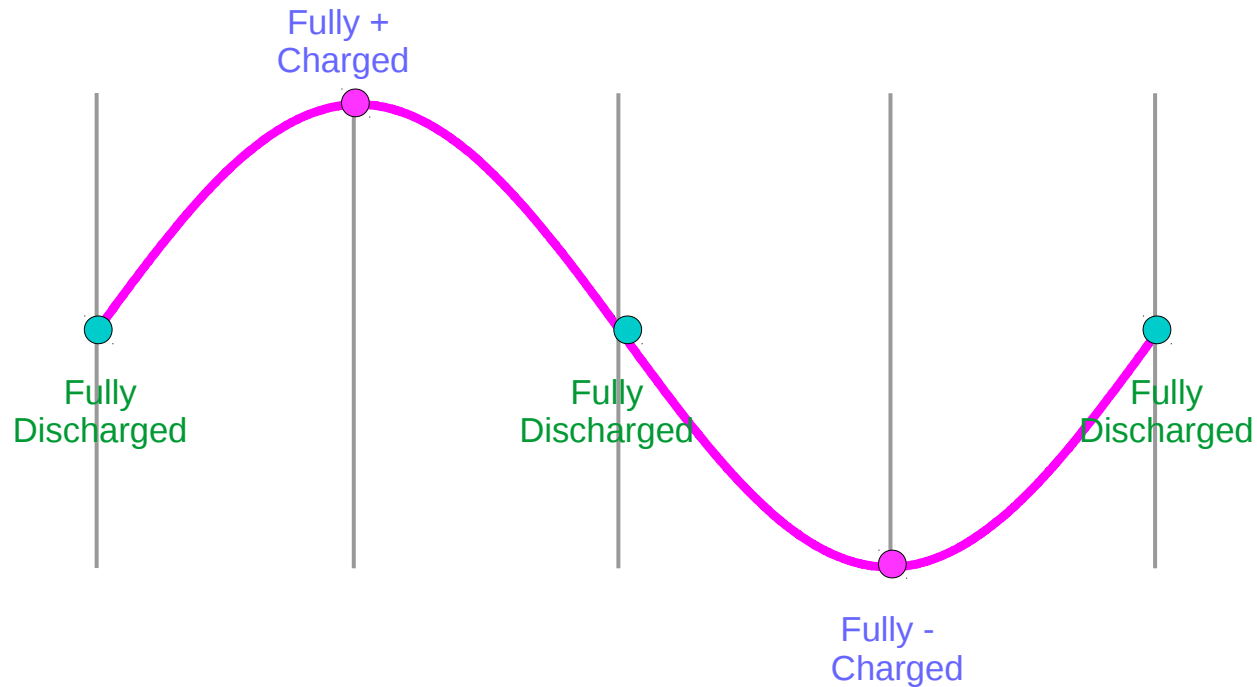
An AC Voltage Source



An AC Voltage Source

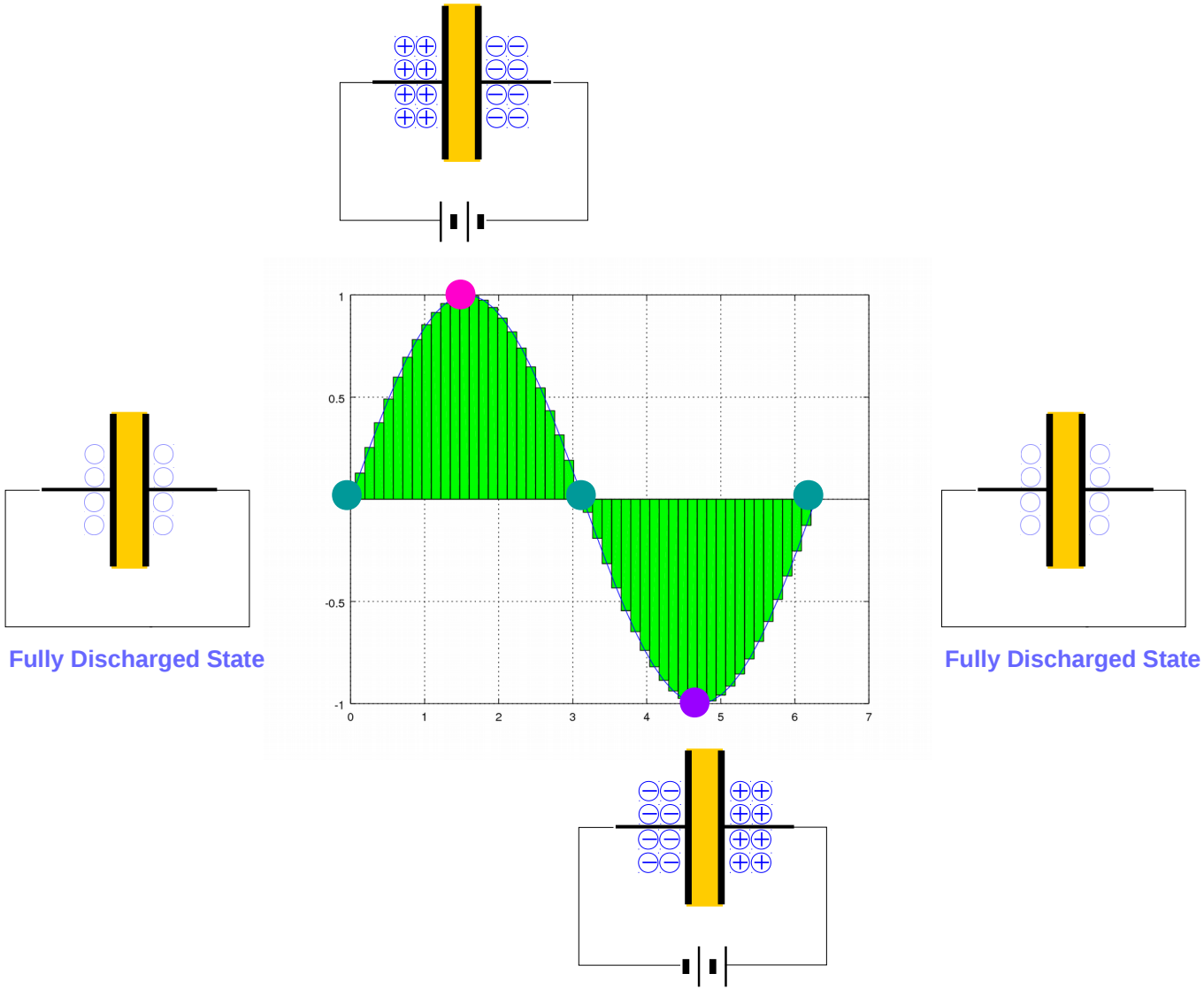


Fully Charged and Fully Discharged

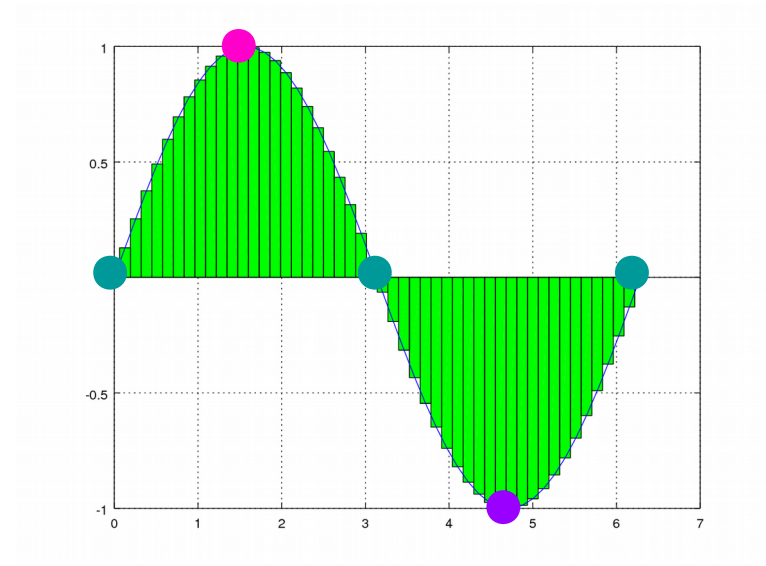
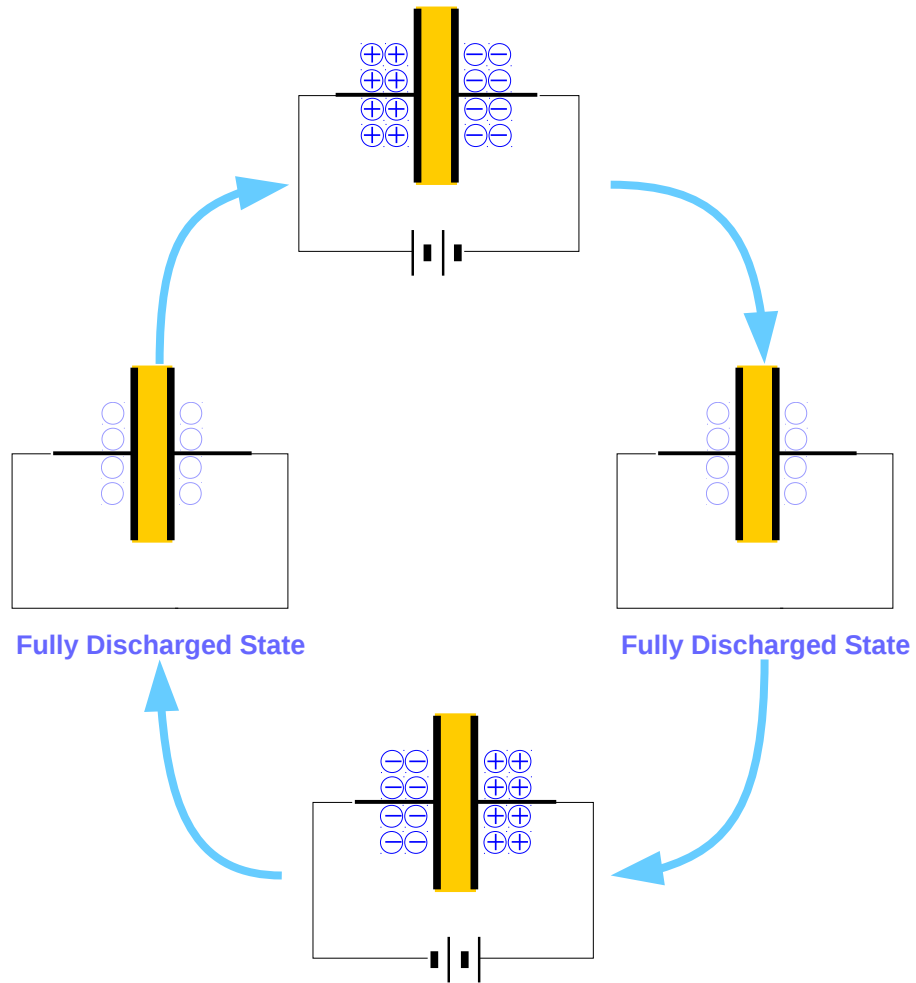


(+) Charging	(-) Charging	(-) Charging	(+) Charging
(+) current	(-) current	(-) current	(+) current
(+) Charging	(+) Discharging	(-) Charging	(-) Discharging

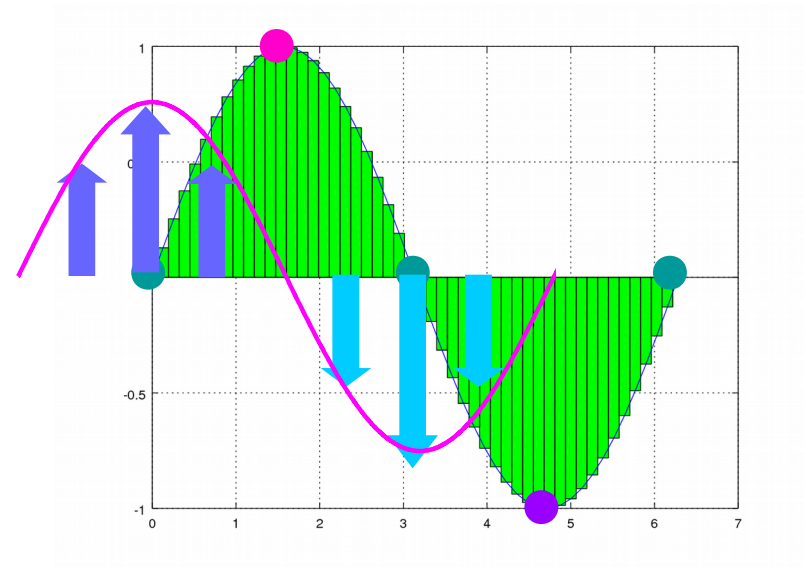
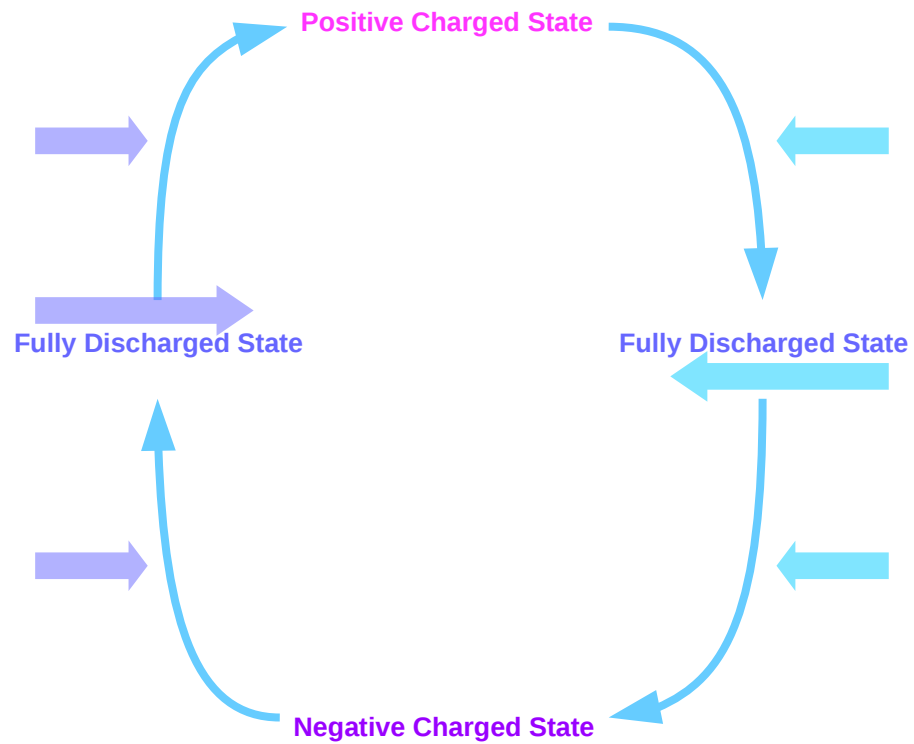
A Cycle



State Transition Diagram



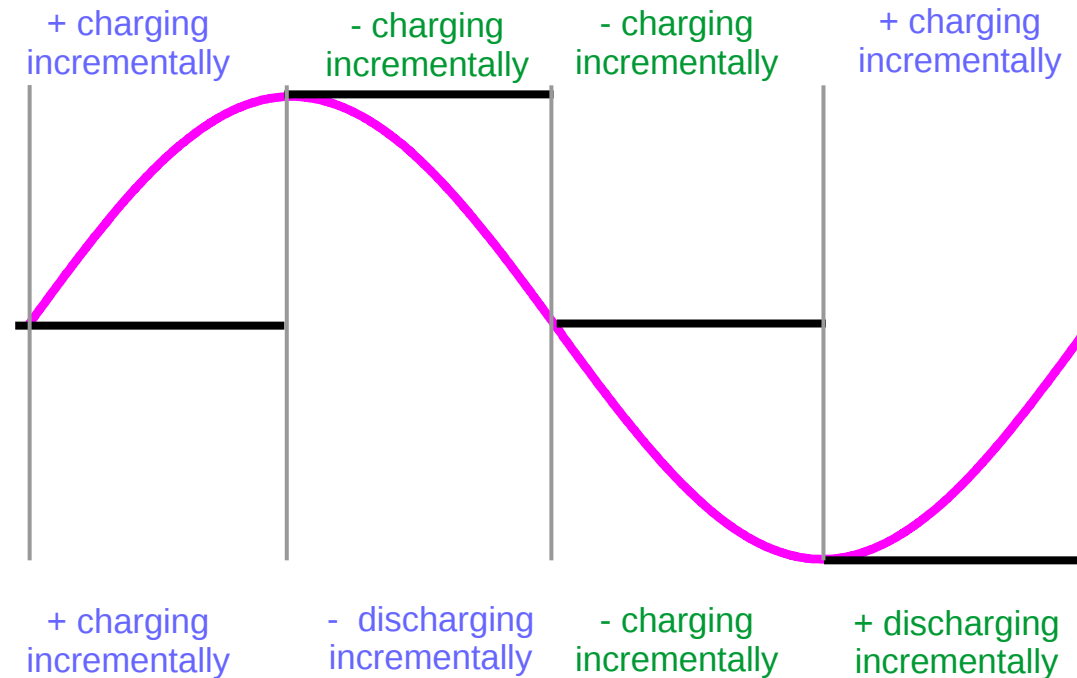
Current Flow



Continuous Charging and Discharging Operations

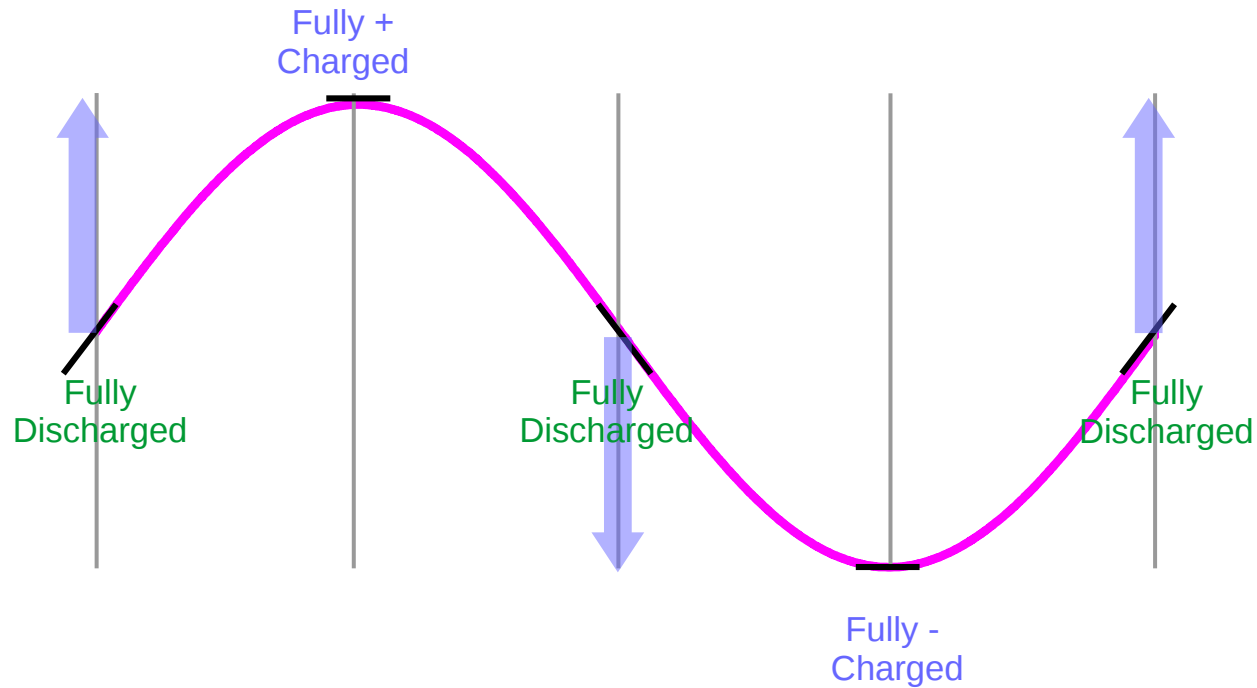
Incremental Voltage Increment \rightarrow + Charging incrementally

Incremental Voltage Decrement \rightarrow - Charging incrementally

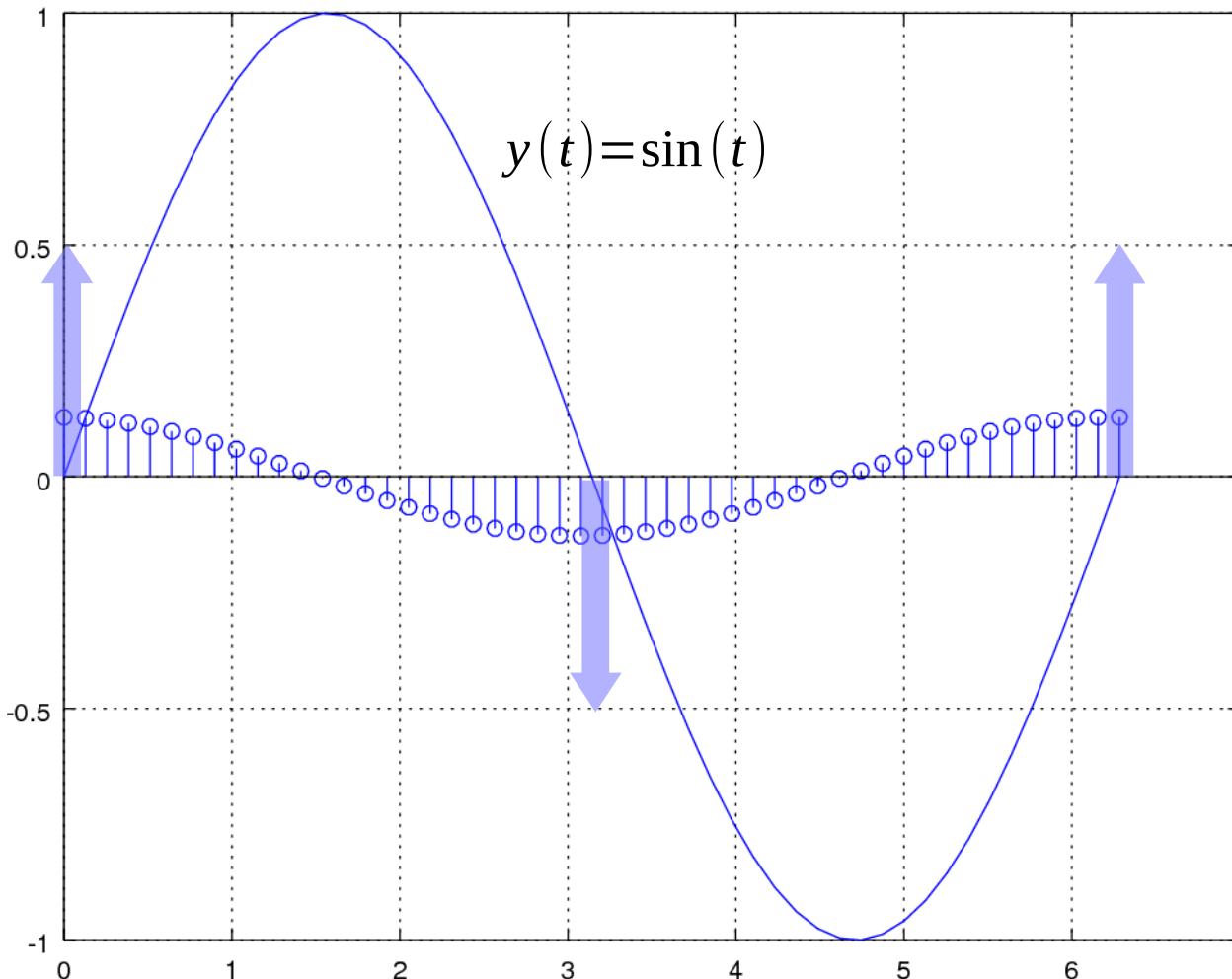


Fully Discharged : Large Current

Incremental Voltage Increment → Continuous Charging
Incremental Voltage Decrement → Continuous Discharging



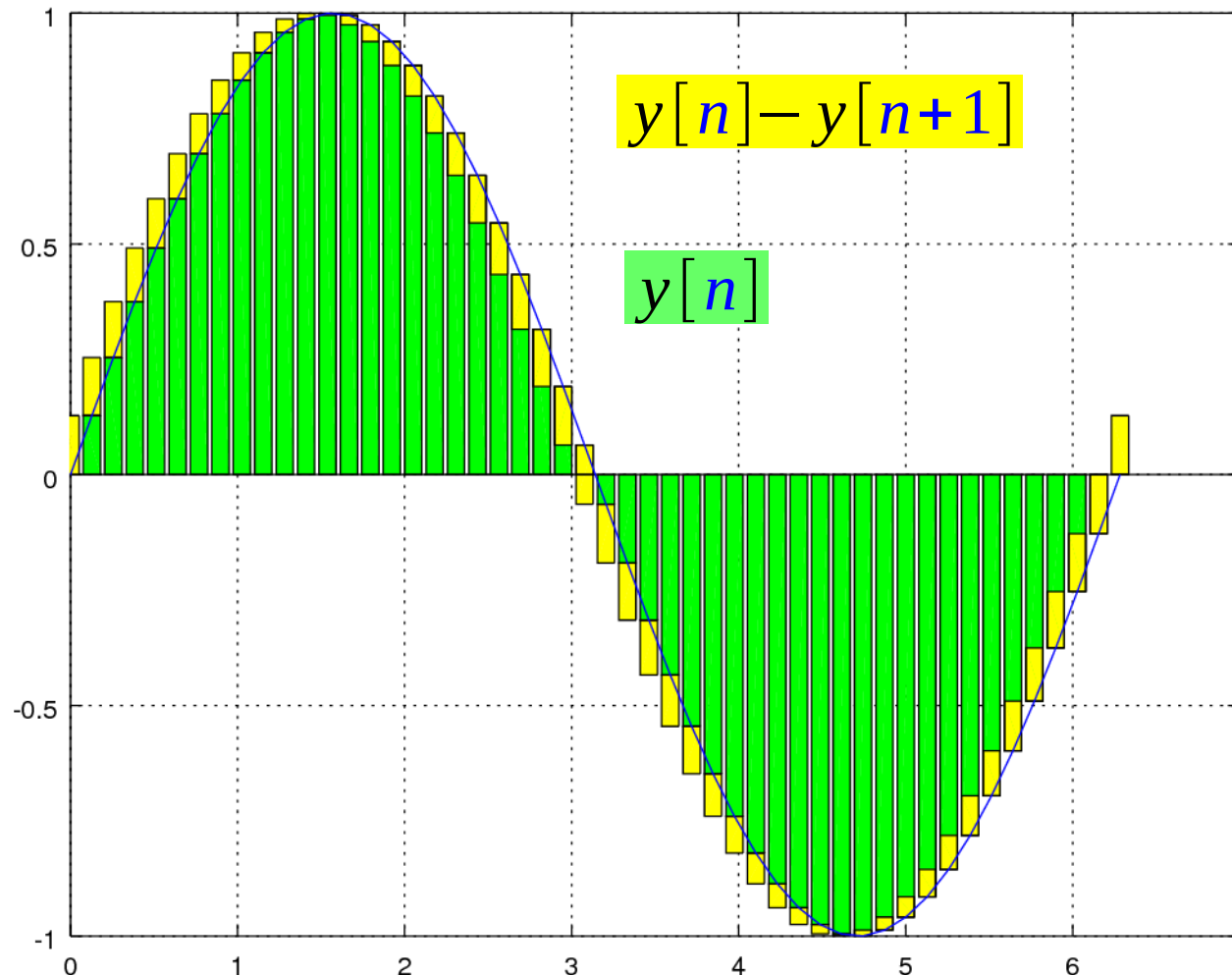
$y[n+1] - y[n]$



```
t = linspace(0, pi*2, 50);  
t1 = t;  
t2 = t + t(2);  
y1 = sin(t1);  
y2 = sin(t2) - sin(t1);  
stem(t1, y2)  
hold on  
plot(t1, y1)
```

$$y[n] - y[n+1] = y(nT) - y((n+1)T) = \sin(nT) - \sin((n+1)T)$$

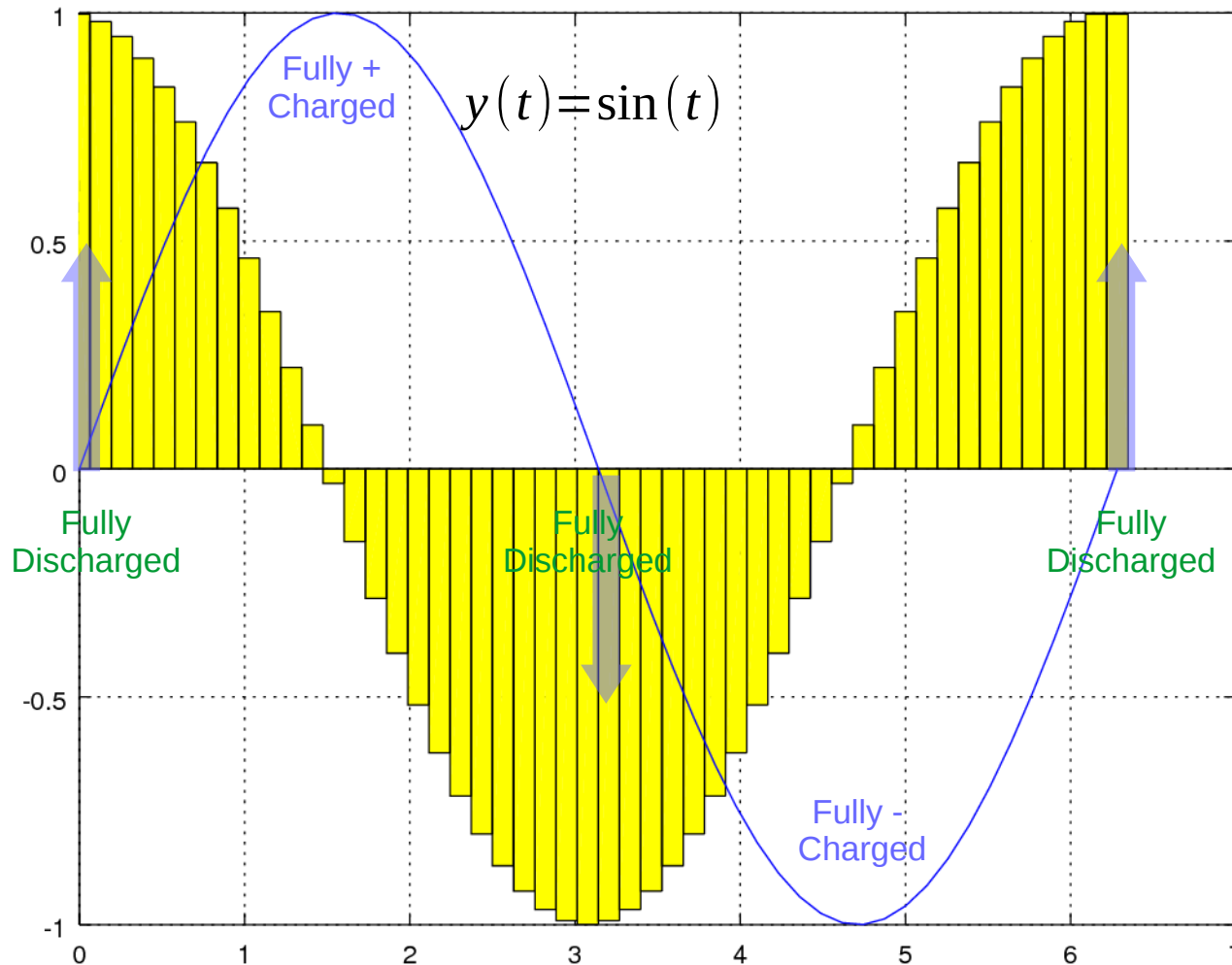
Fully Charged and Fully Discharged



```
h = bar(t1, [y1' y2'],  
"stacked")  
set(h(1), "facecolor", "g");  
set(h(2), "facecolor", "y");  
hold on  
plot(t1, y1)  
axis([0 7 -1 1]);
```

$$y[n] - y[n+1] = y(nT) - y((n+1)T) = \sin(nT) - \sin((n+1)T)$$

Fully Charged and Fully Discharged

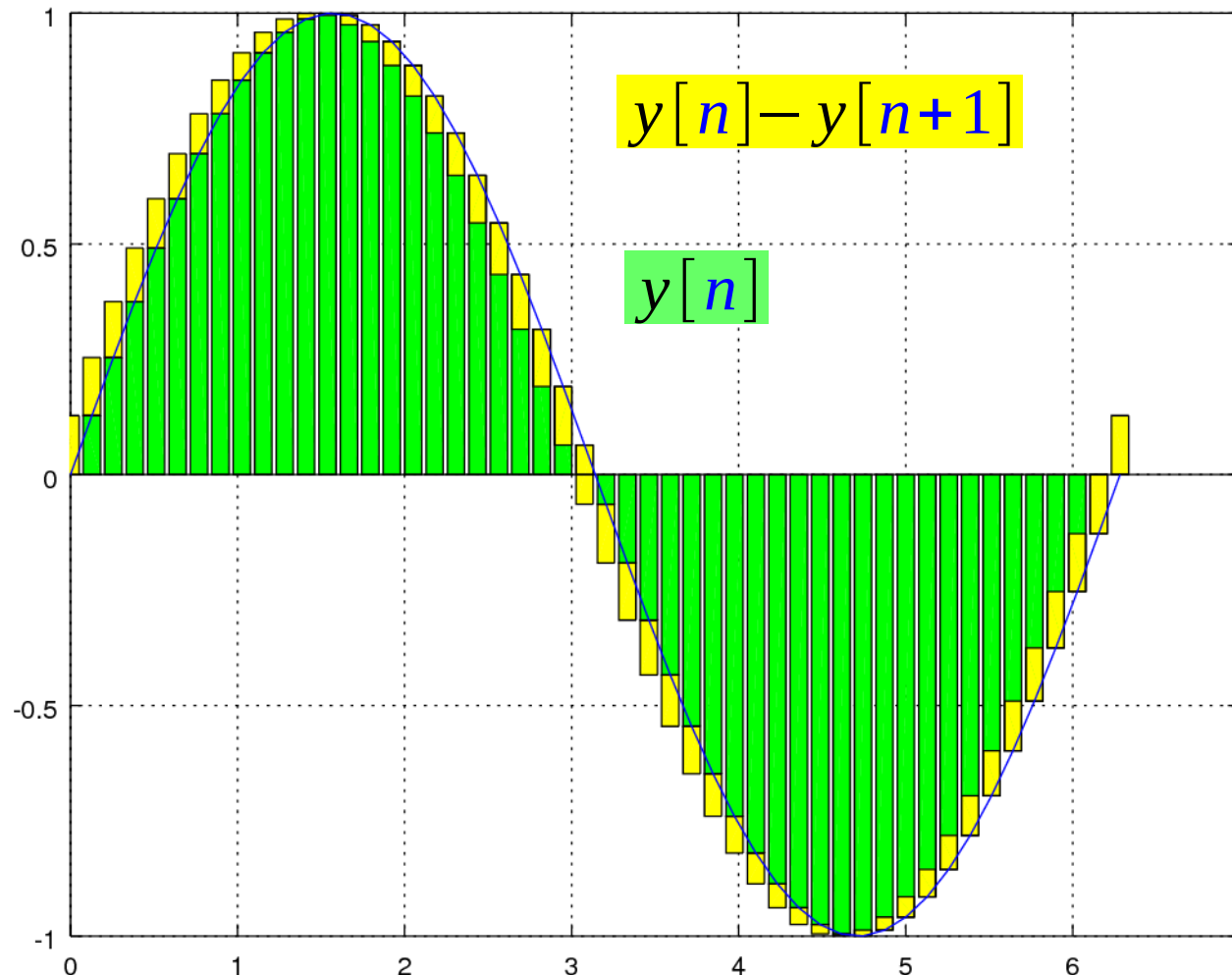


```
h = bar(t1, y2/t(2), "hist")
set(h(1), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 7 -1 1]);
```

$$\frac{y[n] - y[n+1]}{T}$$

$$\propto \frac{dy}{dt}$$

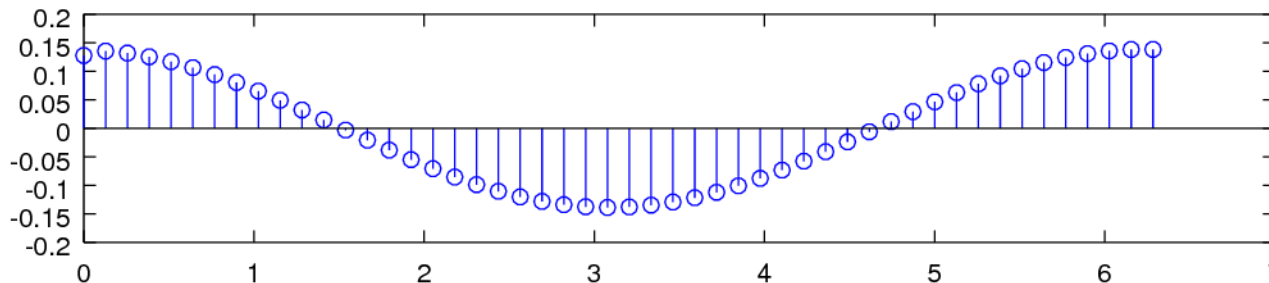
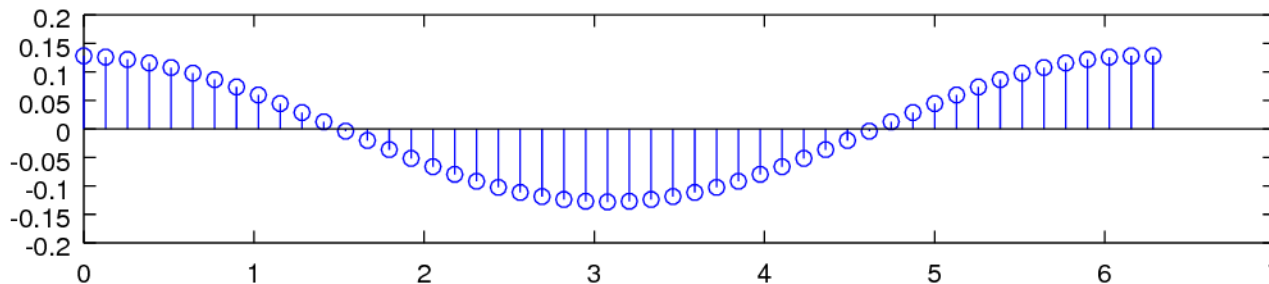
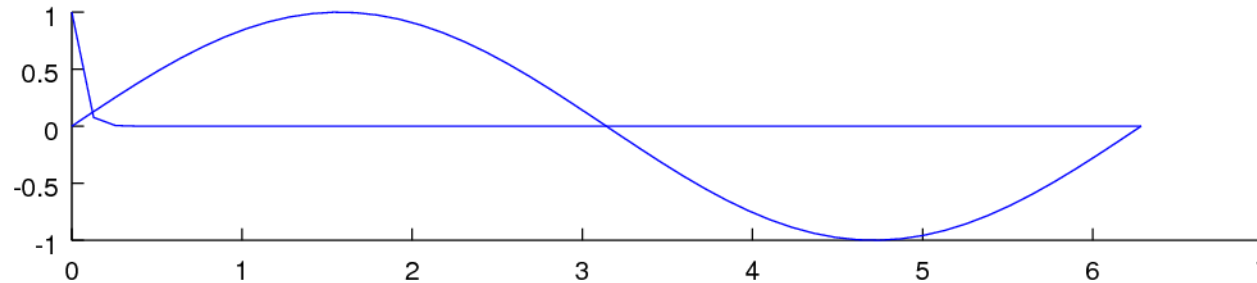
Fully Charged and Fully Discharged



```
h = bar(t1, [y1' y2'],  
"stacked")  
set(h(1), "facecolor", "g");  
set(h(2), "facecolor", "y");  
hold on  
plot(t1, y1)  
axis([0 pi]);
```

$$y[n] - y[n+1] = y(nT) - y((n+1)T) = \sin(nT) - \sin((n+1)T)$$

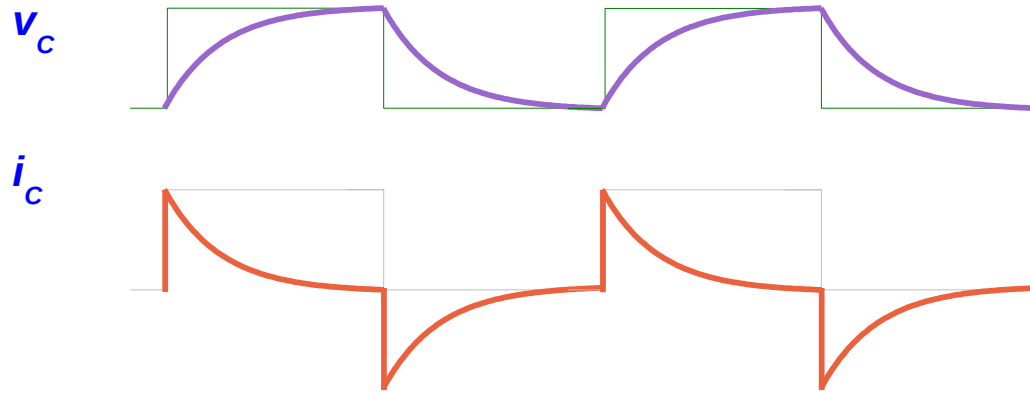
Fully Charged and Fully Discharged



```
clf
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
y3 = e.^(-20*t);
y4 = conv(y2, y3);
y5 = y4([1:length(t1)]);
```

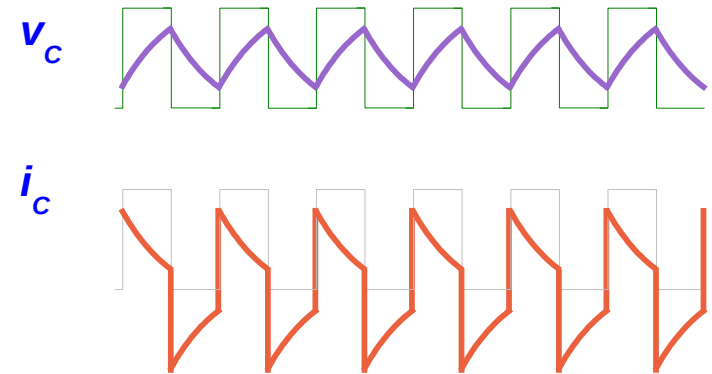
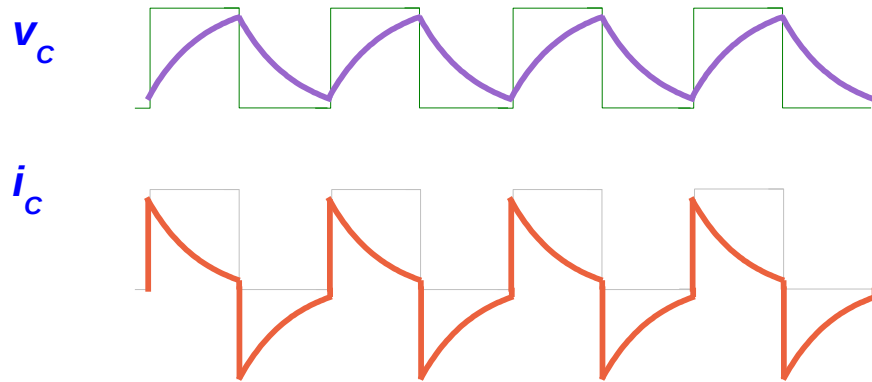
```
subplot(3, 1, 2);
stem(t1, y2)
subplot(3, 1, 1);
hold on
plot(t1, y1);
plot(t1, y3);
subplot(3, 1, 3);
stem(t1, y5);
```

Pulse



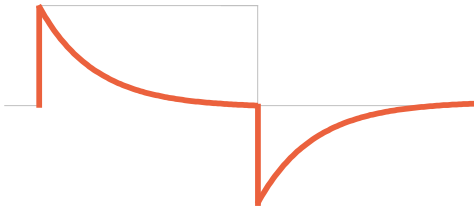
$$i_c = C \frac{dv_c}{dt}$$

ω ↑ i_c ↑ X_c ↓

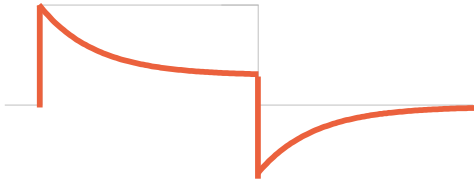


Time Constants

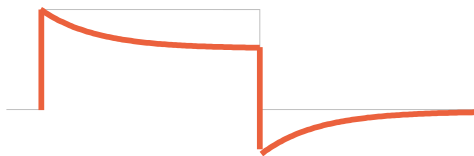
i_c



$\tau = RC$ *small time constant*

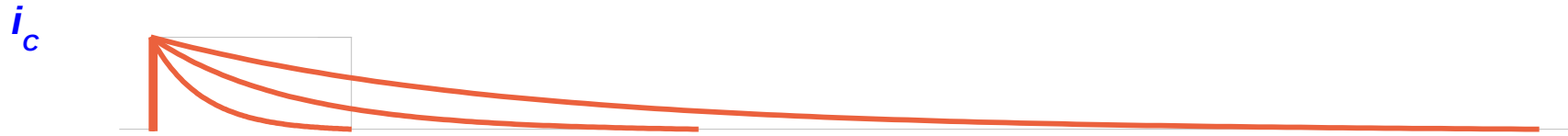


$\tau = RC$ *medium time constant*



$\tau = RC$ *large time constant*

Time Constants



$$\tau_1 < \tau_2 < \tau_3$$

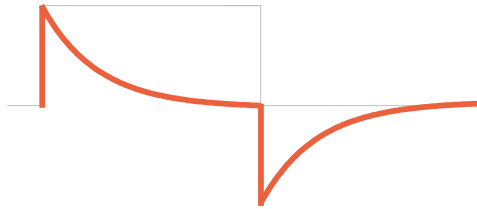
$$a_1 > a_2 > a_3$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}} = e^{-at}$$

$$\tau = RC = \frac{1}{a}$$

Time Constants

i_c



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ
small C

$$\text{large } \frac{1}{\omega C} \gg R$$

Fully Capacitive



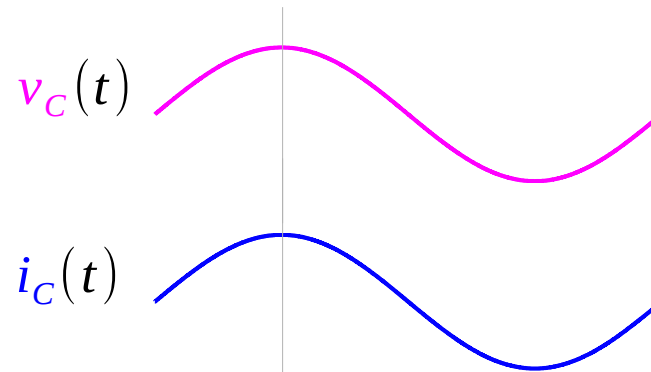
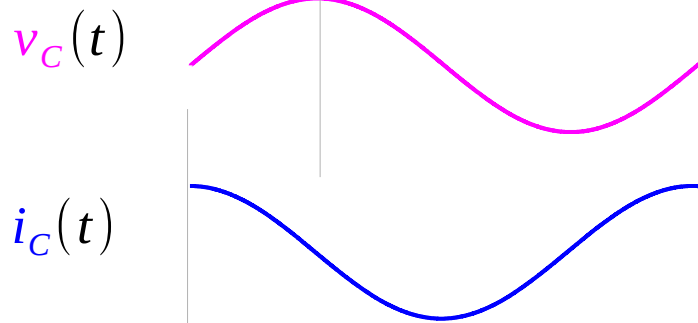
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ
large C

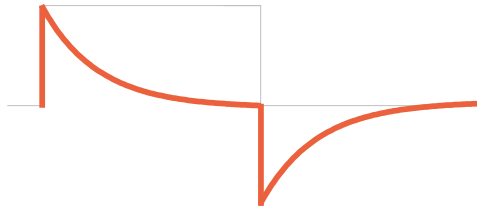
$$\text{small } \frac{1}{\omega C} \ll R$$

Fully Resistive



Time Constants

i_c

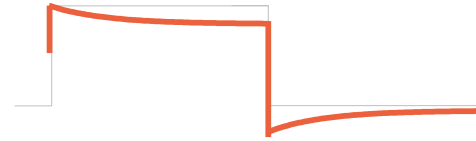


$$\tau = RC$$
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ
small C

$$\text{large } \frac{1}{\omega C} \gg R$$

Fully Capacitive



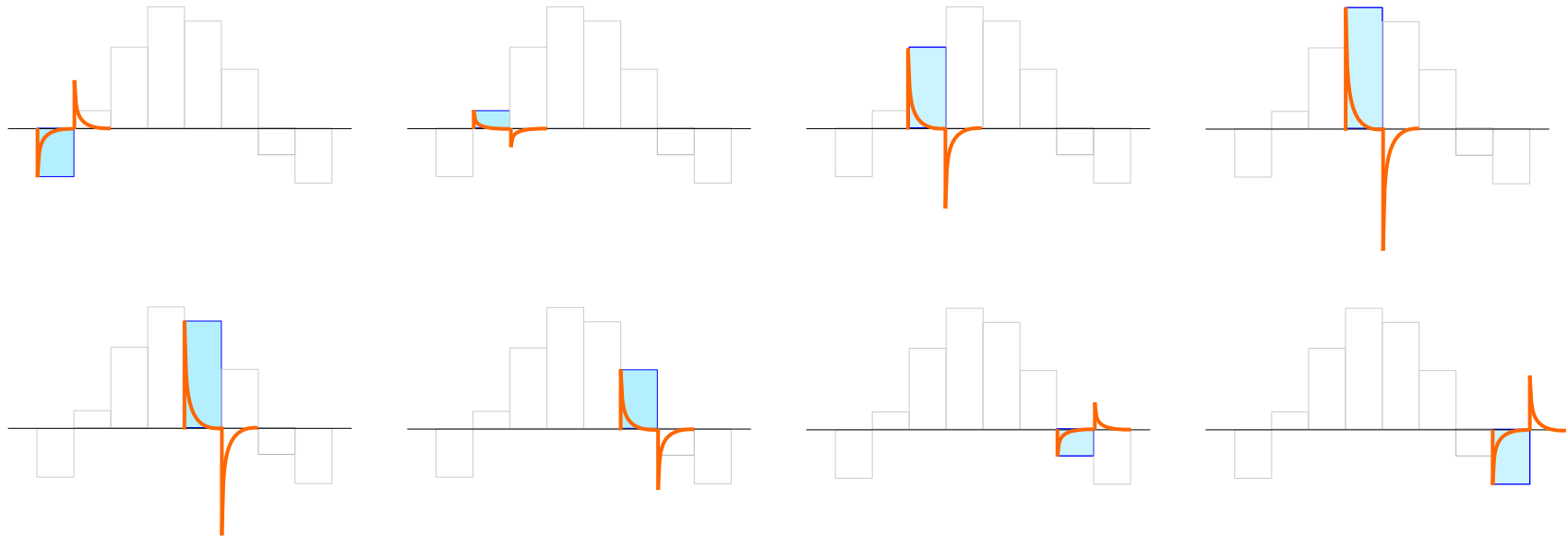
$$\tau = RC$$
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ
large C

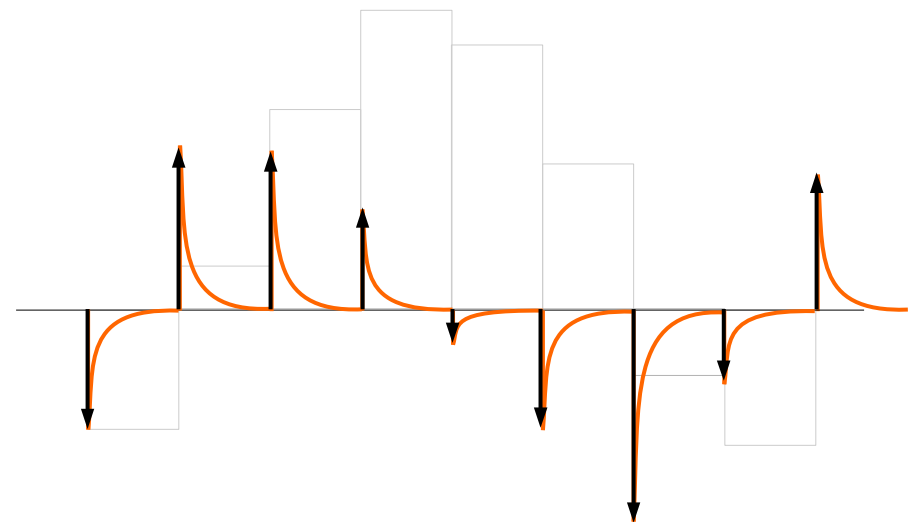
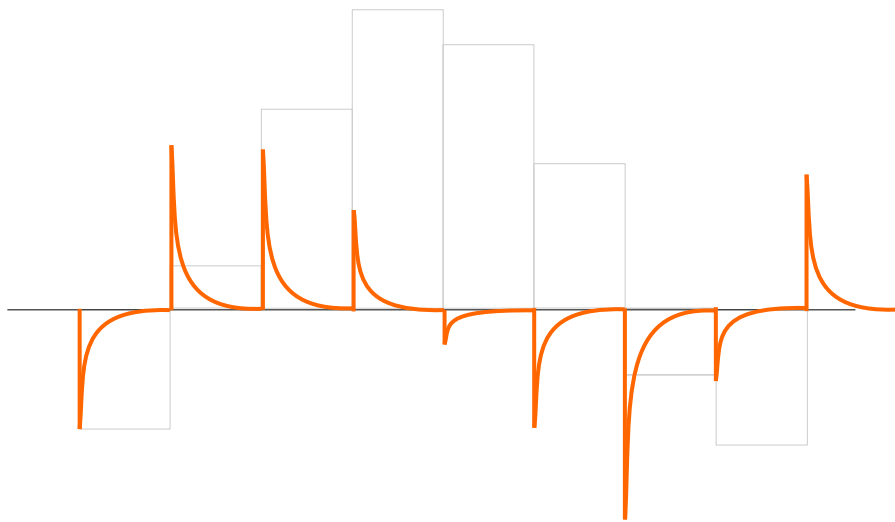
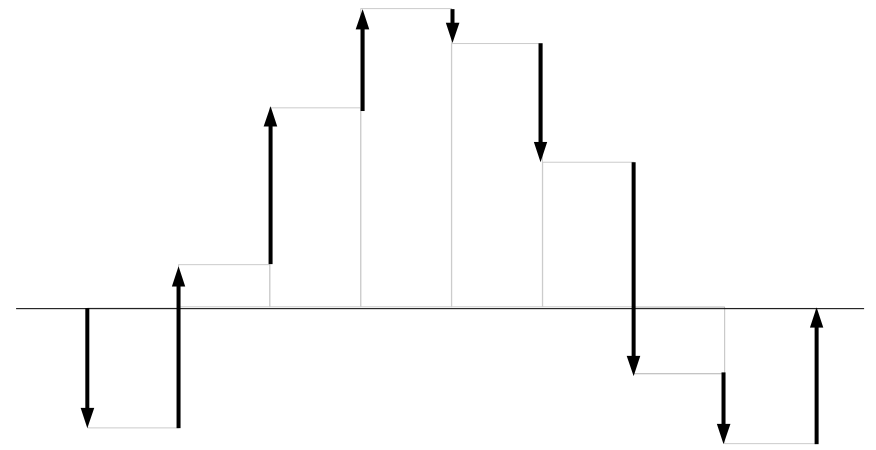
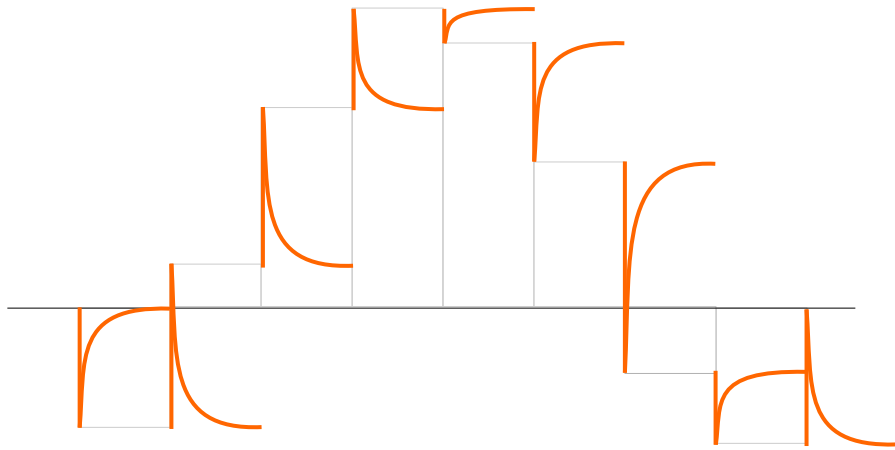
$$\text{small } \frac{1}{\omega C} \ll R$$

Fully Resistive

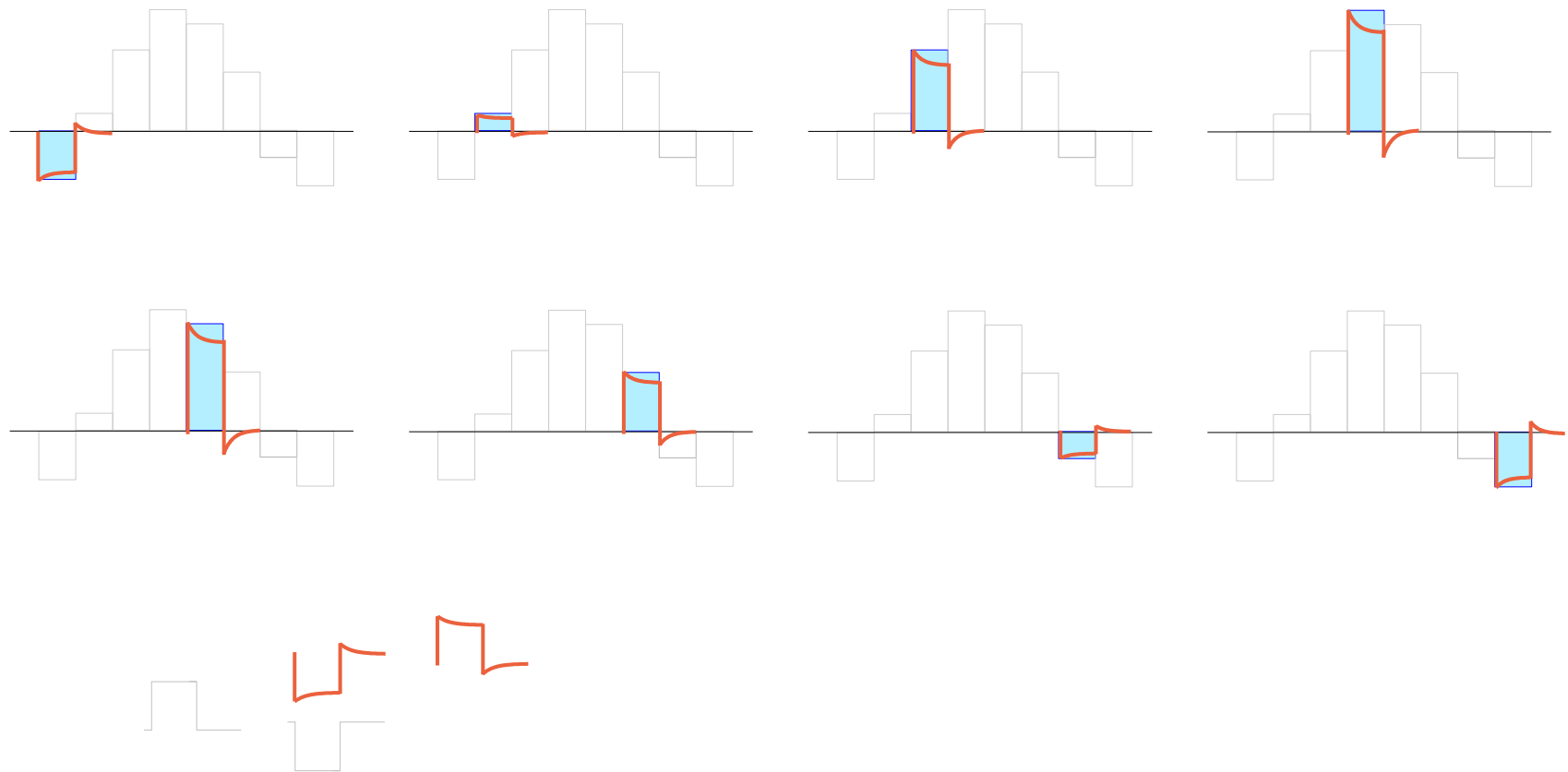
Superposition - Small Time Constant



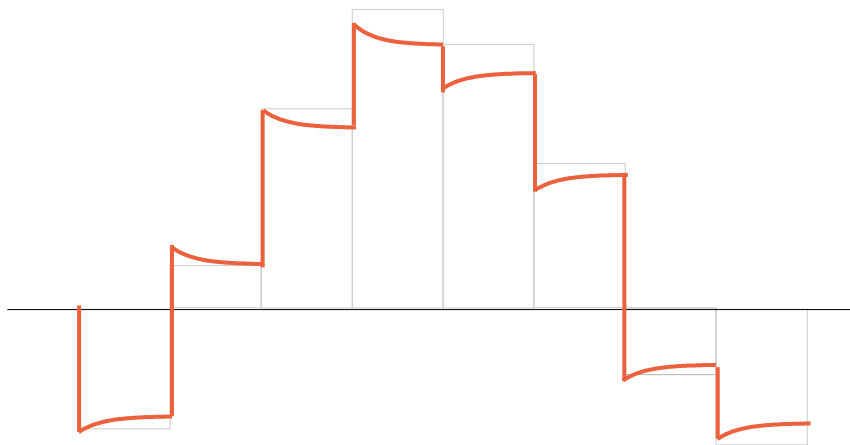
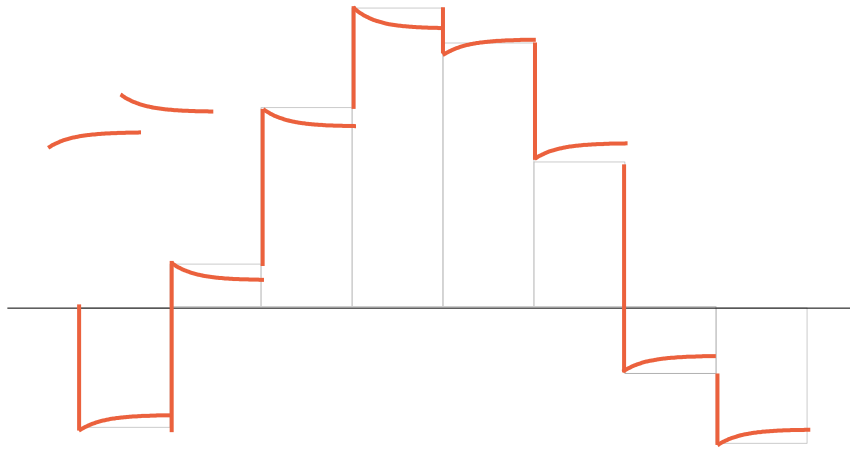
Small Time Constants



Superposition - Large Time Constant

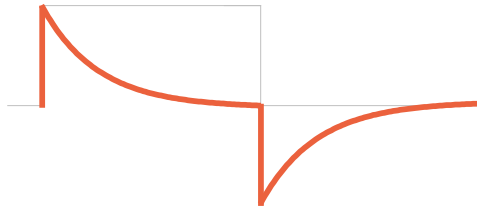


Large Time Constants



Time Constants

i_c



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ
small C

$$\text{large } \frac{1}{\omega C} \gg R$$

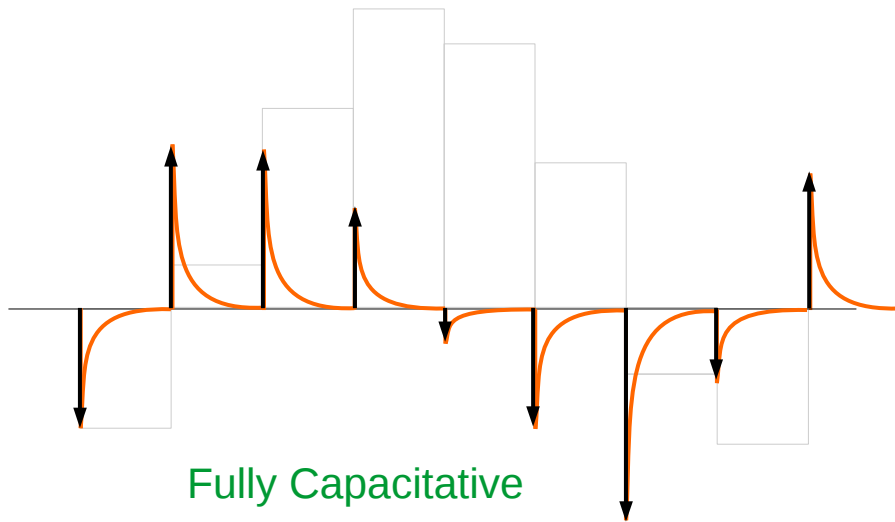


$$\tau = RC$$

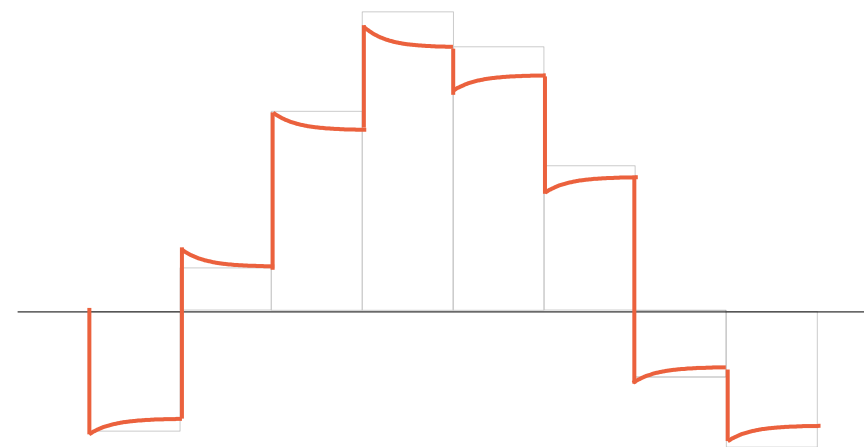
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ
large C

$$\text{small } \frac{1}{\omega C} \ll R$$



Fully Capacitive



Fully Resistive

Plotting superposition results

```
clf
t = linspace(0, pi*2, 50);
tt= linspace(0, pi*2, 500);
N = length(t);
NN= length(tt);

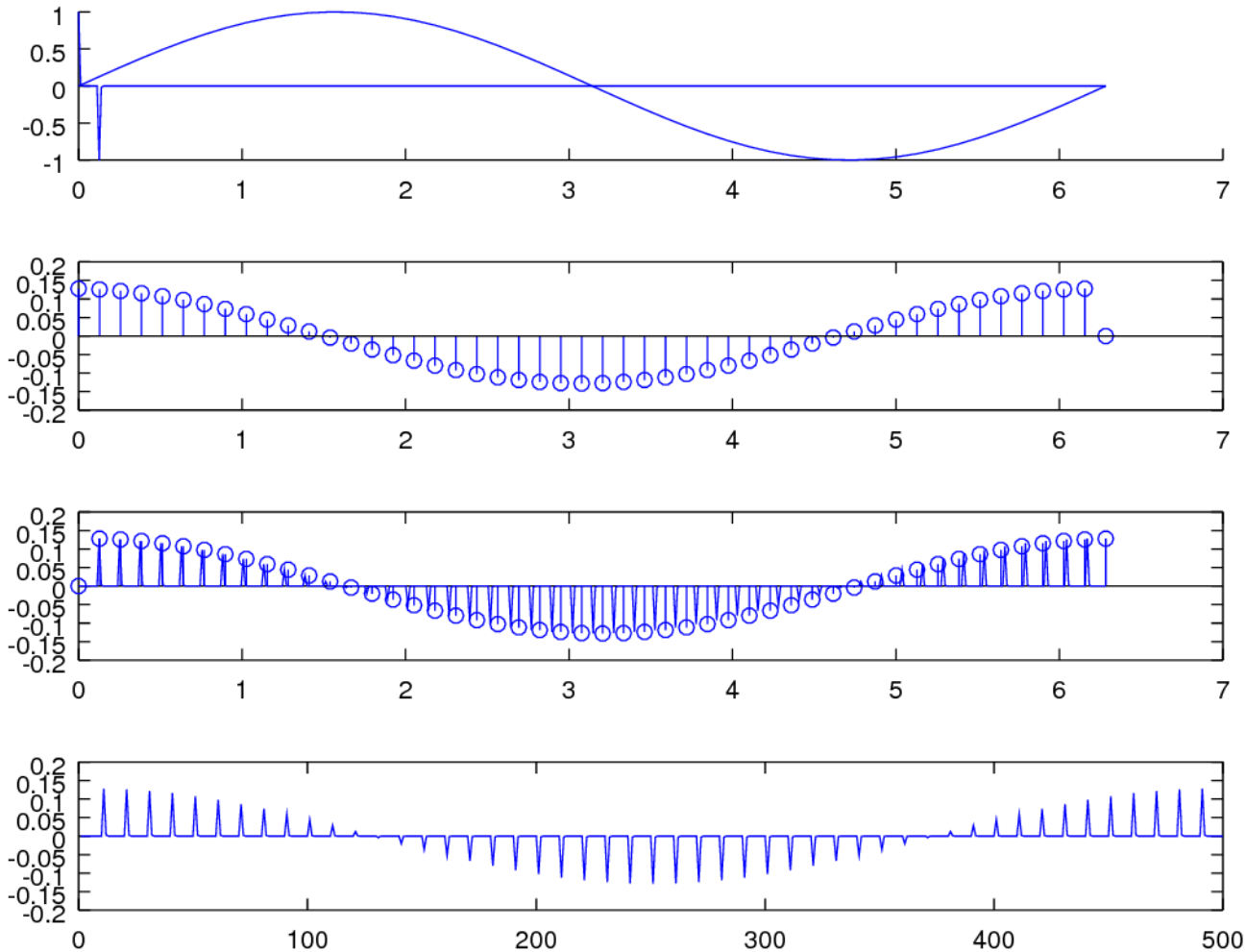
t1 = t;
t2 = [t(2:N), t(N)];
y1 = sin(t1);
y2 = sin(t2) - sin(t1);

yy = [y1; zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 - [zeros(1, NN/N),
e.^(-a*tt)](1:NN);

svec = zeros(1, NN);
for i = 1:NN;
    tvec = zeros(1, NN);
    tvec = [zeros(1, i-1), yy3];
    tvec = yy2(i) * tvec(1:NN);
    svec = svec + tvec;
endfor
yy4 = svec;
% yy4= conv(yy2, yy3);
y5 = yy4([1:NN/N:NN]);
yy5= yy4([1:NN]);

subplot(4, 1, 2);
stem(t1, y2)
subplot(4, 1, 1);
hold on
plot(t1, y1);
plot(tt, yy3);
subplot(4, 1, 3);
stem(t1, y5); hold on
plot(tt, yy5)
subplot(4, 1, 4);
plot(yy4);
```

Small Time Constant



```
yy = [y1;  
zeros(NN/N-1, N)];  
yy2= yy(:)';  
a = 300;  
yy3= e.^(-a*tt);  
yy3 =yy3 -  
[zeros(1, NN/N),  
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

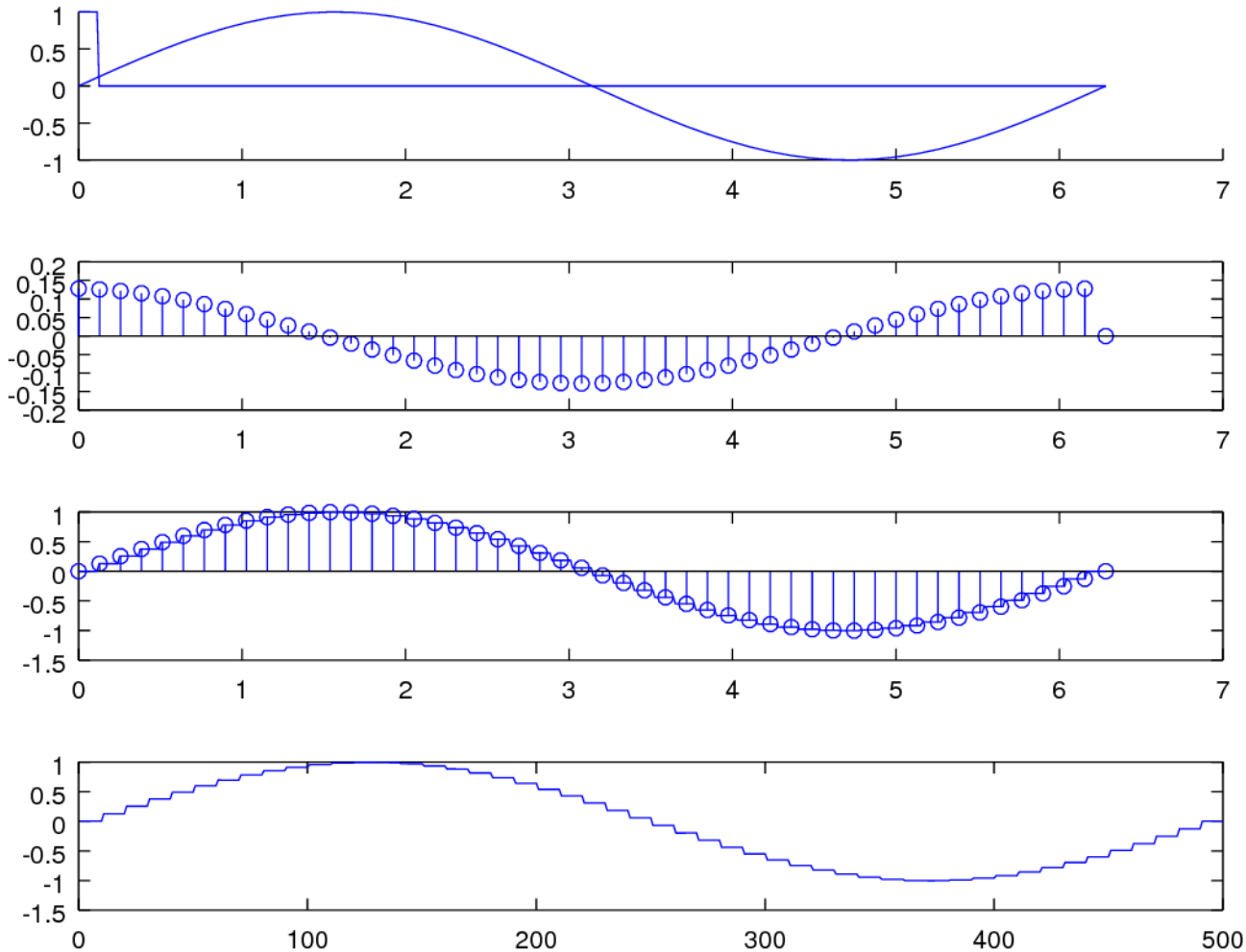
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C}$

Large Time Constant



```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

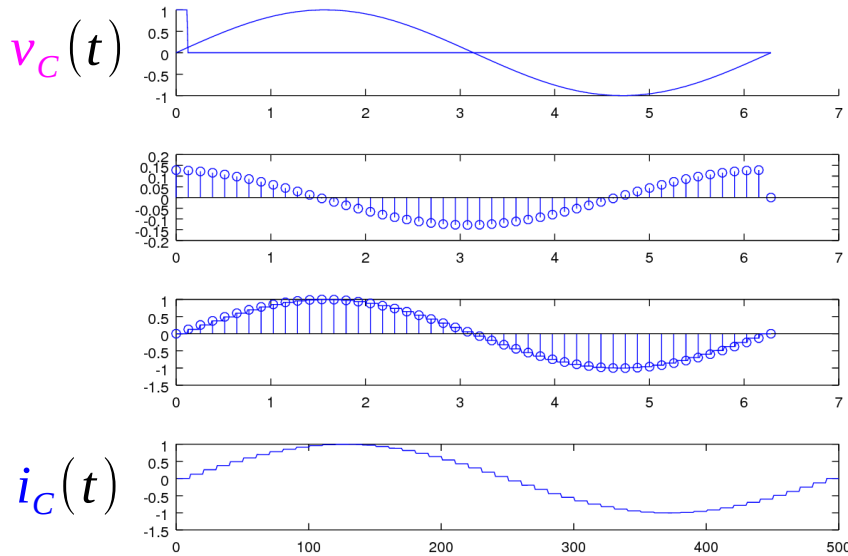
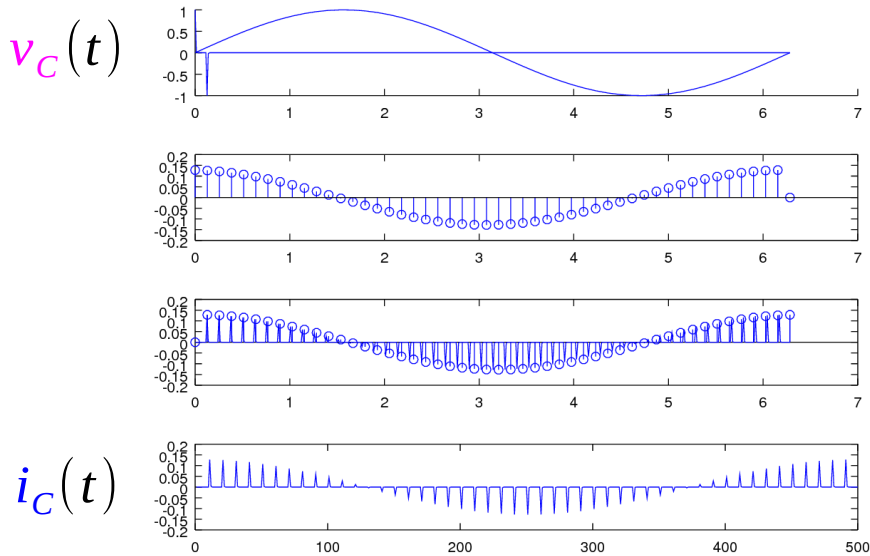
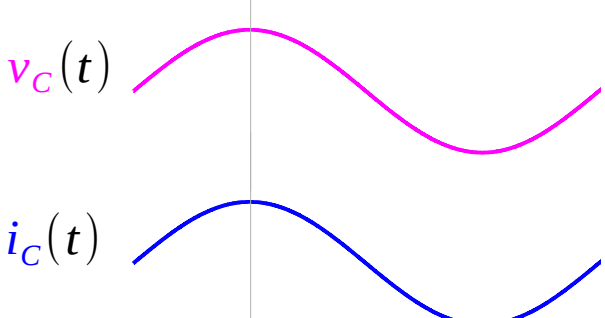
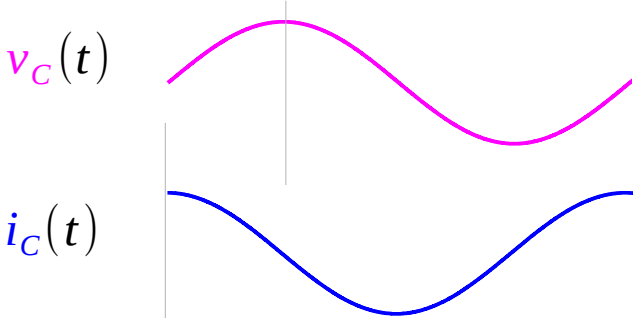
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ

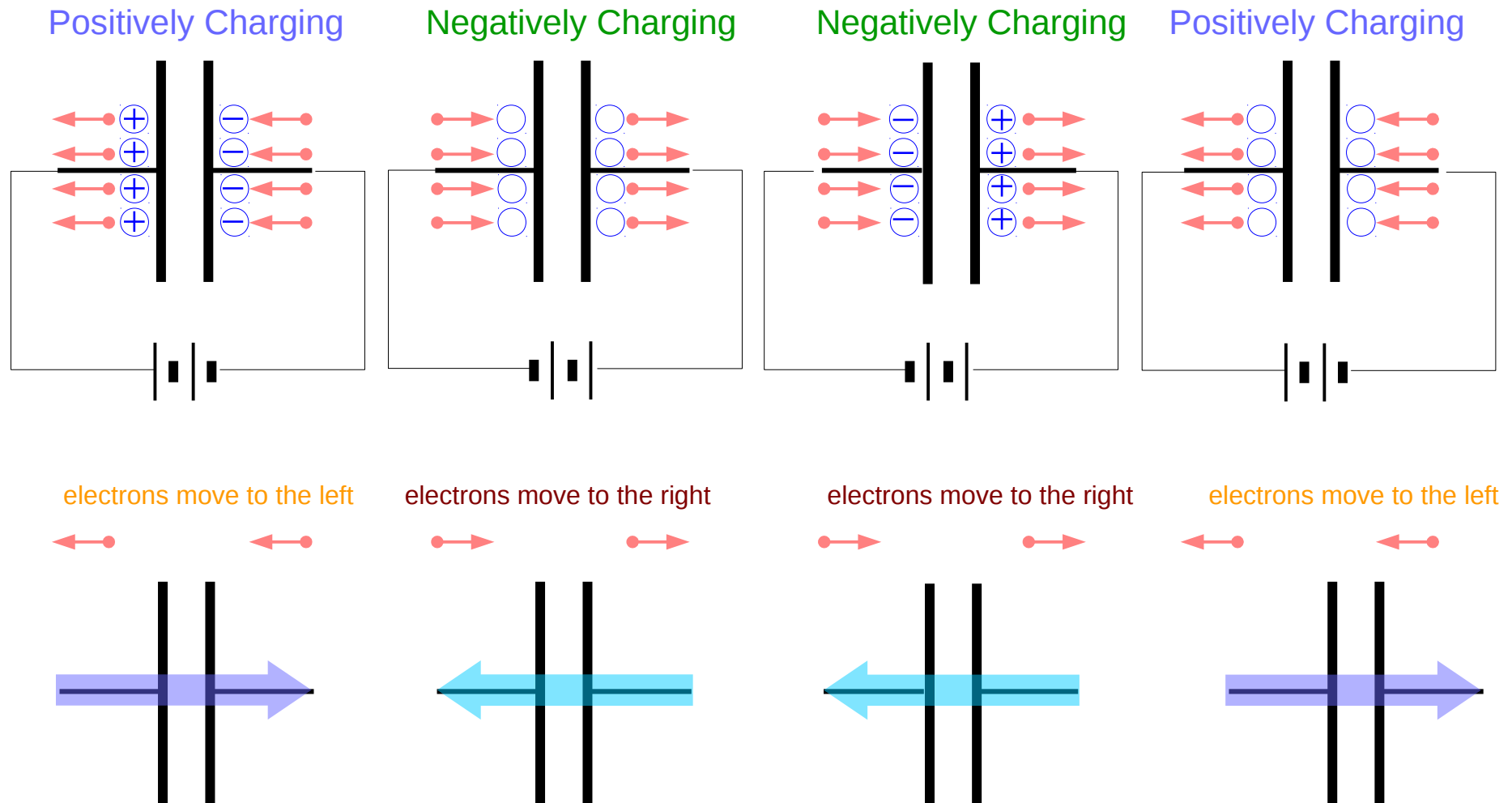
large C

small $\frac{1}{\omega C}$

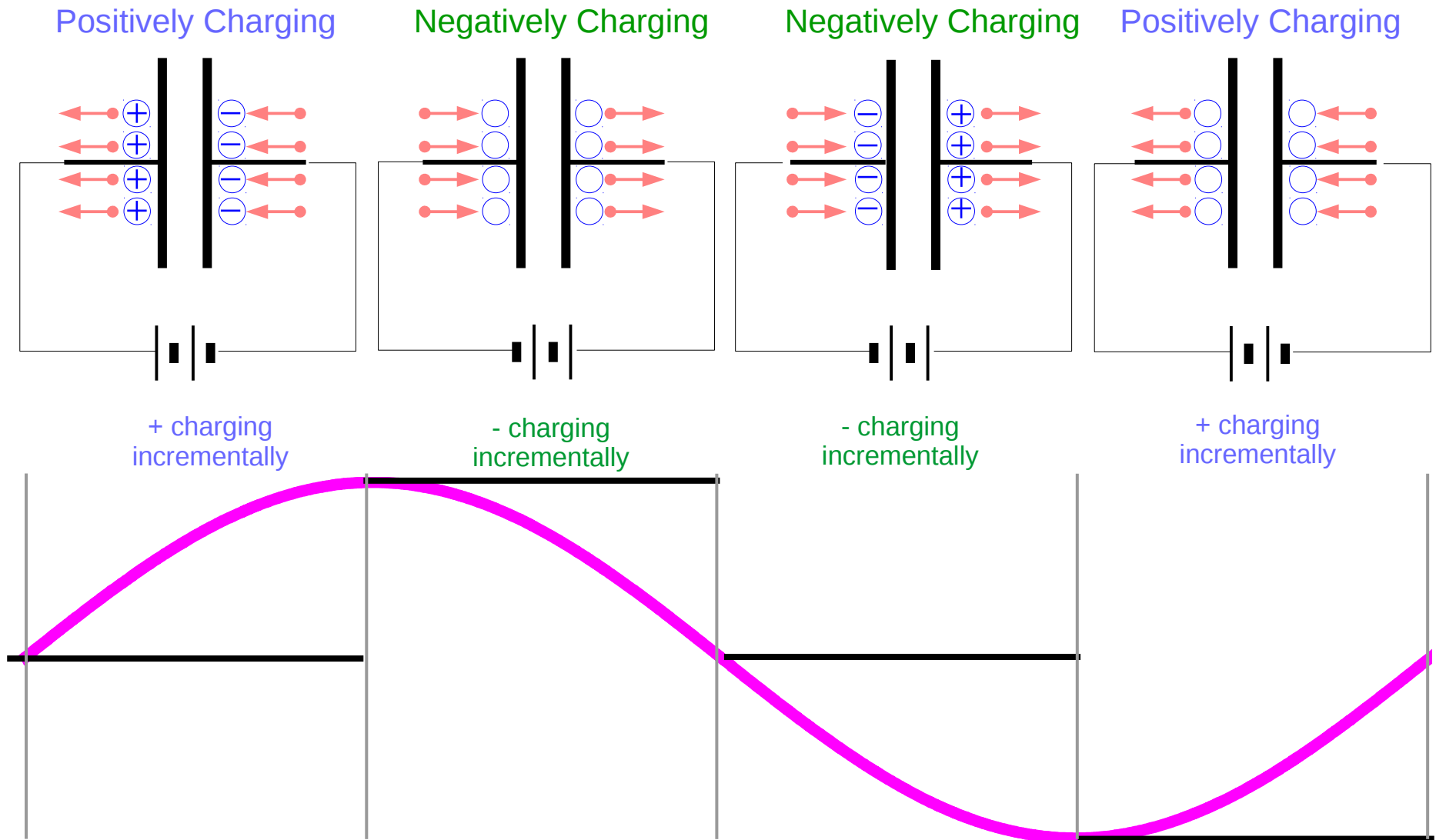
Time Constants



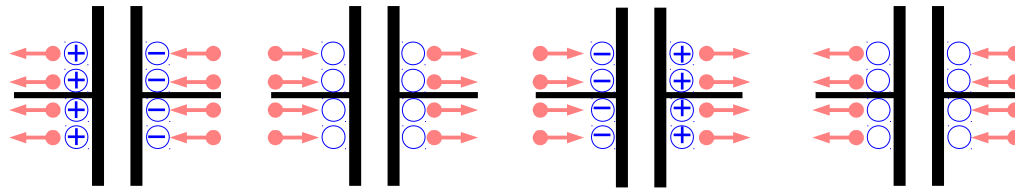
Evercharging signal pairs



Continuous Charging and Discharging Operations

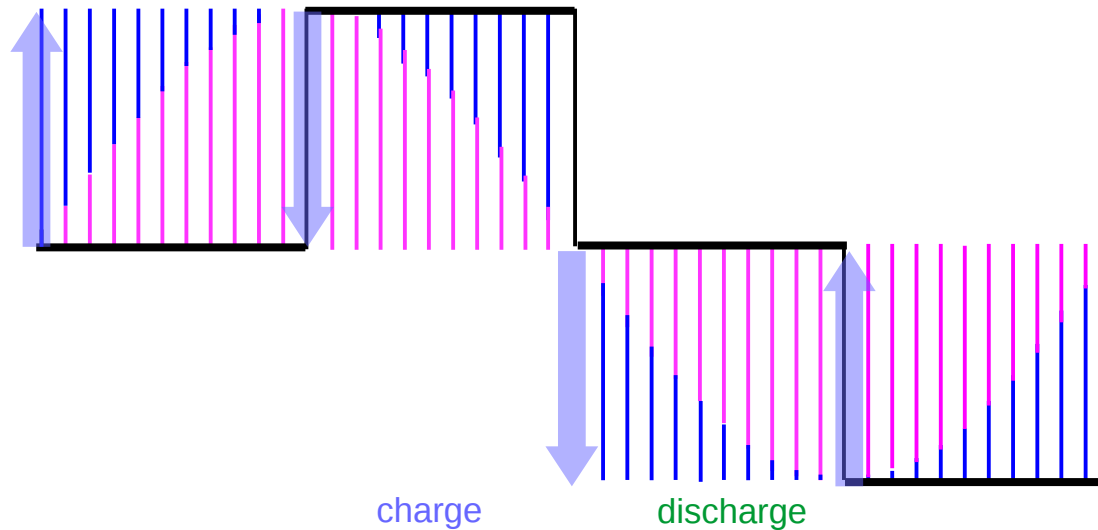


Everchanging signal pairs



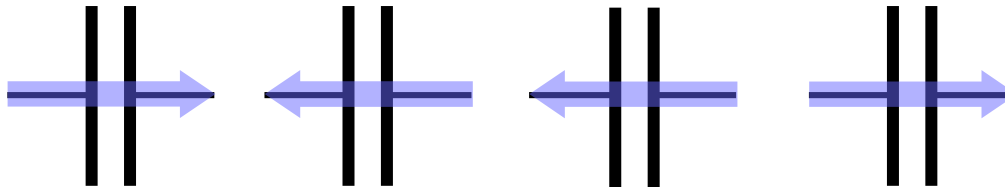
charge

discharge

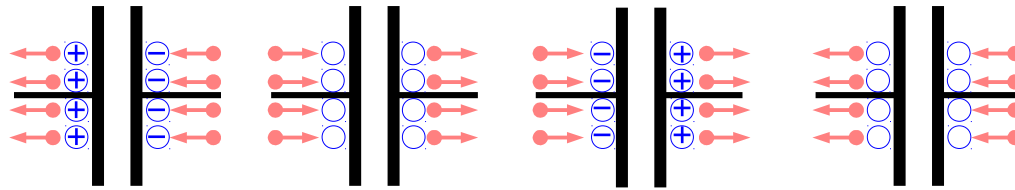


charge

discharge

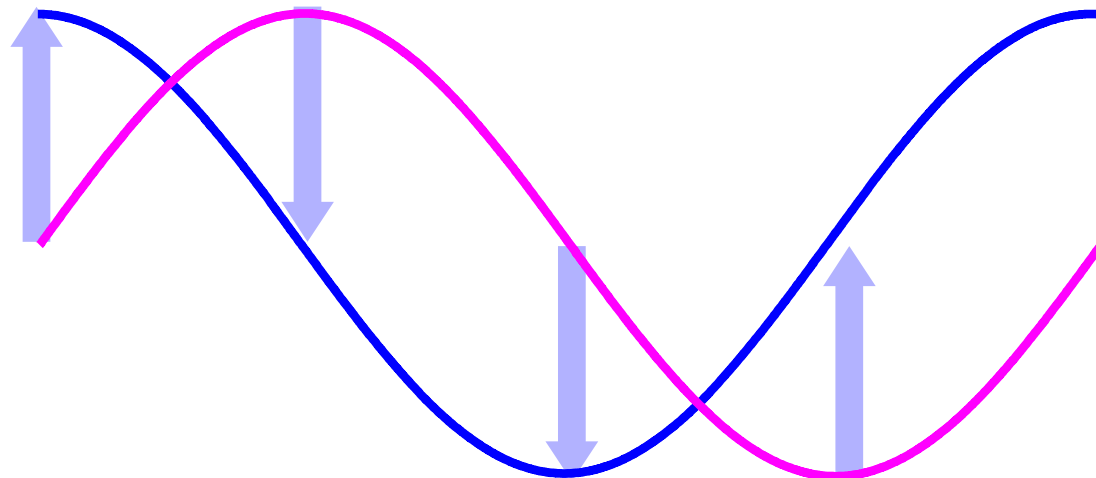


Everchanging signal pairs



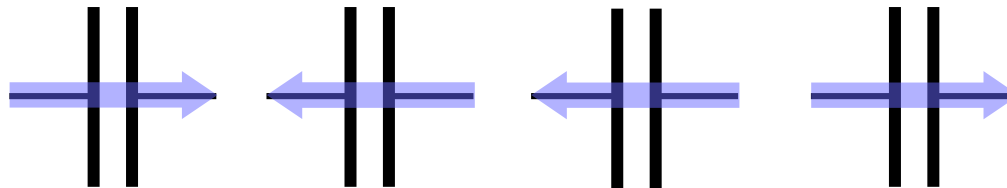
charge

discharge

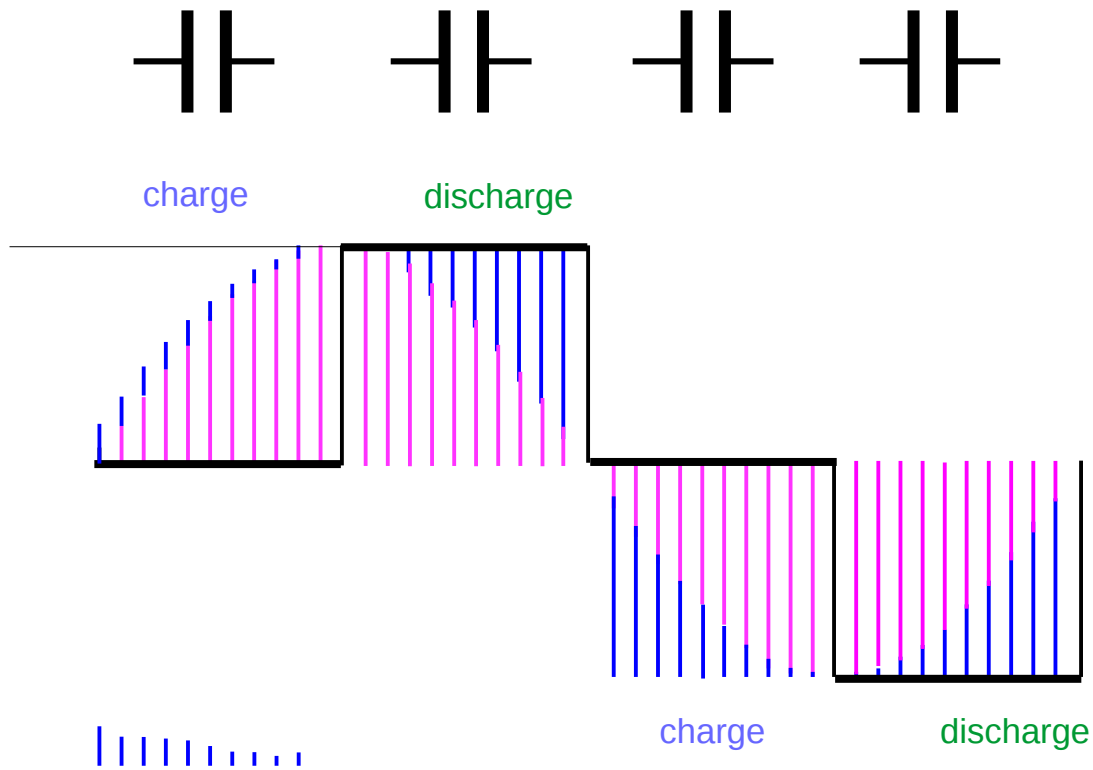


charge

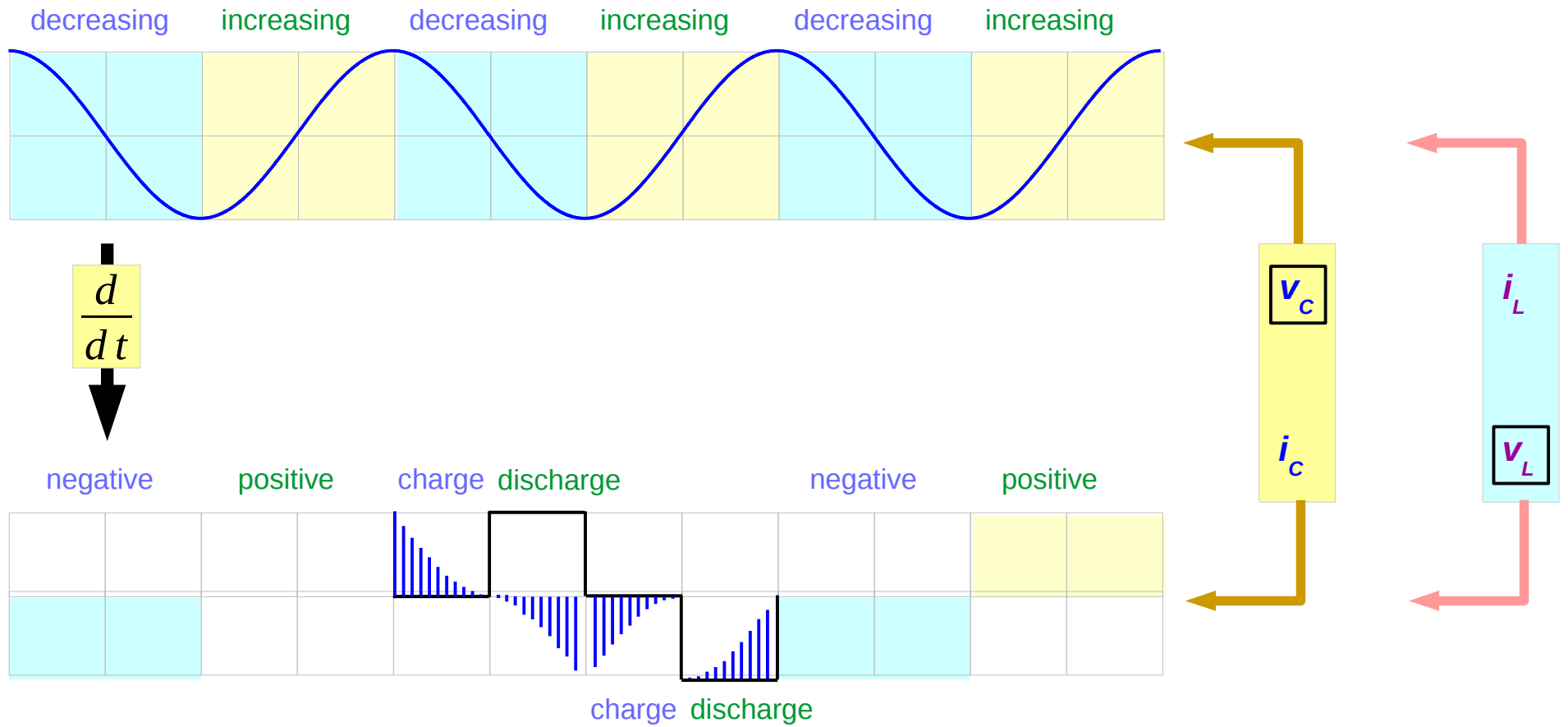
discharge



Everchanging signal pairs

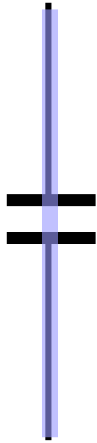


Everchanging signal pairs



I leads V by 90°

Initial charge

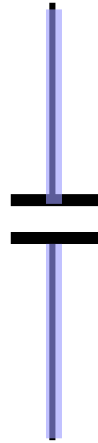


SHORT

V = 0

I : peak

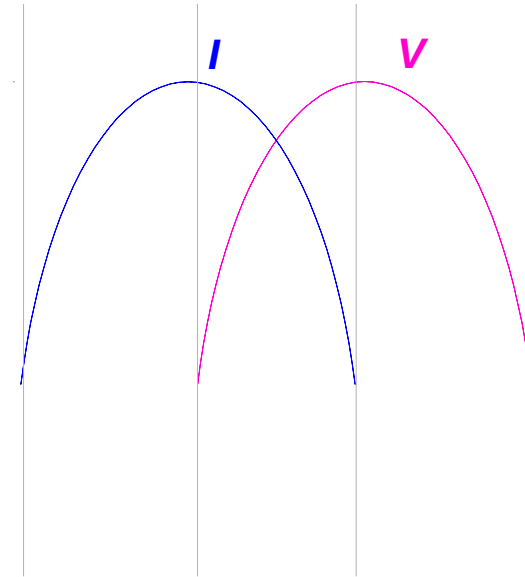
Full charge



OPEN

I = 0

V : peak



References

[1] <http://en.wikipedia.org/>

[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003